in practice, the method from this paper appears to be a good starting point in stable adaptive control design for a more general class of nonlinearly parameterized plants.

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Global Positioning of Robot Manipulators via PD Control Plus a Class of Nonlinear Integral Actions

Rafael Kelly

***Abstract*—This paper deals with the position control of robot manipulators. Proposed is a simple class of robot regulators consisting of a linear proportional–derivative (PD) feedback plus an integral action of a nonlinear function of position errors. By using Lyapunov's direct method and LaSalle's invariance principle, the authors characterize a class of such nonlinear functions, and they provide explicit conditions on the regulator gains to ensure global asymptotic stability. These regulators offer an attractive alternative to global regulation compared with the well-known partially model-based PD control with gravity compensation and PD control with desired gravity compensation.**

***Index Terms*—Manipulators, position control, stability.**

I. Introduction

Position control of robot manipulators, also called regulation of robots, may be recognized as the simplest aim in robot control and at the same time one of the most relevant issues in the practice of manipulators. The goal of global position control is to move the manipulator from any initial state to a fixed desired configuration. It is well known that many applications of robots moving freely in their workspace can be well performed by position controllers [1].

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The proportional–derivative (PD) control plus gravity compensation together with the PD control plus desired gravity compensation are the simplest global regulators for robot manipulators. The best feature of these controllers is that the tuning procedure to achieve global asymptotic stability reduces to select the proportional and derivative gains in a straightforward manner [2]. However, a drawback of both control strategies is that the knowledge of the gravitational torque vector of the robot dynamics which depends on some parameters as mass of the payload, usually uncertain, is required. To overcome parametric uncertainties on the gravitational torque vector, adaptive versions of above controllers have been introduced in [3]–[5]. However, two minor weaknesses remain for these approaches; first, the structure of the gravitational torque vector has to be known, and second, the parameters of the controllers have to be chosen satisfying complex inequalities. On the other hand, the common practice of using the linear proportional-integral-derivative (PID) control in most industrial robots, which does not require any component of the robot dynamics into its control law, lacks a global asymptotic stability proof [6]–[8]. Recently, a semiglobally stable linear regulator without gravity compensation called $\mathrm{PI}^2\mathrm{D}$ has been proposed to solve the robot position goal [9].

The first global regulator for robot manipulators removing the use of the gravitational torque vector in the control law was proposed in [10]. The controller structure incorporates a PI term driven by a bounded nonlinear function of the position error and a linear PD feedback loop. It has been shown that there exist suitable parameters of the controller which achieve positioning in a global sense. The controller proposed in [10] has been the main motivation and starting point of this paper.

In this paper we introduce a new class of global position controllers for robots which do not include their dynamics in the control laws. Motivated by the controller introduced in [10], which aims at modifying the potential energy of the closed–loop system and the injection of the required dissipation, we develop a new class of regulators leading to a linear PD feedback plus an integral action driven by a class of nonlinear functions of the position error. We characterize the class of function and give simple explicit conditions on the controller parameters which guarantee global positioning.

Throughout this paper, we use the notation $\lambda_{\mathrm{m}}\{A\}$ and $\lambda_{\mathrm{M}}\{A\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \Re^n$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$, and that of matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_{\mathrm{M}}\{A^TA\}}$.

The organization of this paper is as follows. Section II summarizes the robot model, its main properties, and the controller introduced in [10]. Our main results are presented in Section III, where we propose a class of PD controller with nonlinear integral action and we provide conditions on the controller gains to ensure global asymptotic stability. Finally, we offer some concluding remarks in Section IV.

II. Robot Dynamics and Motivation

In the absence of friction or other disturbances, the dynamics of a serial n-link rigid robot manipulator can be written as [13]

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied joint torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of centripetal and Coriolis torques, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$ due to gravity. We assume the links are connected with revolute joints, and matrix $C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is defined using the Christoffel symbols.

The equation of motion (1) has the following important properties.

Property 1 [14]: The matrix

$$\tfrac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{2}$$

is skew-symmetric. □

Property 2: The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in \mathbb{R}^n$ [15]. In addition, there exists a positive constant k_g satisfying [3], [16]

$$k_g > \left\| \frac{\partial \boldsymbol{g}(\boldsymbol{x})}{\partial \boldsymbol{x}} \right\|, \qquad \forall \boldsymbol{x} \in \mathbb{R}^n$$

and

$$\|\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{y})\| \le k_g \|\boldsymbol{x} - \boldsymbol{y}\|, \qquad \forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n. \tag{3}$$

□

Property 3: There exists a positive constant k_{C1} such that

$$\|C(\boldsymbol{q}, \boldsymbol{x})\boldsymbol{y}\| \le k_{C1} \|\boldsymbol{x}\| \|\boldsymbol{y}\|, \qquad \forall\, \boldsymbol{q}, \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n. \tag{4}$$

□

Property 4: (See the Appendix.) For any constant vector $\boldsymbol{q}_d \in \mathbb{R}^n$, the function

$$\mathcal{U}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) - \mathcal{U}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d)^T \tilde{\boldsymbol{q}} + \frac{k_g}{2}\|\tilde{\boldsymbol{q}}\|^2$$

is globally positive definite with respect to $\tilde{\boldsymbol{q}} \in \mathbb{R}^n$. □

Now we recall that the global position control problem is to design a controller to evaluate the torque $\boldsymbol{\tau} \in \mathbb{R}^n$ applied to the joints so that the robot joint displacements $\boldsymbol{q}$ tend asymptotically to a constant desired joint displacement $\boldsymbol{q}_d$ regardless the initial conditions $\boldsymbol{q}(0)$ and $\dot{\boldsymbol{q}}(0)$.

Motivated by the energy-shaping methodology and passivity theory, a simple position (set-point) controller for robot manipulator has been proposed in [10]–[12]. The controller structure is composed by a saturated, proportional, and differential (SP-D) feedback plus a PI controller driven by a linear sum of velocity and saturated position errors. The control law can be written as

$$\boldsymbol{\tau} = K_p \tilde{\boldsymbol{q}} - K_v \dot{\boldsymbol{q}} + K_D \boldsymbol{f}(\tilde{\boldsymbol{q}}) + K_i \int_0^t \boldsymbol{f}[\tilde{\boldsymbol{q}}(s)]\, ds \tag{5}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the joint position error, and K_p, K_v, K_D, and K_i are suitable $n \times n$ matrices. The entries of the nonlinear vector function $\boldsymbol{f}(\tilde{\boldsymbol{q}}) = [f(\tilde{q}_1) \quad f(\tilde{q}_2) \cdots f(\tilde{q}_n)]^T$ are given by

$$f(x) = \begin{cases} \sin(x), & \text{if } |x| < \pi/2 \\ 1, & \text{if } x \ge \pi/2 \\ -1, & \text{if } x \le -\pi/2. \end{cases} \tag{6}$$

Hence, the control law (5) is constituted by a PI term driven by the nonlinear function $\boldsymbol{f}(\tilde{\boldsymbol{q}})$ of the position error $\tilde{\boldsymbol{q}}$ and a linear PD feedback. One important feature of controller (5) is that its structure does not depend on the robot dynamics. In [10]–[12], it has been proven that there exists a suitable choice of the controller parameters so that the overall closed-loop system is globally asymptotically stable.

In this paper we extend the work of [10]–[12] in two directions. First, we show that global asymptotic stability is still possible without the nonlinear position error feedback $K_D \boldsymbol{f}(\tilde{\boldsymbol{q}})$. This leads to a linear PD control plus an integral term of a nonlinear function of $\tilde{\boldsymbol{q}}$.

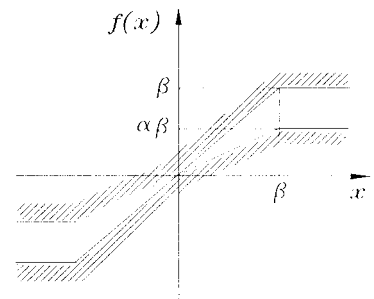

Fig. 1. $\mathcal{F}(\alpha, \beta, x)$ functions.

Second, we characterize a class of nonlinear functions $\boldsymbol{f}(\tilde{\boldsymbol{q}})$ which yields, under a suitable selection of the controller gains, a globally asymptotically stable closed-loop system.

III. A Class of PD Controllers with Nonlinear Integral Action

Most of the present industrial robots are controlled through local PID controllers [1]. The textbook version of the PID controller can be described by the equation

$$\boldsymbol{\tau} = K_p \tilde{\boldsymbol{q}} - K_v \dot{\boldsymbol{q}} + K_i \int_0^t \tilde{\boldsymbol{q}}(\sigma)\, d\sigma$$

where K_p, K_v, and K_i are suitable positive definite diagonal $n \times n$ matrices, and $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector. Although the PID controller has been shown in practice to be effective for position control of robot manipulators, unfortunately it lacks, until now, a global asymptotic stability proof [6]–[8].

In this paper we propose a modification to the integral term of the PID controller which leads to a new class of controllers which yield globally asymptotically stable systems. This modification follows the idea presented in [10] and [4] where global position control is guaranteed by using controllers whose integral term is driven by a saturated position error.

For the purpose of this paper, it is convenient to introduce the following.

Definition 1: $\mathcal{F}(\alpha, \beta, \boldsymbol{x})$ with $1 \ge \alpha > 0$, $\beta > 0$, and $\boldsymbol{x} \in \mathbb{R}^n$ denotes the set of all continuous differentiable increasing functions $\boldsymbol{f}(\boldsymbol{x}) = [f(x_1) \quad f(x_2) \cdots f(x_n)]^T$ such that

- $|x| \ge |f(x)| \ge \alpha |x|, \qquad \forall\, x \in \mathbb{R} : |x| < \beta;$
- $\beta \ge |f(x)| \ge \alpha\beta, \qquad \forall x \in \mathbb{R} : |x| \ge \beta;$
- $1 \ge (d/dx) f(x) \ge 0;$

where $|\cdot|$ stands for the absolute value. □

Fig. 1 depicts the region allowed for functions belonging to set $\mathcal{F}(\alpha, \beta, \boldsymbol{x})$. For instance, the function considered in [10], whose entries are given by (6), belongs to set $\mathcal{F}(\sin(1), 1, \boldsymbol{x})$. Another example is the tangent hyperbolic function

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

which belongs to $\mathcal{F}(\tanh(1), 1, \boldsymbol{x})$.

Two important properties of functions $\boldsymbol{f}(\boldsymbol{x})$ belonging to $\mathcal{F}(\alpha, \beta, \boldsymbol{x})$ are now established.

Property 5: The Euclidean norm of $\boldsymbol{f}(\boldsymbol{x})$ satisfies for all $\boldsymbol{x} \in \mathbb{R}^n$

$$\|\boldsymbol{f}(\boldsymbol{x})\| \ge \begin{cases} \alpha\|\boldsymbol{x}\|, & \text{if } \|\boldsymbol{x}\| < \beta \\ \alpha\beta, & \text{if } \|\boldsymbol{x}\| \ge \beta \end{cases}$$

and

$$\|\boldsymbol{f}(\boldsymbol{x})\| \le \begin{cases} \|\boldsymbol{x}\|, & \text{if } \|\boldsymbol{x}\| < \beta \\ \sqrt{n}\beta, & \text{if } \|\boldsymbol{x}\| \ge \beta. \end{cases}$$

□

Property 6: The function $\boldsymbol{f}(\boldsymbol{x})^T\boldsymbol{x}$ satisfies for all $\boldsymbol{x} \in \mathbb{R}^n$

$$\boldsymbol{f}(\boldsymbol{x})^T\boldsymbol{x} \geq \begin{cases} \alpha\|\boldsymbol{x}\|^2, & \text{if } \|\boldsymbol{x}\| < \beta \\ \alpha\beta\|\boldsymbol{x}\|, & \text{if } \|\boldsymbol{x}\| \geq \beta. \end{cases}$$

□

A. PD Control with Nonlinear Integral Action

Let us propose the following control law:

$$\boldsymbol{\tau} = K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} + K_i \int_0^t \boldsymbol{f}[\tilde{\boldsymbol{q}}(\sigma)]\, d\sigma \tag{7}$$

where K_p, K_v, and K_i are diagonal positive definite $n \times n$ matrices, and $\boldsymbol{f}(\tilde{\boldsymbol{q}}) \in \mathcal{F}(\alpha, \beta, \tilde{\boldsymbol{q}})$. The above control law is composed by a linear PD term plus an integral action of the nonlinear function $\boldsymbol{f}(\tilde{\boldsymbol{q}})$.

The closed-loop system dynamics is obtained by substituting the control action τ from (7) into the robot dynamic model (1). By defining z as

$$z(t) = \int_0^t \boldsymbol{f}(\tilde{\boldsymbol{q}}(\sigma))\, d\sigma - K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d) \tag{8}$$

we can describe the closed-loop system by (9), as shown at the bottom of the page, which is an autonomous nonlinear differential equation whose origin $\begin{bmatrix}\tilde{\boldsymbol{q}}^T & \dot{\boldsymbol{q}}^T & z^T\end{bmatrix}^T = \boldsymbol{0} \in \mathbb{R}^{3n}$ is the unique equilibrium.

The objective is now to provide conditions on the controller gains K_p, K_v, and K_i guaranteeing global asymptotic stability of the unique equilibrium. This would mean that the global position control with regulator (7) is ensured for any $\boldsymbol{f}(\tilde{\boldsymbol{q}}) \in \mathcal{F}(\alpha, \beta, \tilde{\boldsymbol{q}})$. This is established in the following.

Proposition 1: Consider the robot dynamics (1) together with control law (7) where $\boldsymbol{f}(\tilde{\boldsymbol{q}}) \in \mathcal{F}(\alpha, \beta, \tilde{\boldsymbol{q}})$. If

$$\lambda_{\rm m}\{K_i\} > 0 \tag{10}$$

$$\lambda_{\rm m}\{K_v\} > \lambda_{\rm M}\{M\} + k_{C1}\beta\sqrt{n} \tag{11}$$

$$\lambda_{\rm m}\{K_p\} > k_g\frac{\sqrt{n}}{\alpha} + \lambda_{\rm M}\{M\} + \lambda_{\rm M}\{K_i\} \tag{12}$$

then, the equilibrium $[\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T \quad z^T]^T = \boldsymbol{0} \in \mathbb{R}^{3n}$ of (9) is globally asymptotically stable. □

Proof: To carry out the stability analysis, we consider the following Lyapunov function candidate:

$$\begin{aligned} V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) = {} & \tfrac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \tfrac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\boldsymbol{f}(\tilde{\boldsymbol{q}}) \\ & - \tfrac{1}{2}\boldsymbol{f}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \tfrac{1}{2}[z + \tilde{\boldsymbol{q}}]^T K_i[z + \tilde{\boldsymbol{q}}] \\ & + \tfrac{1}{2}\tilde{\boldsymbol{q}}^T[K_p - K_i]\tilde{\boldsymbol{q}} + \int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{f}(\boldsymbol{x})^T K_v\, d\boldsymbol{x} \\ & + \mathcal{U}(\boldsymbol{q}) - \mathcal{U}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} \end{aligned} \tag{13}$$

where

$$\int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{f}(\boldsymbol{x})^T K_v\, d\boldsymbol{x} = \int_0^{\tilde{q}_1} f(x_1)k_{v1}\, dx_1 + \cdots + \int_0^{\tilde{q}_n} f(x_n)k_{vn}\, dx_n$$

with $K_v = \mathrm{diag}\{k_{v1}, \cdots, k_{vn}\}$.

Concerning the Lyapunov function candidate, we digress momentarily to give the following explanation. Let us recall that the PD control with desired gravity compensation given by [2]

$$\boldsymbol{\tau} = K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d)$$

yields the closed-loop equation

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q}_d)] \end{bmatrix}. \tag{14}$$

It is worth noting that the structure of the closed-loop system (9) becomes (14) if we only consider the state variables $\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}$, and $K_i = 0$. By using the following Lyapunov function proposed in [4]:

$$\begin{aligned} V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) = {} & \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} - \frac{\delta}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\boldsymbol{h}(\tilde{\boldsymbol{q}}) - \frac{\delta}{2}\boldsymbol{h}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ & + \frac{1}{2}\tilde{\boldsymbol{q}}^T K_p\tilde{\boldsymbol{q}} + \mathcal{U}(\boldsymbol{q}) - \mathcal{U}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} \end{aligned} \tag{15}$$

where

$$\boldsymbol{h}(\tilde{\boldsymbol{q}}) = \frac{1}{1 + \|\tilde{\boldsymbol{q}}\|}\tilde{\boldsymbol{q}}$$

and $\delta > 0$, the global asymptotic stability of (14) can be shown [4]. Note that these terms, but with $\boldsymbol{f}(\tilde{\boldsymbol{q}})$ instead of $\boldsymbol{h}(\tilde{\boldsymbol{q}})$ and $\delta = 1$, are all included in the Lyapunov function candidate (13) which contains the additional terms

$$\tfrac{1}{2}z^T K_i z + \tilde{\boldsymbol{q}}^T K_i z + \int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{f}(\boldsymbol{x})^T K_v\, d\boldsymbol{x}.$$

The first two terms are included in the Lyapunov function candidate (13) to take into account the state variable z induced by the integral action. The remaining integral term, which is not a key term, turns out to be useful to cancel cross terms in the time derivative of the Lyapunov function. It should be pointed out that Lyapunov function candidate (15) was motivated from [2] and [17].

Now we show that under assumption (12) on K_p, the Lyapunov function candidate (13) is a positive definite function. This Lyapunov function candidate (13) can be written as

$$\begin{aligned} V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) = {} & \tfrac{1}{2}[\dot{\boldsymbol{q}} - \boldsymbol{f}(\tilde{\boldsymbol{q}})]^T M(\boldsymbol{q})[\dot{\boldsymbol{q}} - \boldsymbol{f}(\tilde{\boldsymbol{q}})] + \tfrac{1}{2}[z + \tilde{\boldsymbol{q}}]^T \\ & \cdot K_i[z + \tilde{\boldsymbol{q}}] + \int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{f}(\boldsymbol{x})^T K_v\, d\boldsymbol{x} + \tfrac{1}{2}\tilde{\boldsymbol{q}}^T \\ & \cdot [K_p - K_i]\tilde{\boldsymbol{q}} - \tfrac{1}{2}\boldsymbol{f}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\boldsymbol{f}(\tilde{\boldsymbol{q}}) + \mathcal{U}(\boldsymbol{q}) \\ & - \mathcal{U}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}}. \end{aligned} \tag{16}$$

The first term is a nonnegative function of $\tilde{\boldsymbol{q}}$ and $\dot{\boldsymbol{q}}$, while the second is a nonnegative function of $\tilde{\boldsymbol{q}}$ and z. It can be shown that the third term satisfies

$$\int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{f}(\boldsymbol{x})^T K_v\, d\boldsymbol{x} > 0, \qquad \forall\tilde{\boldsymbol{q}} \neq \boldsymbol{0} \in \mathbb{R}^n \tag{17}$$

because K_v is a diagonal positive definite matrix, $\boldsymbol{f}(\boldsymbol{0}) = \boldsymbol{0}$, and the entries of $\boldsymbol{f}(\boldsymbol{x})$ are increasing functions. Therefore, this term is positive definite with respect to $\tilde{\boldsymbol{q}}$. Now, we prove that the remaining terms yield a positive definite function with respect to $\tilde{\boldsymbol{q}}$. To this end, notice that

$$-\tfrac{1}{2}\boldsymbol{f}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\boldsymbol{f}(\tilde{\boldsymbol{q}}) \geq -\tfrac{1}{2}\lambda_{\rm M}\{M\}\|\tilde{\boldsymbol{q}}\|^2 \tag{18}$$

where we have used $\|\tilde{\boldsymbol{q}}\|^2 \geq \boldsymbol{f}(\tilde{\boldsymbol{q}})^T\tilde{\boldsymbol{q}} \geq \|\boldsymbol{f}(\tilde{\boldsymbol{q}})\|^2$. From this and using Property 4 we have

$$\begin{aligned} & \tfrac{1}{2}\tilde{\boldsymbol{q}}^T[K_p - K_i]\tilde{\boldsymbol{q}} - \tfrac{1}{2}\boldsymbol{f}(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\boldsymbol{f}(\tilde{\boldsymbol{q}}) + \mathcal{U}(\boldsymbol{q}) - \mathcal{U}(\boldsymbol{q}_d) \\ & \quad + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} \\ & \geq \tfrac{1}{2}[\lambda_{\rm m}\{K_p\} - \lambda_{\rm M}\{K_i\} - \lambda_{\rm M}\{M\} - k_g]\|\tilde{\boldsymbol{q}}\|^2 \end{aligned}$$

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \\ z\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[K_p\tilde{\boldsymbol{q}} - K_v\dot{\boldsymbol{q}} + K_i z - C(q, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q}_d)] \\ \boldsymbol{f}(\tilde{\boldsymbol{q}}) \end{bmatrix} \tag{9}$$

which is a positive definite function with respect to $\tilde{\boldsymbol{q}}$ because of selection (12) of K_p; this implies that the Lyapunov function candidate (13) is in turn a positive definite function.

After some simplifications and using Property 1, the time derivative of the Lyapunov function candidate (13) along the trajectories of the closed-loop system (9) can be written as

$$\begin{aligned}\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) = &- \dot{\boldsymbol{q}}^T[K_v - M(\boldsymbol{q})F(\tilde{\boldsymbol{q}})]\dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}^T C(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{f}(\tilde{\boldsymbol{q}}) \\ &- \boldsymbol{f}(\tilde{\boldsymbol{q}})^T[K_p - K_i]\tilde{\boldsymbol{q}} + [\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_d)]^T \boldsymbol{f}(\tilde{\boldsymbol{q}}) \qquad (19)\end{aligned}$$

where $\dot{\boldsymbol{f}}(\tilde{\boldsymbol{q}}) = -F(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}}$, with $F(\tilde{\boldsymbol{q}})$ being a diagonal matrix whose entries $\partial f(\tilde{q}_i)/\partial \tilde{q}_i$ are nonnegative and smaller than or equal to one.

By using Properties 3 and 5 we have

$$-\dot{\boldsymbol{q}}^T C(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{f}(\tilde{\boldsymbol{q}}) \le k_{C1}\sqrt{n}\beta\|\dot{\boldsymbol{q}}\|^2.$$

On the other hand, it is easy to show

$$\dot{\boldsymbol{q}}^T M(\boldsymbol{q})F(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} \le \lambda_{\mathrm{M}}\{M\}\|\dot{\boldsymbol{q}}\|^2.$$

Therefore, the time derivative of the Lyapunov function candidate (19) satisfies

$$\begin{aligned}\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) \le &- \gamma\|\dot{\boldsymbol{q}}\|^2 - \boldsymbol{f}(\tilde{\boldsymbol{q}})^T[K_p - K_i]\tilde{\boldsymbol{q}} \\ &+ [\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_d)]^T \boldsymbol{f}(\tilde{\boldsymbol{q}}) \qquad (20)\end{aligned}$$

where $\gamma = \lambda_{\mathrm{m}}\{K_v\} - \lambda_{\mathrm{M}}\{M\} - k_{C1}\sqrt{n}\beta$ is a positive constant because of the selection of K_v in (11). The second term of the above equation is bounded by

$$-\boldsymbol{f}(\tilde{\boldsymbol{q}})^T[K_p - K_i]\tilde{\boldsymbol{q}} \le -[\lambda_{\mathrm{m}}\{K_p\} - \lambda_{\mathrm{M}}\{K_i\}]\boldsymbol{f}(\tilde{\boldsymbol{q}})^T\tilde{\boldsymbol{q}}$$

because K_p and K_i are diagonal positive definite matrices and $\tilde{q}_i f(\tilde{q}_i) \ge 0$. Hence, the second and third right-hand side terms of (20) satisfy

$$\begin{aligned}&- \boldsymbol{f}(\tilde{\boldsymbol{q}})^T[K_p - K_i]\tilde{\boldsymbol{q}} + [\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_d)]^T \boldsymbol{f}(\tilde{\boldsymbol{q}}) \\ &\quad \le -[\lambda_{\mathrm{m}}\{K_p\} - \lambda_{\mathrm{M}}\{K_i\}]\boldsymbol{f}(\tilde{\boldsymbol{q}})^T\tilde{\boldsymbol{q}} + k_g\|\tilde{\boldsymbol{q}}\|\|\boldsymbol{f}(\tilde{\boldsymbol{q}})\| \qquad (21)\end{aligned}$$

where we have used Property 2. Taking into account Properties 5 and 6, we obtain

$$\begin{aligned}&- [\lambda_{\mathrm{m}}\{K_p\} - \lambda_{\mathrm{M}}\{K_i\}]\boldsymbol{f}(\tilde{\boldsymbol{q}})^T\tilde{\boldsymbol{q}} + k_g\|\tilde{\boldsymbol{q}}\|\|\boldsymbol{f}(\tilde{\boldsymbol{q}})\| \\ &\quad \le \begin{cases} -\delta\|\tilde{\boldsymbol{q}}\|^2, & \text{if } \|\tilde{\boldsymbol{q}}\| < \beta \\ -\delta\beta\|\tilde{\boldsymbol{q}}\|, & \text{if } \|\tilde{\boldsymbol{q}}\| \ge \beta \end{cases} \qquad (22)\end{aligned}$$

where $\delta = [\lambda_{\mathrm{m}}\{K_p\} - \lambda_{\mathrm{M}}\{K_i\}]\alpha - k_g\sqrt{n}$. The choice of K_p in (12) ensures $\delta > 0$.

Therefore, incorporating (21) and (22) into (20), we get

$$\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, z) \le \begin{cases} -\gamma\|\dot{\boldsymbol{q}}\|^2 - \delta\|\tilde{\boldsymbol{q}}\|^2, & \text{if } \|\tilde{\boldsymbol{q}}\| < \beta \\ -\gamma\|\dot{\boldsymbol{q}}\|^2 - \delta\beta\|\tilde{\boldsymbol{q}}\|, & \text{if } \|\tilde{\boldsymbol{q}}\| \ge \beta \end{cases}$$

which is a globally negative semidefinite function.

Using the fact that the Lyapunov function candidate (13) is a globally positive definite function and its time derivative is a globally negative semidefinite function, we conclude that the equilibrium of the closed-loop system (9) is stable. Finally, by invoking the LaSalle's invariance principle, the global asymptotic stability of the equilibrium is proven straightforward.

IV. Concluding Remarks

In this paper we have characterized a class of global regulators for robot manipulators. The main feature of these regulators is the simple structure based on a linear PD feedback plus an integral action driven by a nonlinear function of the joint position error. This nonlinear function has been characterized, and we provide explicit conditions on the regulator gains given in terms of some information extracted from the robot dynamics and the nonlinear function characterization to ensure global asymptotic stability of the overall closed-loop system.

Appendix

This Appendix presents a proof of Property 4 following ideas reported in [3]. Let us define the twice continuously differentiable function $\mathcal{G}(\tilde{\boldsymbol{q}}) : \mathbb{R}^n \to \mathbb{R}$ as

$$\mathcal{G}(\tilde{\boldsymbol{q}}) = \frac{k_g}{2}\|\tilde{\boldsymbol{q}}\|^2 + \mathcal{U}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) - \mathcal{U}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}}$$

where $\boldsymbol{q}_d \in \mathbb{R}^n$ is a constant vector.

First notice that $\mathcal{G}(\boldsymbol{0}) = 0$. Using $\boldsymbol{g}(\boldsymbol{q}) = \partial\mathcal{U}(\boldsymbol{q})/\partial\boldsymbol{q}$, we have that the gradient of $\mathcal{G}(\tilde{\boldsymbol{q}})$ with respect to $\tilde{\boldsymbol{q}}$ given by

$$\frac{\partial\mathcal{G}(\tilde{\boldsymbol{q}})}{\partial\tilde{\boldsymbol{q}}} = k_g\tilde{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}_d)$$

vanishes at $\tilde{\boldsymbol{q}} = \boldsymbol{0}$. Thus, $\mathcal{G}(\tilde{\boldsymbol{q}})$ has a critical point at $\tilde{\boldsymbol{q}} = \boldsymbol{0}$. The Hessian matrix of $\mathcal{G}(\tilde{\boldsymbol{q}})$ with respect to $\tilde{\boldsymbol{q}}$ is given by

$$k_g I + \frac{\partial\boldsymbol{g}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}})}{\partial[\boldsymbol{q}_d - \tilde{\boldsymbol{q}}]}$$

where I is the identity matrix. Using Property 2 on k_g we have the conclusion that the Hessian is a positive definite matrix for all $\boldsymbol{q}_d - \tilde{\boldsymbol{q}} \in \mathbb{R}^n$. Therefore, function $\mathcal{G}(\tilde{\boldsymbol{q}})$ is a globally strictly convex function vanishing at the unique global minimum $\tilde{\boldsymbol{q}} = \boldsymbol{0}$. This implies that $\mathcal{G}(\tilde{\boldsymbol{q}})$ is a globally positive definite function which holds for any constant $\boldsymbol{q}_d$.

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Robust Estimation Without Positive Real Condition

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***Abstract*—The strictly positive real (SPR) condition on the noise model is necessary for a discrete-time linear stochastic control system with unmodeled dynamics, even so for a time-invariant ARMAX system, in the past robust analysis of parameter estimation. However, this condition is hardly satisfied for a high-order and/or multidimensional system with correlated noise. The main work in this paper is to show that for robust parameter estimation and adaptive tracking, as well as closed-loop system stabilization, the SPR condition is replaced by a stable matrix polynomial. The main method is to design a "two-step" recursive least squares algorithm with or without a weighted factor and with a fixed lag regressive vector and to define an adaptive control with bounded external excitation and with randomly varying truncation.**

***Index Terms*—Adaptive control, least squares, robust estimation, stochastic system, unmodeled dynamics.**

I. Introduction

It is well known that when the quantity of raw input and output data from a complicated system has much oscillation, the model describing a true system should be established as an autoregressive and moving average model with extraneous input (ARMAX), which means that the system noise is correlated. Further, in the past theory work for convergent estimation and/or adaptive control was necessary to impose the following strictly positive real (SPR) condition:

$$C^{-1}(e^{i\omega}) + C^{-\tau}(e^{-i\omega}) - I > 0, \qquad \forall\, \omega \in [0,\, 2\pi], \qquad i = \sqrt{-1} \tag{1}$$

on the true system (cf., [2]–[7], [12], [23], and [24]), where $C(z)$ is a matrix polynomial

$$C(z) = I + C_1 z + \cdots + C_r z^r. \tag{2}$$

This condition cannot obviously be verified *a priori*. Of course, it is automatically satisfied in the case of $r = 0$, i.e., for uncorrelated system noise. Condition (1) also implies that $\sum_{i=1}^{r} C_i^2 < 1$ and is implied by $\sum_{i=1}^{r} |C_i| < 1$ in the case of $r > 0$ and of one-dimensional system noise, i.e., $m = 1$ in (3) where $C_i (i = 1, \cdots, r)$ are unknown scalar parameters [11]. Hence, it is not satisfied if $\sum_{i=1}^{r} C_i^2 \geq 1$. This means that the existent theory results are only obtained for a special class of discrete-time linear stochastic systems. However, it is also known that if the SPR condition is not valid, counterexamples can be constructed such that the estimation of least squares (LS) algorithm does not converge to a true parameter vector [1]. Even so, some endeavors are taken to relax the SPR condition for the usual time-invariant ARMAX system, e.g., the prefilter [2], [5], the overparameterization [6], the "prewhitening" [4], [13], the ARMA model described by stationary processes [8]–[10], and so on. Recently, by use of the limit theorems for double array martingale [15] and a "two-step" LS algorithm where an increasing (but nonrecursive) lag regressive sequence is defined, the strong consistency of parameter estimates is established in [14] for the time-invariant ARMAX system where the SPR condition on the noise model is weakened to a stable noise polynomial.

However, a real system usually contains unmodeled dynamics which may cause many adaptive control algorithms to go unstable if other precautions are not taken [16], [17]. Therefore, it is most important to analyze the influence of the unmodeled dynamics upon the system stability and the adaptive control. The SPR condition is necessary for guaranteeing the robust estimation and the robust adaptive control in [18]–[20]. Naturally, it is more difficult to weaken the SPR condition for the stochastic system with the unmodeled dynamics than for the time-invariant ARMAX system.

In this paper, we design the "two-step" recursive algorithm. The estimates for the noise process are generated by a fixed lag LS algorithm with or without a weighted factor in the first step. The estimates for all unknown parameters in the stochastic system are thus generated by the other LS algorithms with or without the weighted factor in the second step, where the regressive vector sequence is obtained by use of the noise estimates in the first step, and the weighted factor is chosen the same as in the "two-step" algorithm.

This paper is organized as follows. We state the considered system and present the "two-step" recursive algorithm in Section II. In Section III, we first design the adaptive control both with the bounded external excitation and with the randomly varying truncation. Second, we establish the results of robust parameter estimation, robust adaptive tracking, and closed-loop system stability. The robust proofs are given in Section IV and Appendixes A and B.

II. System and Algorithm

Let us consider the following stochastic systems with the unmodeled dynamics η_n:

$$\begin{aligned} A(z)y_{n+1} &= B(z)u_{n+1} + C(z)w_{n+1} + \eta_n, \qquad n \geq 0 \\ y_n &= w_n = \eta_n = 0,\ u_n = 0,\ n < 0 \end{aligned} \tag{3}$$

where y_n, u_n, and w_n are m-dimensional output, input, and noise sequences, respectively, $A(z)$, $B(z)$, and $C(z)$ are matrix polynomials in backward-shift operator z

$$\begin{aligned} A(z) &= I + A_1 z + \cdots + A_p z^p, \qquad p \geq 0 \\ B(z) &= B_1 z + B_2 z^2 + \cdots + B_q z^q, \qquad q \geq 1 \\ C(z) &= I + C_1 z + \cdots + C_r z^r, \qquad r \geq 0 \end{aligned}$$

with the known upper bound of the orders p, q, and r, and with the unknown parameter matrix

$$\theta = [-A_1 \cdots -A_p,\ B_1 \cdots B_q,\ C_1 \cdots C_r]^\tau. \tag{4}$$

The unmodeled dynamics η_n is $\mathcal{F}_n$-measurable, satisfying

$$\|\eta_n\| \leq \varepsilon \sum_{i=0}^{n} a^{n-i}(\|y_i\| + \|u_i\| + \|w_i\| + 1) \tag{5}$$

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