Global Currency Hedging

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ABSTRACT

Over the period 1975 to 2005, the US dollar (particularly in relation to the Canadian dollar) and the euro and Swiss franc (particularly in the second half of the period) have moved against world equity markets. Thus these currencies should be attractive to risk-minimizing global equity investors despite their low average returns. The risk-minimizing currency strategy for a global bond investor is close to a full currency hedge, with a modest long position in the US dollar. There is little evidence that risk-minimizing investors should adjust their currency positions in response to movements in interest differentials.

What role should foreign currency play in a diversified investment portfolio? In practice, many investors appear reluctant to hold foreign currency directly, perhaps because they see currency as an investment with high volatility and low average return. At the same time, many investors hold indirect positions in foreign currency when they buy foreign equities or bonds without hedging the currency exposure implied by the foreign asset holding. Such investors receive the foreign-currency excess return on their foreign assets, plus the return on foreign currency.

In this paper we consider an investor with an exogenous portfolio of equities or bonds and ask how the investor can use foreign currency to manage the risk of the portfolio. We assume that the investor's domestic money market is riskless in real terms, and use mean-variance analysis to find the foreign currency positions that minimize the risk of the total portfolio. We consider seven major developed-market currencies, the dollar, euro, Japanese yen, Swiss franc, pound sterling, Canadian dollar and Australian dollar, over the period 1975 to 2005. Any of these can be the investor's domestic currency or can be available as a foreign currency. We assume a one-quarter investment horizon, but obtain similar results for horizons ranging from one month to one year.

Although our framework is standard, and has been applied to US holdings of foreign currencies by Glen and Jorion (1993), our implementation and empirical results are novel in several respects. Following Glen and Jorion, we start by considering an equity investor who chooses fixed currency weights to minimize the unconditional variance of her portfolio. Such an investor wishes to hold currencies that are negatively correlated with equities. Our first novel result is that our seven currencies fall along a spectrum. At one extreme, the Australian dollar and the Canadian dollar are positively correlated with local-currency returns on equity markets around the world, including their own domestic markets. At the other extreme, the euro and the Swiss franc are negatively correlated with world stock returns and their own domestic stock returns. The Japanese yen, the British pound, and the US dollar fall in the middle, with the yen and the pound more similar to the Australian and Canadian dollars, and the US dollar more similar to the euro and the Swiss franc.

When we consider currencies in pairs, we find that risk-minimizing equity investors should short those currencies that are more positively correlated with equity returns and should hold long positions in those currencies that are more negatively correlated with returns. When we consider all seven currencies as a group, we find that optimal currency positions tend to be long the US dollar, the Swiss franc, and the euro, and short the other currencies. A long position in the US-Canadian exchange rate is a particularly effective hedge against equity risk.

We obtain a second novel result when we consider the risk-minimization problem of global bond investors rather than global equity investors. We find that most currency returns are almost uncorrelated with bond returns and thus risk-minimizing bond investors should avoid holding currencies; that is, they should fully currency-hedge their international bond positions. This is consistent with common practice of institutional investors, although global bond mutual funds are available with or without currency hedging. The US dollar is an exception to the general pattern in that it tends to appreciate when bond prices fall, that is when interest rates rise, around the world. This generates a modest demand for US dollars by risk-minimizing bond investors.

In capital market equilibrium, one might expect that average currency returns would reflect the risk characteristics of currencies. Specifically, if reserve currencies are attractive to risk-minimizing global equity investors, these currencies might offer lower returns in equilibrium. We analyze the historical average returns on currency pairs and obtain a third novel result, that high-beta pairs have delivered higher average returns. However the historical reward for taking equity beta risk in currencies has been quite modest, and much smaller than the historical average excess return on a global stock index.

Another way to find a risk-return relation in foreign currencies is to condition upon currency characteristics that are known to predict currency returns, and to ask whether these characteristics predict currency risks. Following an extensive literature on the predictive power of interest differentials for currency returns, we use deviations of interest differentials from their time-series averages as conditioning variables, imposing that they have the same effects on currency covariances regardless of the particular currency pair under consideration. Our fourth novel result is that increases in interest rates have only modest effects on currency-equity covariances. Over the full sample period, and particularly the first half of the sample, increases in interest differentials are, if anything, associated with decreases in these covariances. This implies that risk-minimizing equity investors should tilt their portfolios towards currencies that have temporarily high interest rates, amplifying the speculative "carry trade" demands for such currencies rather than offsetting them.

The organization of the paper is as follows. Section I sets the stage by briefly reviewing the related literature. Section II describes our data and conducts preliminary statistical analysis of stock, bond, and currency returns. Section III lays out the analytical framework we use for our empirical analysis and presents unconditionally optimal currency hedges for equity portfolios, while Section IV reports the analogous results for bond portfolios. Section V discusses the relation between unconditional risks of currencies and their unconditional average returns. Section VI introduces the possibility of conditional hedging, varying currency positions in response to interest differentials. Section VII quantifies the risk reductions that are achievable with an unconditional or conditional currency hedging strategy, and discusses the effects of currency hedging on the Sharpe ratios of equity and bond portfolios. Section VIII concludes. The Internet Appendix presents analytical details and additional empirical results.[1]

I. Literature Review

The academic finance literature has explored a number of reasons why investors might want to hold foreign currency. These can be divided into risk management demands, resulting from covariances of foreign currency with the state variables that determine investors' marginal utility, and speculative demands, resulting from positive expected excess returns on foreign currency over domestic safe assets.[2]

One type of risk management demand arises if there is no domestic asset that is riskless in real terms, for example because only nominal bills are available and there is uncertainty about the rate of inflation. In this case, the minimum-variance portfolio may contain foreign currency (Adler and Dumas 1983). This effect can be substantial in countries with extremely volatile inflation, such as some emerging markets, but is quite small in developed countries over short time intervals. Campbell, Viceira, and White (2003) show that it can be more important for investors with long time horizons, because nominal bills subject investors to fluctuations in real interest rates, while nominal bonds subject them to inflation uncertainty which is relatively more important at longer horizons. If domestic inflation-indexed bonds are available, however, they are riskless in real terms if held to maturity and thus drive out foreign currency from the minimum-variance portfolio.

Another type of risk management demand for foreign currency arises if an investor holds other assets for speculative reasons, and foreign currency is correlated with those assets. For example, an investor may wish to hold a globally diversified equity portfolio. If the foreign-currency excess return on foreign equities is negatively correlated with the return on the foreign currency (as would be the case, for example, if stocks are real assets and the shocks to foreign currency are primarily related to foreign inflation), then an investor holding foreign equities can reduce portfolio risk by holding a long position in foreign currency. This type of risk management currency demand is the subject of this paper.

Many international investors think not about the foreign currency positions they would like to hold, but about the currency hedging strategy they should follow.[3]

An unhedged position in international equity, for example, corresponds to a long position in foreign currency equal to the equity holding, while a fully hedged position corresponds to a net zero position in foreign currency. When currencies and equities are uncorrelated, risk management demands for foreign currencies are zero, implying that in the absence of speculative demands, full hedging is optimal (Solnik 1974). Empirically, Perold and Schulman (1988) find that US investors can reduce volatility by fully hedging the currency exposure implicit in internationally diversified equity and bond portfolios.

We derive optimal hedging strategies for global equity and bond investors. Like Glen and Jorion (1993), we find that optimal currency hedging substantially reduces risk for equity investors. The benefits of optimal hedging are large even in countries like Canada where full hedging is actually riskier than no hedging at all. We report results for quarterly returns, but these results are robust to variation in the investment horizon between one month and one year. Froot (1993) studies the dollar and the pound over a longer sample period and finds that risk-minimizing foreign currency positions increase with the investment horizon, implying that long-horizon equity investors should not hedge their currency risk. We do not find this horizon effect in our post-1975 dataset.

Speculative currency demands require that investors perceive positive expected excess returns on foreign currencies. A unique feature of currencies is that investors in every country can simultaneously perceive positive expected excess returns on foreign currencies over their own domestic currencies. That is, a US investor can perceive a positive expected excess return on euros over dollars, while a European investor can at the same time perceive a positive expected excess return on dollars over euros. This possibility arises from Jensen's inequality and is known as the Siegel paradox (Siegel 1972). It can explain symmetric speculative demand for foreign currency by investors based in all countries.

In practice, however, the speculative currency demand generated by this effect is quite modest. If currency movements are lognormally distributed and the expected excess *log* return on foreign currency over domestic currency is zero (a condition that can be satisfied for all currency pairs simultaneously), then the expected excess *simple* return on foreign currency is one-half the variance of the foreign currency return. With a foreign currency standard deviation of about 10% per year, the expected excess foreign currency return is 50 basis points and the corresponding Sharpe ratio is only 0.05. A Sharpe ratio this small generates little demand from investors with typical levels of risk aversion.

A more important source of speculative currency demand arises from expected excess returns on particular currencies, as opposed to all currencies simultaneously. Although unconditional average excess returns on currencies are close to zero, there is evidence that conditional expected excess returns on currencies undergo short-term fluctuations which constitute the basis for active currency strategies. Most obviously, the literature on the forward premium puzzle (Hansen and Hodrick 1980, Fama 1984, Hodrick 1987, Engel 1996) shows that currencies with high short-term interest rates deliver high returns on average. The currency carry trade, which exploits this phenomenon by holding high-rate currencies and shorting low-rate currencies, was extremely profitable in and beyond our sample period until 2008 (Burnside et al. 2007, Brunnermeier, Nagel, and Pedersen 2008).

It is natural to ask whether the carry trade has risk characteristics that offset its profitability. Farhi and Gabaix (2008) argue that high-rate currencies are exposed to the risk of rare economic disasters, while Lustig and Verdelhan (2007) find that high-rate currencies, in a sample that includes emerging-market currencies, have higher sensitivity to US consumption growth. We use sensitivity to world stock returns as a measure of risk, and find that developed-market currencies with high unconditional average interest rates do have somewhat higher betas with the world stock market, but that there is no tendency for a currency whose interest rate is temporarily high to have a temporarily higher beta. Over our full sample period and in the first half of the sample, currency betas even have a weak tendency to decline when interest rates increase. Thus risk considerations might deter equity investors from implementing an unconditional form of the carry trade, but a conditional form of the trade, that invests in currencies with temporarily high interest rates, should be attractive even to risk-averse equity investors.

II. Data and Summary Statistics

Our empirical analysis uses stock return data from Morgan Stanley Capital International, and data on exchange rates, short-term interest rates, and long-term bond yields from the International Financial Statistics database published by the International Monetary Fund.[4] We calculate log bond returns from yields on long-term bonds using the approximation suggested in Campbell, Lo and MacKinlay (1997).[5] These data series are available at a monthly frequency, and we report results for several different investment horizons in the appendix, but our basic analysis assumes a one-quarter horizon and therefore runs monthly regressions of overlapping quarterly excess returns. We report results for seven developed economies: Australia, Canada, Euroland, Japan, Switzerland, the UK and the US. The sample period starts in 1975:7, the earliest date for which we have data available for all variables and for all seven markets, and ends in 2005:12.

We define "Euroland" as a value-weighted stock basket that includes Germany, France, Italy, and the Netherlands. These are the countries in the euro zone for which we have the longest record of stock total returns, interest rates, and exchange rates. For simplicity, we will refer to the Euroland stock portfolio as a "country" stock portfolio when describing our empirical results, even though this is not literally correct. With regard to currencies, prior to 1999 we refer to a basket of currencies from those countries, with weights given by their relative stock market capitalization, as the "euro".

Of course, our definition of Euroland implies some look-ahead bias, since in 1975 it would not have been obvious whether a European monetary union would occur, and which countries from the region would have been part of that union. However, one can reasonably argue that these countries would have been candidates, and that from the perspective of today's investors, it probably makes sense to consider these markets as a single market. We have also conducted our analysis including only Germany in Euroland, and using the deutschemark to proxy for the euro before 1999; this procedure gives very similar results.

Table I reports the full-sample annualized mean and standard deviation of short-term nominal interest rates, log stock and bond returns in excess of their local short-term interest rates, changes in log exchange rates with respect to the US dollar, and log currency excess returns with respect to the dollar. The means of log excess returns (geometric averages) are adjusted for Jensen's Inequality by adding one half their variance to convert them into mean simple excess returns (arithmetic averages).

Annualized average nominal short-term interest rates differ across countries. They are lowest for Switzerland and Japan, and highest for Australia, Canada, and the UK.[6] Short-term rates exhibit very low annualized volatility, under 2% for all countries.

Average changes in exchange rates with respect to the US dollar over this period are negative for the Australian dollar, close to zero for the Canadian dollar, the British pound, and the euro, and positive for the Swiss franc and the yen, reflecting an appreciation of these currencies with respect to the US dollar over this period. Exchange rate volatility relative to the dollar is in the range 10-13% for all currencies except the Canadian dollar, which moves closely with the US dollar giving a bilateral volatility of only 5.4%.

Excess returns to currencies are small on average and exhibit annual volatility similar to that of exchange rates, a result of the stability of short-term interest rates. Using the usual formula for the mean of a serially uncorrelated random variable, it is easy to verify that unconditional mean excess returns on currencies are insignificantly different from zero.

Table II reports full-sample quarterly correlations of foreign-currency excess returns (Panel A), and fully currency-hedged excess returns on stocks (Panel B) and bonds (Panel C). For simplicity, we report in Panel A the average correlation of each currency pair across all possible base currencies in our set, giving results for individual base currencies in the appendix. Panel A shows that all currency returns are positively cross-correlated. Currency correlations are large—almost all correlation coefficients are above 30%—but they are far from perfect, implying that there is significant cross-sectional variation in the dynamics of exchange rates.

Four correlations stand out as unusually large. The Canadian dollar exhibits a very high correlation with the US dollar (89%) and also with the Australian dollar (70%). The high correlation of the Canadian dollar with both the US dollar and the Australian dollar reflects the dual role of the Canadian economy as a resource-dependent economy that is simultaneously highly integrated with the US. Similarly, the euro is highly correlated with the Swiss franc (89%) and to a lesser extent the British pound (68%), reflecting the integration of European economies. Importantly, these correlations are high not only on average, but also individually, regardless of the base currency used to measure them.

The correlation coefficients between stock market returns shown in Panel B are all between 40% and 70%, with two important exceptions. The Canadian stock market is highly correlated with the US stock market (75%), and the Swiss stock market is highly correlated with the Euroland stock market (80%). The Canadian stock market also exhibits a relatively large correlation of 66% with the Australian stock market. These correlations demonstrate again the dual role of the Canadian economy and the integration of the Swiss economy with the European economy.

While significant, the stock market correlations are still small enough to suggest the presence of substantial benefits of international diversification in this sample period. Not surprisingly, the Japanese stock market exhibits the lowest cross-sectional correlation with all other markets. This is a reflection of the prolonged period of low or negative stock market returns in Japan during the 1990's, at a time when most other markets delivered large positive returns.

Long-term bond market correlations are generally somewhat smaller than stock market correlations. Panel C in Table III shows that, with some important exceptions, these correlations are all in the range of 30-60%. The exceptions are the Euroland bond market, which is highly correlated with both the Swiss bond market (65%)

and the US bond market (62%), and the Canadian bond market, which is highly correlated with the US bond market (79%). Overall, these results imply that there are meaningful benefits to international diversification in bond market investing.

III. Unconditional Currency Risk Management for Equity Investors

A. Estimating Unconditional Currency Demands

Our empirical analysis is based on the estimation of risk-minimizing currency demands for a set of stock and bond portfolios and currencies. We begin by establishing some notation. We use $R_{c,t+1}$ to denote the gross return in currency c from holding country c assets from the beginning to the end of period $t+1$, and $\omega_{c,t}$ to denote the weight of those assets at time t in the investor's portfolio. $S_{c,t+1}$ denotes the spot exchange rate in units of domestic currency per unit of foreign currency c at the end of period $t+1$, and $I_{c,t}$ denotes the nominal short-term nominal interest rate on bills denominated in currency c.

By convention, we index the domestic country by $c = 1$ or d and the n foreign countries by $c = 2, ..., n+1$. Of course, the domestic exchange rate is constant over time and equal to 1: $S_{1,t+1} = 1$ for all t. For convenience, throughout this section we set the domestic country to be the US, and hence refer to the domestic investor as a US investor, and to the domestic currency as the dollar. We also use small caps to denote log (or continuously compounded) returns, exchange rates, and interest rates. That is, $r_{c,t+1} = \log(R_{c,t+1})$, $s_{c,t+1} = \log(S_{c,t+1})$, and $i_{c,t} = \log(1 + I_{c,t})$

An investor holding an arbitrary portfolio of domestic and foreign assets can alter the currency exposure of her portfolio by overlaying a zero investment portfolio of domestic and foreign bills or, equivalently, by entering into an appropriate number of forward currency contracts. This is intuitive. Since the investor is already fully invested in other assets, she can alter the currency exposure implied by those assets only by borrowing—or equivalently, by shorting bonds—in some currencies, and using the proceeds to buy bonds denominated in other currencies.

For convenience we work with net currency exposures relative to full currency hedging, which we denote by $\psi_{c,t}$, instead of currency hedging demands. Of course, there is direct correspondence between them: $\psi_{c,t} = 0$ corresponds to a fully hedged currency position, in which the investor does not hold any exposure to currency c. A positive value of $\psi_{c,t}$ means that the investor holds exposure to currency c, or equivalently that the investor does not fully hedge the currency exposure implicit in her stock position in country c. When $\psi_{c,t} = \omega_{c,t}$, the portfolio is completely unhedged.

The internet appendix shows that with arbitrary currency exposure, the log portfolio excess return over the domestic interest rate is approximately equal to

$$r^h_{p,t+1} - i_{1,t} = \mathbf{1}'\boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) + \boldsymbol{\Psi}'_t \left(\Delta\mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}^d_t\right) + \frac{1}{2}\Sigma^h_t, \tag{1}$$

where $\boldsymbol{\omega}_t$ is the $(n+1 \times n+1)$ diagonal matrix of portfolio weights, $\mathbf{r}_{t+1}$ is the vector of log nominal asset returns in local currencies, $\boldsymbol{\Psi}_t$ is the vector of net currency exposures, $\Delta\mathbf{s}_{t+1}$ is the vector of the changes in log spot exchange rates, $\mathbf{i}_t$ is the vector of log short-term nominal interest rates, $\mathbf{i}^d_t = \log\left(1 + I_{1,t}\right)\mathbf{1}$, and $\mathbf{1}$ is a vector of ones. All the vectors have dimension $(n+1 \times 1)$.

Equation (1) provides an intuitive decomposition of the portfolio excess return. The first term represents the excess return on a fully hedged portfolio which has no exposure to currency risk. The second term involves only the vector of excess returns on currencies, $\Delta\mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}^d_t$, and thus represents pure currency exposure, given by $\boldsymbol{\Psi}_t$. The third term is a Jensen's Inequality correction given in the appendix.

Since $\boldsymbol{\Psi}_t$ represents the weights in a zero investment portfolio of domestic and foreign bills, we must have that $\Psi'_t\mathbf{1} = \mathbf{0}$, or that the domestic currency exposure $\psi_{1,t}$ is automatically determined once we determine the vector of foreign currency demands, which we denote by $\widetilde{\boldsymbol{\Psi}}_t = \left(\psi_{2,t}, ..., \psi_{n+1,t}\right)'$.

We show in the appendix that the vector $\widetilde{\boldsymbol{\Psi}}_t$ that minimizes the one-period conditional global variance of the log excess return on the hedged portfolio is equal to

$$\widetilde{\boldsymbol{\Psi}}_{RM,t}^{*}=-\operatorname{Var}_t\left(\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^{d}\right)^{-1}\left[\operatorname{Cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\Delta\mathbf{s}}_{t+1}+\widetilde{\mathbf{i}}_t-\widetilde{\mathbf{i}}_t^{d}\right)\right)\right], \tag{2}$$

where we have added the subscript RM to emphasize that equation (2) describes risk management currency demands. In the rest of the paper we will refer to this risk management component of currency demand simply as optimal currency demand or currency exposure.

Equation (2) writes $\widetilde{\Psi}_{RM,t}^{*}$ as a vector of multiple regression coefficients of portfolio stock returns on currency returns. If stock returns and exchange rates are uncorrelated, the risk management currency demand is zero. In this case holding currency exposure adds volatility to the investor's portfolio and, unless this volatility is compensated, the investor is better off holding no currency exposure at all or, equivalently, fully hedging her portfolio. If stock returns and exchange rates are positively correlated, the foreign currency tends to depreciate when the stock market falls. Thus the investor can reduce portfolio return volatility by over-hedging, that is, by shorting foreign currency in excess of what would be required to fully hedge the currency exposure implicit in her stock portfolio. Conversely, a negative correlation between stock returns and exchange rates implies that the foreign currency appreciates when the stock market falls. Then the investor can reduce portfolio return volatility by under-hedging, that is, by holding foreign currency.

A useful property of the optimal currency demands in (2), proven in the appendix, is that for a given stock portfolio, they are invariant to changes in the base currency, provided that a riskless real asset is available in each base currency and that the set of available currencies (which always includes an investor's own domestic currency) does not change. If we restrict the set of available currencies to a pair, for example the US dollar and the euro, this means that residents of both the US and Germany will have the same optimal demands for dollars and euros corresponding to a given equity portfolio. Residents of a third country, however, have another domestic currency available to them and so they will not necessarily have the same demands for dollars and euros even if they hold the same equity portfolio. If we allow a larger set of available currencies, then residents of all the countries in the set will have the same vector of optimal currency demands for a given equity portfolio.

In this section and the next we present estimation results which are based on an unconditional version of equation (2). The unconditional version of our model follows immediately from the conditional version by simply assuming constant risk premia and constant second moments of returns. We relax this assumption in section VI, which examines conditional hedging policies where the covariances of portfolio returns with currency excess returns vary over time as a function of interest rate differentials.

Equation (2) with constant second moments implies that we can compute optimal currency exposures by regressing portfolio excess returns $\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)$ onto a constant and the vector of currency excess returns $\widetilde{\Delta\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t$, and switching the sign of the slopes. In our empirical analysis we consider several practically relevant cases. First, we consider an investor who is fully invested in a domestic stock portfolio and optimally decides how much exposure to a single currency c to hold in order to minimize total portfolio return volatility. The optimal currency demand is the negative of the slope coefficient in a regression of the domestic excess stock return onto a constant and the excess return on currency c.

Second, we consider an investor who is fully invested in a domestic stock portfolio, but who uses the whole range of available currencies to minimize total portfolio return volatility. In that case the vector of optimal currency demands is given by the negative of the slopes of a multiple regression of the excess stock return on the domestic market onto a constant and the vector of currency excess returns.

Third, we consider a case where the investor holds an equally weighted global equity portfolio, using the whole vector of available currencies to minimize total portfolio return volatility. Finally, in the appendix we also consider value weighted and home biased global equity portfolios.

B. Single-Country Stock Portfolios

We start our empirical analysis by examining the case of an investor who is fully invested in a single-country equity portfolio and is considering whether exposure to other currencies would help reduce the volatility of her quarterly portfolio return.

Table III reports optimal currency exposures for the case in which the investor is considering one currency at time (Panel A), and that in which she is considering multiple currencies simultaneously (Panel B). In both panels, the reference stock market is reported at the left of each row, while the currency under consideration is reported at the top of each column. In all tables we report Newey-West heteroskedasticity and autocorrelation consistent standard errors in parentheses below each optimal currency exposure. Following standard convention, we mark with one, two, or three stars coefficients for which we reject the null of zero at a 10%, 5%, and 1% significance level, respectively.

Panel A in Table III considers an investor who is deciding how much to hedge of the currency exposure implicit in an investment in a specific national stock market, in isolation from other investments and currencies this investor might hold. To facilitate interpretation, it is useful to discuss an example in detail. The first non-empty cell in the first column of the table, which corresponds to the Australian stock market and the euro, has a value of 0.39. This means that a risk-minimizing Euroland investor who is fully invested in the Australian stock market and has access to the Australian dollar and the euro should buy a portfolio of euro-denominated bills worth 1.39 euros per euro invested in the Australian stock market, financing this long position with a short position in Australian bills—i.e., by borrowing Australian dollars. That is, the investor should over-hedge the Australian dollar exposure implicit in her Australian stock market investment, and hold a net long 39% exposure to the euro.

Panel A of Table III shows that optimal demands for foreign currency are large, positive and statistically significant for two stock markets (rows of the table), those of Australia and Canada. Investors in the Australian and Canadian stock markets are keen to hold foreign currency, regardless of the particular currency under consideration, because the Australian and Canadian dollars tend to depreciate against all currencies when their stock markets fall; thus any foreign currency serves as a hedge against fluctuations in these stock markets. The long positions in euros, Swiss francs, or US dollars are particularly large and statistically significant.

At the opposite extreme, it is optimal for investors in the Swiss stock market to hold economically and statistically large short positions in all currencies, implying that the Swiss franc tends to appreciate against all currencies when the Swiss stock market falls. Results are similar for the Euroland stock market, except that this market is hedged by a long position in the Swiss franc. The Japanese and UK stock markets generate large positive demands for the Swiss franc and the euro, and negative or small positive demands for all other currencies. The British stock market generates significant negative demands for the Australian dollar and the Canadian dollar.

The last row of this panel describes individual optimal currency demands for a portfolio fully invested in US stocks. Most of these demands are economically small and statistically insignificant, but there are two important exceptions to this pattern. The first exception is a modest positive demand for the Swiss franc, which tends to appreciate when the US stock market falls. The euro generates a similar demand, though it is statistically significant only at the 10% significance level. The second exception is a large negative demand for the Canadian dollar, reflecting the fact that the Canadian dollar tends to depreciate when the US stock market falls.

Panel B of Table III reports optimal currency demands for single-country stock portfolios considering all currencies simultaneously. That is, each row of Panel B reports the unconditional version of (2) when $\mathbf{r}_{t+1}$ is unidimensional and equal to the stock market shown on the leftmost column. Note that the the numbers in each row must add up to zero, since the domestic currency exposure must offset the vector of foreign currency demands.

When single-country stock market investors consider investing in all currencies simultaneously, they almost always choose positive exposures to the US dollar, the euro and the Swiss franc, and negative exposures to the Australian dollar, Canadian dollar, British pound, and Japanese yen. Relative to panel A, the optimal currency demands are generally larger and statistically more significant for the US dollar, and less statistically significant for the euro and the Swiss franc. This reflects two features of the multiple-currency analysis. First, a position that is long the US dollar and short the Canadian dollar is a highly effective hedge against stock market declines. Thus allowing investors to use both North American currencies increases the risk management demand for the US dollar. Second, the euro and Swiss franc are both good hedges but they are highly correlated; thus the demand for each currency is less precisely estimated when investors are allowed to take positions in both currencies. In this sense the euro and the Swiss franc are substitutes for one another.

C. Global Equity Portfolios

Thus far we have considered only investors who are fully invested in a single country stock market, and use currencies to hedge the risk of that stock market. In this section we consider risk-minimizing investors with internationally diversified stock portfolios.

We focus our analysis on investors who are equally invested in the seven stock markets included in our analysis: Euroland, Australia, Canada, Japan, Switzerland, the UK, and the US. We have already noted that, in the multiple-currency case, optimal currency demands generated by a given global portfolio are the same regardless of the currency base. Accordingly, we only need to report one set of currency demands, which add up to zero as in panel B of Table III.

Panel A of Table IV considers the case in which investors have access to all seven currencies from the countries included in the equally-weighted stock portfolio. Panel B considers a case in which investors do not have close currency substitutes available for investment. Specifically, Panel B excludes Canada and Switzerland from the analysis because the Canadian stock market is highly correlated with the US stock market, and the Canadian dollar is also highly correlated with the US dollar; similarly, there is a very high positive correlation between the Swiss stock market and the Euroland market, and between the Swiss franc and the euro. Comparison of the results in panel A and panel B clarifies the roles of the Canadian dollar and the US dollar, and the euro and the Swiss franc, in investors' portfolios. Both panels report estimates of optimal currency demands based first on our full sample, and then on two subperiods, 1975 to 1989 (Subperiod I) and 1990 to 2004 (Subperiod II). We discuss subperiod results in part D of this section.

The optimal currency portfolio in Panel A has a large, statistically significant exposure of 40% to the US dollar, and an even larger negative exposure of -61% to the Canadian dollar. These two positions are not independent of each other: Panel B shows that, once we exclude the Canadian dollar from the menu of currencies available to the investor, the optimal exposure to the US dollar becomes small and statistically insignificant. Just as in the previous analysis of single-country stock portfolios, a position that is long the US dollar and short the Canadian dollar helps investors hedge against global stock market movements. The 3-month annualized return on this position, driven by the movements of the bilateral US/Canadian exchange rate, is plotted in Figure 1 together with the 3-month annualized excess return on the equally weighted, fully currency-hedged global equity index. The figure clearly shows the tendency of the US dollar to appreciate relative to the Canadian dollar in periods of stock market weakness.

The optimal currency portfolio in Panel A also has positive exposures to the euro and the Swiss franc. These exposures are not economically very large individually, and statistically significant only at a 10% level. This lack of individual economic and statistical significance is because the euro and the Swiss franc are close substitutes. Panel B shows that when the Swiss franc is excluded from the menu of currencies, the demand for the other currency in the pair, the euro, increases dramatically to 56% and is statistically significant at the 1% level. Figure 2 plots the 3-month annualized return of the euro against an equally weighted basket of other currencies, together with the currency-hedged excess global equity return.

In addition to the optimal negative exposure to the Canadian dollar already discussed, the optimal exposures to the Australian dollar, the Japanese yen, and the British pound are also negative. These short positions are small and statistically insignificant for the pound, but larger and statistically significant for the yen. The Australian dollar short position is small if the Canadian dollar is included in the set of currencies, but becomes larger and statistically significant in panel B when the Canadian dollar is excluded. These two currencies, which are unusually highly correlated with one another, are substitutes in investors' currency portfolios.

Once again, it is useful to review the exact meaning of the numbers we report. The numbers shown in Table IV are optimal currency exposures. If it is optimal for all investors to fully hedge the currency exposure implicit in their stock portfolios or, equivalently, to hold no currency exposure, the optimal currency demands shown in Table IV should be equal to zero everywhere. To obtain optimal currency hedging demands from optimal currency exposures, we need only compute the difference between portfolio weights—which in this case are 14.3% for each country stock market—and the optimal currency exposure corresponding to that country.

The results in Panel A imply that, say, a risk-minimizing Euroland investor holding our equally-weighted seven-country equity portfolio would invest 99 euro cents in currencies for each euro invested in the stock portfolio. These 99 euro cents would be invested in US Treasury bills worth 40 euro cents, Euroland (say, German) bills worth 32 euro cents, and Swiss bills worth 27 euro cents. These purchases would be financed with proceeds from borrowing Australian dollars (11 euro cents per euro invested in the stock portfolio), Canadian dollars (61 cents), yen (17 cents) and British pounds (10 cents). Note that the similarity of the 99 euro cents invested in long and short currency positions to the 100 euro cents invested in equities is merely an arbitrary feature of this particular example, not a general property of the optimal hedging strategy.

We can easily restate these results in terms of hedging demands. The Euroland investor would underhedge her exposure to the US dollar and the Swiss franc, and overhedge her exposure to the Australian dollar, the Canadian dollar, the yen and the British pound. More precisely, this Euroland investor would not only leave unhedged the 14 euro cent exposures to the US dollar and the Swiss franc implied by each euro of the stock portfolio, she would also enter into forward contracts to buy US dollars worth 26 euro cents and Swiss francs worth 13 euro cents. She would simultaneously enter into forward contracts to sell Australian dollars, Canadian dollars, yen and British pounds worth, respectively, 25, 75, 31, and 24 euro cents per euro invested in the stock portfolio.

We show in the appendix that the results of Table IV are robust to reasonable variations in specification. Varying the investment horizon between 1 and 3 months has little impact on the results, although at horizons of 6 or 12 months a long position in the Swiss franc drives out both the bilateral US/Canadian position and the euro in the optimal currency portfolio, and a short position in the Japanese yen becomes large and statistically significant. Results for a value-weighted global equity portfolio are qualitatively and quantitatively similar to those for the equally-weighted portfolio. This derives from the fact that, with the exception of the US stock market, no single stock market dominates the market capitalization of the overall portfolio.[7] Results remain similar when we consider a "home-biased" portfolio that is 75% invested in the domestic stock market and 25% invested in a value-weighted world portfolio that excludes this market.

In summary, the risk-minimizing strategy for a global equity investor involves long exposure to the US dollar and the euro (or a combination of the euro and the Swiss franc), a large short position in the Canadian dollar, and smaller short positions in all other major currencies. That is, investors in global equities want to underhedge their exposure to the dollar, the euro, and the Swiss franc, and overhedge their exposure to the other currencies. This strategy minimizes the volatility of overall portfolio returns, because the euro, Swiss franc, and US dollar tend to appreciate when international stock markets decline.

D. Stability Across Subperiods

The sample period for which we have estimated optimal currency exposures includes an early period of global high inflation and interest rates, with exceptional performance of the Japanese stock market relative to other stock markets, followed by another subperiod of global lower inflation and interest rates, with extremely poor performance of the Japanese stock market. The second subperiod also saw the reunification of Germany and the creation of the euro as a common European currency. It is reasonable to examine if the results we have shown for the full sample hold across these two markedly different subperiods, so we divide our sample period into the periods 1975 to 1989 and 1990 to 2005.

The bottom two rows in each panel of Table IV report subsample results for an investor holding an equally-weighted global stock portfolio, and using the vector of available currencies to manage risk. The results are generally familiar, with long positions for the US dollar, Swiss franc, and euro, and short positions for other currencies. It is striking, however, that US dollar positions tend to fall between the first subperiod and the second, while the sum of euro and Swiss franc positions (in Panel A) or the euro position (in Panel B) strongly increase. The time-series plot in Figure 2 also shows that the euro began to move more consistently against world stock markets in the second subsample.

In summary, an important change occurred between the periods 1975 to 1989 and 1990 to 2005: The Swiss franc and the euro became much more competitive with the US dollar as desirable currencies for risk-minimizing global equity investors. This change is not an artefact of our use of a composite currency to proxy for the euro in the period before the creation of a common European currency, because we obtain similar results when we use the deutschemark as our euro proxy. Rather, it is likely to reflect the fact that the euro has found growing acceptance as a reserve currency for international investors.

IV. Unconditional Currency Risk Management for Bond Investors

We now consider the risk-minimizing currency exposures implied by an equally-weighted global bond portfolio. Table V, whose structure is identical to Table IV, reports optimal currency exposures at a one-quarter horizon in the multiple currency case for our full sample period and for the subperiods 1975 to 1989 and 1990 to 2005.

Risk-minimizing currency demands for internationally diversified bond market investors are generally very small and not statistically significant. The US dollar is an exception. The optimal demand for the US dollar is positive and statistically significant, regardless of whether the dollar is the only currency available for investment, or just one of many. But these dollar exposures are economically small. They are largest in the first subperiod, and almost zero in the second subperiod. Here again we see evidence of a decline over time in the attractiveness of the US dollar for risk-minimizing global investors.

We have also considered single-country bond portfolios. This is relevant to many investors, since "home bond bias" is even more prevalent among investors than "home equity bias", and in most countries relatively few mutual funds offer international bonds. The results are shown in the appendix. We find modest positive demands for the US dollar, consistent with the results in Table V. We also find that an investor holding UK bonds will hold all foreign currencies, reflecting the fact that the British pound tends to depreciate when the British bond market declines.

Overall, our results imply that international bond investors should fully hedge the currency exposure implicit in their bond portfolios, with possibly a small long bias towards the US dollar. Interestingly, full currency hedging is more common among international bond mutual funds than among international equity funds, and is also frequently practiced by US institutions investing in international bonds.

V. Unconditional Currency Risk and Return

In section III we showed that currencies systematically differ in their comovements with global stock markets. Excess returns on reserve currencies—the US dollar, the euro, and the Swiss franc—covary negatively with global stock market returns, while excess returns on other currencies—particularly those of commodity-dependent Australia and Canada—covary positively. These correlations generate positive risk management demands for reserve currencies, and negative risk management demands for other currencies.

However, in equilibrium investors must be willing to hold all currencies (Black 1990). This suggests that average excess returns on currencies might adjust to generate speculative currency demands that offset the risk management demands we have identified. In global capital market equilibrium, investors may be willing to receive lower compensation for holding US dollar, euro, and Swiss franc denominated bills because of the hedging properties of these currencies, while they may demand higher compensation for holding bills denominated in other currencies. In fact, we saw in Table I that the US and Switzerland have had the lowest currency returns in our sample, and together with Euroland have had the lowest interest rates with the exception of Japan. If this is a systematic phenomenon, it suggests that a country benefits from having a reserve currency not only because international demand for its monetary base generates seigniorage revenue, but also because international demand for its Treasury bills reduces the interest cost of financing the government debt.[8]

We now explore the equilibrium consequences of risk management demand for currencies by looking at the relation between currencies' average excess returns and their betas with a global stock index. We consider all possible non-redundant pairs (or exchange rates) in our cross section of currencies, and treat each one as a long-short portfolio of bills. For example, the excess return on the Canadian dollar with respect to the US dollar is the return on a portfolio long Canadian bills and short US Treasury bills.

For each of these portfolios, we compute the average log currency excess return and its beta with respect to the currency-hedged excess return on a value-weighted global stock portfolio, and we plot all these mean returns and betas together in a single figure.[9] To simplify our plot, we choose the ordering of the pairs so that their betas are all positive.

Figure 3 shows the mean-beta diagram based on our full sample. This figure plots full-sample annualized average excess currency returns on the vertical axis, and currency betas in the horizontal axis. The points marked with triangles refer to long-short currency portfolios with euro-denominated bills on the short side of the portfolio. The square corresponds to the portfolio long Canadian dollars and short US dollars, and the circles correspond to all other non-redundant currency pairs.

The figure also plots a regression line of currency excess returns on currency betas, with the intercept restricted to equal zero. We can interpret this line as the security market line generated from a global CAPM using currencies as assets. The slope of this line is 3.2%, and the R^2 is reasonably large at 48%; adding a free intercept has little effect on these estimates. The slope of the security market line reflects the equilibrium world market premium implied by currency returns. At 3.2% per annum, this premium is smaller than the ex-post average excess return on world stock markets over this period, which Table I shows is about 7%. However, this estimate is close to the ex-ante equity risk premium that others have estimated from US equity returns over a longer period in which the ex-post equity premium has also been very large (Fama and French, 2002).

The point in the figure that lies furthest to the right corresponds to the portfolio long Canadian dollars and short US dollars. We have shown that a position long US dollars and short Canadian dollars is a particularly effective hedge against fluctuations in global equity markets. Conversely, a portfolio long Canadian dollars and short US dollars is particularly risky, because it is highly positively correlated with global stock markets. Figure 3 shows that this is the portfolio with the largest full-sample beta, above 0.6. It provides investors with an average positive return of about 1.2% per annum (see Table I) which, though positive, is located below the fitted security market line and is well below the average return on other portfolios with lower betas.

We have shown that portfolios which are long euro-denominated bills also help investors attenuate fluctuations in global stock portfolios, because the euro tends to covary negatively with global equity returns. Conversely, portfolios which are short euros and long other currencies are positively correlated with global equities. These are the points corresponding to the euro pairs shown in Figure 6. As expected, these portfolios all exhibit positive betas. While their average excess returns exhibit a positive relation with betas, they tend to lie below the fitted security market line.

Overall, we do see differences in average realized currency returns that correlate with currency risks. However, these average return differences are quite modest, particularly in the case of the US dollar and the euro. Also, it is important to keep in mind that sample average currency returns are noisy estimates of true mean currency returns. None of the currencies in our sample have excess returns relative to the US dollar that are significantly different from zero at the 5% significance level.

Figure 4 repeats the exercise shown in Figure 3, except that it treats all currency pairs in each of the subperiods we considered in section III.D (1975 to 1989 and 1990 to 2005) as separate assets. Consistent with the results of section III.D, portfolios which are short the euro tend to be significantly riskier in the second subsample, reflecting the increasing tendency of the euro to move as a reserve currency. Also consistent with our earlier results, the portfolio which is long the Canadian dollar and short the US dollar is somewhat less risky in the second subperiod. In general, currency pairs show a much wider dispersion in betas in the second subperiod. The security market lines have modest positive slopes in both subperiods, but of course the precision of mean currency return estimates is even lower in the subperiods than in the full sample.

VI. Conditional Currency Risk Management

In previous sections of this paper we have estimated unconditional risk management demands for currencies, and have found that reserve currencies tend to have positive risk management demands and low average returns.

Along with these cross-sectional patterns, there could also be systematic variation in currency risks over time. It is well known that interest differentials predict excess returns on currencies, even controlling for cross-currency differences in average returns. Contrary to the predictions of the uncovered interest parity hypothesis, currencies whose short-term interest rates are higher than normal tend to appreciate relative to currencies whose short-term interest rates are lower than normal. This behavior contributes to the profits of the currency carry trade, which takes long positions in high-interest-rate currencies and short positions in low-interest-rate currencies. Some of these profits come from long-run differences in average currency returns and interest rates, of the sort summarized in Table I, but carry trade profits also result from time-variation in currency returns along with interest differentials.

It is natural to ask if this conditional component of the carry trade is attractive to risk-minimizing investors, or if such investors should avoid currencies with temporarily high interest differentials. To explore this question, we consider a conditional model for risk management currency demand which depends linearly on interest differentials:

$$\widetilde{\boldsymbol{\Psi}}^*_{RM,t} = \widetilde{\boldsymbol{\Psi}}^*_0 \cdot \left(\overline{\mathbf{i}_t - \mathbf{i}^d_t}\right) + \widetilde{\boldsymbol{\Psi}}^*_1 \cdot \left(\mathbf{i}_t - \mathbf{i}^d_t - \left(\overline{\mathbf{i}_t - \mathbf{i}^d_t}\right)\right) \tag{3}$$

where $\left(\overline{\mathbf{i}_t - \mathbf{i}^d_t}\right)$ is the unconditional expectation of the vector of interest differentials, $\widetilde{\boldsymbol{\Psi}}^*_0 = (\psi_{0,2}, ..., \psi_{0,n+1})'$, $\widetilde{\boldsymbol{\Psi}}^*_1 = (\psi_{1,2}, ..., \psi_{1,n+1})'$, and $\cdot$ is the element-by-element product operator. Thus $\widetilde{\boldsymbol{\Psi}}^*_{RM,0} \cdot \left(\overline{\mathbf{i}_t - \mathbf{i}^d_t}\right)$ represents the vector of average risk management demands, and $\widetilde{\boldsymbol{\Psi}}^*_{RM,1}$ the vector of sensitivities of those demands to movements in interest differentials over time.

Using (2), we can estimate currency risk management demands (3) by estimating the simple regression

$$\begin{aligned} \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) &= \gamma_0 - \sum_{c=2}^{n+1} \psi_{0,c}(\Delta s_{t+1} - i^d_t + i_{c,t})\left(\overline{i_{c,t} - i_{1,t}}\right) \\ &\quad - \sum_{c=2}^{n+1} \psi_{1,c}(\Delta s_{t+1} - i^d_t + i_{c,t})\left(i_{c,t} - i_{1,t} - \left(\overline{i_{c,t} - i_{1,t}}\right)\right) + \varepsilon_{t+1}, \end{aligned} \tag{4}$$

where the left hand side variable is the excess return on a fully hedged portfolio. We are primarily interested in estimating the vector of slopes $\widetilde{\boldsymbol{\Psi}}^*_1 = (\psi_{1,2}, ..., \psi_{1,n+1})'$ and testing whether they are zero. Note that when they are zero, the regression model (4) recovers exactly the unconditional risk management demands that we estimated in sections III and IV.

We estimate two sets of conditional risk management currency demands, and report the results in Table VI. First, we consider the case where only one single foreign currency is available for investment, in addition to the domestic or base currency; second, we consider the case where all currencies are available for investment simultaneously. Table VI reports results for both a global equity portfolio and a global bond portfolio.

To increase the power of our tests, we estimate (4) imposing the constraint that all slopes are equal, i.e., that $\widetilde{\mathbf{\Psi}}_1^* = \psi_1 \mathbf{1}$. Table VI reports the constrained estimate of the slope ψ_1 for each base currency, its Newey-West standard error, and the p-value of the null hypothesis that this slope is the same across all currencies. We omit from the table the estimated average risk management demands $\widetilde{\mathbf{\Psi}}_{RM,0}^* \cdot \overline{\mathbf{i}_t - \mathbf{i}_t^d}$, because they are very similar both economically and statistically to the unconditional risk management demands reported in sections III and IV. It is important to note here that the coefficients $\psi_{0,c}$ and $\psi_{1,c}$ in model (3) are not invariant to the base currency of the investor, even in the multiple currency case.[10] Accordingly, we report results for each possible base currency in the rows of the table.

The left panel of Table VI shows results for the equally-weighted global equity portfolio. In the single-foreign-currency case, we find almost no evidence that risk management currency demands vary with interest differentials. The estimated slopes are all positive, but they are close to zero and imprecisely estimated. In the multiple-foreign-currency case we find positive and statistically significant coefficients whenever the base currency is a reserve currency—the euro, the Swiss franc, or the US dollar. The slopes are insignificantly different from zero for other base currencies.[11]

These results imply that the currency carry trade is attractive not only to currency speculators, but also to risk-minimizing equity investors, provided that their base currency is a reserve currency and that they can also hold fixed positions in foreign currencies that are unrelated to interest differentials. The time-series pattern shown in Table VI, that increases in a currency's interest rate improve its risk characteristics, contradicts the cross-sectional pattern illustrated in Figures 6 and 7, that normally high-rate currencies have poor risk characteristics.

Although this is a striking result, it is important not to exaggerate its significance. It applies only when the base currency is a reserve currency, and even in these cases the effect is modest. The reserve-currency slope coefficients reported in Table VI are in the range 4 to 6, which imply standard deviations of foreign currency weights of about 4-6%, given that the standard deviations of interest differentials are about 1%.[12] Such standard deviations are quite small relative to the average risk management currency demands reported in Table IV. Also, the appendix shows that within subsamples, the reserve-currency slope coefficients are positive and significant only in the second of our two subsamples, and only for euroland and Switzerland.

The right panel of Table VI looks at the risk-management properties of the carry trade from the point of view of a global bond market investor. None of the slope coefficients are statistically significant except for an Australian bond investor trading in multiple currencies, who finds that increasing interest rates make foreign currencies riskier.

To better relate the time-series evidence to our earlier cross-sectional findings, we have also estimated our unconditional risk management currency demands adding an additional synthetic currency to the base set of currencies. Following Lustig and Verdelhan (2007), this synthetic currency is a zero-investment portfolio of currencies which at the start of every month in our sample period goes long an equally weighted portfolio of the three currencies with the largest nominal short-term interest rates, and short an equally weighted portfolio with the currencies with the smallest nominal short-term interest rates. Thus its returns mimic one of the most popular implementations of the currency carry trade.

In the appendix, we report that the equally-weighted global equity portfolio generates a positive and statistically significant risk management demand for the synthetic currency when all other currencies are available for investment, but a negative and significant risk management demand when only the synthetic currency is available. When risk-minimizing equity investors can hold fixed long positions in reserve currencies and short positions in commodity-dependent currencies, they are willing to hold an additional position in the synthetic carry-trade portfolio, but this portfolio is unattractive in isolation because on average it tends to short reserve currencies and hold commodity-dependent currencies. This finding reproduces the contrast between the cross-sectional pattern illustrated in Figure 6 and the time-series evidence reported in Table VI.[13]

Table VI uses nominal interest differentials, the usual basis for the carry trade, as conditioning information. It is also possible to condition on lagged ex post real interest differentials. When we do this, in the appendix, we find only one case where a risk-minimizing investor would want to condition currency demand on real interest differentials, and this case (a UK equity investor with access to multiple currencies) again has the demand for each currency increasing in its interest differential.

In conclusion, we find relatively weak evidence that interest differentials provide important conditioning information for currency risk management. When we allow risk-minimizing positions in our seven currencies to move with interest differentials, we find statistically significant time-variation only when the base currency is the euro, Swiss franc, or US dollar. This time-variation is quite small relative to the unconditional average currency demands we estimated earlier. Interestingly, it increases the demand for currencies with temporarily high interest rates, amplifying the speculative demand rather than offsetting it. We obtain similar results when we add a synthetic carry-trade currency to our unconditional analysis.

VII. Risk Reduction from Currency Hedging

We have argued that fixed positions in foreign currencies, possibly with some additional time-variation in currency positions in response to changing interest differentials, can reduce portfolio risks for global equity and bond investors. But how large are the feasible risk reductions?

Table VII reports the portfolio standard deviations that investors can achieve by combining their global stock and bond portfolios with risk-minimizing currency exposures. For each base currency, we report the annualized standard deviations of quarterly returns given several alternative currency hedging strategies. The first three columns report volatilities for unhedged, half hedged, and fully hedged portfolios. Half hedging is a compromise strategy that is popular with some institutional investors. Next we report the volatilities of various types of optimally hedged portfolios. The baseline case adopts fixed, unconditionally optimal currency positions, as in sections III and IV. We compare this with constrained conditional hedging in the manner of Table VI. The volatilities are independent of the base currency for the cases with full or unconditionally optimal hedging, but not for the cases with no hedging, half hedging, or constrained conditionally optimal hedging. The right hand part of the table reports F-statistics and p-values to test the statistical significance of the risk reductions achieved by unconditionally optimal currency hedging in relation to full hedging and zero hedging, and by constrained conditional hedging in relation to unconditionally optimal hedging.

Full sample results, in Table VII, show that the benefit of full currency hedging depends sensitively on an investor's base currency. It is particularly large for Euroland and Swiss investors, because these investors have a risk-reducing base currency so they gain by hedging back to that currency and out of foreign currencies. The volatility reduction from full currency hedging is particularly small for Australian and Canadian investors, because the home currency for these investors is risky in the sense that it is positively correlated with their equity positions. In fact, full currency hedging actually increases risk for a Canadian investor.

Optimal hedging, of course, reduces risk for all investors, including Australians and Canadians. The important point is that the benefit of optimal hedging is substantial, both economically and statistically. The big gain comes from adopting an unconditionally optimal currency hedging strategy with fixed currency positions. Relative to full hedging, this strategy reduces the standard deviation of an equally-weighted global equity portfolio by 135 basis points. This difference is statistically highly significant, with p-values that are well below 1%.

There is only a small additional reduction from conditional currency hedging that responds to time-varying interest differentials. The largest additional risk reductions come for Swiss investors (11 basis points, significant at the 2% level) and Euroland and US investors (6 basis points each, significant at around the 6% level). Unconditional hedging with a synthetic carry-trade currency delivers a comparably modest risk reduction of 8 basis points relative to unconditionally optimal currency hedging.

The gains from currency hedging are also substantial for global bond investors, but in this case almost all the gains can be achieved by full currency hedging. Table VII shows that relative to full hedging, unconditionally optimal currency hedging reduces the standard deviation of an equally-weighted global bond portfolio by only 19 basis points. This difference is statistically significant at almost the 1% level. As in the equity case, the additional benefits of conditional currency hedging are extremely small.

In the appendix we report subperiod results. The main difference between the first and the second subperiod is that the benefit of unconditionally optimal hedging for equity investors, relative to a simple policy of full hedging, increases from 62 basis points in the first subperiod to 267 basis points in the second. The benefit of unconditionally optimal hedging for bond investors is much smaller and relatively stable across subperiods, and the additional benefits of conditional hedging are always modest for both equity and bond investors.

The optimal currency hedging policies that we have estimated allow global equity and bond investors to achieve economically and statistically significant reductions in portfolio return volatility. A question of practical importance is whether these volatility reductions come at the cost of lower expected return per unit of portfolio risk. To examine this question we have computed the realized Sharpe ratios of global equity and bond portfolios under the same currency hedging policies considered in Table VII. The results are reported and discussed in detail in the appendix.

Overall, the Sharpe ratios on currency strategies depend sensitively on the average returns realized on different base currencies. On average across base currencies, Sharpe ratios are higher for fully and unconditionally optimally hedged portfolios than for unhedged portfolios, and higher again for conditionally optimally hedged portfolios. The increases in Sharpe ratios are particularly large for US, Euroland, and Swiss equity investors implementing constrained conditional hedging strategies. These are the base currencies for which we found the strongest effect of interest differentials on equity investors' currency hedging demands in Table VI. These results should be interpreted with caution because they are calculated using sample average currency returns, which are noisy estimates of true mean currency returns. Even over the full sample, there is no currency whose average excess log return, relative to the US dollar, is significantly different from zero at the 5% level.

VIII. Conclusion

In this paper we have studied the correlations of foreign exchange rates with stock and bond returns over the period 1975 to 2005 and have drawn out the implications for risk management by international equity and bond investors. We have found that many currencies—in particular the Australian dollar, Canadian dollar, Japanese yen, and British pound—are positively correlated with world stock markets. The euro, the Swiss franc, and the bilateral US-Canadian exchange rate, however, are negatively correlated with the world equity market. These patterns imply that international equity investors can minimize their equity risk by taking short positions in the Australian and Canadian dollars, Japanese yen, and British pound, and long positions in the US dollar, euro, and Swiss franc. For risk-minimizing US equity investors, the implication is that the currency exposures of international equity portfolios should be at least fully hedged, and probably overhedged, with the exception of the euro and Swiss franc which should be partially hedged.

These results are robust to variation in the investment horizon between one month and one year. We obtain similar results when we consider the 1970's and 1980's in one subsample and the 1990's and 2000's in another, except that risk-minimizing equity investors should hold more euros and Swiss francs in the later period and slightly fewer dollars. Over the full sample period the optimal currency hedging strategy reduces the standard deviation of a global equity portfolio by 135 basis points relative to a strategy of fully hedging all currency risk, and by over 250 basis points (for a US investor) relative to a strategy of leaving currency risk unhedged.

We have found that bond investors' risk management demands for currencies are small or zero, regardless of the investors' home country, and regardless of whether they hold only domestic bonds or an international bond portfolio. These optimal zero currency demands reflect correlations close to zero between bond excess returns and currency excess returns. The only exception is a weak negative correlation of bond returns with excess returns on the dollar relative to other currencies. This correlation implies a small positive allocation to the dollar by most bond investors. Our results thus provide support for the practice prevalent among international bond investors to fully hedge the currency exposures implicit in their international bond holdings.

Campbell, Viceira, and White (2003) show that long-term investors interested in minimizing real interest rate risk using international portfolios of bills—or equivalently, currency exposures—also have large demands for bills denominated in euros and US dollars, because these two currencies have had relatively stable interest rates. Their results suggest that these two currencies are attractive stores of value for international money market investors. Our results add to this evidence, by showing that the US dollar and the euro tend to appreciate when international stock markets fall. This negative correlation generates demands for US dollar and euro denominated bills as a way to reduce the volatility of international stock portfolios. In other words, the US dollar and the euro are attractive stores of value for international equity investors.

One might expect that in equilibrium, those currencies that are attractive for risk management purposes would offer lower average returns. Indeed, there is a positive relation between average currency returns in our sample and the betas of currencies with a currency-hedged world stock index, although the reward for taking beta exposure through currencies has been quite modest in our sample, certainly well below the historical equity premium. To the extent that international investors are willing to receive lower compensation for holding US dollar and euro denominated bills because of the hedging properties of these currencies, a country benefits from having a reserve currency not only because international demand for its monetary base generates seigniorage revenue, but also because international demand for its Treasury bills reduces the interest cost of financing the government debt.

We have also explored whether movements in interest differentials over time, which are known to predict excess currency returns, predict changes in currency risks as might be expected in capital market equilibrium. Here we obtain a perverse result that, if anything, currencies become more attractive to risk-minimizing global equity investors when their interest rates increase. The effect on risk-management currency demands is small, but certainly we find no evidence that the forward premium puzzle can be explained by changing covariances of currencies with global stock returns.

Our findings raise the interesting question why currencies are so heterogeneous in their correlations with equity markets. The Australian and Canadian economies are unusually commodity-dependent, and the positive correlations of their currencies with world stock markets are consistent with the idea that fluctuations in world economic growth drive equity and commodity prices in the same direction. It is more challenging to explain why the US dollar, euro, and Swiss franc behave differently from other currencies. One possible explanation is that they attract flows of capital at times when bad news arrives about the world economy, or when investors become more risk averse. This "flight to quality" drives up the dollar, euro, and Swiss franc at times when the prices of risky financial assets decline. This explanation takes as given that these currencies are regarded as safe assets and therefore benefit from a flight to quality.[14] It is consistent with the role of the US dollar and the euro as reserve currencies in the international financial system. Our finding that the risk-minimizing demand for euros has increased over time suggests that the euro has partially displaced the dollar as a reserve currency.

The analysis of this paper can be extended in several ways. First, we can consider emerging-market currencies jointly with developed-market currencies, as Lustig and Verdelhan (2007) and Walker (2008) do. Second, we can allow for short-term inflation risk that makes short-term nominal assets risky in real terms. The results of Cooper and Kaplanis (1994) and Campbell, Viceira, and White (2003) suggest that this has only a minor effect on optimal demands for long-term assets and currencies in developed markets, but it may be more important in emerging markets. Third, we can integrate the speculative and risk-management components of currency demand by solving the optimal portfolio choice problem for an investor choosing currency positions jointly with stock and bond positions.[15] Given the high historical returns to the currency carry trade, foreign currencies are likely to play an important role in such a portfolio choice analysis.

Fourth, we can measure the equity and bond covariances of foreign currencies more accurately by using high-frequency data. To the extent that these covariances change predictably over time, covariance forecasts based on high-frequency data can be used to calculate dynamic hedging strategies that may further reduce portfolio risk. Finally, we can conduct an out-of-sample analysis to see whether correlations of currencies with bond and stock returns are stable enough that historically estimated risk management demands actually reduce portfolio risk in the future.

The financial crisis of 2008 is not part of our sample period, and currency movements in 2008 can be used as an informal out-of-sample assessment of our results. Reserve currencies, particularly the US dollar, and currencies with low unconditional average interest rates such as the Japanese yen, have tended to strengthen against other currencies during the stock market declines of 2008, while commodity-dependent currencies such as the Australian dollar have weakened. This confirms the attractiveness of reserve currencies as hedges for equity investors. Government bonds have moved opposite stocks in 2008, and thus the currency positions that have minimized risk for bond investors are quite different and much closer to the conventional strategy of fully hedging all currency risk.

Endnotes

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1. This appendix is available online at http://www.afajof.org/supplements.asp.

2. Risk management demands are more commonly called hedging demands, but this can create confusion in the context of foreign currency because hedging a foreign currency corresponds to taking a short position to cancel out an implicit long position in that currency. In this paper we use foreign currency terminology and avoid the use of the term hedging demand for assets.

3. For a discussion of currency hedging from a practitioner's perspective, see Jorion (1989) or the other papers collected in Thomas (1990).

4. In the case of the Swiss short-term interest rate, our data source is the OECD. We use euro-money rates up to 1989, and LIBOR rates afterwards, as published by the OECD.

5. This approximation to the log return on a coupon bond is

$$r_{c,n,t+1} = D_{cn} y_{cnt} - (D_{cn} - 1)\, y_{c,n-1,t+1},$$

where $r_{c,n,t+1}$ denotes the log return on a coupon bond with coupon rate c and n periods to maturity, $y_{cnt} \equiv \log(1 + Y_{cnt})$ denotes the log yield on this bond at time t, and D_{cn} is its duration, which we approximate as

$$D_{cn} = \frac{1 - (1 + Y_{cnt})^{-n}}{1 - (1 + Y_{cnt})^{-1}}.$$

In our computations we treat all bonds as having a maturity of 10 years, and assume that bonds are issued at par, so that the coupon rate equals the yield on the bond. We also assume that the yield spread between a 9 years and 11 months bond and a 10-year bond is zero.

6. If we include only Germany in Euroland, this region also exhibits one of the lowest average short-term nominal interest rates, similar to those of Japan and Switzer-

land.

7. On average, the US stock market represents 49.3% of total market capitalization (and 54.5% at the end of our sample period). The Japanese, Euroland and British stock markets follow with weights of 20.6% (11.3%), 12.9% (13.7%), and 9.7% (11.3%), respectively. The Australian, Canadian and Swiss markets are much smaller, respectively representing 1.7% (2.5%), 3.2% (3.5%) and 2.6% (3.3%) of our seven countries' market capitalization. These weights are fairly stable over our sample period, with the exception of the Japanese stock market whose market capitalization share grew rapidly in the late 1980's before collapsing in the 1990's.

8. In a similar spirit, Lustig and Verdelhan (2007) show that currencies with high interest rates have high covariances with US consumption growth. The connection between liquidity preference (the demand for safe assets with low returns) and risk was first made explicitly by Tobin (1958).

9. There are no meaningful differences if we use the log of average currency excess returns, or calculate betas with respect to a value-weighted global stock portfolio. Also, note that currency betas with respect to the global stock market portfolio are proportional to the negative of the currency demands that we find in section III for the case with a global stock portfolio and a single available currency pair.

10. To obtain an invariance result similar to the one we have applied in sections 4 and 5, we would need to include the cross-product of each currency excess return with all interest rate differentials in the regression (4). This would force us to estimate an excessive number of parameters.

11. All these results are obtained conditional on the restriction that slopes are equal across foreign currencies. We find no evidence against this restriction in the single-foreign-currency case, but the evidence is mixed in the multiple-foreign-currency case, where we reject the restriction at the 5% level for the US base currency, and the 10% level for the Australian and Japanese base currencies.

12. Interest differentials do take some extreme values, up to 5 standard deviations in either direction, implying larger effects on optimal currency positions at one or two points in the sample. However these extreme values are not driving the results reported in Table 7. If we winsorize differentials at 3 or 4 standard deviations before including them in the system, we obtain similar results to those reported.

13. A caveat is that this contrast is due to the early part of our sample period. Subperiod results in the appendix show that even with fixed currency positions available, the synthetic currency has become unattractive to risk-minimizing equity investors in the period since 1990.

14. Similarly, Campbell, Sunderam, and Viceira (2008) argue that long-term US Treasury bonds benefit from a flight to quality only at times when their fundamentals make them negatively correlated with equities. Pavlova and Rigobon (2007) model fundamentals that affect the correlation between a country's stock market and its exchange rate.

15. A simple approach to this problem would use mean-variance analysis as in Cooper and Kaplanis (1994), but if the investor has a long-term objective function, a more appropriate framework is the long-term portfolio choice model of Merton (1971), as implemented for example by Campbell, Chan, and Viceira (2003) and Jurek and Viceira (2006). Long-horizon mean-variance analysis ignores the fact that investors can rebalance their portfolios over time, and the alternative framework takes this into account.

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Table I
Summary Statistics

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Log interest rates							
Average	6.42	8.49	7.39	3.69	3.24	8.14	5.83
Standard Deviation	1.39	1.78	1.78	1.52	1.14	1.57	1.44
Excess log stock returns in local currency							
Adjusted Average	7.52	6.99	5.24	4.97	8.77	6.93	7.06
Standard Deviation	18.02	18.75	17.40	18.65	17.67	16.82	14.63
Excess log bond returns in local currency							
Adjusted Average	1.12	2.10	2.55	3.00	2.16	2.85	2.84
Standard Deviation	5.05	8.80	7.89	7.97	5.58	7.84	8.59
Change in log exchange rate							
Adjusted Average	0.93	-1.33	-0.28	3.80	3.15	-0.07	.
Standard Deviation	11.12	10.30	5.26	12.23	12.88	11.23	.
Excess log currency returns							
Adjusted Average	1.66	1.42	1.28	1.74	0.70	2.36	.
Standard Deviation	11.23	10.51	5.39	12.64	13.14	11.45	.

Note. Stock market returns are from the Morgan Stanley Capital International database. All other variables are from the IMF's IFS database. Data are monthly. Coverage extends from 1975:7 to 2005:12. Unless otherwise specified, all following tables use data from the full period.

Interest rates are log 3-month government bill rates, with the exception of Switzerland (see footnote 4 in text)

Excess log stock (bond) excess returns are the returns on foreign stocks (bonds) to a fully hedged investor, i.e. local currency returns, in excess of the local log nominal interest rate.

The currency excess return is the log return to a US investor of borrowing in dollars to hold foreign currency.

Rows labeled Adjusted Average report the mean of each variable plus one-half its variance, in percentage points per annum. The mean gives an estimate of the mean simple excess return.

Table II
Cross-country return correlations

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A: Currencies							
Euroland	1.00						
Australia	0.38	1.00					
Canada	0.45	0.70	1.00				
Japan	0.51	0.34	0.35	1.00			
Switzerland	0.89	0.20	0.24	0.54	1.00		
UK	0.68	0.38	0.44	0.40	0.50	1.00	
US	0.47	0.54	0.89	0.42	0.27	0.47	1.00
Panel B : Stocks							
Euroland	1.00						
Australia	0.55	1.00					
Canada	0.61	0.66	1.00				
Japan	0.50	0.35	0.44	1.00			
Switzerland	0.80	0.54	0.56	0.42	1.00		
UK	0.69	0.55	0.55	0.44	0.66	1.00	
US	0.71	0.61	0.75	0.41	0.68	0.65	1.00
Panel B : Bonds							
Euroland	1.00						
Australia	0.45	1.00					
Canada	0.60	0.44	1.00				
Japan	0.51	0.36	0.43	1.00			
Switzerland	0.65	0.35	0.46	0.35	1.00		
UK	0.50	0.33	0.45	0.31	0.39	1.00	
US	0.62	0.43	0.79	0.44	0.45	0.40	1.00

Note. This table presents cross-country correlations of log excess returns on currencies, stocks and bonds.
Each cell of Panel A reports the correlation of currency log excess returns ($s_{c,t}+i_{c,t}-i_{d,t}$, where c indexes the base country), between the row and column currencies. The correlation reported is an average across all possible base countries.
Panels B and C report correlations of hedged stock market excess returns ($r_{c,t}-i_{c,t}$, see Table I note) and hedged bond excess return ($rb_{c,t}-i_{c,t}$, see Table I note) between the row and column countries.
All correlations are computed using overlapping monthly observations of quarterly returns.

Table III

Optimal currency exposure for single-country stock portfolios: single and multiple currency cases

Stock market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		-0.43***	-0.57***	-0.32***	0.37*	-0.37***	-0.52***
		(0.11)	(0.14)	(0.09)	(0.20)	(0.12)	(0.14)
Australia	0.39***		0.13	0.18*	0.32***	0.16	0.29**
	(0.12)		(0.13)	(0.11)	(0.11)	(0.14)	(0.12)
Canada	0.42***	-0.02		0.13	0.35***	0.14	0.96***
	(0.11)	(0.12)		(0.10)	(0.10)	(0.12)	(0.22)
Japan	0.34***	-0.09	-0.06		0.36***	0.17*	0.04
	(0.12)	(0.11)	(0.12)		(0.12)	(0.10)	(0.13)
Switzerland	-0.51***	-0.37***	-0.44***	-0.27***		-0.32***	-0.43***
	(0.17)	(0.08)	(0.10)	(0.09)		(0.09)	(0.11)
UK	0.26*	-0.26**	-0.32**	-0.10	0.26**		-0.24*
	(0.13)	(0.11)	(0.13)	(0.08)	(0.10)		(0.13)
US	0.19*	-0.14	-0.77***	-0.03	0.19**	0.09	
	(0.11)	(0.09)	(0.17)	(0.10)	(0.09)	(0.11)	
Panel B : Multiple currencies							
Euroland	0.42	-0.10	-0.40*	-0.19	0.34	-0.09	0.02
	(0.26)	(0.12)	(0.24)	(0.12)	(0.21)	(0.14)	(0.24)
Australia	0.55**	-0.20	-0.66***	-0.11	0.11	-0.31*	0.62**
	(0.24)	(0.14)	(0.20)	(0.13)	(0.22)	(0.17)	(0.25)
Canada	0.34	-0.06	-1.00***	-0.21*	0.32	-0.31**	0.92***
	(0.23)	(0.11)	(0.24)	(0.11)	(0.23)	(0.15)	(0.23)
Japan	0.38*	-0.18	-0.58**	-0.27*	0.16	0.03	0.46*
	(0.21)	(0.16)	(0.26)	(0.15)	(0.20)	(0.13)	(0.25)
Switzerland	0.12	-0.15	-0.20	-0.02	0.40**	-0.01	-0.13
	(0.24)	(0.10)	(0.21)	(0.12)	(0.19)	(0.14)	(0.23)
UK	0.34	-0.13	-0.49**	-0.18*	0.27	-0.01	0.20
	(0.25)	(0.11)	(0.22)	(0.10)	(0.22)	(0.16)	(0.22)
US	0.09	0.04	-0.91***	-0.23**	0.31*	-0.01	0.71***
	(0.21)	(0.09)	(0.18)	(0.10)	(0.17)	(0.12)	(0.20)

Note. This table considers an investor holding a portfolio composed of equity from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the equity being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country stock market onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock market onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table IV

Optimal currency exposure for an equally-weighted global equity portfolio: multiple-currency case

Time Period	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Full period	0.32*	-0.11	-0.61***	-0.17*	0.27*	-0.10	0.40**
	(0.17)	(0.09)	(0.16)	(0.09)	(0.15)	(0.11)	(0.18)
Subperiod I	0.14	-0.05	-0.63**	-0.20	0.22	-0.09	0.62*
	(0.21)	(0.12)	(0.26)	(0.14)	(0.18)	(0.15)	(0.35)
Subperiod II	0.44	-0.17	-0.65***	-0.08	0.37	-0.12	0.22
	(0.28)	(0.14)	(0.21)	(0.10)	(0.23)	(0.14)	(0.19)
Panel B : 5 country optimization							
Full period	0.56***	-0.27***		-0.14*		-0.09	-0.06
	(0.11)	(0.10)		(0.08)		(0.11)	(0.14)
Subperiod I	0.35**	-0.15		-0.15		-0.10	0.05
	(0.17)	(0.11)		(0.11)		(0.15)	(0.20)
Subperiod II	0.79***	-0.47***		-0.06		-0.11	-0.15
	(0.13)	(0.12)		(0.11)		(0.13)	(0.17)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Panel A considers a case where all 7 currencies are available, whereas Panel B excludes the Canadian dollar and the Swiss franc.

Within each Panel, rows indicate the time period over which the optimization is computed, columns the currencies used to manage risk. The full period runs from 1975 to 2005, the first subperiod covers years 1975 through 1989 and the second subperiod covers the rest of the sample.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *3*-months returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table V

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time Period	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Full period	-0.03	0.04	-0.10	-0.07*	-0.03	0.01	0.18***
	(0.07)	(0.04)	(0.08)	(0.04)	(0.07)	(0.05)	(0.06)
Subperiod I	-0.02	0.04	-0.30**	-0.16***	-0.01	0.02	0.43***
	(0.10)	(0.05)	(0.14)	(0.06)	(0.08)	(0.06)	(0.12)
Subperiod II	-0.17	0.13**	-0.08	-0.01	0.06	0.04	0.04
	(0.12)	(0.06)	(0.11)	(0.05)	(0.11)	(0.06)	(0.06)
Panel B : 5 country optimization							
Full period	-0.06	0.01		-0.08*		0.01	0.11***
	(0.05)	(0.03)		(0.04)		(0.05)	(0.04)
Subperiod I	-0.03	-0.04		-0.14**		0.01	0.20***
	(0.08)	(0.04)		(0.07)		(0.05)	(0.06)
Subperiod II	-0.13**	0.12***		0.00		0.03	-0.02
	(0.06)	(0.04)		(0.05)		(0.06)	(0.05)

Note. This table considers an investor holding a portfolio composed of long term bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Panel A considers a case where all 7 currencies are available, whereas Panel B excludes the Canadian dollar and the Swiss franc.

Within each Panel, rows indicate the time period over which the optimization is computed, columns the currencies used to manage risk. The full period runs from 1975 to 2005, the first subperiod covers years 1975 through 1989 and the second subperiod covers the rest of the sample.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *3* -months returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table VI

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	0.08	1.00	5.94*	0.64	0.04	0.99	0.86	0.01
	(1.02)		(3.15)		(0.22)		(1.33)	
Australia	0.04	0.99	-0.35	0.10	-0.02	1.00	-1.49**	0.77
	(0.47)		(1.79)		(0.16)		(0.58)	
Canada	0.31	1.00	1.74	0.13	0.03	1.00	0.28	0.22
	(0.70)		(2.63)		(0.17)		(1.59)	
Japan	0.05	1.00	0.95	0.10	-0.01	1.00	0.66	0.09
	(0.36)		(2.98)		(0.16)		(1.21)	
Switz.	0.27	1.00	4.49**	0.67	0.04	0.99	0.46	0.00
	(0.78)		(1.91)		(0.15)		(0.73)	
UK	0.00	1.00	1.10	0.22	0.09	0.99	2.25	0.08
	(0.54)		(2.83)		(0.21)		(1.41)	
US	0.22	1.00	4.78*	0.07	0.06	1.00	1.38	0.09
	(0.57)		(2.50)		(0.22)		(1.06)	

Note. This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of the base country). Yet, we impose the constraints that the slopes of the optimal positions with respect to the interest rate differential be equal across foreign currencies.

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.

Table VII

Standard deviations of hedged global equity and bond portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge: Baseline	Optimal hedge: Conditional hedging (constrained)	Tests of significance: Baseline vs. full hedge F-Stat	Baseline vs. full hedge P-value	Baseline vs. no hedge F-Stat	Baseline vs. no hedge P-value	Conditional vs. Baseline F-Stat	Conditional vs. Baseline P-value
Equity											
Euroland	17.67	15.47	13.86	12.51	12.45	7.98	0.00	33.36	0.00	3.55	6.05
Australia	15.00	13.52	13.86	12.51	12.51	7.98	0.00	20.09	0.00	0.04	84.37
Canada	13.74	13.22	13.86	12.51	12.50	7.98	0.00	6.49	0.00	0.44	50.85
Japan	17.08	14.67	13.86	12.51	12.50	7.98	0.00	31.32	0.00	0.10	74.89
Switzerland	19.19	16.09	13.86	12.51	12.40	7.98	0.00	41.75	0.00	5.54	1.91
UK	16.78	14.74	13.86	12.51	12.50	7.98	0.00	25.47	0.00	0.15	69.90
US	15.05	13.91	13.86	12.51	12.45	7.98	0.00	15.20	0.00	3.67	5.63
Bonds											
Euroland	8.39	6.10	5.40	5.21	5.21	2.76	1.23	54.14	0.00	0.42	51.99
Australia	12.08	7.85	5.40	5.21	5.19	2.76	1.23	210.02	0.00	6.57	1.08
Canada	10.18	7.12	5.40	5.21	5.21	2.76	1.23	85.17	0.00	0.03	86.00
Japan	10.86	6.85	5.40	5.21	5.21	2.76	1.23	87.07	0.00	0.30	58.52
Switzerland	9.93	6.52	5.40	5.21	5.21	2.76	1.23	85.62	0.00	0.40	52.89
UK	10.35	6.98	5.40	5.21	5.19	2.76	1.23	87.03	0.00	2.53	11.23
US	10.53	7.36	5.40	5.21	5.20	2.76	1.23	127.73	0.00	1.68	19.53

Note. This table reports the standard deviation of portfolios featuring different uses of currency for risk-management.

We present results for equally-weighted global portfolios, for equity and bonds as respectively described in Table IV and Table VI. Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge"

Reported standard deviations are annualized, and measured in percentage points.

All results presented are computed considering returns at a quarterly horizon.

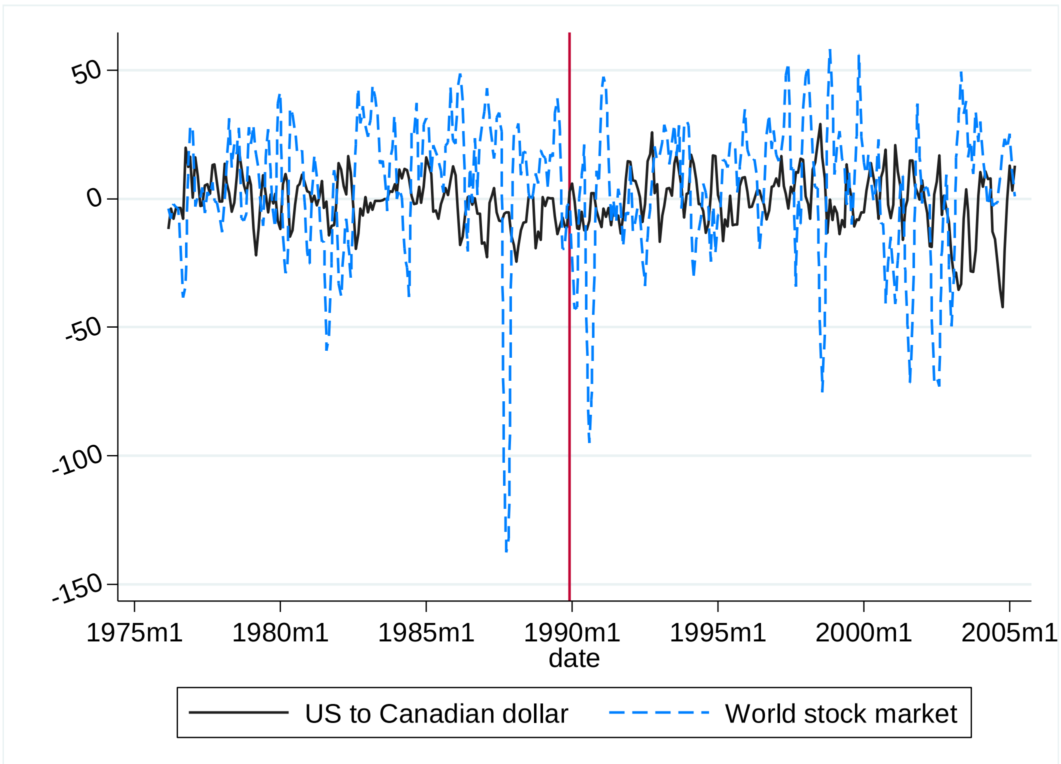

Figure 1: Annualized Excess Returns on US Dollar Long, Canadian Dollar Short Portfolio and the Hedged World Stock Market.

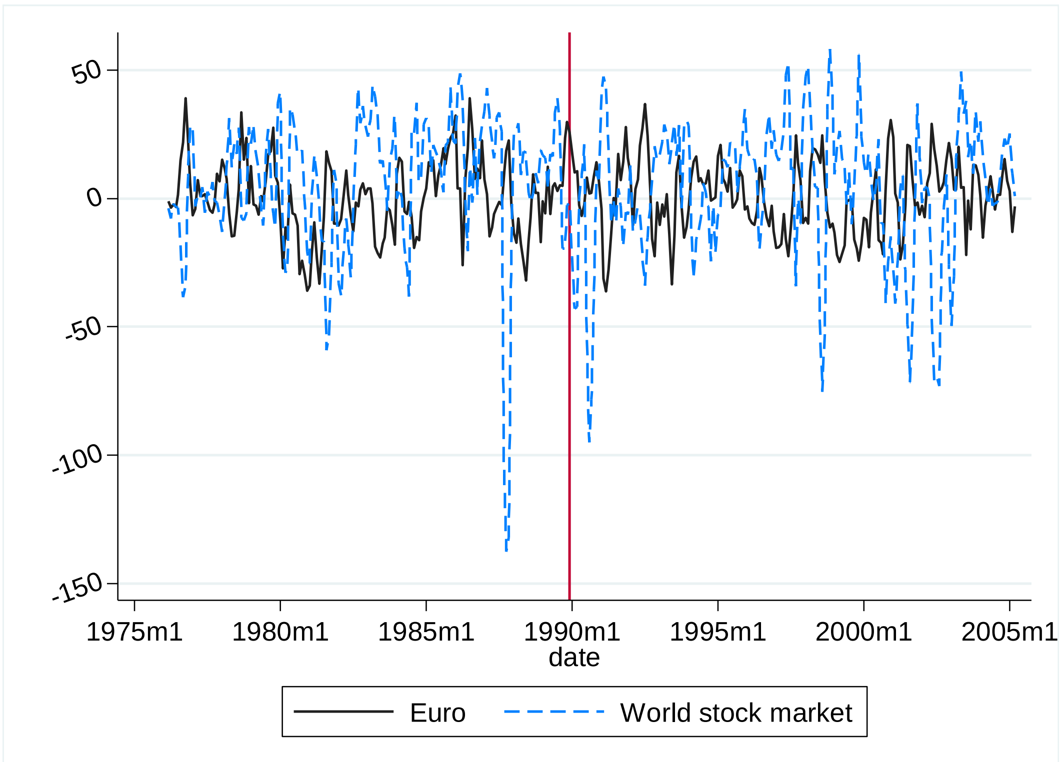

Figure 2: Excess Returns on the Euro and the Hedged World Stock Market.

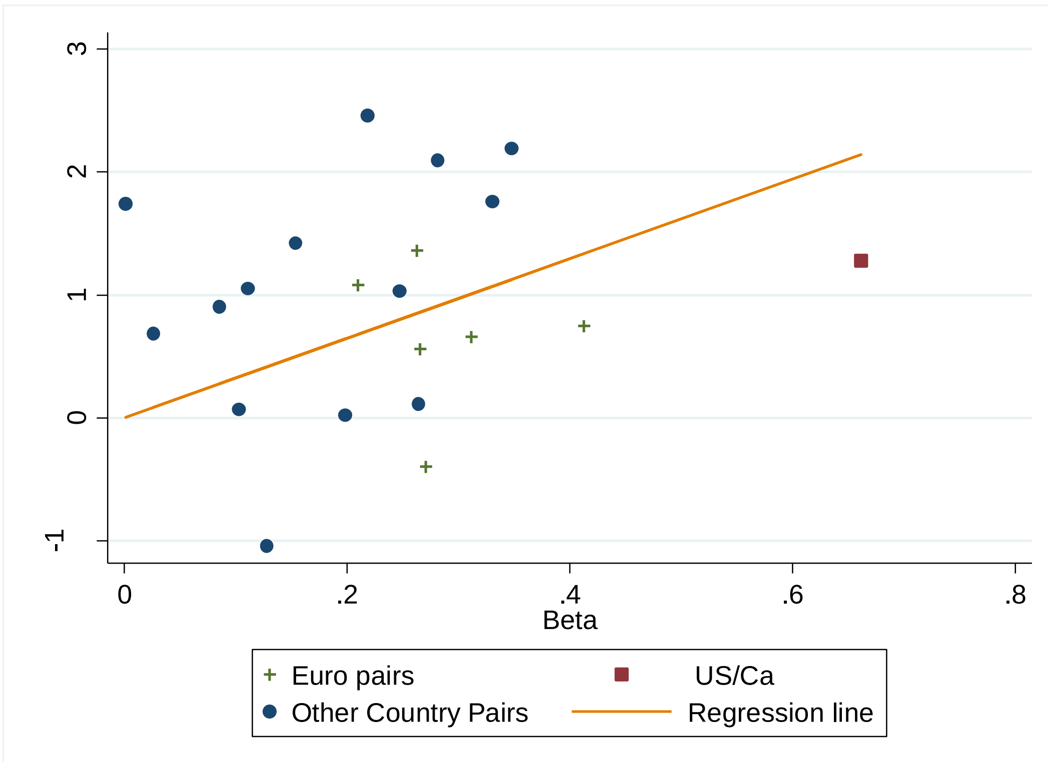

Figure 3: Mean-Beta Diagram for Currency Pairs, Full Sample Period (1975-2005). Betas are calculated with respect to a value-weighted, currency-hedged world market portfolio. Regression of mean returns on betas imposes a zero intercept.

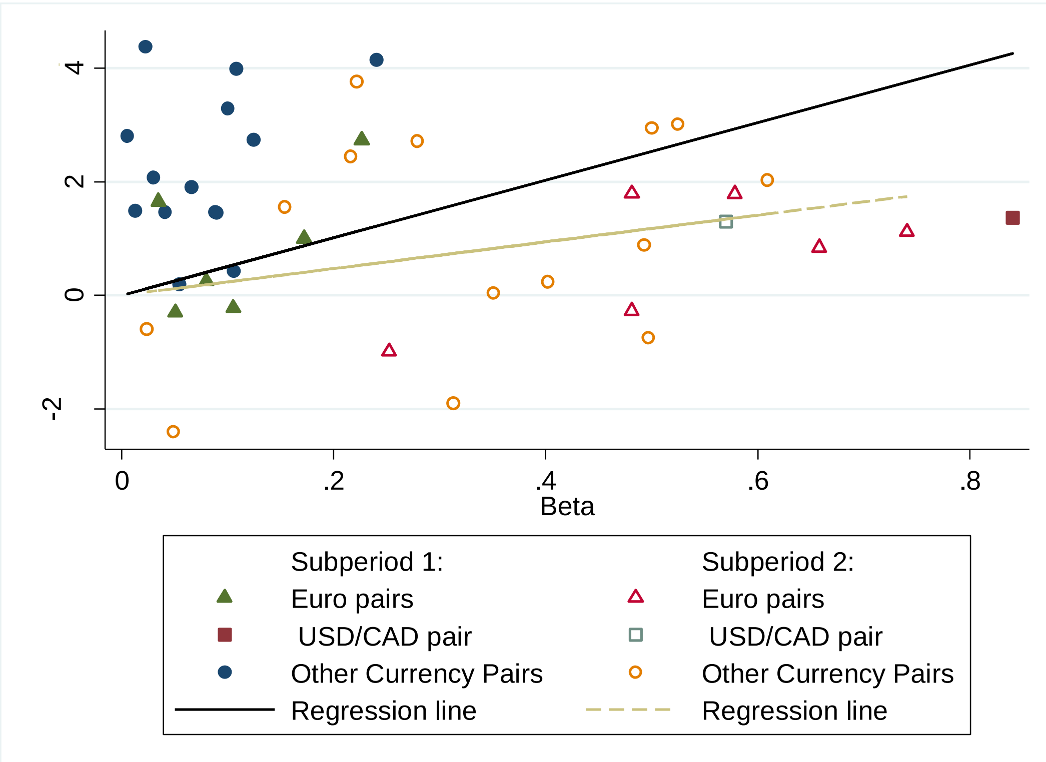

Figure 4: Mean-Beta Diagram for Currency Pairs, Subperiods (1975-1989 and 1990-2005). Betas are calculated with respect to a value-weighted, currency-hedged world market portfolio. Regression of mean returns on betas imposes a zero intercept.

Internet Appendix for "Global Currency Hedging"*

A. Hedged Portfolio Return

Let $R_{c,t+1}$ denote the gross return in currency c from holding country c stocks from the beginning to the end of period $t+1$, and let $S_{c,t+1}$ denote the spot exchange rate in dollars per foreign currency c at the end of period $t+1$. By convention, we index the domestic country by $c = 1$ and the n foreign countries by $c = 2, ..., n+1$. Of course, the domestic exchange rate is constant over time and equal to 1: $S_{1,t+1} = 1$ for all t.

At time t, the investor exchanges a dollar for $1/S_{c,t}$ units of currency c in the spot market which she then invests in the stock market of country c. After one period, stocks from country c return $R_{c,t+1}$, which the US investor can exchange for $S_{c,t+1}$ dollars, to earn an unhedged gross return of $R_{c,t+1}S_{c,t+1}/S_{c,t}$. For an arbitrarily weighted portfolio, the unhedged gross portfolio return is given by

$$R_{p,t+1}^{uh} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right),$$

where $\boldsymbol{\omega}_t = \text{diag}\left(\omega_{1,t}, \omega_{2,t}, ..., \omega_{n+1,t}\right)$ is the $(n+1 \times n+1)$ diagonal matrix of weights on domestic and foreign stocks at time t, $\mathbf{R}_{t+1}$ is the $(n+1 \times 1)$ vector of gross nominal stock returns in local currencies, $\mathbf{S}_{t+1}$ is the $(n+1 \times 1)$ vector of spot exchange rates, and $\div$ denotes the element-by-element ratio operator, so that the c-th element of $(\mathbf{S}_{t+1} \div \mathbf{S}_t)$ is $S_{c,t+1}/S_{c,t}$. The weights add up to 1 in each period t:

$$\sum_{c=1}^{n+1} \omega_{c,t} = 1 \qquad \forall t. \tag{A1}$$

We next consider the hedged portfolio. Let $F_{c,t}$ denote the one-period forward exchange rate in dollars per foreign currency c, and $\theta_{c,t}$ the dollar value of the amount of forward exchange rate contracts for currency c the investor enters into at time t per dollar invested in her stock portfolio.[1] At the end of period $t+1$, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of the foreign-currency denominated return $R_{c,t+1}\omega_{c,t}/S_{c,t}$ back into dollars at an exchange rate $F_{c,t}$. She then exchanges the rest, which amounts to $(R_{c,t+1}\omega_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency c, at the spot exchange rate $S_{c,t+1}$. Collecting returns for all countries leads to a hedged portfolio return $R^h_{p,t+1}$ of

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \mathbf{\Theta}'_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \mathbf{\Theta}'_t \left(\mathbf{F}_t \div \mathbf{S}_t\right), \tag{A2}$$

where $\mathbf{F}_t$ is the $(n+1\times1)$ vector of forward exchange rates, and $\mathbf{\Theta}_t = (\theta_{1,t}, \theta_{2,t}, ..., \theta_{n,t}, \theta_{n+1,t})'$. Of course, since $S_{1t} = F_{1,t} = 1$ for all t, the choice of domestic hedge ratio $\theta_{1,t}$ is arbitrary. For convenience, we set it so that all hedge ratios add up to 1:

$$\theta_{1,t} = 1 - \sum_{c=2}^{n+1} \theta_{c,t}. \tag{A3}$$

Under covered interest parity, the forward contract for currency c trades at $F_{c,t} =$

$S_{c,t}(1+I_{1,t})/(1+I_{c,t})$, where $I_{1,t}$ denotes the domestic nominal short-term riskless interest rate available at the end of period t, and $I_{c,t}$ is the corresponding country c nominal short-term interest rate. Thus the hedged dollar portfolio return (A2) can be written as

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t\left[\left(\mathbf{1}+\mathbf{I}^d_t\right) \div \left(\mathbf{1}+\mathbf{I}_t\right)\right], \qquad \text{(A4)}$$

where $\mathbf{I}_t = (I_{1,t}, I_{2,t}..., I_{n+1,t})$ is the $(n+1\times 1)$ vector of nominal short-term interest rates and $\mathbf{I}^d_t = I_{1,t}\mathbf{1}$.

Equation $(A4)$ shows that selling currency forward—i.e., setting $\theta_{c,t} > 0$—is analogous to a strategy of shorting foreign bonds and holding domestic bonds, i.e. borrowing in foreign currency and lending in domestic currency.[2]

To capture the fact that the investor can alter the currency exposure implicit in her foreign stock position using forward contracts or lending and borrowing, we now define a new variable $\psi_{c,t}$ as $\psi_{c,t} \equiv \omega_{c,t} - \theta_{c,t}$. A fully hedged portfolio, in which the investor does not hold any exposure to currency c, corresponds to $\psi_{c,t} = 0$. A positive value of $\psi_{c,t}$ means that the investor wants to hold exposure to currency c, or equivalently that the investor does not want to fully hedge the currency exposure implicit in her stock position in country c. Of course, a completely unhedged portfolio corresponds to $\psi_{c,t} = \omega_{c,t}$. Thus $\psi_{c,t}$ is a measure of currency demand or currency exposure. Accordingly we refer to $\psi_{c,t}$ as currency demand or currency exposure indistinctly.

For convenience, we now rewrite equation (A4) in terms of currency demands:

$$\begin{aligned} R^h_{p,t+1} &= \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \mathbf{1}'\boldsymbol{\omega}_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right] \\ &\quad + \boldsymbol{\Psi}'_t\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \end{aligned} \tag{A5}$$

where $\boldsymbol{\Psi}_t = \left(\psi_{1,t}, \psi_{2,t}, ..., \psi_{n+1,t}\right)'$.

Note that $\boldsymbol{\Psi}_t = \boldsymbol{\omega}_t\mathbf{1} - \boldsymbol{\Theta}_t$. Given the definition of $\psi_{c,t}$, equations (A1) and (A3) imply that

$$\psi_{1,t} = -\sum_{c=2}^{n+1}\psi_{c,t}. \tag{A6}$$

or $\Psi'_t\mathbf{1} = \mathbf{0}$, so that $\psi_{1,t}$ indeed represents the domestic currency exposure. That currency demands must add to zero is intuitive. Since the investor is fully invested in stocks, she can achieve a long position in a particular currency c only by borrowing—or equivalently, by shorting bonds—in her own domestic currency, and investing the proceeds in bonds denominated in that currency. Thus the currency portfolio is a zero investment portfolio.

B. Log Portfolio Returns Over Short Time Intervals

Assuming log-normality of the hedge returns, the derivation of the optimal Ψ

requires an expression for the log-return on the hedged portfolio, $r_{p,t+1}^{hedge}$. We compute this log hedged return as a discrete-time approximation to its continuous-time counterpart. In order to do this, we need to specify, in continuous time, the return processes for stocks $P_{c,t}$, for currencies $X_{c,t}$ and for interest rates $B_{c,t}$.We assume that they all follow a geometric brownian motions:

$$\frac{dP_{c,t}}{P_{c,t}} = \mu_{P_c} dt + (\sigma_{P_c})_t \, dW_t^{P_c}, \qquad c = 1...n+1 \tag{B1}$$

$$\frac{dB_{c,t}}{B_{c,t}} = \mu_{B_c} dt, \qquad c = 1...n+1 \tag{B2}$$

$$\frac{dX_{c,t}}{X_{c,t}} = \mu_{X_c} dt + (\sigma_{X_c})_t \, dW_t^{X_c}, \qquad c = 1...n+1, \tag{B3}$$

where $W_t^{P_c}$, $W_t^{B_c}$ and $W_t^{X_c}$ are diffusion processes. $\frac{dP_{c,t}}{P_{c,t}}$ represents the stock return, $\frac{dB_{c,t}}{B_{c,t}}$ the nominal return to holding a riskless bond from country and $\frac{dX_{c,t}}{X_{c,t}}$ the return to holding foreign currency c.

For notational simplicity, in what follows, we are momentarily dropping time subscripts for the standard deviations.

Using Ito's lemma, the log returns on each asset are given by:

$$\begin{aligned} d \log P_{c,t} &= \frac{dP_{c,t}}{P_{c,t}} - \frac{1}{2}\sigma_{P_c}^2 dt \\ d \log B_{c,t} &= \frac{dB_{c,t}}{B_{c,t}} - \frac{1}{2}\sigma_{B_c}^2 dt \\ d \log X_{c,t} &= \frac{dX_{c,t}}{X_{c,t}} - \frac{1}{2}\sigma_{X_c}^2 dt. \end{aligned}$$

Note that, because country 1 is the domestic country, which has a fixed exchange rate of 1, we have $d \log X_{1,t} = 0$. This implies $\mu_{X_1} = \sigma_{X_1} = 0$.

The domestic currency return on foreign stock is then given by $\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}$. To derive an expression for this return, we will note that the return dynamics above, by standard calculations, imply :

$$\begin{aligned}\log P_{c,t}X_{c,t} &= \log P_{c,0}X_{c,0} + \left(\mu_{P_c} + \mu_{X_c} - \frac{1}{2}\sigma_{P_c}^2 - \frac{1}{2}\sigma_{X_c}^2\right) t \\ &\quad + \sigma_{P_c}\left(W_t^{P_c} - W_0^{P_c}\right) + \sigma_{X_c}\left(W_t^{X_c} - W_0^{X_c}\right)\end{aligned}$$

Differentiating, and then applying Ito's lemma, yields :

$$\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} = \frac{dP_{c,t}}{P_{c,t}} + \frac{dX_{c,t}}{X_{c,t}} + \sigma_{P_c}\sigma_{X_c}\rho_{P_c,X_c}dt \tag{B4}$$

$$\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} = d\log P_{c,t} + d\log X_{c,t} + \frac{1}{2}\mathrm{Var}_t\left(p_{c,t} + x_{c,t}\right)dt, \tag{B5}$$

where $x_{c,t} = d\log X_{c,t}$ and $p_{c,t} = dlogP_{c,t}$. Note that for c=1, the formula does yield the simple stock return as $\frac{dP_{1,t}X_{1,t}}{P_{1,t}X_{1,t}} = \frac{dP_{1,t}}{P_{1,t}} + \frac{dX_{1,t}}{X_{1,t}} + \sigma_{P_1}\sigma_{X_1}\rho_{P_1,X_1}dt = \frac{dP_{1,t}}{P_{1,t}}$.

A similar calculation yields the following dynamics for the return of the strategy consisting in holding the domestic bond and shorting the foreign one :

$$\frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} = d\log B_{1,t} - d\log B_{c,t} \tag{B6}$$

We note V_t the value of the portfolio.The log return on the portfolio, by Ito's lemma, is :

$$d \log V_t = \frac{dV_t}{V_t} - \frac{1}{2}\left(\frac{dV_t}{V_t}\right)^2 .$$

We can now derive each of the right-hand side terms:

$$\frac{dV_t}{V_t} = \sum_{c=1}^{n+1} \omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right) + \sum_{c=1}^{n+1} \theta_c \omega_{c,t} \frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} - \sum_{c=1}^{n+1} \theta_c \omega_{c,t} \frac{dX_{c,t}}{X_{c,t}},$$

which follows from our convention regarding the domestic country.

Using expressions $(B3)$, $(B5)$, and $(B6)$ to substitute and moving to matrix notation, we get :

$$\begin{aligned} \frac{dV_t}{V_t} &= \mathbf{1}'\boldsymbol{\omega}\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right) - \boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t\right) \\ &\quad + \frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right)\right) - \boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\, \mathbf{x}_{t+1}\right)\right] dt \quad , \end{aligned}$$

where $\mathbf{p}_{t+1} = \left(d \log P_{1,t},\ d \log P_{2,t} ..., d \log P_{n+1,t}\right)'$, $\mathbf{x}_{t+1} = \left(d \log X_{1,t},\ d \log X_{2,t} ..., d \log X_{n+1,t}\right)'$, $\mathbf{b}_t^d = \left(d \log B_{1,t}\right)\mathbf{1}$, $\mathbf{b}_t = \left(d \log B_{1,t},\ d \log B_{2,t} ..., d \log B_{n+1,t}\right)'$ and $diag\left(X\right)$ denotes, for a symmetric $(n \times n)$ matrix X, the $(n \times 1)$ vector of its diagonal terms.

Then,

$$\left(\frac{dV_t}{V_t}\right)^2 = \mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+o\left(dt\right)$$

$$= \begin{bmatrix} \mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\boldsymbol{\omega}_t\iota \\ -2\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{cov}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1},\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t \\ +\boldsymbol{\Theta}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t \end{bmatrix} dt+o\left(dt\right).$$

So, finally,

$$\begin{aligned} d\log V_t &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right) \\ &+\frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\right)-\boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\,\mathbf{x}_{t+1}\right)\right]dt \\ &-\frac{1}{2}\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+\mathrm{o}\left(dt\right) \end{aligned} \tag{B7}$$

Now, we get the approximation for $r_{p,t+1}^h$ by computing the previous expression for $dt = 1$, replacing $d\log X_{c,t} = \Delta s_{c,t+1}$, $dlogP_{c,t} = r_{c,t+1}$, and $dlogB_{c,t} = i_{c,t}$ and neglecting the higher order terms. Noting, for any variable, $\mathbf{z}_t$, the $(n+1\times 1)$ vector $\left(z_{1,t}, z_{2,t}..z_{n+1,t}\right)$, this is equivalent to replacing in equation $(B7)$ $\mathbf{p}_{t+1}$ by $\mathbf{r}_{t+1}$, $\mathbf{x}_{t+1}$ by $\boldsymbol{\Delta}\mathbf{s}_{t+1}$, $\mathbf{b}_t^d$ by $\mathbf{i}_t^d$ and $\mathbf{b}_t$ by $\mathbf{i}_t$.

$$r_{p,t+1}^h \simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h$$

where Σ_{t+1}^h is equal to :

$$\Sigma_t^h = \mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1} + \mathbf{\Delta s}_{t+1}\right)\right) - \mathbf{\Theta}_t' diag\left(\mathrm{Var}_t\, \mathbf{\Delta s}_{t+1}\right)$$
$$- \mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} + \mathbf{\Delta s}_{t+1}\right) - \mathbf{\Theta}_t'\left(\mathbf{\Delta s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\right]$$

where, for any variable z, $\mathbf{z}_t$ denotes the vector of country observations $(z_{1,t}, z_{2,t}...z_{n+1,t})'$ and small case letters denote logs in the following fashion : $r_{c,t+1} = \log\left(R_{c,t+1}\right)$, $s_{t+1} = \log\left(S_{t+1}\right)$, $i_t^d = \log\left(1 + I_{1,t}\right)\mathbf{1}$ and $i_{c,t} = \log\left(1 + I_{c,t}\right)$.

We can now rewrite the portfolio return as a function of $\mathbf{\Psi}_t$ by substituting for $\mathbf{\Theta}_t$. This yields :

$$\begin{aligned} r_{p,t+1}^h &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} + \mathbf{i}_t^d - \mathbf{i}_t\right) + \mathbf{\Psi}_t'\left(\mathbf{\Delta s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right) + \frac{1}{2}\Sigma_t^h \\ &= i_{1,t}^d + \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) + \mathbf{\Psi}_t'\left(\mathbf{\Delta s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right) + \frac{1}{2}\Sigma_t^h, \end{aligned}$$

where:

$$\begin{aligned} \Sigma_t^h &= \mathbf{1}'\boldsymbol{\omega}_t \,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1} + \mathbf{\Delta s}_{t+1}\right)\right) - \left(-\mathbf{\Psi}_t + \boldsymbol{\omega}_t\mathbf{1}\right)' \mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right) \quad \text{(B8)} \\ &\quad - \mathrm{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} + \mathbf{i}_t^d - \mathbf{i}_t\right) + \mathbf{\Psi}_t'\left(\mathbf{\Delta s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\right). \end{aligned}$$

C. Equivalence Between Forward Contracts and Foreign Currency Borrowing and Lending

With the same notations and assumptions as above, when the investor uses forward contracts to hedge currency risk, the portfolio return is:

$$R_{p,t+1}^{h} = \mathbf{R}_{t+1}^{\prime}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}_t^{\prime}\left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}_t^d\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right]$$

Another natural view is one in which the investor borrows in foreign currency and lends in domestic currency to hedge currency risk. Then, the portfolio return is:

$$R_{p,t+1}^{BL} = \mathbf{R}_{t+1}^{\prime}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}^{\prime}\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right)\left(1 + \mathbf{I}_t\right) + \boldsymbol{\Theta}^{\prime}\left(1 + \mathbf{I}_t^d\right)$$

Then, with V_t^{BL} the value of the portfolio with borrowing and lending, we have in continuous time:

$$\begin{aligned}
\frac{dV_t^{BL}}{V_t^{BL}} &= \sum_{c=1}^{n+1}\omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right) - \sum_{c=1}^{n+1}\Theta_{c,t}\frac{dX_{c,t}B_{c,t}}{X_{c,t}B_{c,t}} + \sum_{c=1}^{n+1}\Theta_{c,t}\frac{dB_{1,t}}{B_{1,t}} \\
&= \sum_{c=1}^{n+1}\omega_{c,t}\left(\log P_{c,t} + \log X_{c,t} + \frac{1}{2}\operatorname{Var}_t\left(p_{c,t} + x_{c,t}\right)dt\right) \\
&\quad -\sum_{c=1}^{n+1}\Theta_{c,t}\left(\log\left(X_{c,t}\right) + \log\left(B_{c,t}\right) + \frac{1}{2}\operatorname{Var}_t\left(x_{c,t}\right)dt\right) \\
&\quad +\sum_{c=2}^{n+1}\Theta_{c,t}\log\left(B_{1,t}\right) \\
&= \mathbf{1}^{\prime}\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right) - \Theta^{\prime}\left(\mathbf{x}_{t+1} + \mathbf{b}_t - \mathbf{b}_t^d\right) + \frac{1}{2}\mathbf{1}^{\prime}\boldsymbol{\omega}_t\operatorname{diag}\operatorname{Var}_t\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right)dt \\
&\quad -\frac{1}{2}\Theta^{\prime}\operatorname{diag}\operatorname{Var}_t\left(\mathbf{x}_{t+1}\right)dt
\end{aligned}$$

and

$$\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2 = \mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\Theta'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right).$$

So

$$\begin{aligned}
d\log V_t^{BL} &= \frac{dV_t^{BL}}{V_t^{BL}}-\frac{1}{2}\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2 \\
&= \boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)+\frac{1}{2}\boldsymbol{\omega}_t'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)dt \\
&\quad -\frac{1}{2}\boldsymbol{\Theta}'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}\right)dt \\
&\quad -\frac{1}{2}\mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right)
\end{aligned}$$

We now go to the limit of $dt = 1$ and get :

$$\begin{aligned}
r_{p,t+1}^{BL} &\simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}+\mathbf{i}_t-\mathbf{i}_t^d\right)+\frac{1}{2}\Sigma_t^h \\
&= r_{p,t+1}^h
\end{aligned}$$

D. Mean-Variance Optimization

D.1. Unconstrained Hedge Ratio

In the general case, $r_{p,t+1}^h - i_{1,t}^d = \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h$, and the Lagrangian is:

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\left(1-\lambda\right)\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right] \\
&\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\frac{1}{2}\Sigma_t^h\right]
\end{aligned}$$

Substituting for Σ_t^h using equation $(B8)$, this expression is equivalent to :

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\
&\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)-\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\right] \\
&\quad -\frac{\lambda}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\left(\boldsymbol{\omega}_t\mathbf{1}-\Psi_t\right)'\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right]
\end{aligned}$$

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\operatorname{Var}_t\left(\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\lambda\,\mathrm{E}_t\left(\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\
&\quad -\frac{\lambda}{2}\Psi_t'\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right) \\
&\quad +\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right) \\
&\quad +\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\,\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right) \\
&\quad +\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right] \\
&\quad +\lambda\mu_H
\end{aligned}$$

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\Psi_t'\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\Psi_t-\lambda\Psi_t'\begin{bmatrix}\mathrm{E}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\end{bmatrix} \\
&\quad +\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\Psi_t \\
&\quad +K\left(\lambda\right)
\end{aligned}$$

where

$$\begin{aligned}
K\left(\lambda\right) &= \lambda\mu_H+\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\,\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right) \\
&\quad +\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right]
\end{aligned}$$

$K(\lambda)$ is independent of $\widetilde{\Psi}_t$.

Now, we need to solve only for $\widetilde{\Psi}_t$ as Ψ_1, the demand for domestic currency, is given once the other currency demands are determined. We rewrite the Lagrangian in terms of $\widetilde{\Psi}_t$:

$$\begin{aligned} \pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\widetilde{\Psi}_t' \operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\widetilde{\Psi}_t - \lambda\widetilde{\Psi}_t' \begin{bmatrix} \mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right) \\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right) \end{bmatrix} \\ &\quad + \operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\right)\widetilde{\Psi}_t \\ &\quad + K(\lambda) \end{aligned}$$

The F.O.C. gives the following expression for the optimal $\widetilde{\Psi}_t$:

$$\begin{aligned} 0 &= \operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\right) \\ &\quad + \operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\widetilde{\boldsymbol{\Psi}}_t^* - \lambda\left[\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right) + \frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\right] \end{aligned}$$

Finally, the optimal vector of currency demands is :

$$\begin{aligned} \widetilde{\boldsymbol{\Psi}}_t^*(\lambda) &= \lambda \operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\left[E_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right) + \frac{1}{2}diag\left(\operatorname{Var}_t \widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right] \\ &\quad - \operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\left[\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\right)\right] \end{aligned}$$

D.2. Constrained Hedge Ratio

In the case where $\widetilde{\Psi}_t = \psi_t \widetilde{\mathbf{1}}$ (where $\widetilde{\mathbf{1}}$ denotes an $n \times 1$ vector of ones), we note ψ_t^* the optimal scalar constrained hedge ratio and we have :

$$\begin{aligned} \pounds\left(\psi_t\right) &= \frac{1}{2}\psi_t^2 \widetilde{\mathbf{1}}' \operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}} - \lambda\psi_t \widetilde{\mathbf{1}}' \left[\begin{array}{c} \mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right) \\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right) \end{array}\right] \\ &\quad +\psi_t \operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}} \\ &\quad +K\left(\lambda\right) \end{aligned}$$

and

$$\begin{aligned} \psi_t^* &= \frac{\lambda\widetilde{\mathbf{1}}'\left[\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right) + \frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\right]}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}} \\ &\quad -\frac{\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}}}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}_t^d + \widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}} \end{aligned}$$

In this case, ψ_t^* can equivalently be written in terms of the full matrices :

$$\begin{aligned} \psi_t^* &= \frac{\lambda\mathbf{1}'\left[\mathrm{E}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right) + \frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right]}{\mathbf{1}'\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\mathbf{1}} \\ &\quad -\frac{\mathbf{1}'\operatorname{cov}_t\left(\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\boldsymbol{\Delta}\mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\right)\mathbf{1}}{\mathbf{1}'\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\mathbf{1}} \end{aligned}$$

This case corresponds to a domestic investor hedging the same ratio of his foreign stock holdings for all foreign currencies.

E. Invariance of Optimal Currency Demand With Respect to Base Country

In the system of n^2 bilateral exchange rates, there are really only n free parameters as all exchange rates can be backed out of the n bilateral rates for one base domestic country. We use this fact to show that, for a portfolio of stocks from the $n+1$ countries in our model, the optimal hedge ratios on stocks from country c, Ψ_c^{j*} is the same for any base country j. Let us now use the subscript j to index the domestic country.

We assume for this derivation that weights on international stocks are the same for investors from all countries so that $\boldsymbol{\omega}_t^j = \boldsymbol{\omega}_t$. In terms of our empirical tests, this result will hence apply to the cases of an equally weighted or a value weighted world portfolios, in which weights do not vary with the base country. They do not hold for a home biased porftolio, in which weights by definition vary with base country.

Let us think of country 1 as our base country, and write the optimal vector of foreign currency demand assuming that $\lambda^j = 0$ for all values of j. We have :

$$\begin{aligned}\widetilde{\boldsymbol{\Psi}}_{RM}^{1*} &= -\operatorname{Var}_t\left(\widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1}\right)^{-1}\left[\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1}\right)\right] \\ &= -\operatorname{Var}_t\left(\widetilde{x}_{t+1}^{1}\right)^{-1}\left[\operatorname{cov}_t\left(\mathbf{y}_{t+1}^{W}, \widetilde{x}_{t+1}^{1}\right)\right]\end{aligned}$$

where $x_{t+1}^{1} = \Delta s_{t+1}^{1} - i_t^{1,d} + i_t^{1}$ and $\mathbf{y}_{t+1}^{W} = \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right)$.

Now, let us consider exchange rates from the perspective of country 2. By definition of the exchange rate between countries 1 and 2, it follows that $s_{t+1,1}^{2} = -s_{t+1,2}^{1}$.

Also, by definition of the exchange rates, $S_{t+1,3}^{2}$ units of currency 2 can be exchanged into one unit of currency 3. And one unit of currency 3 is equivalent to $S_{t+1,3}^{1}$ units of currency 1, which is equivalent to $S_{t+1,3}^{1}/S_{t+1,2}^{1}$ units of currency 2. So, the absence of arbitrage implies the equality: $S_{t+1,3}^{2} = S_{t+1,3}^{1}/S_{t+1,2}^{1}$. In logs, $s_{t+1,3}^{2} = s_{t+1,3}^{1} - s_{t+1,2}^{1}$. More generally, the following equality can be derived from the absence of arbitrage:

$$s_{t+1,c}^{2} = s_{t+1,c}^{1} - s_{t+1,2}^{1} \qquad c = 3...n+1$$

In matrix notation, this amounts to a linear relationship between $\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}^{2}$ and $\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}^{1}$:

$$\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}^{2} = A_2 \cdot \widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}^{1}$$

where $A_2 = \begin{pmatrix} \text{-1} & 0 & .. & .. & 0 \\ \text{-1} & 1 & 0 & .. & .. \\ \text{-1} & 0 & 1 & 0 & .. \\ \text{-1} & 0 & 0 & .. & 0 \\ \text{-1} & 0 & .. & 0 & 1 \end{pmatrix}$.

Given our notations :

$$\widetilde{\mathbf{i}}_t^{1,d} - \widetilde{\mathbf{i}}_t^1 = (i_{t,2} - i_{t,1}, i_{t,3} - i_{t,1}, ..i_{t,n+1} - i_{t,1})'$$

and

$$\widetilde{\mathbf{i}}_t^{2,d} - \widetilde{\mathbf{i}}_t^2 = (i_{t,1} - i_{t,2}, i_{t,3} - i_{t,2}, .., i_{t,n+1} - i_{t,2})'.$$

It follows that: $\widetilde{\mathbf{i}}_t^{2,d} - \widetilde{\mathbf{i}}_t^2 = A\left(\widetilde{\mathbf{i}}_t^{1,d} - \widetilde{\mathbf{i}}_t^1\right)$.

Similarly, we have the following linear relationship between $\widetilde{\mathbf{x}}_{t+1}^2$ and $\widetilde{\mathbf{x}}_{t+1}^1$:

$$\widetilde{\mathbf{x}}_{t+1}^2 = A\widetilde{\mathbf{x}}_{t+1}^1 \qquad , \tag{E1}$$

Let us substitute equation $(E1)$, the formula for $\widetilde{\mathbf{x}}_{t+1}^2$, into equation (**??**), the formula for the optimal hedge ratio. We use the properties of matrix second moments that

$\operatorname{Var}(AX) = A\operatorname{Var}(X)A'$, $\operatorname{cov}(AX,Y) = Acov(X,Y)$, and the property of inverse matrices that $(AB)^{-1} = B^{-1}A^{-1}$. Also, we note that $A_2 = (A_2)^{-1}$ and $(A_2')^{-1} = A_2'$. Substitution yields:

$$\begin{aligned}
\widetilde{\mathbf{\Psi}}_{RM}^{2*} &= -\operatorname{Var}_t\left(\widetilde{\mathbf{x}}_{t+1}^{2}\right)^{-1}\left[\operatorname{cov}_t\left(\mathbf{y}_{t+1}^{W},\widetilde{\mathbf{x}}_{t+1}^{2}\right)\right] \\
&= -\left(A_2'\right)^{-1}\operatorname{Var}_t\left(\widetilde{\mathbf{x}}_{t+1}^{1}\right)^{-1}(A_2)^{-1}\left[A_2\operatorname{cov}_t\left(\widetilde{\mathbf{x}}_{t+1}^{1},\mathbf{y}_{t+1}^{W}\right)\right] \\
\widetilde{\mathbf{\Psi}}_{RM}^{2*}\left(\lambda^2\right) &= -\left(A_2'\right)^{-1}\operatorname{Var}_t\left(\widetilde{\mathbf{x}}_{t+1}^{1}\right)^{-1}\operatorname{cov}_t\left(\widetilde{\mathbf{x}}_{t+1}^{1},\mathbf{y}_{t+1}^{W}\right)
\end{aligned}$$

$$\widetilde{\mathbf{\Psi}}^{2*} = A_2'\widetilde{\mathbf{\Psi}}^{1*}$$

We write out the vector $\widetilde{\mathbf{\Psi}}_{RM}^{2*}$:

$$\widetilde{\mathbf{\Psi}}_{RM}^{2*} = \left(-\sum_{c=2}^{n+1}\Psi_c^{1*},\Psi_3^{1*},\Psi_4^{1*},..,\Psi_{n+1}^{1*}\right)$$

Given the property that $\sum_{c=1}^{n+1}\Psi_c^{j*} = 1$ for $j = 1..n+1$, $\Psi_1^{1*} = -\sum_{c=2}^{n+1}\Psi_c^{1*}$ so that $\widetilde{\mathbf{\Psi}}_{RM}^{2*} = \left(\Psi_1^{1*},\Psi_3^{1*},\Psi_4^{1*},..,\Psi_{n+1}^{1*}\right)$. Applying this same property twice, $\Psi_2^{2*} = -\sum_{c\neq 2}^{n+1}\Psi_c^{2*} = -\sum_{c\neq 2}^{n+1}\Psi_c^{1*} = \Psi_2^{1*}$, so that: $\Psi_{RM}^{2*} = \left(\Psi_1^{1*},\Psi_2^{1*},\Psi_3^{1*},\Psi_4^{1*},..,\Psi_{n+1}^{1*}\right) = \Psi_{RM}^{1*}$. Finally, the vector of optimal currency positions is the same for investors based in country 2 as that of country 1 investors.

Similar results hold for $j = 3...n+1$, where $A_3 = \begin{pmatrix} 1 & \text{-1} & .. & .. & 0 \\ 0 & \text{-1} & 0 & .. & .. \\ 0 & \text{-1} & 1 & 0 & .. \\ 0 & .. & 0 & .. & 0 \\ 0 & \text{-1} & .. & 0 & 1 \end{pmatrix}$, $A_4 =$ $\begin{pmatrix} 1 & 0 & \text{-1} & .. & 0 \\ 0 & 1 & .. & .. & .. \\ 0 & 0 & \text{-1} & 0 & .. \\ 0 & .. & .. & 1 & 0 \\ 0 & \text{-1} & \text{-1} & 0 & 1 \end{pmatrix}$, etc...

This analysis justifies dropping the base-country subscript j and interpreting the $(n+1 \times 1)$ vector $\mathbf{\Psi}^* = \left(-\sum_{c=2}^{n+1} \Psi_c^{1*}, \Psi_2^{1*}, \Psi_3^{1*}, .., \Psi_{n+1}^{1*} \right)'$ as a common vector of foreign currency demands that is independent of the country of origin.

A situation in which investors from all countries are hedged perfectly corresponds to $\mathbf{\Psi}^* = (0, 0, .., 0)'$.

A situation in which investors from country 1 are not hedged at all corresponds to $\mathbf{\Psi}^* = \left(-1, \omega_2^1, \omega_3^1 .., \omega_{n+1}^1\right)'$. That is, investors from country i undo the hedge of the fully hedged portfolio by taking long positions in each foreign currency proportional to the weight of each foreign country in their stock portfolio. (The perfectly hedged portfolio obtains by shorting each foreign currency by that same amount.) They need to borrow one unit of domestic currency to finance that.

Finally, note that this proof relies on the fact that all relevant exchange rates for an investor in a given base country are linear combinations of the relevant exchange rates for each other base country. In other words, the assumption is that all investors optimize over the same set of currencies.

F. Computation of Sharpe Ratios

Table A14 reports in-sample Sharpe Ratios generated by the set of currency hedging strategies for global portfolios of stocks and bonds considered in the paper. The denominator of the Sharpe ratio is given by the standard deviations of log portfolio returns reported in Table VII of the main text.

The numerator of the Sharpe ratio is given by the log of the mean gross return on each of the portfolios. We compute a time series of gross returns for each strategy using equation (A5), where $\boldsymbol{\Psi}_t$ is resplaced by the vector of fixed or time-varying currency demands that corresponds to each currency hedging strategy—for example, $\boldsymbol{\Psi}_t$ is a vector of zeroes for the "Full Hedge"strategy. Next we average the time series of gross returns, and take the natural log of the arithmetic mean.

Thus our Sharpe ratio is computed as

$$\frac{\log\left(\mathrm{E}\left[R^h_{p,t+1}\right]\right)}{\sqrt{\mathrm{Var}\left(r^h_{p,t+1}\right)}},$$

which at high frequency observation of returns or under lognormality is equivalent to

$$\frac{\mathrm{E}\left[r^h_{p,t+1}\right] + \frac{1}{2}\,\mathrm{Var}\left(r^h_{p,t+1}\right)}{\sqrt{\mathrm{Var}\left(r^h_{p,t+1}\right)}}.$$

Internet Appendix Endnotes

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1. That is, at the end of month t, the investor can enter into a forward contract to sell one unit of currency c at the end of month $t+1$ for a forward price of $F_{c,t}$ dollars.

2. Note, however, that the two strategies are not completely equivalent except in the continuous time limit. We show in Appendix B that, in continuous time, the two strategies are exactly equivalent.

Table A1
Currency return correlations

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Base country: Euroland							
Euroland	.						
Australia	.	1.00					
Canada	.	0.70	1.00				
Japan	.	0.35	0.35	1.00			
Switzerland	.	-0.09	-0.11	0.20	1.00		
UK	.	0.31	0.34	0.24	-0.02	1.00	
US	.	0.63	0.87	0.40	-0.07	0.38	1.00
Base country: Australia							
Euroland	1.00						
Australia	.	.					
Canada	0.53	.	1.00				
Japan	0.67	.	0.46	1.00			
Switzerland	0.92	.	0.47	0.69	1.00		
UK	0.78	.	0.51	0.59	0.72	1.00	
US	0.59	.	0.85	0.55	0.54	0.58	1.00
Base country: Canada							
Euroland	1.00						
Australia	0.23	1.00					
Canada	.	.	.				
Japan	0.59	0.25	.	1.00			
Switzerland	0.91	0.20	.	0.62	1.00		
UK	0.71	0.24	.	0.50	0.65	1.00	
US	0.32	0.11	.	0.35	0.31	0.34	1.00
Base Country: Japan							
Euroland	1.00						
Australia	0.46	1.00					
Canada	0.55	0.74	1.00				
Japan	.	.	.	.			
Switzerland	0.87	0.33	0.39	.	1.00		
UK	0.72	0.49	0.56	.	0.60	1.00	
US	0.55	0.67	0.90	.	0.41	0.58	1.00
Base Country: Switzerland							
Euroland	1.00						
Australia	0.47	1.00					
Canada	0.52	0.77	1.00				
Japan	0.31	0.46	0.47	1.00			
Switzerland	.	.	.	.	.		
UK	0.56	0.49	0.53	0.37	.	1.00	
US	0.51	0.71	0.91	0.51	.	0.55	1.00
Base Country: UK							
Euroland	1.00						
Australia	0.35	1.00					
Canada	0.42	0.71	1.00				
Japan	0.50	0.42	0.44	1.00			
Switzerland	0.84	0.25	0.30	0.52	1.00		
UK	.	.	.	.	.	.	
US	0.42	0.64	0.88	0.48	0.32	.	1.00
Base Country: US							
Euroland	1.00						
Australia	0.26	1.00					
Canada	0.18	0.43	1.00				
Japan	0.55	0.25	0.10	1.00			
Switzerland	0.90	0.21	0.12	0.58	1.00		
UK	0.69	0.25	0.14	0.44	0.61	1.00	
US	.	.	.	.	.	.	.

Note. This table presents cross-country correlations of foreign currency log excess returns $s_{c,t}+i_{c,t}-i_{d,t}$, where d indexes the base country. Correlations are presented separately for investors from each base country. They are computed using monthly returns.

Table A2

Optimal currency exposure for an equally-weighted global equity portfolio: single-currency case

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Euroland		-0.37***	-0.45***	-0.25***	0.28*	-0.30***	-0.33***
		(0.09)	(0.10)	(0.07)	(0.15)	(0.09)	(0.11)
Australia	0.37***		0.02	0.14*	0.33***	0.21**	0.16**
	(0.09)		(0.08)	(0.07)	(0.07)	(0.09)	(0.08)
Canada	0.45***	-0.02		0.15*	0.38***	0.25**	0.55***
	(0.10)	(0.08)		(0.09)	(0.09)	(0.11)	(0.16)
Japan	0.25***	-0.14*	-0.15*		0.32***	0.05	-0.06
	(0.07)	(0.07)	(0.09)		(0.08)	(0.06)	(0.09)
Switzerland	-0.28*	-0.33***	-0.38***	-0.32***		-0.29***	-0.30***
	(0.15)	(0.07)	(0.09)	(0.08)		(0.07)	(0.09)
UK	0.30***	-0.21**	-0.25**	-0.05	0.29***		-0.13
	(0.09)	(0.09)	(0.11)	(0.06)	(0.07)		(0.11)
US	0.33***	-0.16**	-0.55***	0.06	0.30***	0.13	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a position in one foreign currency at a time to minimize the variance of his portfolio. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return to the global equity portfolio onto the excess return of the column country currency to an investor based in the row country. All regressions include an intercept.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A3

Optimal currency exposure for an equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.17	-0.16	-0.61*	-0.11	0.23	-0.11	0.60*
	(0.15)	(0.11)	(0.14)	(0.07)	(0.12)	(0.08)	(0.15)
2 months	0.29	-0.13	-0.63*	-0.19*	0.26	-0.11	0.51*
	(0.15)	(0.09)	(0.15)	(0.07)	(0.13)	(0.09)	(0.15)
3 months	0.32	-0.11	-0.61*	-0.17	0.27	-0.10	0.40*
	(0.17)	(0.09)	(0.16)	(0.09)	(0.15)	(0.11)	(0.18)
6 months	0.20	-0.05	-0.38	-0.25*	0.35	-0.06	0.19
	(0.26)	(0.14)	(0.25)	(0.12)	(0.20)	(0.16)	(0.28)
12 months	-0.20	0.21	-0.22	-0.41*	0.67*	-0.20	0.15
	(0.40)	(0.20)	(0.36)	(0.17)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.37*	-0.29*		-0.08		-0.10	0.11
	(0.11)	(0.11)		(0.07)		(0.08)	(0.08)
2 months	0.50*	-0.27*		-0.15*		-0.09	0.01
	(0.11)	(0.09)		(0.07)		(0.09)	(0.11)
3 months	0.56*	-0.27*		-0.14		-0.09	-0.06
	(0.11)	(0.10)		(0.08)		(0.11)	(0.14)
6 months	0.53*	-0.21		-0.21*		-0.02	-0.09
	(0.14)	(0.13)		(0.10)		(0.15)	(0.18)
12 months	0.44*	0.05		-0.34*		-0.16	0.01
	(0.19)	(0.17)		(0.15)		(0.19)	(0.22)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A4

Subperiod analysis

Equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Subperiod I : 1975-1989							
1 month	0.15	-0.11	-0.73***	-0.06	0.08	-0.06	0.73***
	(0.20)	(0.16)	(0.23)	(0.12)	(0.13)	(0.11)	(0.24)
3 months	0.14	-0.05	-0.63**	-0.20	0.22	-0.09	0.62*
	(0.21)	(0.12)	(0.26)	(0.14)	(0.18)	(0.15)	(0.35)
12 months	-0.62	0.23	-0.15	-0.31	0.57*	-0.04	0.33
	(0.45)	(0.22)	(0.61)	(0.23)	(0.33)	(0.23)	(0.61)
Suberiod II : 1990-2005							
1 month	0.10	-0.25**	-0.49***	-0.15	0.51**	-0.20	0.48***
	(0.27)	(0.12)	(0.18)	(0.09)	(0.23)	(0.13)	(0.18)
3 months	0.44	-0.17	-0.65***	-0.08	0.37	-0.12	0.22
	(0.28)	(0.14)	(0.21)	(0.10)	(0.23)	(0.14)	(0.19)
12 months	0.56	-0.17	-0.31	-0.23	0.47	-0.22	-0.11
	(0.52)	(0.29)	(0.37)	(0.23)	(0.49)	(0.25)	(0.37)
Panel B : 5 country optimization							
Subperiod I : 1975-1989							
1 month	0.21	-0.22		-0.06		-0.06	0.13
	(0.19)	(0.16)		(0.12)		(0.11)	(0.10)
3 months	0.35**	-0.15		-0.15		-0.10	0.05
	(0.17)	(0.11)		(0.11)		(0.15)	(0.20)
12 months	-0.10	0.14		-0.20		-0.02	0.18
	(0.22)	(0.20)		(0.15)		(0.21)	(0.24)
Suberiod II : 1990-2005							
1 month	0.56***	-0.40***		-0.08		-0.20*	0.12
	(0.12)	(0.12)		(0.08)		(0.11)	(0.13)
3 months	0.79***	-0.47***		-0.06		-0.11	-0.15
	(0.13)	(0.12)		(0.11)		(0.13)	(0.17)
12 months	1.02***	-0.40*		-0.22		-0.20	-0.20
	(0.21)	(0.23)		(0.19)		(0.25)	(0.32)

Note. This table replicates Table 5 for two subperiods, respectively extending from 1975:7 to 1989:12 and from 1990:1 to 2005:12. Time horizons include 1, 3 and 12 months only.

Table A5

Optimal currency exposure for a value-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.13	-0.09	-0.70***	-0.13*	0.22*	-0.09	0.66***
	(0.17)	(0.10)	(0.15)	(0.08)	(0.13)	(0.08)	(0.15)
2 months	0.22	-0.07	-0.73***	-0.22***	0.26*	-0.06	0.60***
	(0.16)	(0.09)	(0.15)	(0.08)	(0.13)	(0.09)	(0.16)
3 months	0.22	-0.04	-0.76***	-0.23**	0.30**	-0.03	0.55***
	(0.17)	(0.09)	(0.17)	(0.10)	(0.15)	(0.11)	(0.19)
6 months	0.11	0.01	-0.60***	-0.32**	0.39**	0.03	0.39
	(0.24)	(0.14)	(0.22)	(0.12)	(0.19)	(0.15)	(0.26)
12 months	-0.29	0.25	-0.49	-0.46**	0.72**	-0.09	0.36
	(0.39)	(0.22)	(0.36)	(0.18)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.29***	-0.25***		-0.08		-0.08	0.12
	(0.11)	(0.09)		(0.07)		(0.08)	(0.09)
2 months	0.42***	-0.25**		-0.16**		-0.05	0.04
	(0.11)	(0.10)		(0.08)		(0.09)	(0.11)
3 months	0.46***	-0.24**		-0.17*		-0.03	-0.03
	(0.11)	(0.10)		(0.09)		(0.11)	(0.14)
6 months	0.45***	-0.20		-0.24**		0.06	-0.07
	(0.13)	(0.13)		(0.11)		(0.15)	(0.18)
12 months	0.39*	0.01		-0.32**		-0.08	0.00
	(0.20)	(0.20)		(0.16)		(0.21)	(0.23)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with constant value weights (reflecting the end-of-period 2005:12 weights as reported in Table 7), who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A6

Optimal currency exposure for a home-biased global equity portfolio: single and multiple currency cases

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
PANEL A : Single currency							
Euroland		-0.40***	-0.52***	-0.31***	0.34*	-0.32***	-0.45***
		(0.10)	(0.12)	(0.08)	(0.18)	(0.11)	(0.13)
Australia	0.37***		0.09	0.16*	0.31***	0.17	0.25**
	(0.10)		(0.11)	(0.09)	(0.09)	(0.12)	(0.10)
Canada	0.42***	-0.01		0.12	0.35***	0.17	0.88***
	(0.10)	(0.10)		(0.09)	(0.09)	(0.11)	(0.19)
Japan	0.31***	-0.09	-0.08		0.35***	0.15*	0.02
	(0.10)	(0.09)	(0.10)		(0.10)	(0.08)	(0.11)
Switzerland	-0.45***	-0.35***	-0.42***	-0.29***		-0.29***	-0.38***
	(0.15)	(0.08)	(0.09)	(0.09)		(0.08)	(0.10)
UK	0.25**	-0.24**	-0.30***	-0.10	0.25***		-0.21*
	(0.11)	(0.10)	(0.12)	(0.07)	(0.09)		(0.12)
US	0.23**	-0.14*	-0.71***	-0.01	0.22**	0.11	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	
Panel B : Multiple currencies at once							
Euroland	0.36	-0.08	-0.50**	-0.20*	0.33*	-0.08	0.17
	(0.23)	(0.11)	(0.21)	(0.11)	(0.18)	(0.13)	(0.22)
Australia	0.47**	-0.16	-0.68***	-0.14	0.15	-0.24*	0.60***
	(0.20)	(0.12)	(0.17)	(0.11)	(0.19)	(0.15)	(0.22)
Canada	0.30	-0.05	-0.94***	-0.22**	0.31	-0.23*	0.83***
	(0.20)	(0.10)	(0.21)	(0.10)	(0.20)	(0.13)	(0.21)
Japan	0.34*	-0.14	-0.63***	-0.25**	0.20	0.01	0.48**
	(0.17)	(0.13)	(0.21)	(0.12)	(0.16)	(0.12)	(0.21)
Switzerland	0.14	-0.12	-0.35*	-0.07	0.37**	-0.02	0.04
	(0.21)	(0.09)	(0.19)	(0.11)	(0.17)	(0.13)	(0.21)
UK	0.30	-0.11	-0.56***	-0.20**	0.28	-0.02	0.30
	(0.21)	(0.10)	(0.19)	(0.09)	(0.18)	(0.13)	(0.20)
US	0.15	0.00	-0.83***	-0.22**	0.30*	-0.03	0.62***
	(0.18)	(0.09)	(0.17)	(0.09)	(0.15)	(0.11)	(0.19)

Note. This table considers an investor holding a home-biased portfolio of global equity. The portfolio is constructed by assigning a 75% weight to the home country of the investor, and distributing the remaining 25% over the four other countries according to their value weights. The investor chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return on the row country home biased global equity portfolio onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return on the row country portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A7

Optimal currency exposure for single-country bond portfolios: single and multiple currency cases

Bond market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		0.04	0.05	-0.03	-0.02	0.09**	0.06*
		(0.03)	(0.03)	(0.04)	(0.06)	(0.04)	(0.04)
Australia	-0.02		0.04	0.00	-0.01	0.02	0.06
	(0.05)		(0.07)	(0.05)	(0.05)	(0.04)	(0.05)
Canada	-0.07	0.12**		-0.07	-0.08*	-0.07	0.24***
	(0.05)	(0.05)		(0.05)	(0.04)	(0.06)	(0.08)
Japan	0.05	0.09**	0.14***		0.01	0.10**	0.16***
	(0.05)	(0.04)	(0.05)		(0.05)	(0.05)	(0.05)
Switzerland	0.08	0.02	0.05	-0.02		0.09**	0.05
	(0.07)	(0.02)	(0.03)	(0.04)		(0.04)	(0.03)
UK	0.22***	0.07**	0.11**	0.02	0.13***		0.12***
	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)		(0.04)
US	-0.21***	0.03	-0.21**	-0.15***	-0.18***	-0.09*	
	(0.05)	(0.05)	(0.09)	(0.05)	(0.04)	(0.05)	
Panel B : Multiple currencies							
Euroland	-0.10	0.01	-0.01	-0.07*	0.03	0.08*	0.07
	(0.08)	(0.03)	(0.07)	(0.04)	(0.07)	(0.05)	(0.06)
Australia	-0.13	-0.02	-0.07	0.00	0.03	0.06	0.14
	(0.15)	(0.08)	(0.13)	(0.06)	(0.12)	(0.07)	(0.10)
Canada	0.03	0.18***	-0.35***	-0.08	-0.08	-0.07	0.36***
	(0.12)	(0.05)	(0.11)	(0.06)	(0.11)	(0.08)	(0.08)
Japan	-0.05	-0.02	0.00	-0.12*	-0.07	0.07	0.18*
	(0.10)	(0.05)	(0.10)	(0.06)	(0.08)	(0.05)	(0.09)
Switzerland	-0.03	-0.03	0.05	-0.06	-0.04	0.10**	0.01
	(0.08)	(0.04)	(0.08)	(0.05)	(0.08)	(0.05)	(0.07)
UK	0.28**	0.01	-0.05	-0.10	-0.04	-0.23***	0.13
	(0.13)	(0.06)	(0.14)	(0.06)	(0.11)	(0.06)	(0.10)
US	-0.22*	0.19***	-0.30**	-0.10	-0.02	0.09	0.36***
	(0.11)	(0.06)	(0.12)	(0.06)	(0.09)	(0.07)	(0.09)

Note. This table considers an investor holding a portfolio composed of long-term bonds from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the bond being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country bond onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock bond onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A8

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.02	0.00	-0.12*	-0.06*	-0.04	-0.01	0.22*
	(0.05)	(0.02)	(0.05)	(0.03)	(0.04)	(0.03)	(0.05)
2 months	-0.01	0.03	-0.14*	-0.08*	-0.03	0.00	0.23*
	(0.07)	(0.03)	(0.06)	(0.03)	(0.05)	(0.04)	(0.05)
3 months	-0.03	0.04	-0.10	-0.07	-0.03	0.01	0.18*
	(0.07)	(0.04)	(0.08)	(0.04)	(0.07)	(0.05)	(0.06)
6 months	-0.08	0.13*	-0.05	-0.10	0.00	0.06	0.05
	(0.11)	(0.05)	(0.10)	(0.06)	(0.10)	(0.07)	(0.08)
12 months	-0.26	0.17	0.03	-0.11	0.11	0.14	-0.08
	(0.17)	(0.09)	(0.16)	(0.08)	(0.13)	(0.11)	(0.11)
Panel B : 5 country optimization							
1 month	-0.02	-0.03		-0.07*		-0.01	0.13*
	(0.03)	(0.02)		(0.03)		(0.03)	(0.03)
2 months	-0.04	-0.01		-0.09*		-0.01	0.15*
	(0.04)	(0.03)		(0.03)		(0.03)	(0.04)
3 months	-0.06	0.01		-0.08		0.01	0.11*
	(0.05)	(0.03)		(0.04)		(0.05)	(0.04)
6 months	-0.08	0.11*		-0.10		0.06	0.01
	(0.08)	(0.04)		(0.06)		(0.07)	(0.06)
12 months	-0.14	0.18*		-0.10		0.12	-0.06
	(0.12)	(0.06)		(0.07)		(0.11)	(0.07)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global bond portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A9- Subperiod analysis

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Subperiod I: 1975-1989							
1 month	0.08	0.01	-0.24**	-0.11***	-0.06	0.00	0.33***
	(0.07)	(0.04)	(0.10)	(0.04)	(0.05)	(0.03)	(0.09)
3 months	-0.02	0.04	-0.30**	-0.16***	-0.01	0.02	0.43***
	(0.10)	(0.05)	(0.14)	(0.06)	(0.08)	(0.06)	(0.12)
12 months	-0.29*	0.24**	-0.14	-0.25***	0.12	0.20**	0.13
	(0.16)	(0.11)	(0.19)	(0.08)	(0.10)	(0.09)	(0.14)
Subperiod 2: 1990-2005							
1 month	-0.07	0.00	-0.05	-0.04	0.04	0.00	0.12**
	(0.09)	(0.03)	(0.06)	(0.03)	(0.07)	(0.04)	(0.06)
3 months	-0.17	0.13**	-0.08	-0.01	0.06	0.04	0.04
	(0.12)	(0.06)	(0.11)	(0.05)	(0.11)	(0.06)	(0.06)
12 months	-0.42**	0.24**	-0.14	0.04	0.25*	0.12	-0.10
	(0.21)	(0.12)	(0.22)	(0.09)	(0.15)	(0.11)	(0.10)
Panel B : 5 country optimization							
Subperiod I: 1975-1989							
1 month	0.02	-0.04		-0.11**		-0.01	0.15***
	(0.06)	(0.03)		(0.04)		(0.03)	(0.04)
3 months	-0.03	-0.04		-0.14**		0.01	0.20***
	(0.08)	(0.04)		(0.07)		(0.05)	(0.06)
12 months	-0.18	0.16*		-0.21***		0.20**	0.03
	(0.12)	(0.09)		(0.07)		(0.09)	(0.11)
Subperiod 2: 1990-2005							
1 month	-0.05	-0.01		-0.04		0.00	0.10**
	(0.04)	(0.03)		(0.03)		(0.04)	(0.04)
3 months	-0.13**	0.12***		0.00		0.03	-0.02
	(0.06)	(0.04)		(0.05)		(0.06)	(0.05)
12 months	-0.16	0.20***		0.04		0.08	-0.16*
	(0.13)	(0.05)		(0.08)		(0.11)	(0.10)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A10 - Subperiod I

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-0.10	1.00	-7.57	0.11	0.10	0.99	1.92	0.00
	(0.94)		(7.55)		(0.34)		(3.62)	
Australia	-0.08	1.00	4.19	0.32	0.01	1.00	2.75***	0.61
	(0.51)		(2.58)		(0.28)		(0.84)	
Canada	-0.02	1.00	5.72	0.83	0.04	1.00	3.22	0.54
	(0.49)		(3.83)		(0.36)		(2.92)	
Japan	0.00	1.00	2.20	0.07	0.05	1.00	-0.07	0.03
	(0.07)		(4.81)		(0.44)		(2.07)	
Switz.	0.24	1.00	-8.22	0.04	0.11	1.00	1.74	0.03
	(0.87)		(5.53)		(0.35)		(2.92)	
UK	-0.05	1.00	2.77	0.11	0.12	1.00	-1.21	0.06
	(0.38)		(3.75)		(0.32)		(1.90)	
US	0.02	1.00	-1.88	0.21	0.07	1.00	-0.69	0.09
	(0.34)		(4.48)		(0.57)		(1.99)	

Note. This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each b

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base c

Table A10 - Subperiod II

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-1.24	0.99	8.48**	0.23	0.04	1.00	-3.32*	0.58
	(2.90)		(3.62)		(0.39)		(1.95)	
Australia	-0.71	1.00	8.96**	0.91	0.00	1.00	0.42	0.41
	(1.79)		(3.81)		(0.30)		(1.50)	
Canada	-1.37	0.95	15.96**	0.42	0.04	1.00	-0.79	0.14
	(3.19)		(6.44)		(0.21)		(2.79)	
Japan	0.10	1.00	3.59	0.52	-0.01	1.00	-0.03	0.89
	(1.31)		(4.53)		(0.15)		(2.48)	
Switz.	-1.49	0.97	8.40***	0.11	0.00	1.00	-0.37	0.41
	(2.56)		(2.70)		(0.24)		(1.08)	
UK	-0.71	0.99	8.14	0.03	0.06	1.00	-0.89	0.16
	(2.34)		(7.16)		(0.20)		(3.27)	
US	-0.77	0.98	1.90	0.19	0.03	1.00	-1.84	0.42
	(1.78)		(4.46)		(0.20)		(1.74)	

Note. This table reports optimal currency exposure conditional on interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log interest rate differential (interest rate of the foreign country minus that of

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each b

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base c

Table A11

Optimal synthetic carry-trade currency exposure for equally-weighted global equity and bond portfolios

	Multiple Currencies								Single currency
	Euroland	Australia	Canada	Japan	Switz.	UK	US	Synthetic	Synthetic
Panel A: Stocks									
Full period	0.33**	-0.17*	-0.68***	-0.08	0.34**	-0.18	0.27*	0.27*	-0.23**
	(0.16)	(0.09)	(0.17)	(0.10)	(0.15)	(0.13)	(0.14)	(0.14)	(0.12)
Subperiod I	0.22	-0.25	-0.91***	0.00	0.48**	-0.23	0.69**	0.73*	-0.13
	(0.21)	(0.18)	(0.22)	(0.20)	(0.22)	(0.14)	(0.33)	(0.38)	(0.14)
Subperiod II	0.36	-0.14	-0.57***	-0.17*	0.38*	0.02	0.12	-0.26	-0.37*
	(0.30)	(0.14)	(0.21)	(0.10)	(0.23)	(0.20)	(0.21)	(0.17)	(0.20)
Panel B: Bonds									
Full period	-0.02	0.02	-0.14	-0.04	0.00	-0.02	0.20***	0.11	0.13***
	(0.07)	(0.04)	(0.09)	(0.05)	(0.07)	(0.06)	(0.07)	(0.08)	(0.05)
Subperiod I	0.00	-0.02	-0.39***	-0.10	0.08	-0.03	0.45***	0.24*	0.19***
	(0.09)	(0.07)	(0.15)	(0.07)	(0.09)	(0.07)	(0.12)	(0.14)	(0.07)
Subperiod II	-0.16	0.12**	-0.10	0.02	0.07	0.00	0.06	0.07	0.07
	(0.12)	(0.06)	(0.13)	(0.06)	(0.11)	(0.08)	(0.08)	(0.09)	(0.06)

Note: The first eight columns of this table consider an investor holding a global, equally weighted, stock (Panel A) or bond portfolio (Panel B) who chooses a vector of positions in available currencies to minimize the variance of his portfolio. Available currencies include all foreign currencies as well as a synthetic currency. At each point in time, the synthetic currency return is the average of the return of holding the currencies of the three highest interest rates countries and financing the position using the currencies of the three lowest interest rate countries. The time t return is based on currencies chosen using time t-1 interest rates.

The last column considers the same investor now choosing an optimal position in only one currency: the synthetic currency to minimize the variance of his portfolio.

Table A12

Standard deviations of hedged global equity and bond portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge: Baseline	Optimal hedge: Conditional hedging (constrained)	Optimal hedge: Synthetic currency	Baseline vs. full hedge: F-Stat	Baseline vs. full hedge: P-value	Baseline vs. no hedge: F-Stat	Baseline vs. no hedge: P-value	Conditional vs. Baseline: F-Stat	Conditional vs. Baseline: P-value
Panel A: Full period												
Equity												
Euroland	17.67	15.47	13.86	12.51	12.45	12.43	7.98	0.00	33.36	0.00	3.55	6.05
Australia	15.00	13.52	13.86	12.51	12.51	12.43	7.98	0.00	20.09	0.00	0.04	84.37
Canada	13.74	13.22	13.86	12.51	12.50	12.43	7.98	0.00	6.49	0.00	0.44	50.85
Japan	17.08	14.67	13.86	12.51	12.50	12.43	7.98	0.00	31.32	0.00	0.10	74.89
Switzerland	19.19	16.09	13.86	12.51	12.40	12.43	7.98	0.00	41.75	0.00	5.54	1.91
UK	16.78	14.74	13.86	12.51	12.50	12.43	7.98	0.00	25.47	0.00	0.15	69.90
US	15.05	13.91	13.86	12.51	12.45	12.43	7.98	0.00	15.20	0.00	3.67	5.63
Bonds												
Euroland	8.39	6.10	5.40	5.21	5.21	5.19	2.76	1.23	54.14	0.00	0.42	51.99
Australia	12.08	7.85	5.40	5.21	5.19	5.19	2.76	1.23	210.02	0.00	6.57	1.08
Canada	10.18	7.12	5.40	5.21	5.21	5.19	2.76	1.23	85.17	0.00	0.03	86.00
Japan	10.86	6.85	5.40	5.21	5.21	5.19	2.76	1.23	87.07	0.00	0.30	58.52
Switzerland	9.93	6.52	5.40	5.21	5.21	5.19	2.76	1.23	85.62	0.00	0.40	52.89
UK	10.35	6.98	5.40	5.21	5.19	5.19	2.76	1.23	87.03	0.00	2.53	11.23
US	10.53	7.36	5.40	5.21	5.20	5.19	2.76	1.23	127.73	0.00	1.68	19.53

Note. This table reports the standard deviation of portfolios featuring different uses of currency for risk-management.

We present results for equally-weighted global portfolios, for equity and bonds as respectively described in Table 4 and Table 6. Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge"

Reported standard deviations are annualized, and measured in percentage points.

All results presented are computed considering returns at a quarterly horizon.

Table A12 (continued)
Standard deviations of hedged global equity and bond portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge: Baseline	Optimal hedge: Conditional hedging (constrained)	Optimal hedge: Synthetic currency	Baseline vs. full hedge: F-Stat	Baseline vs. full hedge: P-value	Baseline vs. no hedge: F-Stat	Baseline vs. no hedge: P-value	Conditional vs. Baseline: F-Stat	Conditional vs. Baseline: P-value
Panel B: Subperiod I												
Equity												
Euroland	16.79	14.89	13.74	13.12	13.08	12.85	2.84	1.18	14.87	0.00	1.00	31.76
Australia	16.49	14.09	13.74	13.12	13.03	12.85	2.84	1.18	14.33	0.00	2.64	10.63
Canada	15.31	13.90	13.74	13.12	13.07	12.85	2.84	1.18	6.14	0.00	2.23	13.74
Japan	16.53	14.42	13.74	13.12	13.11	12.85	2.84	1.18	16.94	0.00	0.21	64.78
Switzerland	18.46	15.47	13.74	13.12	13.04	12.85	2.84	1.18	20.15	0.00	2.21	13.94
UK	16.85	14.51	13.74	13.12	13.09	12.85	2.84	1.18	11.63	0.00	0.55	46.13
US	16.16	14.40	13.74	13.12	13.11	12.85	2.84	1.18	8.04	0.00	0.18	67.56
Bonds												
Euroland	8.84	6.48	5.76	5.20	5.19	5.13	3.51	0.27	34.23	0.00	0.28	59.67
Australia	13.15	8.45	5.76	5.20	5.10	5.13	3.51	0.27	104.09	0.00	10.58	0.14
Canada	11.27	7.85	5.76	5.20	5.16	5.13	3.51	0.27	48.74	0.00	1.22	27.08
Japan	9.77	6.40	5.76	5.20	5.20	5.13	3.51	0.27	38.76	0.00	0.00	97.45
Switzerland	10.50	6.80	5.76	5.20	5.19	5.13	3.51	0.27	50.64	0.00	0.35	55.31
UK	11.63	7.65	5.76	5.20	5.19	5.13	3.51	0.27	68.36	0.00	0.41	52.45
US	11.75	8.23	5.76	5.20	5.20	5.13	3.51	0.27	67.75	0.00	0.12	72.89
Panel C: Subperiod II												
Equity												
Euroland	18.38	15.94	13.92	11.25	11.16	11.14	10.70	0.00	37.18	0.00	5.49	2.02
Australia	13.46	12.92	13.92	11.25	11.12	11.14	10.70	0.00	9.92	0.00	5.54	1.97
Canada	12.07	12.50	13.92	11.25	11.04	11.14	10.70	0.00	3.34	0.38	6.15	1.41
Japan	17.72	15.02	13.92	11.25	11.24	11.14	10.70	0.00	32.12	0.00	0.63	42.94
Switzerland	19.71	16.54	13.92	11.25	11.09	11.14	10.70	0.00	44.78	0.00	9.66	0.22
UK	16.46	14.80	13.92	11.25	11.21	11.14	10.70	0.00	28.00	0.00	1.29	25.69
US	13.94	13.41	13.92	11.25	11.25	11.14	10.70	0.00	9.98	0.00	0.18	67.05
Bonds												
Euroland	7.69	5.42	4.82	4.67	4.64	4.66	1.88	8.58	45.10	0.00	2.91	8.98
Australia	11.07	7.23	4.82	4.67	4.67	4.66	1.88	8.58	159.78	0.00	0.08	77.85
Canada	9.03	6.24	4.82	4.67	4.67	4.66	1.88	8.58	67.75	0.00	0.08	77.78
Japan	11.54	7.06	4.82	4.67	4.67	4.66	1.88	8.58	78.46	0.00	0.00	99.12
Switzerland	9.07	5.93	4.82	4.67	4.67	4.66	1.88	8.58	75.94	0.00	0.12	73.43
UK	8.68	6.00	4.82	4.67	4.67	4.66	1.88	8.58	74.16	0.00	0.07	78.60
US	9.21	6.32	4.82	4.67	4.66	4.66	1.88	8.58	100.90	0.00	1.11	29.25

Note. This table reports the standard deviation of portfolios featuring different uses of currency for risk-management.

We present results for equally-weighted global portfolios, for equity and bonds as respectively described in Table 4 and Table 6. Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge"

Reported standard deviations are annualized, and measured in percentage points.
All results presented are computed considering returns at a quarterly horizon.

Table A13

Optimal conditional currency exposure for an equally-weighted global portfolio: single and multiple - currency case using the real interest rate differential

Base Currency	Equity				Bonds			
	Single Currency		Multiple Currencies		Single Currency		Multiple Currencies	
	Slope	P-Value	Slope	P-Value	Slope	P-Value	Slope	P-Value
Euroland	-0.23	1.00	0.73	0.38	0.02	1.00	-0.24	0.09
	(0.62)		(2.22)		0.11		0.94	
Australia	0.00	1.00	0.41	0.10	0.00	1.00	-0.21	0.20
	(0.33)		(0.68)		0.07		0.28	
Canada	0.03	1.00	-2.60	0.06	-0.02	1.00	-1.55	0.19
	(0.37)		(1.98)		0.12		0.88	
Japan	-0.05	0.99	-0.64	0.06	-0.02	1.00	-0.27	0.00
	(0.17)		(1.45)		0.09		0.49	
Switz.	-0.09	1.00	0.48	0.37	0.00	1.00	-1.01	0.66
	(0.48)		(1.86)		0.10		0.73	
UK	0.01	1.00	2.59***	0.63	0.02	1.00	0.17	0.00
	(0.36)		(0.97)		0.07		0.43	
US	0.05	1.00	2.17	0.18	0.00	1.00	0.56	0.49
	(0.34)		(2.22)		0.11		0.98	

Note. This table reports optimal currency exposure conditional on real interest rate. For each base country-currency pair, we now let the optimal currency position vary with the log real interest rate differential (ex-post real interest rate of the foreign country minus that of the base country). Yet, we impose the constraints that the slopes of the optimal positions with respect to the interest rate differential be equal across foreign currencies.

The "Single Currency" columns consider the case of an investor using one currency at a time to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients from a SUR estimation are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.

The "Multiple Currency" columns consider the case of an investor using all foreign currencies simultaenously to manage risk, but still constrain the slopes to be the same across foreign currencies. Resulting slope coefficients are reported for each base country, followed by the P-value of a test of the constraint. A P-value of x% indicates that the constraint can be rejected at the x% level.

Table A14
Sharpe ratios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge		
				Baseline	Conditional hedging (constrained)	Synthetic currency
Panel A: Full period						
Equity						
Euroland	0.41	0.46	0.51	0.47	0.51	0.54
Australia	0.47	0.50	0.47	0.47	0.47	0.54
Canada	0.48	0.50	0.47	0.47	0.49	0.54
Japan	0.42	0.47	0.48	0.47	0.49	0.54
Switzerland	0.45	0.49	0.52	0.47	0.53	0.54
UK	0.38	0.45	0.49	0.47	0.48	0.54
US	0.53	0.52	0.48	0.48	0.55	0.54
Bonds						
Euroland	0.31	0.41	0.44	0.43	0.45	0.50
Australia	0.25	0.35	0.46	0.44	0.41	0.50
Canada	0.26	0.36	0.45	0.43	0.44	0.50
Japan	0.26	0.38	0.43	0.43	0.46	0.50
Switzerland	0.39	0.48	0.44	0.44	0.45	0.50
UK	0.20	0.32	0.45	0.43	0.51	0.50
US	0.36	0.43	0.46	0.44	0.49	0.50

Note. Table 10 reports Sharpe ratios of portfolios featuring different uses of currency for risk-management. Please refer to Table 9 for a detailed description of these portfolios.

The Sharpe Ratio of each portfolio is calculated as the ratio of the log mean gross return on the portfolio divided by the standard deviation of the log return on the portfolio. Please see Appendix for a detailed description of the calculation of the Sharpe Ratio.

All results presented are computed considering returns at a quarterly horizon.

Table 14 (continued)
Sharpe ratios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge: Baseline	Optimal hedge: Conditional hedging (constrained)	Optimal hedge: Synthetic currency
Panel B: Subperiod I						
Equity						
Euroland	0.51	0.57	0.61	0.53	0.49	0.73
Australia	0.61	0.64	0.59	0.54	0.56	0.73
Canada	0.54	0.59	0.59	0.53	0.63	0.73
Japan	0.41	0.52	0.59	0.53	0.58	0.72
Switzerland	0.55	0.60	0.62	0.54	0.41	0.73
UK	0.52	0.59	0.60	0.53	0.59	0.73
US	0.60	0.62	0.61	0.54	0.50	0.73
Bonds						
Euroland	0.14	0.18	0.19	0.05	0.08	0.20
Australia	0.23	0.25	0.20	0.06	0.08	0.21
Canada	0.12	0.16	0.20	0.05	0.19	0.20
Japan	-0.04	0.04	0.16	0.04	0.04	0.19
Switzerland	0.25	0.27	0.18	0.06	0.13	0.21
UK	0.14	0.18	0.19	0.06	-0.01	0.21
US	0.22	0.23	0.22	0.05	0.02	0.20
Panel C: Subperiod II						
Equity						
Euroland	0.32	0.37	0.41	0.39	0.46	0.34
Australia	0.32	0.36	0.35	0.39	0.51	0.33
Canada	0.42	0.40	0.35	0.39	0.50	0.34
Japan	0.43	0.43	0.39	0.40	0.44	0.34
Switzerland	0.37	0.40	0.42	0.40	0.48	0.34
UK	0.26	0.33	0.39	0.39	0.45	0.33
US	0.45	0.43	0.37	0.40	0.43	0.34
Bonds						
Euroland	0.50	0.68	0.74	0.80	0.74	0.84
Australia	0.27	0.47	0.77	0.80	0.81	0.84
Canada	0.42	0.60	0.76	0.80	0.79	0.84
Japan	0.50	0.67	0.75	0.81	0.80	0.85
Switzerland	0.55	0.72	0.75	0.81	0.79	0.84
UK	0.28	0.50	0.75	0.79	0.78	0.84
US	0.53	0.68	0.76	0.81	0.73	0.85