J. Aust. Math. Soc. **97** (2014), 63–77
doi:10.1017/S1446788714000032

GLOBAL DETERMINISM OF CLIFFORD SEMIGROUPS

AIPING GAN and XIANZHONG ZHAO✉

(Received 21 March 2013; accepted 14 December 2013; first published online 16 May 2014)

Communicated by M. Jackson

Abstract

In this paper we shall give characterizations of the closed subsemigroups of a Clifford semigroup. Also, we shall show that the class of all Clifford semigroups satisfies the strong isomorphism property and so is globally determined. Thus the results obtained by Kobayashi ['Semilattices are globally determined', *Semigroup Forum* **29** (1984), 217–222] and by Gould and Iskra ['Globally determined classes of semigroups' *Semigroup Forum* **28** (1984), 1–11] are generalized.

2010 *Mathematics subject classification*: primary 06A12; secondary 20M17.

Keywords and phrases: Clifford semigroup, closed subsemigroup, power semigroup.

1. Introduction and preliminaries

The power semigroup, or global, of a semigroup S is the semigroup $P(S)$ of all nonempty subsets of S equipped with the multiplication

$$AB = \{ab : a \in A, b \in B\} \quad \text{for all } A, B \in P(S).$$

A class $\mathcal{K}$ of semigroups is said to be globally determined if any two members of $\mathcal{K}$ having isomorphic globals must themselves be isomorphic.

Tamura [18] asked in 1967 whether the class of all semigroups is globally determined. The question was negatively answered in the class of all semigroups by Mogiljanskaja [14] in 1973. Crvenković *et al.* [6] proved that involution semigroups are not globally determined in 2001. Also, it is known that the following classes are globally determined: groups [13, 22]; rectangular groups [19]; completely 0-simple semigroups [20]; finite semigroups [21]; lattices and semilattices [10, 12], finite simple semigroups and semilattices of torsion groups in which semilattices are finite [8]; completely regular periodic monoid with irreducible identity [9]; $*$-bands [23]; and

This work is supported by the National Natural Science Foundation of China (11261021) and a grant from the Natural Science Foundation of Shannxi Province (2011JQ1017).

integer semigroups [17]. Also, there are a series of papers in the literature considering power semigroups and related varieties of semigroups (see [1–3, 15, 16]).

In this paper we shall study the question of global determinism of Clifford semigroups and show that the class of all Clifford semigroups satisfies the strong isomorphism property. Recall that a class $\mathcal{K}$ of semigroups is said to satisfy the strong isomorphism property if, for any $S, S' \in \mathcal{K}$, for every isomorphism ψ from $P(S)$ to $P(S')$, $\psi|_S$ (the restriction of ψ to S) is an isomorphism from S to S' [12], where S (respectively, S') is considered to be a subset of $P(S)$ (respectively, S') by identifying an element x of S (respectively, S') with the singleton $\{x\}$. It is proved by Kobayashi in [12] that the class of semilattices satisfies the strong isomorphism property.

For a semigroup S, the set of idempotents of a semigroup S will be denoted $E(S)$, and for each $e \in E(S)$ the maximal subgroup $\mathcal{H}$-class of S containing e will be denoted $H_e(S)$. A singleton member of $P(S)$ will frequently be identified with the element it contains.

The following lemma will be useful to us, which implies that the class of all groups satisfies the strong isomorphism property.

Lemma 1.1 (Lemma 2.1 in [8]). *Let S be a semigroup and e an idempotent element in S. Then $H_e(P(S)) = H_e(S)$.*

Throughout this paper we shall always assume that $S = \bigcup(G_\alpha : \alpha \in E)$ and $S' = \bigcup(G'_\beta : \beta \in E')$ are both semilattice of groups, that is, Clifford semigroups, where E, E' are semilattices and G_α, G'_β are groups. Let ψ be an isomorphism from $P(S)$ onto $P(S')$.

For convenience, we give some notation associated with S and S':

- We identify the semilattice E (respectively, E') with the set of idempotents of S (respectively, S'), that is to say, $E = E(S)$ and $E' = E(S')$.
- The notation $Ch(E)$ (respectively, $Ch(E')$) denotes the set of all subchains of the semilattice E (respectively, E').

In the second section, we shall give the characterizations of the closed subsemigroups of a Clifford semigroup. Starting from the study of closed subsemigroups, we shall show in the third section that the restriction $\psi|_{Ch(E)}$ of ψ to $Ch(E)$ is a mapping from subset $Ch(E)$ of $P(S)$ onto subset $Ch(E')$ of $P(S')$. In the last section we shall show that the class of all Clifford semigroups satisfies the strong isomorphism property and so is globally determined. Thus the results obtained by Kobayashi in [12] and Theorem 2.2 in [8] are generalized.

A few words on notation and terminology:

- For a set A, $|A|$ denotes the cardinal number (or cardinality) of A.
- For a Clifford semigroup $S = \bigcup(G_\alpha : \alpha \in E)$ (respectively, $S' = \bigcup(G'_\beta : \beta \in E')$) and $\alpha \in E$ (respectively, $\beta \in E'$), e_α (respectively, e'_β) denotes the identity element of group G_α (respectively, G'_β). Sometimes, we identify e_α with α, and identify e'_β with β.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

- For $X \in P(S)$ and $\alpha \in E$, X_α denotes the set $X \cap G_\alpha$ and supp X the subset $\{\alpha \in E : X_\alpha \neq \emptyset\}$ of E.

For other notations and terminologies not given in this paper, the reader is referred to the books [4, 5, 11].

2. The closed subsemigroups of a Clifford semigroup

Zhao in [7] and [24] introduced and studied the closed subsemigroups of a semigroup S. To prove our main results in this paper, we shall give some characterizations of closed subsemigroups of a Clifford semigroup. Recall that a subsemigroup C of a semigroup S is said to be closed if

$$sat, sbt \in C \Rightarrow sabt \in C$$

holds for all $a, b \in S$, $s,\ t \in S^1$, where S^1 denotes the semigroup obtained from S by adjoining an identity if necessary. It is easy to see that every subsemilattice of a semilattice is closed. Let S be a semigroup and A a nonempty subset of S. We denote by $\overline{A}$ the closed subsemigroup of S generated by A, that is, the smallest closed subsemigroup of S containing A. In this section, unless stated otherwise, S always denotes a Clifford semigroup $\bigcup(G_\alpha : \alpha \in E)$.

Lemma 2.1 (Theorem 2.3 in [7]). *Let $A \in P(S)$. Then $\overline{A} = \bigcup_{\alpha \in \overline{\text{supp } A}} G_\alpha$, where $\overline{\text{supp } A}$ denotes the (closed) subsemilattice of semilattice $E(S)$ generated by* supp A.

Lemma 2.2. *Let $A \in P(S)$ and $A^2 = A$. Then the following statements are equivalent:*

(i) *$a_\alpha A = b_\alpha A$ for any $\alpha \in$ supp A and any $a_\alpha, b_\alpha \in G_\alpha$;*
(ii) *$a_\alpha A_\alpha = b_\alpha A_\alpha$ for any $\alpha \in$ supp A and any $a_\alpha, b_\alpha \in G_\alpha$;*
(iii) *$A_\alpha = G_\alpha$ for any $\alpha \in$ supp A.*

Proof. (i) $\Rightarrow$ (ii). Suppose that (i) holds. Assume that $\alpha \in$ supp A. Then we have that $e_\alpha A = c_\alpha A \subseteq A$ for any $c_\alpha \in A_\alpha$, where e_α denotes the identity element of group G_α, since $A^2 = A$. Also, it follows that $a_\alpha A_\alpha \subseteq a_\alpha A = b_\alpha A$ for any $a_\alpha,\ b_\alpha \in G_\alpha$.

Thus for any (but fixed) $a \in a_\alpha A_\alpha$, there exists $d_\beta \in A_\beta$ $(\beta \geq \alpha)$ such that $a = b_\alpha d_\beta = b_\alpha(e_\alpha d_\beta) \in b_\alpha A_\alpha$, since $e_\alpha d_\beta \in A \cap G_\alpha = A_\alpha$. This implies that $a_\alpha A_\alpha \subseteq b_\alpha A_\alpha$. Dually, we can show that $b_\alpha A_\alpha \subseteq a_\alpha A_\alpha$. Thus (ii) holds, as required.

(ii) $\Rightarrow$ (iii). Suppose that (ii) holds. Assume that $\alpha \in$ supp A. Then it follows that $A_\alpha = e_\alpha A_\alpha = a_\alpha A_\alpha$ for any $a_\alpha \in G_\alpha$. Also, $A_\alpha^2 \subseteq A_\alpha$ since $A^2 = A$. This implies that A_α is a subgroup of group G_α, and so $A_\alpha = G_\alpha$. We have shown that (iii) holds.

(iii) $\Rightarrow$ (i). Suppose that (iii) holds. Then it follows by Lemma 2 that A is a closed subsemigroup of S since $A^2 = A$. Also, it is easy to prove that

$$a_\alpha A = \bigcup_{\beta \in \text{supp } A,\ \beta \leq \alpha} G_\beta = b_\alpha A$$

for any $\alpha \in$ supp A and any $a_\alpha,\ b_\alpha \in G_\alpha$. We have shown that (i) holds. □

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

By Lemma 2.1 and Lemma 2.2, we have the following result.

Theorem 2.3. *Let $A \in P(S)$. Then A is a closed subsemigroup of S if and only if A satisfies the following two conditions:*

(i) $A^2 = A$;

(ii) $e_\alpha A = g_\alpha A$ *for any* $\alpha \in \operatorname{supp} A$ *and any* $g_\alpha \in G_\alpha$.

Proposition 2.4. *Let $A \in P(S)$. Then SA, AS are both closed subsemigroups of S and $SA = AS = \bigcup_{\gamma\in\Gamma} G_\gamma$, where $\Gamma = \{\gamma \in E : (\exists\, \alpha \in \operatorname{supp} A)\ \gamma \leq \alpha\}$.*

Proof. The proof is routine and is omitted. □

Corollary 2.5. *Let $A \in P(S)$ and let e_α be the identity element of G_α for any $\alpha \in E$. Then*

$$e_\alpha S = AS \Rightarrow e_\alpha A = A.$$

Proof. By Proposition 2.4,

$$\operatorname{supp} A \subseteq \operatorname{supp}(AS) = \operatorname{supp}(e_\alpha S) = \{\gamma \in E : \gamma \leq \alpha\}.$$

Thus $\beta \leq \alpha$ for any $\beta \in \operatorname{supp} A$, and so $e_\alpha A = A$. □

Lemma 2.6. *Let $A, B \in P(S)$. Then* $\operatorname{supp} A \cdot \operatorname{supp} B = \operatorname{supp}(AB)$.

Proof. The proof is routine and is omitted. □

Lemma 2.7. *Let A, $B \in P(S)$ and $A\,\mathcal{H}\,B$. Then* $\operatorname{supp} A = \operatorname{supp} B$.

Proof. Suppose that $A\,\mathcal{H}\,B$ for some A, $B \in P(S)$. Then there exist C, $D \in (P(S))^1$ such that $A = CB$, $B = DA$. Thus by Lemma 2.6,

$$\operatorname{supp} A = \operatorname{supp} C \cdot \operatorname{supp} B \quad \text{and} \quad \operatorname{supp} B = \operatorname{supp} D \cdot \operatorname{supp} A.$$

In the following we will show that $\operatorname{supp} A = \operatorname{supp} B$.

Suppose that $\alpha \in \operatorname{supp} A$. Then there exists $\beta \in \operatorname{supp} B$ such that $\alpha \leq \beta$ since $\operatorname{supp} A = \operatorname{supp} C \cdot \operatorname{supp} B$. Also, $\beta \leq \gamma$ for some $\gamma \in \operatorname{supp} D$ since $\operatorname{supp} B = \operatorname{supp} D \cdot \operatorname{supp} A$. Thus $\alpha \leq \gamma$, and so $\alpha = \gamma\alpha \in \operatorname{supp} D \cdot \operatorname{supp} A = \operatorname{supp} B$. Therefore we have shown that $\operatorname{supp} A \subseteq \operatorname{supp} B$. Dually, we can show that $\operatorname{supp} B \subseteq \operatorname{supp} A$. This shows that $\operatorname{supp} A = \operatorname{supp} B$, as required. □

Lemma 2.8. *Let $A, B \in P(S)$ and $A\,\mathcal{H}\,B$. Then $AS = SA = SB = BS$.*

Proof. Suppose that A, $B \in P(S)$ such that $A\,\mathcal{H}\,B$. Then it follows that $SA = AS$, $SB = BS$, and SA, SB are both closed semigroups of S by Proposition 2.4. To prove that $AS = SA = SB = BS$, it suffices to show that $\operatorname{supp}(SA) = \operatorname{supp}(SB)$ by Lemma 2.1. In fact, by Lemmas 2.6 and 2.7,

$$\operatorname{supp}(SA) = \operatorname{supp} S \cdot \operatorname{supp} A = \operatorname{supp} S \cdot \operatorname{supp} B = \operatorname{supp}(SB).$$

The proof is completed. □

Proposition 2.9. *Let $S = \bigcup(G_\alpha : \alpha \in E)$ and $S' = \bigcup(G'_\beta : \beta \in E')$ be Clifford semigroups and ψ an isomorphism from $P(S)$ onto $P(S')$. Then $\psi(SA)$ (respectively,*

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

$\psi^{-1}(S'B)$) is a closed subsemigroup of S' (respectively, S) for any $A \in P(S)$ (respectively, $B \in P(S')$).

Proof. Let $\beta \in E'$ and $e'_\beta,\ g'_\beta \in G'_\beta$. Then by Lemma 1.1,

$$\begin{aligned} e'_\beta\ \mathcal{H}_{P(S')}\ g'_\beta &\Rightarrow \psi^{-1}(e'_\beta)\ \mathcal{H}_{P(S)}\ \psi^{-1}(g'_\beta) \\ &\Rightarrow \psi^{-1}(e'_\beta)S = \psi^{-1}(g'_\beta)S \quad \text{(by Lemma 2.8)} \\ &\Rightarrow \psi^{-1}(e'_\beta)SA = \psi^{-1}(g'_\beta)SA \\ &\Rightarrow e'_\beta\psi(SA) = g'_\beta\psi(SA). \end{aligned}$$

On the other hand, it follows by Proposition 2.4 that SA is a closed subsemigroup of S, and so $(SA)^2 = SA$. This implies that

$$\psi(SA) = \psi((SA)^2) = \psi(SA)^2.$$

Therefore, we can show by Theorem 2.3 that $\psi(SA)$ is a closed subsemigroup of S'.

By using the above reasoning, we can show that $\psi^{-1}(S'B)$ is a closed subsemigroup of S, since ψ^{-1} is also an isomorphism. □

Corollary 2.10. *Let $S = \bigcup(G_\alpha : \alpha \in E)$ and $S' = \bigcup(G'_\beta : \beta \in E')$ be Clifford semigroups and ψ an isomorphism from $P(S)$ onto $P(S')$. If $A,\ B \in P(S)$ such that* supp A = supp B, *then $A\psi^{-1}(S') = B\psi^{-1}(S')$.*

Proof. Suppose that $A, B \in P(S)$ such that supp A = supp B. Let $\psi^{-1}(C) = A$. Then

$$A\psi^{-1}(S') = \psi^{-1}(C)\psi^{-1}(S') = \psi^{-1}(CS') = \psi^{-1}(S'C).$$

Thus it follows by Proposition 2.9 that $A\psi^{-1}(S')$ is a closed semigroup of S. Similarly, $B\psi^{-1}(S')$ is also a closed semigroup of S. On the other hand, by Lemma 2.6,

$$\text{supp}\ (A\psi^{-1}(S')) = \text{supp}\ (B\psi^{-1}(S'))$$

since supp A = supp B. Thus we have shown that $A\psi^{-1}(S') = B\psi^{-1}(S')$, as required. □

3. On the restriction of ψ to $Ch(E)$

In this section we shall show that the restriction $\psi|_{Ch(E)}$ of ψ to $Ch(E)$ is a mapping from the subset $Ch(E)$ of $P(S)$ onto the subset $Ch(E')$ of $P(S')$. For this aim, the following lemmas are needed.

Lemma 3.1. *Let $D \in Ch(E)$ and $Y \in P(S)$ such that $Y^2 = D$. Then the following statements are true:*

(i) supp $Y = D$ *and* Y^2 = supp Y;
(ii) $Y \cdot$ supp $Y = Y = YD$;
(iii) $Y\ \mathcal{H}\ D$;
(iv) $(\forall \alpha \in \text{supp}\ Y)\quad |Y_\alpha| = 1$.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

Proof. Suppose that $D \in Ch(E)$ and $Y \in P(S)$ such that $Y^2 = D$.

(i) It follows immediately by Lemma 2.6 that

$$\alpha = \alpha^2 \in (\text{supp } Y)^2 = \text{supp } Y^2 = \text{supp } D = D$$

for any $\alpha \in \text{supp } Y$. This shows that $\text{supp } Y \subseteq D$, and so $\text{supp } Y$ is a subchain of D, since $D \in Ch(E)$. Thus $\text{supp } Y = (\text{supp } Y)^2 = D$ and $Y^2 = \text{supp } Y$, as required.

(ii) It is easy to see that $Y \subseteq Y \cdot \text{supp } Y$. To prove that $Y \cdot \text{supp } Y \subseteq Y$, suppose that $a_\beta \in Y_\beta$ and $\alpha \in \text{supp } Y$. Then α and β are comparable since $\text{supp } Y = D$ is a subchain of E. If $\beta \leq \alpha$, then

$$e_\alpha a_\beta = e_\alpha(e_\beta a_\beta) = (e_\alpha e_\beta)a_\beta = e_\beta\, a_\beta = a_\beta \in Y.$$

If $\alpha < \beta$, then $a_\beta a_\beta = e_\beta$ and $y_\alpha a_\beta = e_\alpha$ for any $y_\alpha \in Y_\alpha$, since $Y^2 = \text{supp } Y$. Thus

$$e_\alpha a_\beta = (y_\alpha a_\beta)a_\beta = y_\alpha(a_\beta a_\beta) = y_\alpha e_\beta = y_\alpha \in Y.$$

This shows that $Y \cdot \text{supp } Y \subseteq Y$, and so $Y \cdot \text{supp } Y = Y$ and $YD = Y$, as required.

(iii) Since $Y^2 = D$ and $YD = DY = Y$, it follows immediately that $Y \mathcal{H} D$.

(iv) Suppose that $\alpha \in \text{supp } Y$ and $a_\alpha, b_\alpha \in Y_\alpha$. Then

$$a_\alpha a_\alpha = e_\alpha = a_\alpha b_\alpha$$

since $Y^2 = \text{supp } Y$. This implies that $a_\alpha = b_\alpha$, and so $|Y_\alpha| = 1$, as required. □

Lemma 3.2. *If $D \in Ch(E)$ and $X \in P(S')$ such that $X^2 = \psi(D)$, then the following statements are true:*

(i) *$X \mathcal{H} \psi(D)$;*

(ii) *$X \subseteq \psi(D) \Rightarrow X = \psi(D)$.*

Proof. Suppose that $D \in Ch(E)$ and $X \in P(S')$ such that $X^2 = \psi(D)$.

(i) It follows that there exists $Y \in P(S)$ such that $X = \psi(Y)$, since ψ is an isomorphism. Thus $\psi(Y^2) = \psi(Y)^2 = X^2 = \psi(D)$. This implies that $Y^2 = D$. Therefore, we can conclude by Lemma 3.1 that $Y \mathcal{H} D$, and so $X \mathcal{H} \psi(D)$, as required.

(ii) It is easy to see that $\psi(D)^2 = \psi(D^2) = \psi(D)$. Thus $\psi(D)$ is an idempotent and so the identity element in its $\mathcal{H}$-class.

If $X \subseteq \psi(D)$, then

$$\psi(D) = X^2 \subseteq \psi(D) \cdot X = X \subseteq \psi(D),$$

since $X \in H_{\psi(D)}(P(S'))$. Thus $X = \psi(D)$, as required. □

Lemma 3.3. *If $D \in Ch(E)$, then every $(\psi(D))_\alpha$ ($\alpha \in \text{supp } \psi(D)$) is a periodic subgroup of group G'_α and $\psi(D)$ is a Clifford semigroup.*

Proof. Suppose that $D \in Ch(E)$. Then $D^2 = D$, and so $(\psi(D))^2 = \psi(D)$. This implies that $\psi(D)$ is a subsemigroup of Clifford semigroup S', and so every $(\psi(D))_\alpha$ ($\alpha \in \text{supp } \psi(D)$) is a subsemigroup of G'_α.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

We shall show that every subsemigroup $(\psi(D))_\alpha$ ($\alpha \in \operatorname{supp} \psi(D)$) is periodic; that is, for any $a \in (\psi(D))_\alpha$, there exists a positive integer n such that $a^n = e'_\alpha$, where e'_α is the identity element of group G'_α. Suppose, on the contrary, that the order of element a is infinite. Set $X = \psi(D) \setminus \{a^3\}$. It is clear that $X^2 \subseteq \psi(D)^2 = \psi(D^2) = \psi(D)$. Also, it follows that $\psi(D) \subseteq X^2$. In fact, for any $b \in \psi(D)$, there exist $c,\ d \in \psi(D)$ such that $b = cd$, since $\psi(D) = \psi(D)^2$. To show that $b \in X^2$, we consider the following cases:

- If $c,\ d \in X$, then $b = cd \in X^2$.
- Assume that $c \in X$ and $d = a^3$. Then $b = ca^3 = (ca)a^2 = (ca^2)a$. If $ca = ca^2$, then $ca^3 = ca^2 = ca$, and so $b = cd = ca^3 = ca \in X^2$. Otherwise, $ca \neq ca^2$. Hence, we might as well say that ca^2 is not equal to a^3. Thus $b = (ca^2)a \in X^2$.
- If $d \in X$ and $c = a^3$, we can similarly show that $b = a^3 d \in X^2$.
- If $c = d = a^3$, then $b = cd = a^6 = a^2\, a^4 \in X^2$.

Thus we have shown that $b \in X^2$. That is to say, $\psi(D) \subseteq X^2$. Therefore, it follows that $X^2 = \psi(D)$, contradicting Lemma 3.2. This shows that every $(\psi(D))_\alpha$ ($\alpha \in \operatorname{supp} \psi(D)$) is a periodic subsemigroup of group G'_α and so is a subgroup of group G'_α.

Since every $(\psi(D))_\alpha$ ($\alpha \in \operatorname{supp} \psi(D)$) is a subgroup of group G'_α and $\psi(D)$ is a subsemigroup of Clifford semigroup S', it follows immediately that $\psi(D) = \bigcup((\psi(D))_\alpha : \alpha \in \operatorname{supp} \psi(D))$ is a semilattice of groups. □

Lemma 3.4. *If $D \in Ch(E)$ and $\alpha, \beta \in \operatorname{supp} \psi(D)$ such that $\alpha < \beta$, then $(\psi(D))_\alpha = \{e'_\alpha\}$. In particular, if there is no any maximal element in semilattice $\operatorname{supp} \psi(D)$, then $\psi(D) = \operatorname{supp}(\psi(D))$.*

Proof. Suppose that $\alpha, \beta \in \operatorname{supp} \psi(D)$ such that $\alpha < \beta$. Assume that $X = \psi(D) \setminus \{e'_\alpha\}$. If $(\psi(D))_\alpha \neq \{e'_\alpha\}$, that is, $X_\alpha \neq \emptyset$, then it is easy to verify that $X^2 = \psi(D)$, contradicting Lemma 3.2. The remaining part is easily verified. □

Lemma 3.5. *If $D \in Ch(E)$, then $\operatorname{supp}(\psi(D)) \in Ch(E')$.*

Proof. Suppose, on the contrary, that there exist $\alpha,\ \beta \in \operatorname{supp} \psi(D)$ such that $\alpha\beta$ is neither α nor β. Set $X = \psi(D) \setminus \{e'_{\alpha\beta}\}$. Then we have that $e'_{\alpha\beta} = e'_\alpha e'_\beta \in X^2$. Also, for any $a \in (\psi(D))_{\alpha\beta} \setminus \{e'_{\alpha\beta}\}$, we have

$$a = ae'_{\alpha\beta} = (ae'_\alpha)e'_\beta = ((ae'_{\alpha\beta})e'_\alpha)e'_\beta = (a(e'_{\alpha\beta}e'_\alpha))e'_\beta = (ae'_{\alpha\beta})e'_\beta = ae'_\beta \in X^2,$$

since $a,\ e'_\beta \in X$. This shows that $(\psi(D))_{\alpha\beta} \subseteq X^2$. It is easy to see that X^2 also contains the subgroup $(\psi(D))_\gamma$ of group G'_γ, for all $\gamma \in \operatorname{supp} \psi(D)$ such that $\gamma \neq \alpha\beta$. Thus it follows that $X^2 = \psi(D)$, contradicting Lemma 3.2. We have shown that $\operatorname{supp}(\psi(D)) \in Ch(E')$, as required. □

Lemma 3.6. *If G is a group and $|G| > 2$, then $(G \setminus \{e\})^2 = G$, where e denotes the identity element of G.*

Proof. The proof is omitted. □

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

Proposition 3.7. *$\psi|_{Ch(E)}$ is a mapping from the subset $Ch(E)$ of $P(S)$ onto the subset $Ch(E')$ of $P(S')$.*

Proof. Suppose that $D \in Ch(E)$. Then we know by Lemma 3.5 that $\mathrm{supp}(\psi(D))$ is a subchain of the semilattice E'. If there is no maximal element in the chain $\mathrm{supp}\,\psi(D)$, then by Lemma 3.4 $\psi(D) \in Ch(E')$. Thus we only need to prove that $(\psi(D))_\alpha = \{e'_\alpha\}$ if α is the maximal element in the chain $\mathrm{supp}(\psi(D))$, since $(\psi(D))_\beta = \{e'_\beta\}$ for all $\beta \in \mathrm{supp}\,\psi(D) \setminus \{\alpha\}$ (see Lemma 3.4).

Let α be the maximal element in chain $\mathrm{supp}(\psi(D))$. If $|(\psi(D))_\alpha| > 2$, then it follows immediately by Lemma 3.6 that, for $A = \psi(D) \setminus \{e'_\alpha\}$, we have $A^2 = \psi(D)$, contradicting Lemma 3.2. Thus we have shown that $|(\psi(D))_\alpha| \leq 2$.

Suppose that $(\psi(D))_\alpha = \{e'_\alpha, a_\alpha\} \neq \{e'_\alpha\}$.

Assume that $A = \psi(D) \setminus \{e'_\alpha\}$ and $B = \mathrm{supp}(\psi(D))$. Then it is easy to verify that $A\psi(D) = B\psi(D) = \psi(D)$, and so $\psi^{-1}(A)D = \psi^{-1}(B)D = D$. On the other hand, it follows by Corollary 2.10 that $A\psi(S) = B\psi(S) = \psi(D)\psi(S)$, and so $\psi^{-1}(A)S = \psi^{-1}(B)S = DS$, since $\mathrm{supp}\,A = \mathrm{supp}\,B = \mathrm{supp}(\psi(D))$.

Now, for any (but fixed) $\beta \in \mathrm{supp}\,\psi^{-1}(A)$,

$$\beta \in \mathrm{supp}\,(\psi^{-1}(A)S) = \mathrm{supp}\,(DS),$$

since $\psi^{-1}(A)S = DS$, and so $\beta \leq \delta$ for some $\delta \in D$ by Proposition 2.4. This implies that $b_\beta = b_\beta e_\beta = b_\beta e_\delta$ for any $b_\beta \in (\psi^{-1}(A))_\beta$, and so $b_\beta = b_\beta e_\delta \in \psi^{-1}(A)D = D$. Thus we have shown that $\psi^{-1}(A) \subseteq D$; that is, $\psi^{-1}(A)$ is a subchain of chain D. Similarly, we can show that $\psi^{-1}(B)$ is also a subchain of chain D.

Also, it is easy to verify that $A^2 = B$ and $BA = AB = A$. Thus it follows that $A\,\mathcal{H}\,B$ in $P(S')$, and so $\psi^{-1}(A)\,\mathcal{H}\,\psi^{-1}(B)$ in $P(S)$. This implies that $\mathrm{supp}\,\psi^{-1}(A) = \mathrm{supp}\,\psi^{-1}(B)$, by Lemma 2.7.

Summarizing the above results, we can show that

$$\psi^{-1}(A) = \mathrm{supp}\,\psi^{-1}(A) = \mathrm{supp}\,\psi^{-1}(B) = \psi^{-1}(B),$$

and so $A = B$, which is a contradiction. This shows that $|\psi(D)_\alpha| = 1$, and so $\psi(D) \in Ch(E')$, as required. □

4. Main results

To show that the class of all Clifford semigroups satisfies the strong isomorphism property, we need the following notations:

- $E(P(S)) = \{X \in P(S) : X^2 = X\}$;
- $E(P(E)) = \{X \in P(E) : X^2 = X\}$.

It is clear that $Ch(E) \subseteq E(P(E)) \subseteq E(P(S))$. Define a relation $\leq$ on $E(P(S))$ by

$$X \leq Y \Leftrightarrow X = XY = YX.$$

Then it is easy to see that $\leq$ is a partial ordering relation on $E(P(S))$.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

By identifying an idempotent element e of the semigroup S with the singleton set $\{e\}$, we can find that the restriction $\leq|_E$ of $\leq$ to E is exactly the natural partial order on the semilattice E. That is to say,

$$(\forall e, f \in E) \quad \{e\} \leq \{f\} \Leftrightarrow e \leq f.$$

Recall that for $e, f \in E$ we say that f covers e in the semilattice E if $e < f$ and if there is no $g \in E$ such that $e < g < f$. In such a case we write $e \prec f$. Similarly, for $X,\ Y \in E(P(E))$, we write $X \twoheadrightarrow Y$ (respectively, $X \rightarrowtail Y$) if $X < Y$ and if there is no $Z \in E(P(E))$ (respectively, $Z \in Ch(E)$) such that $X < Z < Y$.

Remark 4.1. It is clear that $X \twoheadrightarrow Y$ implies $X \rightarrowtail Y$.

Remark 4.2. Every singleton member in $P(E)$ is a chain in the semilattice E. However, for any $e, f \in E$, neither $e \twoheadrightarrow f$ nor $e \rightarrowtail f$ holds since if $e < f$, then $e < \{e, f\} < f$.

Proposition 3.7 tells us that $\psi|_{Ch(E)}$ is a bijection from the poset $Ch(E)$ onto the poset $Ch(E')$. Also, it is easy to see that $\psi|_{Ch(E)}$ is order-preserving. The following lemma shows that $\psi|_{Ch(E)}$ is also cover-preserving.

Lemma 4.3. *Let $X,\ Y \in Ch(E)$. If $X \rightarrowtail Y$, then $\psi(X) \rightarrowtail \psi(Y)$.*

Proof. Suppose that $X,\ Y \in Ch(E)$ such that $X \rightarrowtail Y$. If $\psi(X) \leq Z \leq \psi(Y)$ for some $Z \in Ch(E')$, then $X \leq \psi^{-1}(Z) \leq Y$, since $\psi^{-1}|_{Ch(E')}$ is order-preserving. Also, we have by Proposition 3.7 that $\psi^{-1}(Z) \in Ch(E)$. Thus it follows that $\psi^{-1}(Z) = X$ or $\psi^{-1}(Z) = Y$, since $X \rightarrowtail Y$. That is to say, $Z = \psi(X)$ or $Z = \psi(Y)$, as required. □

The following three lemmas are analogous to corresponding statements in Kobayashi in [12]. They will be useful to prove our main result.

Lemma 4.4. *Let $D \in Ch(E)$ and $\alpha \in D$. If α is not the maximal element of D, then $D \twoheadrightarrow D \setminus \{\alpha\}$.*

Proof. Let $D \in Ch(E)$ and $\alpha \in D$. Clearly, $D \setminus \{\alpha\}$ is a subchain of chain D. Suppose that α is not the maximal element of D. Then it is easy to verify that $D(D \setminus \{\alpha\}) = D$, that is, $D < D \setminus \{\alpha\}$. If $D \leq A \leq D \setminus \{\alpha\}$ for some $A \in E(P(E))$, that is,

$$D = DA \quad \text{and} \quad A = (D \setminus \{\alpha\})A,$$

then $A \subseteq D$, since $A = (D \setminus \{\alpha\})A \subseteq DA = D$. Also, we have that for any (but fixed) $d \in D \setminus \{\alpha\}$, there exists $a \in A$ such that $d \leq a$, that is, $d = da$, since $D = DA$. Thus we have shown that $D \setminus \{\alpha\} \subseteq (D \setminus \{\alpha\})A = A \subseteq D$. Therefore, A is equal to either D or $D \setminus \{\alpha\}$. This shows that $D \twoheadrightarrow D \setminus \{\alpha\}$. □

Lemma 4.5. *Let $D \in Ch(E)$ and β be a maximal element of D. If $\beta \prec \gamma$ for some $\gamma \in E$, then $D \rightarrowtail D \cup \{\gamma\}$.*

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

Proof. Let $D \in Ch(E)$ and β be a maximal element of D. Suppose that $\beta \prec \gamma$ for some $\gamma \in E$. Then it is clear that $\gamma \notin D$, since β is a maximal element of D. Also, it is easy to verify that $D < D \cup \{\gamma\}$. If $D \leq A \leq D \cup \{\gamma\}$ for some $A \in Ch(E)$, that is,

$$D = DA \quad \text{and} \quad A = (D \cup \{\gamma\})A,$$

then it follows immediately that D is a subchain of A, since

$$A = (D \cup \{\gamma\})A = (DA) \cup (\gamma A) = D \cup (\gamma A).$$

Assume that $D \neq A$, that is, D is a proper subchain of A. Then there exists an element $\alpha \in A \setminus D$. If $\beta \geq \alpha$, then $\alpha = \beta\alpha \in DA = D$, which is a contradiction. Thus we have that $\beta < \alpha$, since $A \in Ch(E)$. Also, it follows that $\alpha = \gamma\eta$ for some $\eta \in A$, and so $\alpha \leq \gamma$, since $A = D \cup (\gamma A)$. This shows that $\beta < \alpha \leq \gamma$. Therefore, we have that $\alpha = \gamma$, and so $A = D \cup \{\gamma\}$, since $\beta \prec \gamma$. This shows that $D \rightarrowtail D \cup \{\gamma\}$. □

Lemma 4.6. *Let $e \in E$. Then the following statements are true:*

(i) *if $f \in E$ satisfies $e \prec f$, then $e \rightarrowtail \{e, f\}$;*
(ii) *if $Y \in Ch(E)$ satisfies $e \rightarrowtail Y$, then $Y = \{e, f\}$ for some $f \in E$ and $e \prec f$.*

Proof. Let $e \in E$.

(i) Suppose that $f \in E$ satisfies $e \prec f$. Then it is easy to see that $e < \{e, f\}$. If $A \in Ch(E)$ such that $e \leq A \leq \{e, f\}$, that is,

$$e = eA \quad \text{and} \quad A = \{e, f\}A,$$

then it follows immediately that $e \in A$ and $a \leq f$ for any $a \in A$, since

$$A = \{e, f\}A = eA \cup (fA) = \{e\} \cup (fA).$$

Also, we conclude that $e \leq a$ for any $a \in A$, since $e = eA$. This shows that $e \leq a \leq f$. Thus we have shown that $a = e$ or $a = f$, since $e \prec f$. That is to say, $A = \{e\}$ or $A = \{e, f\}$, as required.

(ii) Assume that $Y \in Ch(E)$ such that $e \rightarrowtail Y$. Then $eY = e$ and so $e \leq y$ for any $y \in Y$. Thus it is easy to verify that $e < \{e\} \cup Y \leq Y$. This implies that $\{e\} \cup Y = Y$, since $e \rightarrowtail Y$. Hence, we have that $e \in Y$. Also, it follows immediately that $Y \setminus \{e\} \neq \emptyset$, since $e \rightarrowtail Y$. Thus for any $f \in Y \setminus \{e\}$, setting $Z = \{y \in Y : y \leq f\}$, we conclude that $e < Z \leq Y$. This implies that $Y = Z$, Hence, we have shown that Y is a two-element chain, say $Y = \{e, f\}$.

It remains to prove that $e \prec f$. Suppose that $g \in E$ such that $e < g < f$, then $e < \{e, g\} < \{e, f\} = Y$, contradicting $e \rightarrowtail Y$. This shows that $e \prec f$, as required. □

Let $X, Y, Z, W \in Ch(E)$ and $Y \neq Z$. We use the notions of a topknot which is introduced in [12] and a quasitopknot to describe configurations of arrows as shown on the diagrams below. In such a diagram, the ordinary arrows (between X and Y, say) means (in the 'plain text mode') $X \rightarrowtail Y$.

It is obvious that every topknot of X is a quasitopknot of X.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

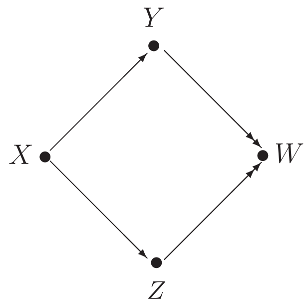

FIGURE 1. A topknot of X.

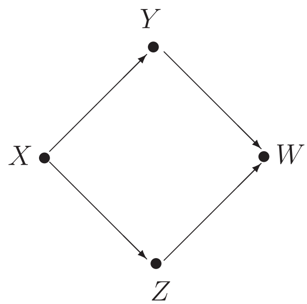

FIGURE 2. A quasitopknot of X.

THEOREM 4.7. *The class of Clifford semigroups satisfies the strong isomorphism property.*

PROOF. Suppose that $S = \bigcup(G_\alpha : \alpha \in E)$ and $S' = \bigcup(G'_\beta : \beta \in E')$ are both Clifford semigroups, and ψ is an isomorphism from $P(S)$ onto $P(S')$. Recall that we may identify the identity e_α of group G_α with α for any $\alpha \in E$, and so E and the set $E(S)$ of all idempotents of S are interchangeable. To show that $\psi|_S$ is an isomorphism of S onto S', we need only to prove by Lemma 1.1 that $\psi(e_\alpha) \in E'$ for any $\alpha \in E$. Assume that $A = \psi(e_\alpha)$ for some $\alpha \in E$. Then we have by Proposition 3.7 that $A \in Ch(E')$. In the following we shall prove that A is a singleton member in $P(E')$.

Claim 1. If $e'_\beta \in A$ and e'_β is not maximal in A, then $|G'_\beta| = 1$.

Let $e'_\beta \in A$. If $|G'_\beta| \geq 2$, then there exists $g'_\beta \in G'_\beta \setminus \{e'_\beta\}$. Let $B = (A \setminus \{e'_\beta\}) \cup \{g'_\beta\}$. Then it is easy to see that supp B = supp A. By Corollary 2.10,

$$\begin{aligned} A\psi(S) = B\psi(S) &\Rightarrow e_\alpha S = \psi^{-1}(B)S \\ &\Rightarrow e_\alpha \psi^{-1}(B) = \psi^{-1}(B) \quad \text{(by Corollary 2.5)} \\ &\Rightarrow AB = B. \end{aligned}$$

If e'_β is not maximal in A, then there exists $e'_\gamma \in A$ such that $e'_\beta < e'_\gamma$. Thus it follows immediately that $e'_\beta = e'_\beta e'_\gamma \in AB = B$, contradicting $e'_\beta \notin B$. The claim is proved.

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

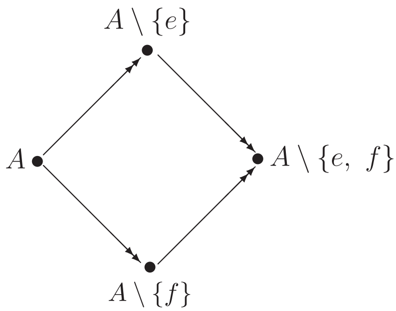

FIGURE 3. A topknot of A.

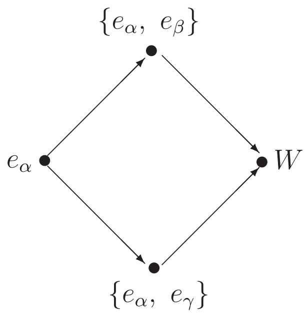

FIGURE 4. A quasitopknot of $\psi^{-1}(A)$.

Claim 2. The second claim is $|A| \le 2$.

Suppose, on the contrary, that $|A| \ge 3$. Then A contains at least three elements, say, e, f, g, such that $e < f < g$. Thus it follows by Lemma 4.4 that A has the topknot (given in Figure 3). Applying ψ^{-1} to Figure 3, we can get the quasitopknot of $\psi^{-1}(A) = e_\alpha$ (see Figure 4) by Proposition 3.7, Lemma 4.3 and Lemma 4.6, where $\psi(\{e_\alpha, e_\beta\}) = A \setminus \{e\}$, $\psi(\{e_\alpha, e_\gamma\}) = A \setminus \{f\}$, $\psi(W) = A \setminus \{e, f\}$ and $e_\alpha \prec e_\beta$, $e_\alpha \prec e_\gamma$.

Since $e_\alpha \prec e_\beta$ and $e_\alpha \prec e_\gamma$, we have that $e_\beta e_\gamma = e_\alpha$, and so

$$
\begin{aligned}
&\{e_\alpha,\ e_\beta\}\{e_\alpha,\ e_\beta,\ e_\gamma\} = \{e_\alpha,\ e_\beta\} \\
&\qquad \Rightarrow (A \setminus \{e\})\ \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e\} \\
&\qquad \Rightarrow (A \setminus \{e\}) \cdot \operatorname{supp} \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e\} \quad \text{(by Lemma 2.6)} \qquad (4.1) \\
&\qquad \Rightarrow A \setminus \{e\} \le \operatorname{supp} \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}). \qquad (4.2)
\end{aligned}
$$

Similarly, we can derive

$$A \setminus \{f\} \le \operatorname{supp} \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}). \tag{4.3}$$

On the other hand, it follows by Figure 4 that

$$\{e_\alpha,\ e_\beta\}W = \{e_\alpha,\ e_\beta\} \quad \text{and} \quad \{e_\alpha,\ e_\gamma\}W = \{e_\alpha,\ e_\gamma\}.$$

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

Thus

$$\begin{aligned}
\{e_\alpha,\ e_\beta,\ e_\gamma\}W &= \{e_\alpha,\ e_\beta,\ e_\gamma\} \\
&\Rightarrow (\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}))\,(A \setminus \{e,\ f\}) = \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) \\
&\Rightarrow [\operatorname{supp}\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\})](A \setminus \{e,\ f\}) = \operatorname{supp}\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) \\
&\qquad (\text{by Lemma 2.6}) \\
&\Rightarrow \operatorname{supp}\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) \leq A \setminus \{e, f\}. \qquad (4.4)
\end{aligned}$$

Summarizing the above, we have

$$\operatorname{supp}\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e, f\}, \qquad (4.5)$$

since $A \setminus \{e\} \twoheadrightarrow A \setminus \{e, f\}$, $A \setminus \{f\} \twoheadrightarrow A \setminus \{e, f\}$. In the following, we shall show that $\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e, f\}$. Consider the following two cases.

Case (i). If A has no the maximal element, then it follows by Claim 1 that $|G'_\delta| = 1$ for any $e'_\delta \in A$. This implies that

$$\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = \operatorname{supp}\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}),$$

and so $\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e, f\}$ by (4.5).

Case (ii). If A has the maximal element e'_ω, then it follows by Claim 1 that $|G'_\delta| = 1$ for any $e'_\delta \in A$ such that $e'_\delta \neq e'_\omega$. So, by (4.5),

$$\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = (A \setminus \{e, f, e'_\omega\}) \cup B_\omega,$$

where B_ω is a subset of G'_ω. Also, for any $b_\omega \in B_\omega$, by (4.1),

$$b_\omega = e'_\omega b_\omega \in (A \setminus \{e\}) \cdot B_\omega \subseteq (A \setminus \{e\}) \cdot \psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e\},$$

and so $b_\omega = e'_\omega$, that is to say, $\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e, f\}$.

Thus we have shown that in either case $\psi(\{e_\alpha,\ e_\beta,\ e_\gamma\}) = A \setminus \{e, f\}$, and so $\{e_\alpha,\ e_\beta,\ e_\gamma\} = \psi^{-1}(A \setminus \{e, f\}) = W$. However, $W = \psi^{-1}(A \setminus \{e, f\}) \in Ch(E)$ by Proposition 3.7, and $\{e_\alpha,\ e_\beta,\ e_\gamma\} \notin Ch(E)$, since $e_\beta e_\gamma = e_\alpha$, which is a contradiction. This shows that the claim is true.

Claim 3. The third claim is $|A| = 1$.

By Claim 2, we have $|A| \leq 2$. Suppose that $|A| = 2$. Then $A = \{e, f\}$ for some $e, f \in E'$ such that $e < f$. It follows immediately by Lemma 4.4 that

$$\begin{aligned}
A \twoheadrightarrow f &\Rightarrow e_\alpha \rightarrowtail \psi^{-1}(f)) \quad (\text{by Lemma 4.3}) \\
&\Rightarrow (\exists\, e_\mu \in E)(e_\alpha \prec e_\mu, \psi^{-1}(f) = \{e_\alpha,\ e_\mu\}) \quad (\text{by Lemma 4.6}) \\
&\Rightarrow \psi^{-1}(f) \twoheadrightarrow e_\mu \quad (\text{by Lemma 4.4}) \\
&\Rightarrow f \rightarrowtail \psi(e_\mu) \quad (\text{by Lemma 4.3}) \\
&\Rightarrow (\exists\, g \in E')(f \prec g,\ \psi(e_\mu) = \{f,\ g\}) \quad (\text{by Lemma 4.6}).
\end{aligned}$$

https://doi.org/10.1017/S1446788714000032 Published online by Cambridge University Press

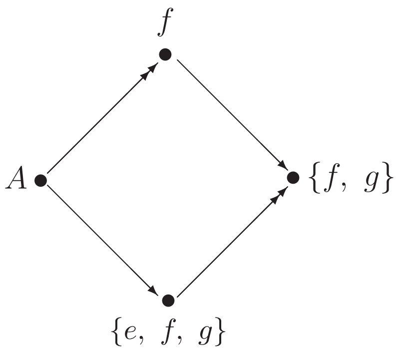

FIGURE 5. A quasitopknot of A.

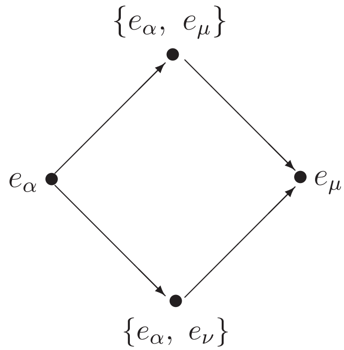

FIGURE 6. A quasitopknot of $\psi^{-1}(A)$.

Thus by Lemmas 4.3–4.6, we get a quasitopknot of A (see Figure 5). Applying ψ^{-1} to Figure 5, we can get a quasitopknot of $\psi^{-1}(A)$ (see Figure 6) by Proposition 3.7 and Lemma 4.6, where $\psi(\{e_\alpha,\ e_\nu\}) = \{e,\ f,\ g\}$, $\psi(\{e_\alpha,\ e_\mu\}) = f$, $\psi(e_\mu) = \{f,\ g\}$, and $f \prec g,\ e_\alpha \prec e_\mu, e_\alpha \prec e_\nu$.

It follows that $e_\mu e_\nu = e_\alpha$, since $e_\alpha \prec e_\mu$ and $e_\alpha \prec e_\nu$. Therefore, we have that $e_\mu\{e_\alpha, e_\nu\} = \{e_\alpha\}$, contradicting $\{e_\alpha, e_\nu\} \rightarrowtail e_\mu$. The proof is completed. □

Acknowledgements

The authors would like to thank Professor Marcel Jackson and the referee for their helping the preparation of the final version of this paper.

References

[1] J. Almeida, 'Power varieties of semigroups *I*', *Semigroup Forum* **33** (1986), 357–373.
[2] J. Almeida, 'Power varieties of semigroups *II*', *Semigroup Forum* **33** (1986), 375–390.
[3] J. Almeida, 'On power varieties of semigroups', *J. Algebra* **120** (1989), 1–17.
[4] J. Almeida, *Finite Semigroups and Universal Algebra*, Series in Algebra, 3 (World Scientific, Singapore, 1994), (translated from the 1992 Portuguese original and revised by the author).

[5] S. Burris and H. P. Sankappanavar, *A Course in Universal Algebra* (Springer, Berlin, 1981).
[6] S. Crvenković, I. Dolinka and M. Vinčić, 'Involution semigroups are not globally determined', *Semigroup Forum* **62** (2001), 477–481.
[7] Y. Y. Fu and X. Z. Zhao, 'The closed subsemigroups of a Clifford semigroup', *Commun. Math. Res.*, to appear.
[8] M. Gould and J. A. Iskra, 'Globally determined classes of semigroups', *Semigroup Forum* **28** (1984), 1–11.
[9] M. Gould and J. A. Iskra, 'Globals of completely regular periodic semigroups', *Semigroup Forum* **29** (1984), 365–374.
[10] M. Gould, J. A. Iskra and C. Tsinakis, 'Globally determined lattices and semilattices', *Algebra Universalis* **19** (1984), 137–141.
[11] J. M. Howie, *Fundamentals of Semigroup Theory* (Clarendon Press, Oxford, 1995).
[12] Y. Kobayashi, 'Semilattices are globally determined', *Semigroup Forum* **29** (1984), 217–222.
[13] D. J. McCarthy and D. L. Hayes, 'Subgroups of the power semigroup of a group', *J. Combin. Theory* Ser A **14** (1973), 173–186.
[14] E. M. Mogiljanskaja, 'Non-isomorphic semigroups with isomorphic semigroups of subsets', *Semigroup Forum* **6** (1973), 330–333.
[15] J. E. Pin, 'Power semigroups and related varieties of finite semigroups', in: *Semigroups and Their Applications* (eds. S. M. Goberstein and P. M. Higgins) (Reidel Publishing Company, Dordrecht, 1987), 139–152.
[16] M. S. Putcha, 'Subgroups of the power semigroup of a finite semigroup', *Canad. J. Math.* **21** (1979), 1077–1083.
[17] M. Sasaki, 'Isomorphism problem of power semigroups of integer semigroup', Annual Report Faculty of Education, Iwate University, Vol. 48(1) (1988), 123–126.
[18] T. Tamura, 'Unsolved problems on semigroups', *Sem. Reports of Math. Sci.* (1967), 33–35.
[19] T. Tamura, 'Power semigroups of rectangular groups', *Math. Japon.* **4** (1984), 671–678.
[20] T. Tamura, 'Isomorphism problem of power semigroups of completely 0-simple semigroups', *J. Algebra* **98** (1986), 319–361.
[21] T. Tamura, 'On the recent results in the study of power semigroups', in: *Semigroups and Their Applications* (eds. S. M. Goberstein and P. M. Higgins) (Reidel Publishing Company, Dordrecht, 1987), 191–200.
[22] T. Tamura and J. Shafer, 'Power semigroups', *Math. Japon.* **12** (1967), 25–32.
[23] M. Vinčić, 'Global determinism of $*$-bands', *Filomat* **33** (2001), 91–97.
[24] X. Z. Zhao, 'Idempotent semirings with a commutative additive reduct', *Semigroup Forum* **64** (2002), 289–296.

AIPING GAN, Department of Mathematics,
Northwest University, Xi'an, Shaanxi,
710127, China
and
College of Mathematics and Information Science,
Jiangxi Normal University,
Nanchang, Jiangxi, 330022, China
e-mail: ganaiping78@163.com

XIANZHONG ZHAO, College of Mathematics and
Information Science, Jiangxi Normal University,
Nanchang, Jiangxi, 330022, China
e-mail: xianzhongzhao@263.net