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GLOBAL EXISTENCE FOR THE THREE-DIMENSIONAL THERMOELASTIC EQUATIONS OF TYPE II

By

YUMING QIN (*Department of Applied Mathematics, Donghua University, Shanghai 201620, People's Republic of China*),

SHUXIAN DENG (*College of Information Science and Technology, Donghua University, Shanghai 201620, People's Republic of China*),

LAN HUANG (*College of Information Science and Technology, Donghua University, Shanghai 201620, People's Republic of China*),

ZHIYONG MA (*College of Information Science and Technology, Donghua University, Shanghai 201620, People's Republic of China*),

AND

XIAOKE SU (*College of Information Science and Technology, Donghua University, Shanghai 201620, People's Republic of China*)

Abstract. In this paper, we shall establish some global existence results for a 3D hyperbolic system arising from Green-Naghdi models of thermoelasticity of type II with a dissipative boundary condition for the displacement. The existence and exponential decay of energy for the linear problem has been solved by Lazzari and Nibbi, Journal of Mathematical Analysis and Applications, 338 (2008), 317–329. Furthermore, we shall establish the global existence of solutions to semilinear and nonlinear thermoelastic systems by using the semigroup approach.

1. Introduction. We consider thermoelastic models based on the theory developed by Green and Naghdi [9]–[11]. Instead of the classical entropy inequality, they used a general entropy balance and, upon introducing a new thermal variable, proposed three models, based on the different material responses, labeled as types I, II and III. The linearized version of the first model leads to the Fourier law, and hence develops the

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E-mail address: yuming_qin@hotmail.com
E-mail address: dshuxian@163.com
E-mail address: huanglan82@hotmail.com
E-mail address: mazhiyong1980@hotmail.com
E-mail address: suxiaoke07@126.com

classical thermoelastic theory; the linearized versions of both type-II and type-III models allow heat transmission at finite speed.

In this paper, we shall study the global existence of solutions to the following thermoelastic model of type II:

$$\rho u_{tt} = \nabla \cdot [C\nabla u(x,t) - \alpha I\theta(x,t)] + f, \qquad (x,t) \in \Omega \times (0,\infty), \tag{1.1}$$

$$c\theta_t = -\nabla \cdot [q(x,t) + \beta u_t(x,t)] + r, \qquad (x,t) \in \Omega \times (0,\infty). \tag{1.2}$$

From the Green-Naghdi law, we have

$$q(x,t) = -k\nabla\tau(x,t), \qquad (x,t) \in \Omega \times (0,\infty) \tag{1.3}$$

where q is the heat flux and τ is a new variable, called the thermal displacement, which satisfies $\tau_t = \theta$.

The body Ω is a bounded open set in $\mathbb{R}^3$ with regular boundary $\partial\Omega$; $u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ represents the displacement vector, and θ denotes the temperature relative to Θ_0, i.e., $\theta = \Theta - \Theta_0$, where Θ represents the absolute temperature. The terms f and r represent external forces, ρ represents the mass density and is a positive constant, and C is a constant, fourth-order, symmetric tensor which is positive definite, i.e., there exist two positive constants k_1 and k_2 such that for all symmetric second-order tensors B,

$$k_1|B|^2 \le CB \cdot B \le k_2|B|^2; \tag{1.4}$$

c and k, as well as $\frac{\alpha}{\beta}$, are positive constants.

Assume that the system (1.1)-(1.3) is subject to the following dissipative boundary condition with memory:

$$T(x,t)n(x,t) = -\gamma_0 v(x,t) - \int_0^\infty \lambda(s)v^t(x,s), \quad x \in \partial\Omega, \tag{1.5}$$

and the Neumann boundary condition for the heat flux, that is,

$$q(x,t) \cdot n(x) = 0, \quad x \in \partial\Omega \tag{1.6}$$

where n is the unit outward normal vector, $v := u_t$ the velocity, $v^t(x,s) := v(x,t-s)$ the history of v, and T the stress tensor, which obeys the constitutive equation

$$T = CE - \alpha I\theta \tag{1.7}$$

where $E = \frac{1}{2}(\nabla u + \nabla u^T)$ is the strain tensor and I is the identity tensor.

For the boundary condition (1.5) with memory terms, several authors have studied the dynamical problem in elasticity (see, e.g., [2, 3, 17]), in electromagnetism [20] and, in thermoelasticity, for the Cattaneo-Maxwell and Gurtin-Pipkin models [8, 14] and the Green-Naghdi model of type II [15]. In this paper, the boundary condition (1.5) guarantees the decay of total energy (mechanical and thermal), because there is no internal dissipation for Green-Naghdi models of type II.

We set the initial conditions to be

$$\begin{cases} u(x,0) = u_0(x), \\ v(x,0) = v_0(x), \\ \tau(x,0) = \tau_0(x), \\ \theta(x,0) = \theta_0(x). \end{cases} \tag{1.8}$$

For the memory kernel $\lambda : \mathbb{R}^+ \to \mathbb{R} \in L^1(\mathbb{R}^+) \cap H^2(\mathbb{R}^+)$, by assuming that $\partial\Omega$ is locally strong dissipative [2], we have

$$\gamma_0 \in \mathbb{R}^+, \ \ \omega \int_0^\infty \lambda'(s) \sin(\omega s)\, ds < 0, \quad \forall \omega \neq 0;$$

furthermore, we assume that

$$\lambda'(s) < 0, \quad \lambda''(s) \geq 0, \quad \forall s \in \mathbb{R}^+. \tag{1.9}$$

Models of boundary conditions that include a memory term which produces damping were proposed in [1] for the study of 1D wave propagation, in [22] for sound evolution in a compressible fluid and in [7] in the context of Maxwell equations.

Let's first recall some previous works in this direction. For thermoelasticity of type I, there are many works (see, e.g., [6], [12]–[13], [16, 18, 29]) on the existence, uniqueness and asymptotic behavior of solutions of the linear system. Racke, Shibata and Zheng [29] obtained the global existence and uniqueness of solutions for the nonlinear thermoelastic system of type I with small initial data; Muñoz Rivera and Qin [18] proved the global existence, uniqueness, and asymptotic behavior of solutions for 1D nonlinear thermoelasticity with thermal memory subject to Dirichlet-Dirichlet boundary conditions.

For the thermoelasticitic model of type II, or without energy dissipation, several results on existence, uniqueness, continuous dependence, spatial decay and wave propagation (see, e.g., [4]–[5], [10, 15, 19], [24]–[27]) have been obtained, among which we would like to mention especially the work by Qin and Muñoz Rivera [24], who studied the global existence and exponential stability of solutions to homogeneous thermoelastic equations of type II with thermal memory. Recently, Qin, Xu and Ma [25] obtained the global existence and exponential stability of solutions to nonhomogeneous thermoelastic equations of type II with thermal memory. Lazzari and Nibbi [15] obtained the exponential decay of total energy for thermoelastic linear inhomogeneous systems of type II (i.e., $f = f(x,t), r = r(x,t)$) with the dissipative boundary condition (1.5). We will in this paper study the global existence of solutions for the semilinear and nonlinear thermoelastic systems of type II (i.e., $f = f(v, \nabla u, \theta, \nabla\tau, a^t)$ and $r = r(v, \nabla u, \theta, \nabla\tau, a^t)$, respectively). To our knowledge, we are the first to use the semigroup approach to study such a problem.

For the thermoelasticitic model of type III, which represents thermal dissipation, there are some interesting results (see, e.g., [15, 23, 28], [30]–[32]); for example, for the Cauchy problem of the linear thermoelastic system of type III, Zhang and Zuazua [32] and Quintanilla and Racke [28] independently studied the decay of energy by using the classical energy method and the spectral method, and they obtained exponential stability in one space dimension, and in two or three space dimensions for radially symmetric situations, while the energy was found to decay polynomially for most domains in two space dimensions. Reissig and Wang [30] studied L^p–L^q decay estimates and propagation of singularities of solutions in one space dimension, and later Yang and Wang [31] studied well-posedness and decay estimates in three space dimensions. Lazzari and Nibbi [15] also studied the asymptotic behavior of the solution of a 3D thermoelastic system of type III with an absorbing boundary. In particular, Qin, Ma and Huang [23] obtained the global existence of solutions for a higher-dimensional linear and nonlinear thermoelastic equations of type III by using the semigroup method.

The notation in this paper will be as follows: L^p $(1 \le p \le +\infty)$, $W^{m,p}$ $(m \in \mathbb{N})$, $H^1 = W^{1,2}$ and $H_0^1 = W_0^{1,2}$ will denote the usual (Sobolev) spaces on Ω. In addition, $\|\cdot\|_B$ denotes the norm in the space B; we also put $\|\cdot\| = \|\cdot\|_{L^2(\Omega)}$. We denote by $C^k(J,B), k \in \mathbb{N}_0$, the space of k-times continuously differentiable functions from $J \subseteq \mathbb{R}$ into a Banach space B, and likewise by $L^p(J,B), 1 \le p \le +\infty$, the corresponding Lebesgue spaces. $C^\beta([0,T],B)$ denotes the Hölder space of B-valued continuous functions with exponent $\beta \in (0,1)$ in the variable t.

From now on we shall drop the x variable whenever no ambiguity arises. In what follows, we shall refer to the problem consisting of (1.1)-(1.3), (1.5)-(1.6) and (1.8) as problem P.

The rest of this paper is organized as follows. In Section 2, we state the main theorems of this paper. In Section 3, we will prove the main theorems via a series of lemmas.

2. Main results.

Using integration by parts for (1.5), we obtain

$$T(x,t)n(x,t) = -\gamma_0 v(x,t) - \int_0^\infty \lambda'(s) w^t(x,s)\, ds, \quad x \in \partial\Omega \tag{2.1}$$

where $w^t(x,s) = u^t(x,s) - u(x,t)$ denotes the past history of u and is defined for $s \in \mathbb{R}^+$.

In order to simplify the notation, we introduce the new variable

$$a^t(s) = -\int_0^\infty \lambda'(\tau + s) w^t(\tau)\, d\tau,$$

so that the boundary condition (1.5) or (2.1) takes the form

$$\check{T}(t)n = T(t)n + \gamma_0 v(t) = a^t(0), \tag{2.2}$$

and introduce the boundary energy function

$$\psi_{\partial\Omega}(t) = -\frac{1}{2}\int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial a^t(s)}{\partial s}\, ds da, \tag{2.3}$$

which satisfies

$$\begin{aligned} \frac{d}{dt}\psi_{\partial\Omega}(t) &= \frac{1}{2}\int_{\partial\Omega} \frac{1}{\lambda'(0)} \frac{\partial a^t(0)}{\partial s} \cdot \frac{\partial a^t(0)}{\partial s}\, da \\ &\quad - \int_{\partial\Omega} \check{T}(t)n \cdot v(t)\, da - \frac{1}{2}\int_{\partial\Omega}\int_0^\infty \frac{\lambda''(s)}{(\lambda'(s))^2} \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial a^t(s)}{\partial s}\, ds da. \end{aligned} \tag{2.4}$$

Thus the energy of a solution to problem P is defined by

$$\begin{aligned} \psi &= \psi_\Omega + \psi_{\partial\Omega} \\ &= \frac{1}{2}\int_\Omega \left[\rho|v|^2 + C\nabla u \cdot \nabla u + \frac{c\alpha}{\beta}\theta^2 + \frac{k\alpha}{\beta}|\nabla\tau|^2\right] dx + \psi_{\partial\Omega}. \end{aligned} \tag{2.5}$$

We further introduce some function spaces. Set

$$H^*(\Omega) = \{\Phi \in L^2(\Omega) :\ \nabla \cdot \Phi \in L^2(\Omega)\}. \tag{2.6}$$

Let the space $\dot{H}^1(\partial\Omega) := \dot{H}^1\left(\partial\Omega \times (0,\infty), \frac{1}{\sqrt{-\lambda'(s)}}\, dsda\right)$ consist of functions $a^t(s)$ on $(0,\infty)$ for which

$$\|a^t\|^2_{\dot{H}^1} = -\int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial a^t(s)}{\partial s}\, dsda. \tag{2.7}$$

Put

$$\mathcal{H} = (L^2(\Omega))^3 \times (L^2(\Omega))^3 \times L^2(\Omega) \times L^2(\Omega) \times \dot{H}^1(\partial\Omega) \tag{2.8}$$

with the energy norm

$$\begin{aligned} \|(v, \nabla u, \theta, \nabla\tau, a^t)\|^2_{\mathcal{H}} &= \int_\Omega \left[\rho|v|^2 + C\nabla u \cdot \nabla u + \frac{c\alpha}{\beta}\theta^2 + \frac{k\alpha}{\beta}|\nabla\tau|^2\right] dx \\ &\quad - \int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial a^t(s)}{\partial s}\, dsda. \end{aligned} \tag{2.9}$$

In order to use the theory of semigroups, we set

$$v(x,t) = u_t(x,t), \quad \tau_t(x,t) = \theta(x,t), \quad a^t(s) = -\int_0^\infty \lambda'(\tau+s)w^t(\tau)\, d\tau.$$

By a straightforward calculation, we obtain

$$\frac{\partial a^t(s)}{\partial s} = \frac{\partial a^t(s)}{\partial t} + \lambda(s)v(x,t). \tag{2.10}$$

Thus we can write problem P as follows:

$$\begin{cases} \rho v_t = \nabla\cdot[C\nabla u(x,t) - \alpha I\theta(x,t)] + f, & (x,t) \in \Omega\times(0,\infty), \\ c\theta_t = -\nabla\cdot[q(x,t) + \beta v(x,t)] + r, & (x,t) \in \Omega\times(0,\infty), \\ q(x,t) = -k\nabla\tau(x,t), & (x,t) \in \Omega\times(0,\infty), \\ T(x,t)n(x,t) = -\gamma_0 v(x,t) - \int_0^\infty \lambda'(s)w^t(x,s)\, ds, & x \in \partial\Omega, \\ q(x,t)\cdot n(x) = 0, & x \in \partial\Omega, \\ w^t(x,s) = u^t(x,s) - u(x,t), & (x,t) \in \Omega\times(0,\infty), \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x \in \Omega, \\ \tau(x,0) = \tau_0(x), \quad \theta(x,0) = \theta_0(x), & x \in \Omega. \end{cases} \tag{2.11}$$

Now we define a linear unbounded operator A on $\mathcal{H}$ by

$$\begin{aligned} &A(v, \nabla u, \theta, \nabla\tau, a^t) \\ &= \left(\frac{1}{\rho}\nabla\cdot(C\nabla u - \alpha I\theta), \nabla v, -\frac{1}{c}\nabla\cdot(\beta - k\nabla\tau), \nabla\theta, \frac{\partial a^t(s)}{\partial s} - \lambda(s)v\right). \end{aligned} \tag{2.12}$$

Put

$$\Phi = (v, \nabla u, \theta, \nabla\tau, a^t), \quad K = (F, 0, G, 0, 0) \tag{2.13}$$

where $F = \frac{1}{\rho}f,\ G = \frac{1}{c}r$. Then system (2.11) can be formulated as an abstract first-order Cauchy problem as follows:

$$\begin{cases} \dfrac{d\Phi}{dt} = A\Phi + K, \\ \\ \Phi(0) = \Phi_0 \end{cases} \tag{2.14}$$

on the Hilbert space $\mathcal{H}$, where $\Phi_0 = (v_0, \nabla u_0, \theta_0, \nabla\tau_0, a^0)$. The domain of A is given by

$$\begin{aligned} D(A) \quad = \quad & \Big\{(v, \nabla u, \theta, \nabla\tau, a^t) \in \mathcal{H} \ : v \in (H^1(\Omega))^3;\ \ \nabla u \in (H^*(\Omega))^3;\ \ \theta \in H^1(\Omega); \\ & \nabla\tau \in H^*(\Omega);\ \ \frac{\partial a^t(s)}{\partial s} - \lambda(s)v \in \dot{H}^1(\partial\Omega);\quad q(x,t)\cdot n(x) = 0,\ \ x \in \partial\Omega; \\ & T(x,t)n(x,t) = -\gamma_0 v(x,t) - \int_0^\infty \lambda'(s) w^t(x,s)\, ds,\ \ x \in \partial\Omega\Big\}. \end{aligned} \tag{2.15}$$

Note that $H^*(\Omega)$ is given by (2.6). It is clear that $D(A)$ is dense in $\mathcal{H}$.

We are now in a position to state our main theorems.

THEOREM 2.1. Suppose that $F = F(\Phi)$ and $G = G(\Phi)$, where $\Phi = (v, \nabla u, \theta, \nabla\tau, a^t)$, and that $K = (F, 0, G, 0, 0)$ satisfies the global Lipschitz condition on $\mathcal{H}$, i.e., there is a positive constant L such that for all $\Phi_1, \Phi_2 \in \mathcal{H}$,

$$\|K(\Phi_1) - K(\Phi_2)\|_{\mathcal{H}} \le L\|\Phi_1 - \Phi_2\|_{\mathcal{H}}. \tag{2.16}$$

Then for any $\Phi_0 = (v_0, \nabla u_0, \theta_0, \nabla\tau_0, a^0) \in \mathcal{H}$, there exists a global mild solution Φ to system (2.11) such that $\Phi \in C([0,\infty), \mathcal{H})$, i.e.,

$$\begin{aligned} & v(t) \in C([0,\infty), (L^2(\Omega))^3); \quad \nabla u(t) \in C([0,\infty), (L^2(\Omega))^3); \\ & \theta(t) \in C([0,\infty), L^2(\Omega)); \qquad \nabla\tau(t) \in C([0,\infty), L^2(\Omega)); \\ & a^t(t) \in C([0,\infty), \dot{H}^1(\partial\Omega)). \end{aligned}$$

THEOREM 2.2. Suppose that $F = F(\Phi)$ and $G = G(\Phi)$, where $\Phi = (v, \nabla u, \theta, \nabla\tau, a^t)$, and that $K = (F, 0, G, 0, 0)$ is a nonlinear operator from $D(A)$ into $D(A)$ which satisfies the global Lipschitz condition on $D(A)$, i.e., there is a positive constant L such that for all $\Phi_1,\ \Phi_2 \in D(A)$,

$$\|K(\Phi_1) - K(\Phi_2)\|_{D(A)} \le L\|\Phi_1 - \Phi_2\|_{D(A)}. \tag{2.17}$$

Then for any $\Phi_0 = (v_0, \nabla u_0, \theta_0, \nabla\tau_0, a^0) \in D(A)$, there exists a unique global classical solution $\Phi = (v, \nabla u, \theta, \nabla\tau, a^t) \in C^1([0,\infty),\ \mathcal{H}) \cap C([0,\infty), D(A))$ to system (2.11), i.e.,

$$\begin{aligned} & v \in C^1([0,\infty), (L^2(\Omega))^3) \cap C([0,\infty), (H^1(\Omega))^3), \\ & \nabla u \in C^1([0,\infty), (L^2(\Omega))^3) \cap C([0,\infty), (H^*(\Omega))^3), \\ & \theta \in C^1([0,\infty), L^2(\Omega)) \cap C([0,\infty), H^1(\Omega)), \\ & \nabla\tau \in C^1([0,\infty), L^2(\Omega)) \cap C([0,\infty), H^*(\Omega)), \\ & a^t \in C^1([0,\infty), \dot{H}^1(\partial\Omega)) \cap C([0,\infty), \dot{H}^2(\partial\Omega)), \end{aligned}$$

where $\dot{H}^2(\partial\Omega) = \dot{H}^2\left(\partial\Omega \times (0,\infty), \frac{1}{\sqrt{-\lambda'(s)}}\, dsda\right)$ with the norm

$$\|a^t\|^2_{\dot{H}^2} = -\int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial^2 a^t(s)}{\partial s^2} \cdot \frac{\partial^2 a^t(s)}{\partial s^2}\, dsda.$$

3. Proofs of Theorems 2.1 and 2.2. In this section we will complete the proofs of Theorems 2.1 and 2.2 by establishing a series of lemmas.

LEMMA 3.1. The operator A defined by (2.12) is dissipative and closed.

Proof. By a straightforward calculation, it follows from (2.9) and (2.2) that for any $(v, \nabla u, \theta, \nabla\tau, a^t) \in D(A)$,

$$\begin{aligned}
&(A(v, \nabla u, \theta, \nabla\tau, a^t),\ (v, \nabla u, \theta, \nabla\tau, a^t)) \\
&= \int_\Omega \Big[\nabla\cdot(C\nabla u(t) - \alpha I\theta(t))\cdot v(t) + C\nabla v(t)\cdot\nabla u(t)\Big]\,dx \\
&\quad + \frac{\alpha}{\beta}\int_\Omega \Big[\nabla\cdot(k\nabla\tau(t) - \beta v(t))\,\theta(t) + k\nabla\theta(t)\cdot\nabla\tau(t)\Big]\,dx \\
&\quad - \int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)}\frac{\partial}{\partial s}\left[\frac{\partial a^t(s)}{\partial s} - \lambda(s)v(t)\right]\cdot\frac{\partial a^t(s)}{\partial s}\,dsda \\
&= \int_{\partial\Omega}(C\nabla u - \alpha I\theta)\,v\cdot n\,da - \frac{\alpha}{\beta}\int_{\partial\Omega}(\beta v - k\nabla\tau)\theta\cdot n\,da \\
&\quad + \alpha\int_\Omega I\theta\cdot\nabla v\,dx + \frac{\alpha}{\beta}\int_\Omega(\beta v - k\nabla\tau)\cdot\nabla\theta\,dx + \frac{\alpha}{\beta}\int_\Omega k\nabla\theta\cdot\nabla\tau\,dx \\
&\quad - \int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)}\frac{\partial^2 a^t(s)}{\partial s^2}\cdot\frac{\partial a^t(s)}{\partial s}\,dsda + \int_{\partial\Omega}\int_0^\infty v(t)\cdot\frac{\partial a^t(s)}{\partial s}\,dsda \\
&= \int_{\partial\Omega} T(t)n\cdot v(t)\,da - \int_{\partial\Omega} v(t)\cdot a^t(0)\,da - \int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)}\frac{\partial^2 a^t(s)}{\partial s^2}\cdot\frac{\partial a^t(s)}{\partial s}\,dsda \\
&= -\int_{\partial\Omega}\gamma_0|v(t)|^2\,da + \frac{1}{2}\int_{\partial\Omega}\frac{1}{\lambda'(0)}\frac{\partial a^t(0)}{\partial s}\cdot\frac{\partial a^t(0)}{\partial s}\,da \\
&\quad - \frac{1}{2}\int_{\partial\Omega}\int_0^\infty \frac{\lambda''(s)}{(\lambda'(s))^2}\frac{\partial a^t(s)}{\partial s}\cdot\frac{\partial a^t(s)}{\partial s}\,dsda \le 0.
\end{aligned} \tag{3.1}$$

Thus, A is dissipative.
To prove that A is closed, let $(v_n, \nabla u_n, \theta_n, \nabla\tau_n, a_n^t) \in D(A)$ be such that

$$(v_n, \nabla u_n, \theta_n, \nabla\tau_n, a_n^t) \to (v, \nabla u, \theta, \nabla\tau, a^t) \quad \text{in} \quad \mathcal{H}$$

and

$$A(v_n, \nabla u_n, \theta_n, \nabla\tau_n, a_n^t) \to (\varphi, z, \xi, \eta, \zeta) \quad \text{in} \quad \mathcal{H}.$$

Then we have

$$v_n \to v \qquad \text{in} \quad (L^2(\Omega))^3, \tag{3.2}$$
$$\nabla u_n \to \nabla u \qquad \text{in} \quad (L^2(\Omega))^3, \tag{3.3}$$
$$\theta_n \to \theta \qquad \text{in} \quad L^2(\Omega), \tag{3.4}$$
$$\nabla\tau_n \to \nabla\tau \qquad \text{in} \quad L^2(\Omega), \tag{3.5}$$
$$a_n^t \to a^t \qquad \text{in} \quad \dot{H}^1(\partial\Omega) \tag{3.6}$$

and

$$\frac{1}{\rho}\nabla\cdot(C\nabla u_n-\alpha I\theta_n)\to\varphi \qquad \text{in} \quad (L^2(\Omega))^3, \tag{3.7}$$

$$\nabla v_n\to z \qquad \text{in} \quad (L^2(\Omega))^3, \tag{3.8}$$

$$-\frac{1}{c}\nabla\cdot(\beta v_n-k\nabla\tau_n)\to\xi \qquad \text{in} \quad L^2(\Omega), \tag{3.9}$$

$$\nabla\theta_n\to\eta \qquad \text{in} \quad L^2(\Omega), \tag{3.10}$$

$$\frac{\partial a_n^t}{\partial s}-\lambda(s)v_n\to\zeta \qquad \text{in} \quad \dot{H}^1(\partial\Omega). \tag{3.11}$$

From (3.2) and (3.8), we deduce that

$$v_n\to v \quad \text{in} \quad (H^1(\Omega))^3 \tag{3.12}$$

and

$$z=\nabla v, \quad v\in(H^1(\Omega))^3. \tag{3.13}$$

Similarly, using (3.4) and (3.10), we deduce that

$$\theta_n\to\theta \qquad \text{in} \quad H^1(\Omega) \tag{3.14}$$

and

$$\eta=\nabla\theta, \quad \theta\in H^1(\Omega). \tag{3.15}$$

From (3.7) and (3.14), we deduce that

$$\nabla\cdot(C\nabla u_n)\to\rho\varphi+\alpha\nabla\cdot I\theta \quad \text{in} \quad (L^2(\Omega))^3, \tag{3.16}$$

and consequently, it follows from (3.3) that

$$\nabla u_n\to\nabla u \quad \text{in} \quad (H^*(\Omega))^3 \tag{3.17}$$

and

$$\varphi=\frac{1}{\rho}\nabla\cdot(C\nabla u-\alpha I\theta), \quad \nabla u\in(H^*(\Omega))^3. \tag{3.18}$$

From (3.9) and (3.12), we deduce that

$$\frac{1}{c}\nabla\cdot(k\nabla\tau_n)\to\xi+\frac{\beta}{c}\nabla\cdot v \qquad \text{in} \quad L^2(\Omega), \tag{3.19}$$

and consequently, it follows from (3.5) that

$$\nabla\tau_n\to\nabla\tau \quad \text{in} \quad H^*(\Omega) \tag{3.20}$$

and

$$\xi=-\frac{1}{c}\nabla\cdot(C\nabla u-\alpha I\theta), \quad \nabla\tau\in H^*(\Omega). \tag{3.21}$$

In addition, it follows from (3.6), (3.11) and (3.12) that

$$\frac{\partial a_n^t}{\partial s}\to\frac{\partial a^t}{\partial s} \quad \text{in} \quad \dot{H}^1(\partial\Omega) \tag{3.22}$$

and

$$\zeta=\frac{\partial a^t}{\partial s}-\lambda(s)v\in\dot{H}^1(\partial\Omega). \tag{3.23}$$

Moreover, it is easy to deduce that

$$T(x,t)n(x,t) = -\gamma_0 v(x,t) - \int_0^\infty \lambda'(s) w^t(x,s), \quad x \in \partial\Omega,$$
$$q(x,t) \cdot n(x) = 0, \qquad x \in \partial\Omega.$$

Thus, using (3.13), (3.15), (3.18), (3.21) and (3.23), we deduce that

$$A(v, \nabla u, \theta, \nabla\tau, a^t) = (\varphi, z, \xi, \eta, \zeta), \quad (v, \nabla u, \theta, \nabla\tau, a^t) \in D(A).$$

Hence, A is closed. □

LEMMA 3.2. The adjoint operator A^* of A is also dissipative.

Proof. Let $\tilde{\Phi} = (\tilde{v}, \nabla\tilde{u}, \tilde{\theta}, \nabla\tilde{\tau}, \tilde{a}^t)$ be in $\mathcal{H}$ and consider the boundary conditions

$$(C\nabla\tilde{u}(t) - \alpha I\tilde{\theta}(t))n = \gamma_0\tilde{u}(t) + \tilde{a}^t(0), \quad \nabla\tilde{\tau}(t) \cdot n = 0, \quad x \in \partial\Omega. \tag{3.24}$$

Denoting by H the Heaviside function and introducing a function $j(\tilde{a}^t)$ such that

$$\frac{\partial}{\partial s} j(\tilde{a}^t)(s) = -\lambda'(s)\frac{\partial}{\partial s}\left(\frac{H(s)}{\lambda'(s)}\right)\frac{\partial\tilde{a}^t}{\partial s},$$

we claim that $A^*\tilde{\Phi}$ is equal to

$$\left(\frac{1}{\rho}\nabla\cdot(\alpha I\tilde{\theta} - C\nabla\tilde{u}), -\nabla\tilde{v}, \frac{1}{c}\nabla\cdot(\beta\tilde{v} - k\nabla\tilde{\tau}), -\nabla\tilde{\theta}, -\frac{\partial\tilde{a}^t(s)}{\partial s} + \lambda(s)\tilde{v} + j(\tilde{a}^t)(s)\right)$$

and that the domain of A^* is

$$\begin{aligned} D(A^*) \;=\; \Big\{ &(\tilde{v}, \nabla\tilde{u}, \tilde{\theta}, \nabla\tilde{\tau}, \tilde{a}^t) \in \mathcal{H} : \tilde{v} \in (H^1(\Omega))^3; \quad \nabla\tilde{u} \in (H^*(\Omega))^3; \quad \tilde{\theta} \in H^1(\Omega); \\ &\nabla\tilde{\tau} \in H^*(\Omega); \quad -\frac{\partial\tilde{a}^t(s)}{\partial s} + \lambda(s)\tilde{v} + j(\tilde{a}^t)(s) \in \dot{H}^1(\Omega); \quad \nabla\tilde{\tau}(t)\cdot n = 0, \\ &x \in \partial\Omega; (C\nabla\tilde{u}(t) - \alpha I\tilde{\theta}(t))n = \gamma_0\tilde{u}(t) + \tilde{a}^t(0), \quad x \in \partial\Omega \Big\}. \end{aligned} \tag{3.25}$$

By a straightforward calculation, we can obtain that for any $\Phi \in D(A)$ and $\tilde{\Phi} \in D(A^*)$,

$$
\begin{aligned}
(A\Phi, \tilde{\Phi}) &= \int_\Omega \Big[\nabla \cdot (C\nabla u(t) - \alpha I\theta(t)) \cdot \tilde{v}(t) + C\nabla v \cdot \nabla \tilde{u}(t)\Big]\, dx \\
&\quad -\frac{\alpha}{\beta} \int_\Omega \Big[\nabla \cdot (\beta v(t) - k\nabla \tau(t))\, \tilde{\theta}(t) + k\nabla\theta(t) \cdot \nabla\tilde{\tau}(t)\Big]\, dx \\
&\quad -\int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial}{\partial s}\left(\frac{\partial a^t(s)}{\partial s} - \lambda(s)v(t)\right) \cdot \frac{\partial \tilde{a}^t(s)}{\partial s}\, ds da \\
&= -\int_\Omega \Big[v(t) \cdot \nabla \cdot \Big(C\nabla\tilde{u}(t) - \alpha I\tilde{\theta}(t)\Big) + C\nabla u(t) \cdot \nabla \tilde{v}(t)\Big]\, dx \\
&\quad +\frac{\alpha}{\beta} \int_\Omega \Big[\nabla \cdot (\beta \tilde{v}(t) - k\nabla \tilde{\tau}(t))\, \theta(t) + k\nabla\tilde{\theta}(t) \cdot \nabla\tau(t)\Big]\, dx \\
&\quad +\int_{\partial\Omega} v(t) \cdot \Big[\Big(C\nabla\tilde{u}(t) - \alpha I\tilde{\theta}(t)\Big) n - \gamma_0 \tilde{v}(t) - \tilde{a}^t(0)\Big]\, da \\
&\quad +\int_{\partial\Omega}\int_0^\infty \frac{1}{\lambda'(s)} \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial}{\partial s}\left(\frac{\partial \tilde{a}^t(s)}{\partial s} - \lambda(s)\tilde{v}(t)\right) ds da \\
&\quad -\frac{k\alpha}{\beta} \int_{\partial\Omega} \theta(t) \nabla\tilde{\tau}(t) \cdot n\, da + \int_{\partial\Omega} \frac{1}{\lambda'(0)} \frac{\partial a^t(0)}{\partial s} \cdot \frac{\partial \tilde{a}^t(0)}{\partial s}\, da \\
&\quad +\int_{\partial\Omega}\int_0^\infty \frac{\partial}{\partial s}\left(\frac{1}{\lambda'(s)}\right) \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial \tilde{a}^t(s)}{\partial s}\, ds da.
\end{aligned}
$$

Note that $\tilde{\Phi} \in D(A^*)$, so $\tilde{\Phi} \in D(A^*)$ satisfies the boundary conditions (3.24), and we have

$$
(A\Phi, \tilde{\Phi}) = (\Phi, -A\tilde{\Phi}) + \int_{\partial\Omega}\int_{-\infty}^\infty \frac{\partial}{\partial s}\left(\frac{H(s)}{\lambda'(s)}\right) \frac{\partial a^t(s)}{\partial s} \cdot \frac{\partial \tilde{a}^t(s)}{\partial s}\, ds da.
$$

Thus

$$
(A^*\tilde{\Phi}, \tilde{\Phi}) = -\int_{\partial\Omega} \gamma_0 |\tilde{v}(t)|^2\, da - \frac{1}{2}\int_{\partial\Omega}\int_0^\infty \frac{\lambda''(s)}{(\lambda'(s))^2} \frac{\partial \tilde{a}^t(s)}{\partial s} \cdot \frac{\partial \tilde{a}^t(s)}{\partial s}\, ds da \le 0.
$$

Hence, A^* is also dissipative. □

LEMMA 3.3. Let A be a densely defined linear operator on a Hilbert space $\mathcal{H}$; if A and A^* (the adjoint of A) are dissipative, then A generates a C_0-semigroup of contractions on $\mathcal{H}$.

Proof. By virtue of the Lumer-Phillips theorem (see, e.g., Pazy [21]), we need to prove that

$$
R(I - A) = \mathcal{H}, \tag{3.26}
$$

i.e., that $I - A$ is surjective. If A is dissipative and closed, then $R(I - A) \subseteq \mathcal{H}$. Suppose that $R(I-A) \neq \mathcal{H}$; then there is a nontrivial element $x^* \in \mathcal{H}^*$ such that for all $x \in D(A)$,

$$
(x^*, x - Ax) = 0. \tag{3.27}
$$

Thus

$$
x^* - A^*x^* = 0. \tag{3.28}
$$

Since A^* is dissipative, we know that $x^* = 0$, a contradiction. Hence the proof is complete. □

From Lemmas 3.1-3.3, we know that the operator A defined by (2.12) generates a C_0-semigroup of contractions on $\mathcal{H}$. In the following, for the sake of convenience, we introduce the definition of a maximal accretive operator (see, e.g., Zheng [33] and Pazy [21]).

Definition. Let B be a linear operator defined in a Banach space $\mathcal{H}$, that is, $B : D(B) \subset \mathcal{H} \mapsto \mathcal{H}$. If for any $x,\ y \in D(B)$ and any $\lambda > 0$,

$$\|x - y\| \leq \|x - y + \lambda(Bx - By)\|, \tag{3.29}$$

then B is said to be an accretive operator. Moreover, if B is a densely defined accretive operator and $I + B$ is surjective, i.e., $R(I + B) = \mathcal{H}$, then B is said to be a maximal accretive operator.

If we choose $B = -A$, then by virtue of Lemmas 3.1-3.3 and the definition of a maximal accretive operator, we know that the operator B is a maximal accretive operator and generates a C_0-semigroup $S(t)$ of contractions on $\mathcal{H}$. Then system (2.11) can be formulated as an abstract first-order Cauchy problem as follows:

$$\begin{cases} \dfrac{d\Phi}{dt} + B\Phi = K, \\ \Phi(0) = \Phi_0 \end{cases} \tag{3.30}$$

where $\Phi_0 = (v_0, \nabla u_0, \theta_0, \nabla \tau_0, a^0)$ and B is a maximal accretive operator defined in a dense subset $D(B) = D(A)$ of a Hilbert space $\mathcal{H}$.

Lemma 3.4. Suppose that $K = K(t)$ and

$$K(t) \in C^1([0, \infty), \mathcal{H}), \quad \Phi_0 \in D(B).$$

Then problem (3.30) admits a unique global classical solution

$$\Phi \in C^1([0, \infty), H) \cap C([0, \infty), D(B)) \tag{3.31}$$

which can be expressed as

$$\Phi(t) = S(t)(\Phi_0) + \int_0^t S(t - \tau)K(\tau)d\tau. \tag{3.32}$$

Proof. Since $S(t)\Phi_0$ satisfies the homogeneous equation and nonhomogeneous initial condition, it suffices to verify that $w(t)$ given by

$$w(t) = \int_0^t S(t - \tau)K(\tau)d\tau \tag{3.33}$$

belongs to $C^1([0,\infty),\mathcal{H})\cap C([0,\infty),D(B))$ and satisfies the nonhomogeneous equation. Consider the difference quotient

$$\begin{aligned}
&\frac{w(t+h)-w(t)}{h}\\
&=\frac{1}{h}\Big(\int_0^{t+h}S(t+h-\tau)K(\tau)d\tau-\int_0^t S(t-\tau)K(\tau)d\tau\Big)\\
&=\frac{1}{h}\int_t^{t+h}S(t+h-\tau)K(\tau)d\tau+\frac{1}{h}\int_0^t(S(t+h-\tau)-S(t-\tau))K(\tau)d\tau\\
&=\frac{1}{h}\int_t^{t+h}S(z)K(t+h-z)dz+\frac{1}{h}\int_0^t S(z)(K(t+h-z)-K(t-z))dz.
\end{aligned}\tag{3.34}$$

As $h\to 0$, the terms in the last line of (3.34) tend to the limit

$$S(t)K(0)+\int_0^t S(z)K'(t-z)dz\in C([0,\infty),\mathcal{H}).\tag{3.35}$$

It turns out that $w\in C^1([0,\infty),\mathcal{H})$ and that the terms in the third line of (3.34) tend to a limit too, which should be

$$S(0)K(t)-Bw(t)=K(t)-Bw(t).\tag{3.36}$$

Thus the proof is complete. □

LEMMA 3.5. Suppose that $K=K(t)$ and

$$K(t)\in C([0,\infty),D(B)),\quad \Phi_0\in D(B).$$

Then problem (3.30) admits a unique global classical solution.

Proof. From the proof of Lemma 3.4, we can obtain that

$$\begin{aligned}
&\frac{w(t+h)-w(t)}{h}\\
&=\frac{1}{h}\int_t^{t+h}S(t+h-\tau)K(\tau)d\tau+\frac{1}{h}\int_0^t(S(t+h-\tau)-S(t-\tau))K(\tau)d\tau\\
&=\frac{1}{h}\int_t^{t+h}S(t+h-\tau)K(\tau)d\tau+\frac{1}{h}\int_0^t S(t-\tau)(\frac{S(h)-I}{h})K(\tau)d\tau
\end{aligned}\tag{3.37}$$

As $h\to 0$, the last terms in the line of (3.37) tend to

$$\begin{aligned}
&S(0)K(t)-\int_0^t S(t-\tau)BK(\tau)d\tau\\
&=S(0)K(t)-B\int_0^t S(t-\tau)K(\tau)d\tau=K(t)-Bw(t)
\end{aligned}\tag{3.38}$$

which, along with the results of Lemma 3.4, proves this lemma. □

Now we give the proofs of the main results.

Proof of Theorem 2.1. We can infer from (2.16) that $K=(F,0,G,0,0)$ satisfies the global Lipschitz condition on $\mathcal{H}$. Therefore, we use the contraction mapping theorem to prove the present theorem. Two key steps in applying the contraction mapping theorem are to figure out a closed set of the Banach space under consideration, and to set up an auxiliary problem so that the nonlinear operator defined by this auxiliary problem maps from the closed set into itself and turns out to be a contraction. In the following we proceed along these lines.

Let

$$\mathcal{E}(\Phi) = S(t)\Phi_0 + \int_0^t S(t-\tau)K(\Phi(\tau))d\tau \tag{3.39}$$

and

$$\mathcal{L} = \left\{ \Phi \in C([0,+\infty), \mathcal{H}) \Big| \ \sup_{t\geq 0} \left(\| \Phi(t) \| e^{-kt} \right) < \infty \right\} \tag{3.40}$$

where k is a positive constant such that $k > L$. In $\mathcal{L}$, we introduce the following norm:

$$\| \Phi \|_{\mathcal{L}} = \sup_{t\geq 0} \left(\| \Phi(t) \| e^{-kt} \right). \tag{3.41}$$

Clearly, $\mathcal{L}$ is a Banach space. We now show that the nonlinear operator $\mathcal{E}$ defined by (3.39) maps $\mathcal{L}$ into itself, and that the mapping is a contraction. Indeed, for $\mathcal{E} \in \mathcal{L}$, we have

$$\begin{aligned}
\| \mathcal{E}(\Phi) \| &\leq \| S(t)\Phi_0 \| + \int_0^t \| S(t-\tau) \| \| K(\Phi) \| d\tau \\
&\leq \| \Phi_0 \| + \int_0^t \| K(\Phi) \| d\tau \leq \| \Phi_0 \| + \int_0^t (L \| \Phi(\tau) \| + \| K(0) \|) d\tau \\
&\leq \| \Phi_0 \| + C_0 t + L \sup_{t\geq 0} \| \Phi(t) \| e^{-kt} \int_0^t e^{k\tau} d\tau \\
&\leq \| \Phi_0 \| + C_0 t + \frac{L}{k} e^{kt} \| \Phi \|_{\mathcal{L}}
\end{aligned} \tag{3.42}$$

where $C_0 = \| K(0) \|$. Thus,

$$\| \mathcal{E}(\Phi) \|_{\mathcal{L}} \leq \sup_{t\geq 0} \left[(\| \Phi_0 \| + C_0 t) e^{-kt} \right] + \frac{L}{k} \| \Phi \|_{\mathcal{L}} < \infty, \tag{3.43}$$

i.e., $\mathcal{E}(\Phi) \in \mathcal{L}$.

For $\Phi_1,\ \Phi_2 \in \mathcal{L}$, we have

$$\begin{aligned}
\| \mathcal{E}(\Phi_1) - \mathcal{E}(\Phi_2) \|_{\mathcal{L}} &= \sup_{t\geq 0} \left(e^{-kt} \| \int_0^t S(t-\tau)(K(\Phi_1(\tau)) - K(\Phi_2(\tau))) d\tau \| \right) \\
&\leq \sup_{t\geq 0} \left(e^{-kt} L \int_0^t \| \Phi_1 - \Phi_2 \| d\tau \right) \\
&\leq \sup_{t\geq 0} \left[e^{-kt} \cdot \frac{L}{k} \cdot (e^{kt} - 1) \right] \| \Phi_1 - \Phi_2 \|_{\mathcal{L}} \\
&\leq \frac{L}{k} \| \Phi_1 - \Phi_2 \|_{\mathcal{L}}.
\end{aligned} \tag{3.44}$$

Therefore, by the contraction mapping theorem, the problem has a unique solution in $\mathcal{L}$. To show that the uniqueness also holds in $C([0,\infty), \mathcal{H})$, let $\Phi_1,\ \Phi_2 \in C([0,\infty), \mathcal{H})$ be two solutions of the problem and let $\Phi = \Phi_1 - \Phi_2$. Then

$$\Phi(t) = \int_0^t S(t-\tau)(K(\Phi_1) - K(\Phi_2))d\tau, \tag{3.45}$$

$$\| \Phi(t) \| \leq L \int_0^t \| \Phi(\tau) \| d\tau. \tag{3.46}$$

By the Gronwall inequality, we immediately conclude that $\Phi(t) = 0$, i.e., the uniqueness in $C([0,\infty), H)$ follows. Thus the proof is complete. □

Proof of Theorem 2.2. It follows from (2.17) that $K = (F, 0, G, 0, 0)$ satisfies the global Lipschitz condition on $D(A)$. Since B is a maximal accretive operator, let

$$A_1 = D(B), \ \ B_1 = B^2 : \ \ D(B_1) = D(B^2) \mapsto A_1. \tag{3.47}$$

Then A_1 is a Banach space and B_1 is a densely defined operator from $D(B^2)$ into A_1. In what follows we prove that B_1 is a maximal accretive operator in $A_1 = D(B)$.

Indeed, for any $x,\ y \in D(B^2)$, since B is accretive in $\mathcal{H}$, we have

$$\begin{aligned} &\| x - y + \lambda(Bx - By) \|_{D(B)} \\ &= \Big(\| x - y + \lambda(Bx - By) \|^2 + \| Bx - By + \lambda(B^2x - B^2y) \|^2 \Big)^{\frac{1}{2}} \\ &\geq \Big(\| x - y \|^2 + \| Bx - By \|^2 \Big)^{\frac{1}{2}} = \| x - y \|_{D(B)}, \end{aligned} \tag{3.48}$$

i.e., B_1 is accretive in A_1. Furthermore, since B is a maximal accretive operator in $\mathcal{H}$, for any $y \in \mathcal{H}$, there is a unique $x \in D(B)$ such that

$$x + Bx = y. \tag{3.49}$$

Now for any $y \in A_1 = D(B)$, equation (3.49) admits a unique solution $x \in D(B)$. It turns out that

$$Bx = y - x \ \in \ D(B). \tag{3.50}$$

Thus $x \in D(B^2)$, i.e., B_1 is a maximal accretive operator in A_1. Let $S_1(t)$ be the semigroup generated by B_1. If $\Phi_0 \in \ D(B^2) = D(B_1)$, then

$$\Phi(t) = S_1(t)\Phi_0 \in C([0,+\infty), D(B^2)) \cap C^1([0,+\infty), D(B))$$

is unique classical solution of the problem. On the other hand, $\Phi(t) = S_1(t)\Phi_0$ is also a classical solution in

$$C([0,+\infty), D(B)) \cap C^1([0,+\infty), \mathcal{H}).$$

This implies that $S_1(t)$ is a restriction of $S(t)$ on A_1. By virtue of the proof of Theorem 2.1, there exists a unique mild solution $\Phi \in C([0,+\infty), A_1)$. Since $S_1(t)$ is a restriction of $S(t)$ on $D(B)$, we infer from $K(\Phi)$ being an operator from $D(B)$ to $D(B)$ and Lemma 3.5 that Φ is a classical solution to the problem. Thus the proof is complete. □

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