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Globalized BM3D using Fast Eigenvalue Filtering

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Outline

- Image denoising
- Previous method
  - Improving method by eigenvalue filtering for denoising
  - Eigenvalue filtering using Chebyshev polynomial approximation
  - BM3D
- Proposed method
- Evaluation
- Conclusion

Image Denoising

Image denoising: estimating the true image from the observed image

Observation model

noise $\mathbf{n} \sim \mathcal{N}(0, \sigma^2)$

$$\mathbf{z} = \mathbf{y} + \mathbf{n}$$

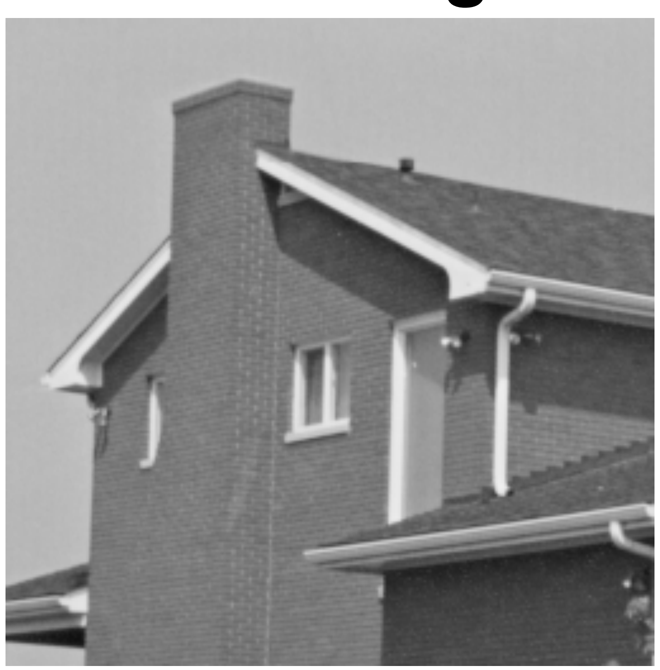
$\mathbf{z} \in \mathbb{R}^N$: Observed image

$\mathbf{y} \in \mathbb{R}^N$: True image

$\mathbf{n} \in \mathbb{R}^N$: Noise signal

N : The number of pixels

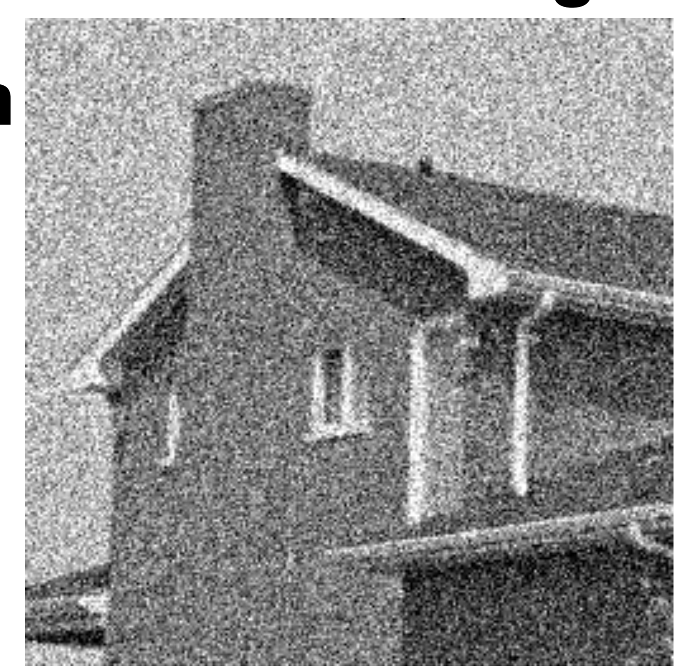
True image



Noise contamination

Denoising

Observed image



Filter Matrix and Its Decomposition

- **Denoising methods can be expressed as** $\mathbf{W} \in \mathbb{R}^{N \times N}$

  Ex.) Gaussian Filter, Bilateral Filter, Non-local means

  Restored image

$$\hat{\mathbf{y}} = \mathbf{W}\mathbf{z}$$

- **The filter matrix is decomposed as**

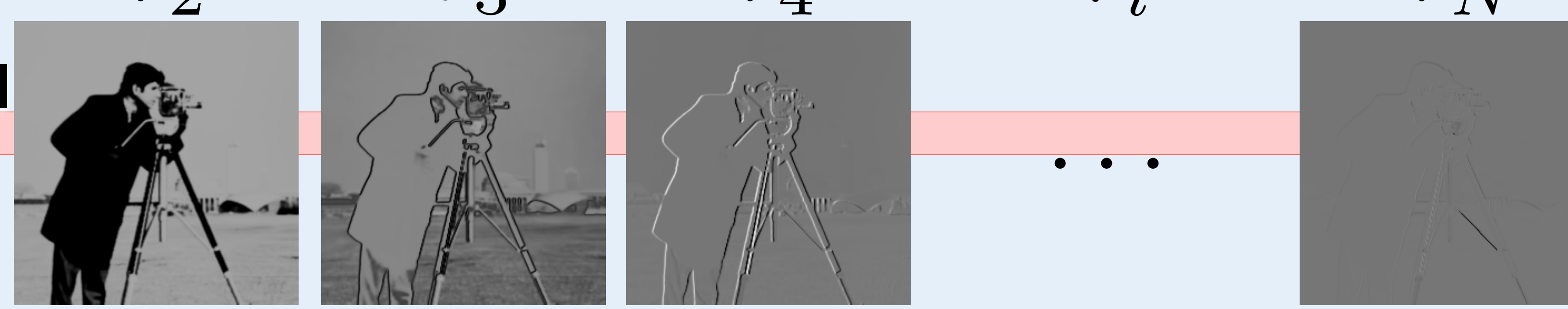
$$\mathbf{W} = \mathbf{V}\mathbf{S}\mathbf{V}^{-1}$$

Eigenvalue matrix
$\mathbf{S} = \mathrm{diag}[\lambda_1 \cdots \lambda_N]$
Eigenvector matrix
$\mathbf{V} = [\mathbf{v}_1 \cdots \mathbf{v}_N]$

Eigenvalue **Large** λ_2 λ_3 λ_4 $\cdots$ λ_i $\cdots$ λ_N **Small**

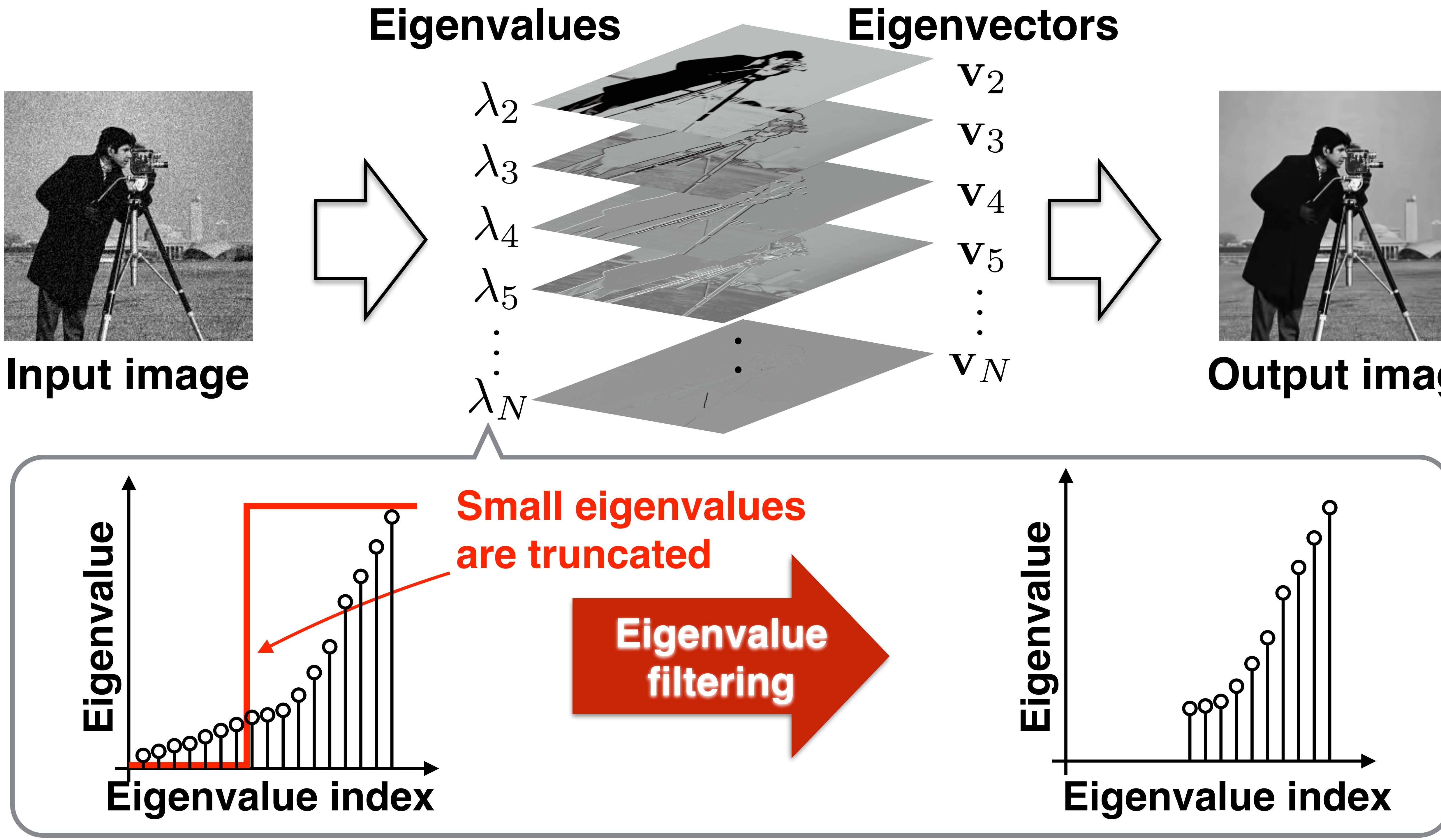
$\mathbf{v}_2$ $\mathbf{v}_3$ $\mathbf{v}_4$ $\cdots$ $\mathbf{v}_i$ $\cdots$ $\mathbf{v}_N$

Elements of eigenvector **Oscillated slowly**



Oscillated rapidly

Eigenvalue Filtering for the Filter Matrix



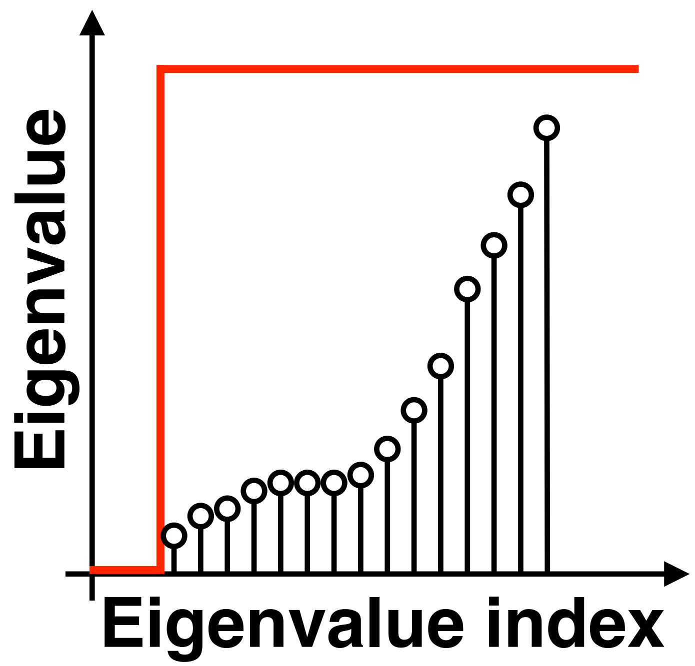
→ **The restored image becomes smoother**

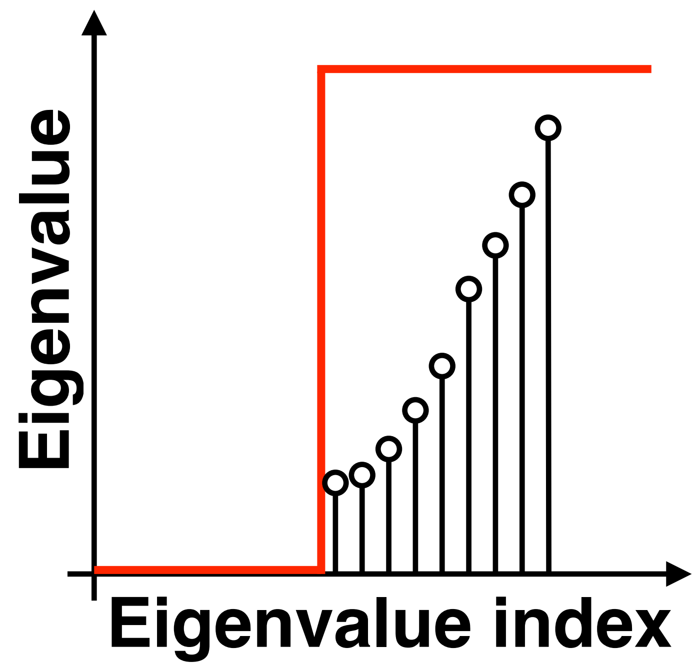
Eigenvalue Filtering for the Filter Matrix

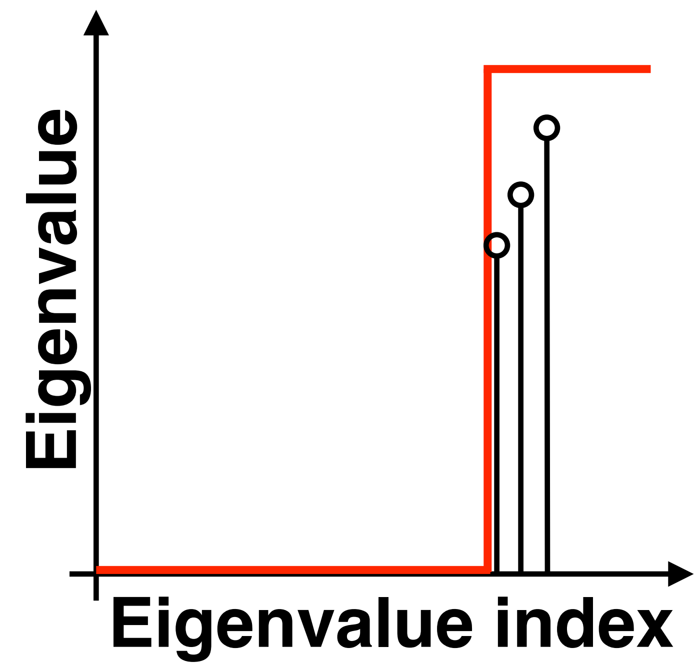
Eigenvalue filtering

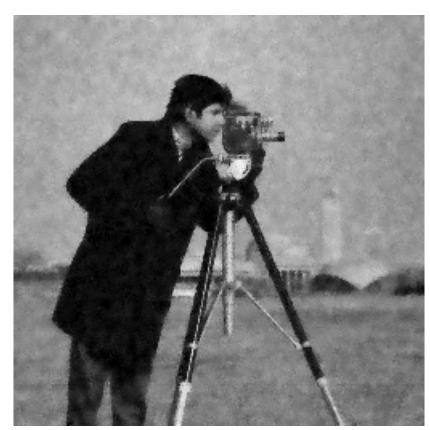
$$\mathcal{H}(\mathbf{W}) = \mathbf{V}\mathrm{diag}(h(\lambda_1), \cdots, h(\lambda_i), \cdots, h(\lambda_N))\mathbf{V}^{-1}$$

$h(\cdot)$: Arbitrary filter kernel



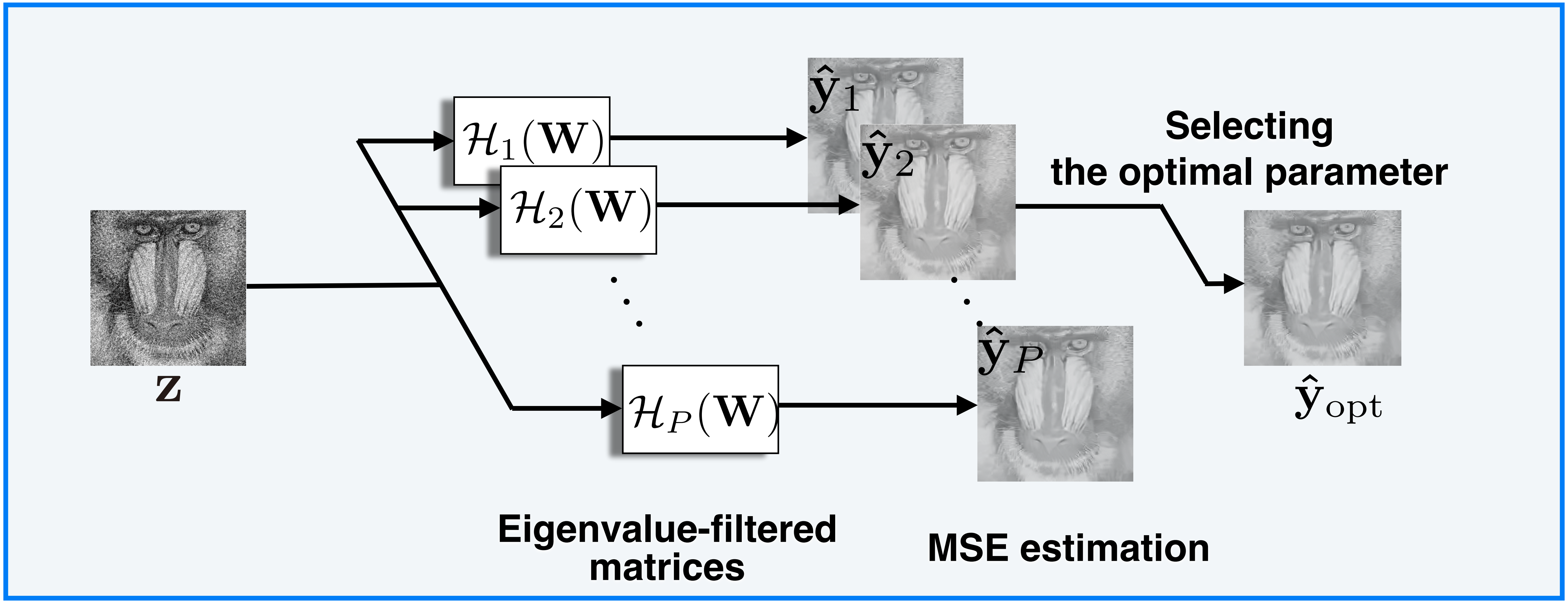






- **The smoothing strength is controlled according to the filter kernel**

Parameter Selection of Eigenvalue Filtering



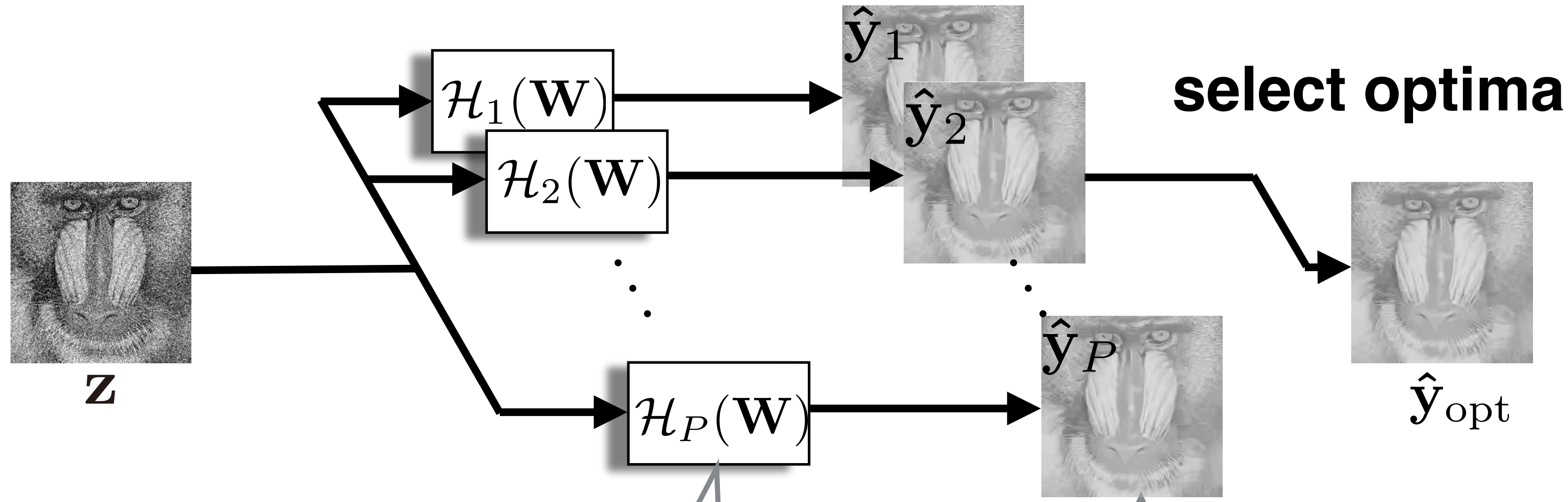
I. Perform eigenvalue filtering using various filter kernels controlled by the parameter

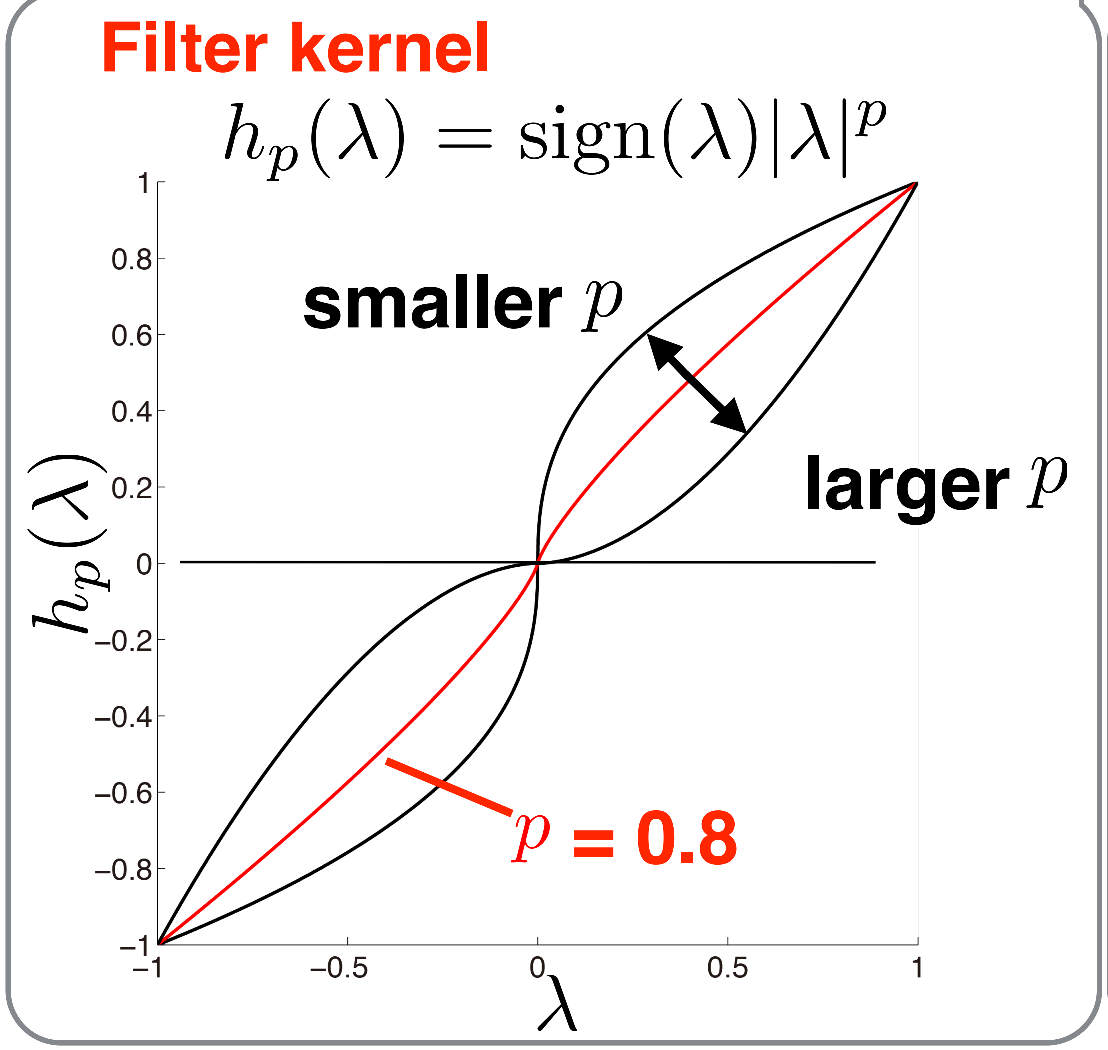
II. Obtain restored images using each eigenvalue-filtered matrices

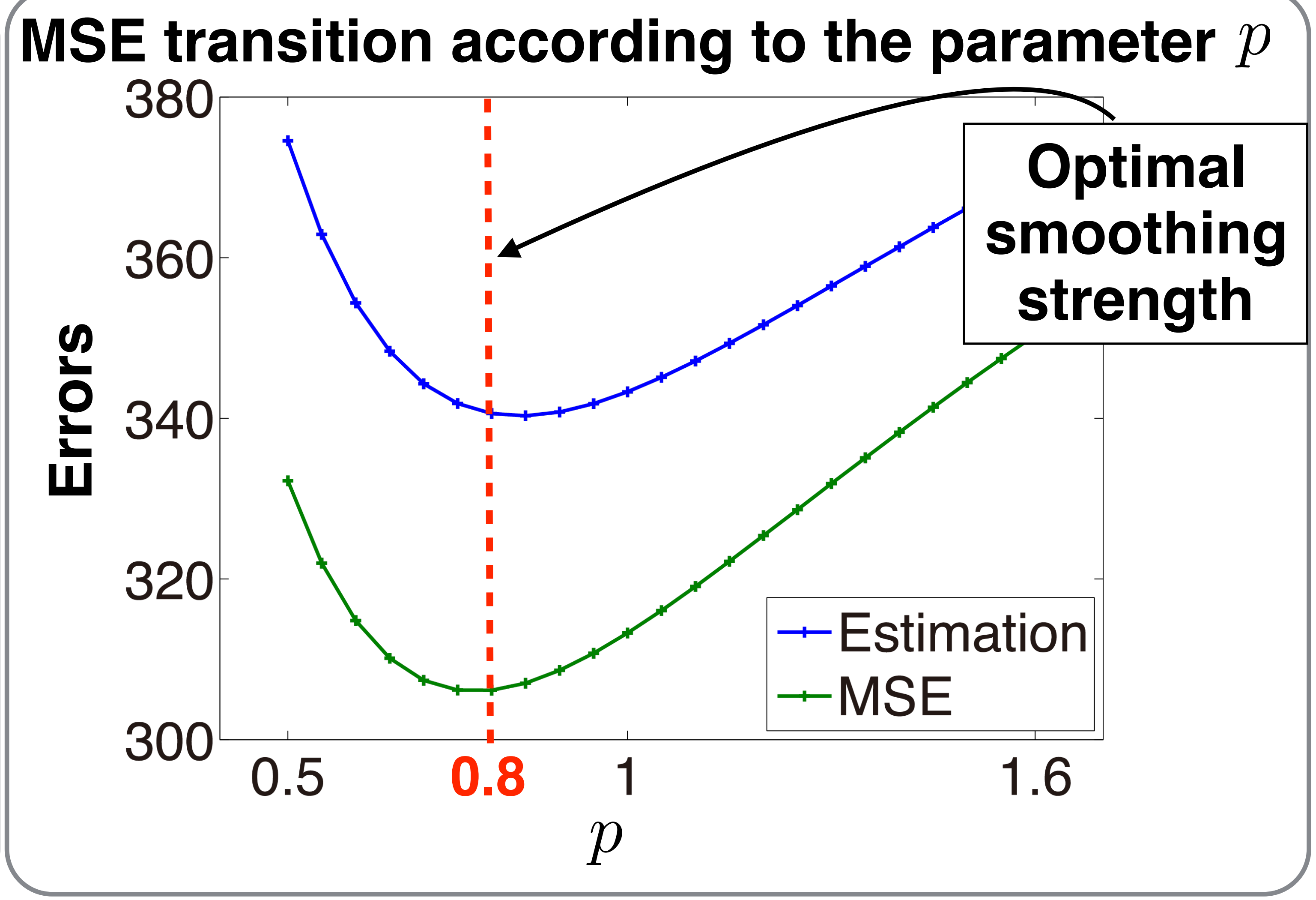
III. Estimate MSEs of each restored image

IV. Select an optimal output (an image having minimum MSE)

Improving Method by Eigenvalue Filtering







Approximation of Filter Kernels by CPA

- **Eigendecomposition takes much computational cost**

➡ **Eigenvalue filtering by Chebyshev polynomial approximation(CPA)** [1]

CPA for scalar function

$$h(y) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty} c_k T_k(y)$$

$h(\cdot)$: Arbitrary function

Chebyshev polynomial

$$T_k(y) = \cos(k \arccos(y))$$

Chebyshev coefficient

$$c_k = \frac{2}{\pi}\int_{-1}^{1} \frac{T_k(y)h(y)}{\sqrt{1-y^2}}\, dy = \frac{2}{\pi}\int_{0}^{\pi} \cos(k\theta)h(\cos\theta)\, d\theta$$

Chebyshev polynomials are obtained by recurrence relation

Recurrence relation	$T_k(y) = 2yT_{k-1}(y) - T_{k-2}(y)$
Initial conditions	$T_0(y) = 1,\ T_1(y) = y$

[1] M. Onuki, S. Ono, K. Shirai, and Y. Tanaka, "Non-local/local image filters using fast eigenvalue filtering," in *Proc. ICIP*, 2015.

Eigenvalue Filtering by CPA

- **Eigenvalue filtering can be realized without eigendecomposition**

$$\hat{\mathbf{y}} = \mathcal{H}(\mathbf{W})\mathbf{z} = \left(\frac{1}{2}c_0\mathbf{I} + \sum_{k=1}^{d} c_k \mathcal{T}_k(\mathbf{W})\right)\mathbf{z}$$

CPA for a filter matrix

$$\mathcal{H}(\mathbf{W}) = \frac{1}{2}c_0\mathbf{I} + \sum_{k=1}^{\infty} c_k \mathcal{T}_k(\mathbf{W})$$

Chebyshev polynomial

$$\mathcal{T}_k(\mathbf{W}) = \mathbf{V}\mathrm{diag}(\cos k\theta_1, \ldots, \cos k\theta_i, \ldots, \cos k\theta_N)\mathbf{V}^{-1}$$

Chebyshev coefficient

$$c_k = \frac{2}{\pi}\int_0^{\pi} \cos(k\theta) h(\cos\theta)\, d\theta$$

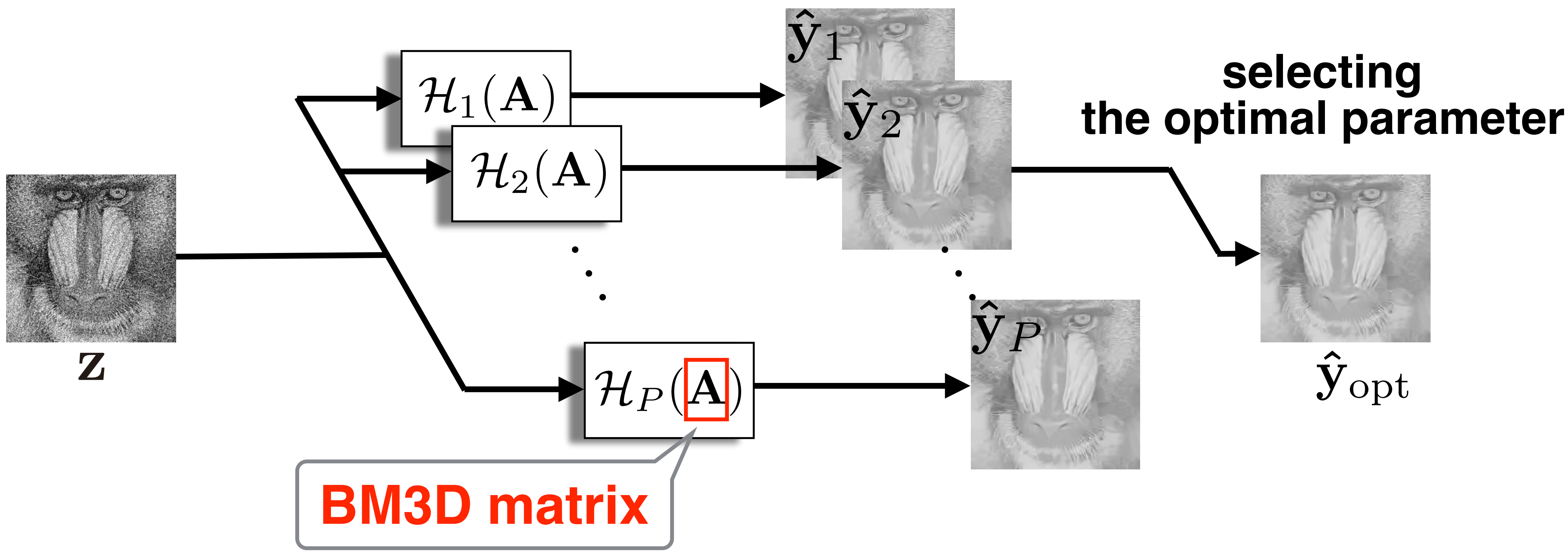
$h(\cdot)$: Arbitrary function

Recurrence relation	$\mathcal{T}_k(\mathbf{W}) = 2\mathbf{W}\mathcal{T}_{k-1}(\mathbf{W}) - \mathcal{T}_{k-2}(\mathbf{W})$
Initial conditions	$\mathcal{T}_0(\mathbf{W}) = \mathbf{I},\ \mathcal{T}_1(\mathbf{W}) = \mathbf{W}$

Purpose of Proposed Method

Purpose **Applying eigenvalue filtering to state-of-the-art methods**

I.e.) BM3D [2]



▶▶ **Next topic**

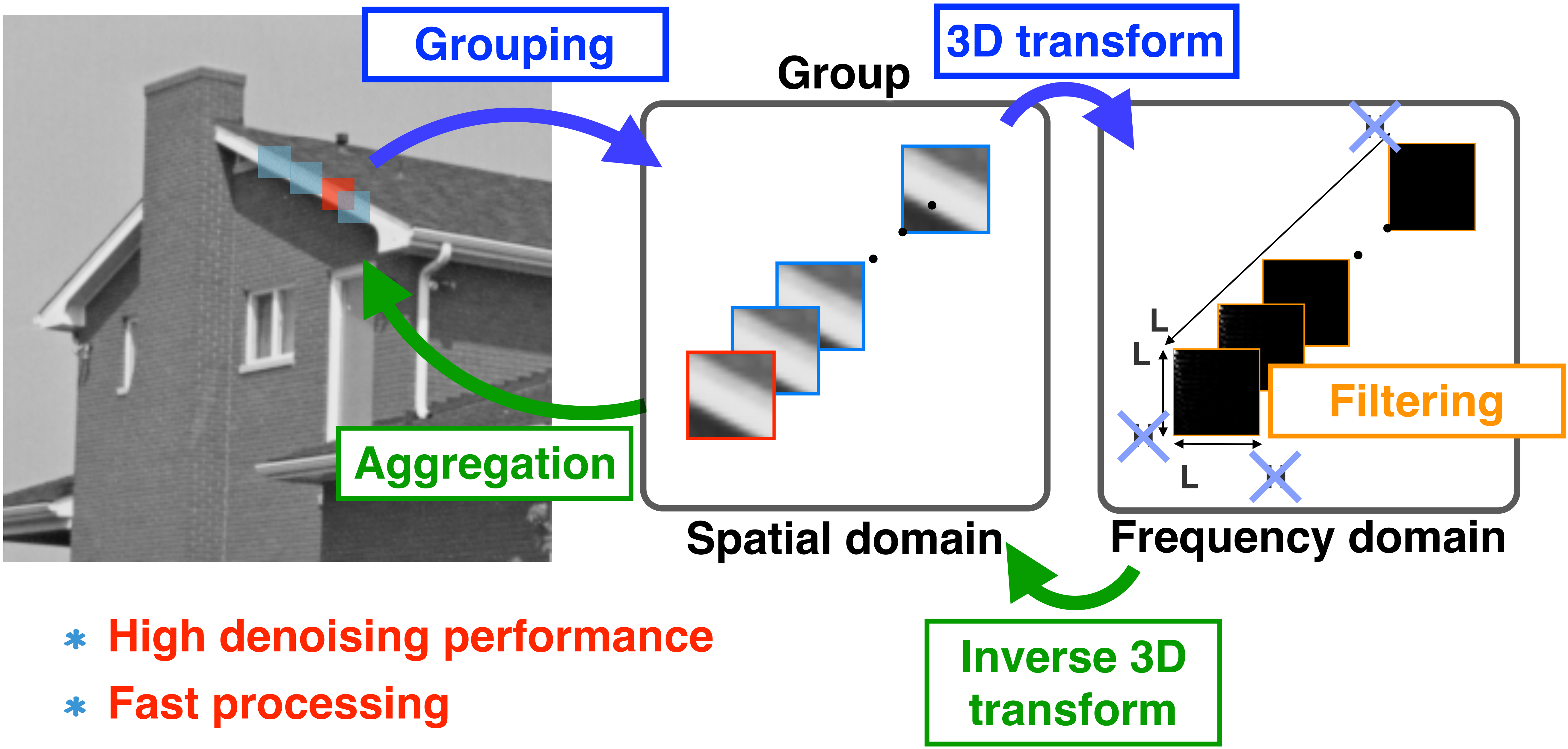
- **BM3D algorithm and its matrix representation**
- **Problem of matrix construction**
- **Solution (Proposed method)**

[2] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-D transform-domain collaborative filtering", *IEEE Trans. Image Process.*, vol. 16, no. 8, pp. 2080–2095, Aug. 2007.

BM3D Algorithm

Block Matching and 3D Filtering (BM3D):

Redundant filtering using similarity among blocks



- **High denoising performance**
- **Fast processing**

Matrix Construction and its problem

- **BM3D is expressed as a filter matrix**

$$\hat{\mathbf{y}} = \mathcal{F}_{\mathrm{BM3D}}(\mathbf{z}) = \mathbf{\Psi\Gamma\Phi z} = \mathbf{Az}$$

- **Construction of $\mathbf{\Phi}$ and $\mathbf{\Psi}$ needs much computational cost**

  Ex.) Image with 1024×1024 pixels

$$\underset{\text{BM3D matrix}}{\boxed{\mathbf{A}}} = \underset{\text{Synthesis}}{\boxed{\mathbf{\Psi}}}\ \underset{\text{Shrinkage}}{\boxed{\mathbf{\Gamma}}}\ \underset{\text{Analysis}}{\boxed{\mathbf{\Phi}}}$$

1 million (columns of $\mathbf{\Phi}$), **12 million** (rows of $\mathbf{\Phi}$)

Synthesis: **Inverse 3D transform**, **Aggregation**

Shrinkage: **Filtering**

Analysis: **Grouping**, **3D transform**

→ **It is hard to construct the BM3D matrix**

Proposed Method

Restored image using eigenvalue filtering by CPA

$$\hat{\mathbf{y}}_p = \mathcal{H}_p(\mathbf{A})\mathbf{z} = \left(\frac{1}{2}c_0\mathbf{I} + \sum_{k=1}^{d} c_k \mathcal{T}_k(\mathbf{A})\right)\mathbf{z} = \frac{1}{2}c_0\mathbf{z} + \sum_{k=1}^{d} c_k \boxed{\mathcal{T}_k(\mathbf{A})\mathbf{z}}$$

Previous method

$$\mathcal{T}_k(\mathbf{A})\mathbf{z} = \underline{2\mathbf{A}\mathcal{T}_{k-1}(\mathbf{A})\mathbf{z}} - \mathcal{T}_{k-2}(\mathbf{A})\mathbf{z}$$

$$\mathcal{T}_0(\mathbf{A})\mathbf{z} = \mathbf{z}\ , \qquad \mathcal{T}_1(\mathbf{A})\mathbf{z} = \underline{\mathbf{A}\mathbf{z}}$$

$$\mathcal{B}_k(\mathbf{z}) = \mathcal{T}_k(\mathbf{A})\mathbf{z}$$

Replacing

$$\mathcal{F}_{\mathrm{BM3D}}(\mathbf{z}) = \mathbf{A}\mathbf{z}$$

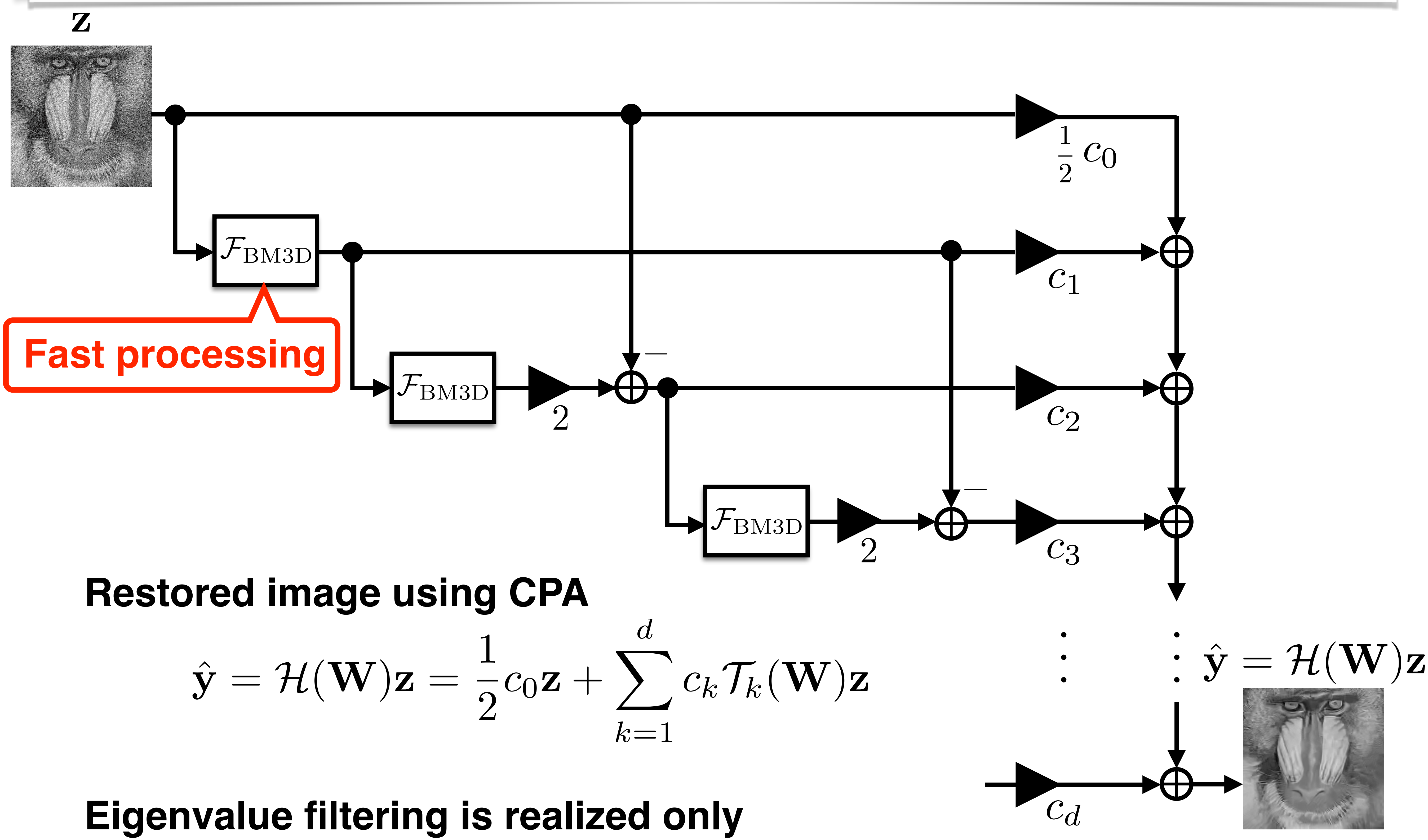
Proposed method

$$\mathcal{T}_k(\mathbf{A})\mathbf{z} \simeq \mathcal{B}_k(\mathbf{z}) = \underline{2\mathcal{F}_{\mathrm{BM3D}}(\mathcal{B}_{k-1}(\mathbf{z}))} - \mathcal{B}_{k-2}(\mathbf{z})$$

$$\mathcal{T}_0(\mathbf{A})\mathbf{z} = \mathcal{B}_0(\mathbf{z}) = \mathbf{z}\ , \quad \mathcal{T}_1(\mathbf{A})\mathbf{z} = \mathcal{B}_1(\mathbf{z}) = \underline{\mathcal{F}_{\mathrm{BM3D}}(\mathbf{z})}$$

→ **Matrix construction is not required**

Fast Eigenvalue Filtering



Restored image using CPA

$$\hat{\mathbf{y}} = \mathcal{H}(\mathbf{W})\mathbf{z} = \frac{1}{2}c_0\mathbf{z} + \sum_{k=1}^{d} c_k \mathcal{T}_k(\mathbf{W})\mathbf{z}$$

Eigenvalue filtering is realized only by using BM3D operators and Chebyshev coefficients

Eigenvalue distribution on each step

Problem：Input-dependency of the BM3D

CPA: $\mathbf{A}$ **must be fixed regardless of the degree of polynomials**

$$\mathcal{T}_k(\mathbf{A})\mathbf{z} = 2\mathbf{A}\mathcal{T}_{k-1}(\mathbf{A})\mathbf{z} - \mathcal{T}_{k-2}(\mathbf{A})\mathbf{z}$$

Needs verification

BM3D: $\mathcal{F}_{\mathrm{BM3D}}$ **is adaptive to the input image**

$$\mathcal{T}_k(\mathbf{A})\mathbf{z} = 2\mathcal{F}_{\mathrm{BM3D}}(\mathcal{T}_{k-1}(\mathbf{A})\mathbf{z}) - \mathcal{T}_{k-2}(\mathbf{A})\mathbf{z}$$

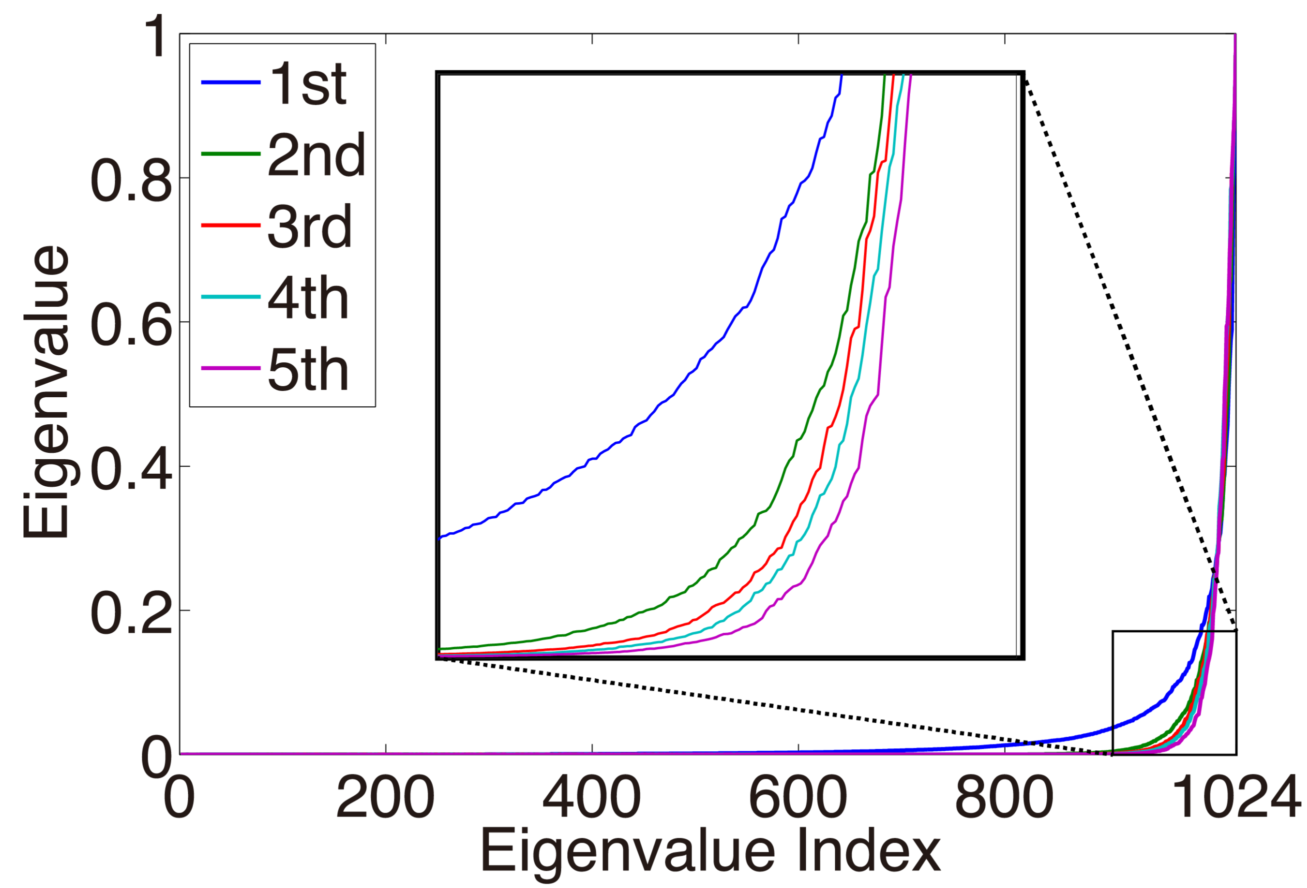
Due to Block matching and filter coefficients

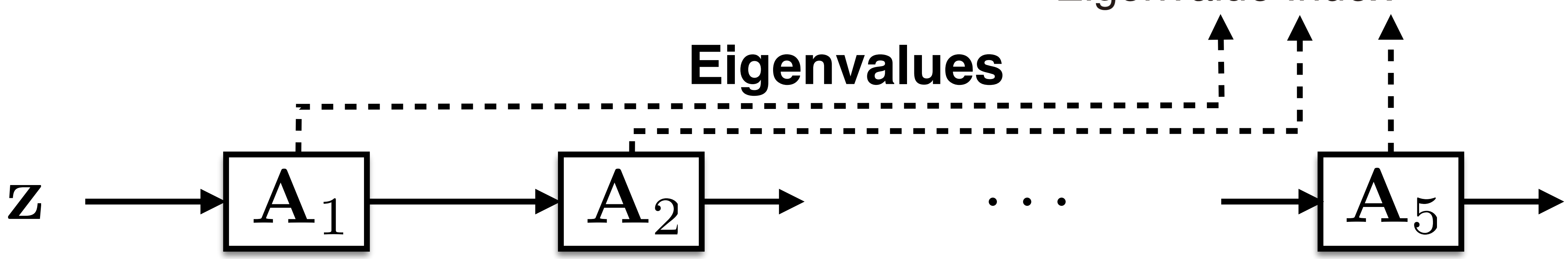
Verification Experiment

- **Verify eigenvalue distributions according to iteration numbers**

Test sub-image $\mathbf{z}$

Mandrill (32×32)

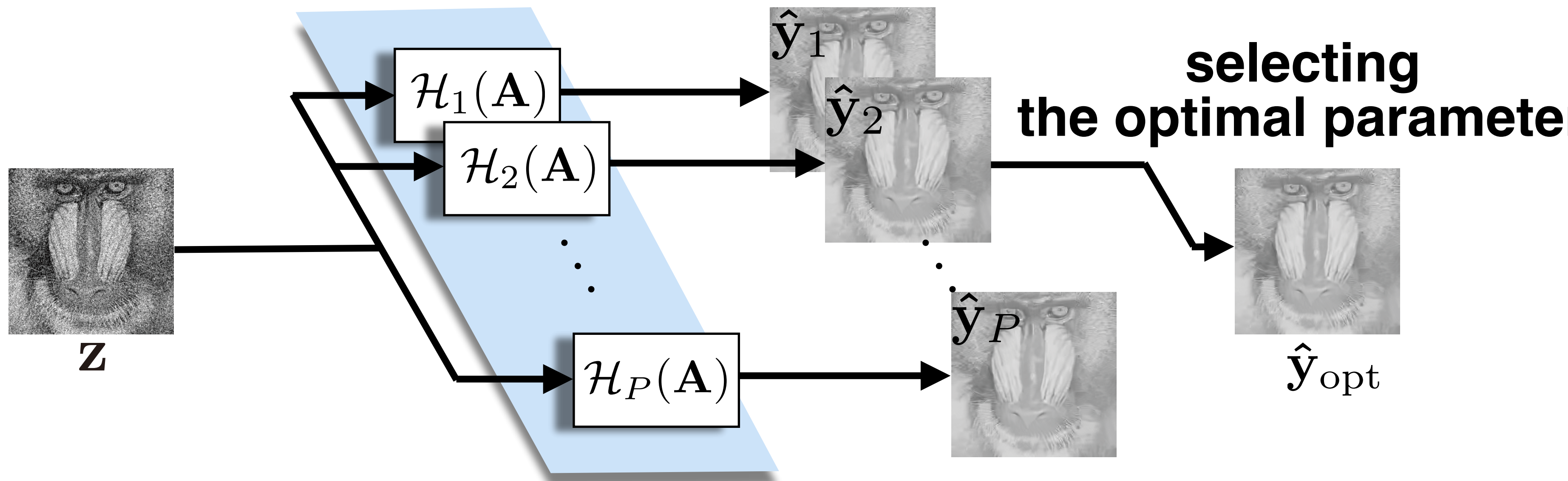




$\mathbf{A}_n$: n th matrix of BM3D given by $\mathcal{T}_{n-1}(\mathbf{A})\mathbf{z}$

- **Eigenvalue distributions could be assumed to be consistent regardless of the iteration number**

Summary of Proposed Method



Eigenvalue filtering by CPA

$$\mathcal{H}(\mathbf{A}) = \frac{1}{2} c_0 \mathbf{I} + \sum_{k=1}^{d} c_k \mathcal{T}_k(\mathbf{A})$$

$$\mathcal{F}_{\mathrm{BM3D}}(\mathbf{z}) = \mathbf{A}\mathbf{z}$$

Previous method

$$\mathcal{T}_k(\mathbf{A}) = 2\mathbf{A}\mathcal{T}_{k-1}(\mathbf{A}) - \mathcal{T}_{k-2}(\mathbf{A})$$
$$\mathcal{T}_0(\mathbf{A}) = \mathbf{I} \ , \quad \mathcal{T}_1(\mathbf{A}) = \mathbf{A}$$

Proposed method

$$\mathcal{B}_k(\mathbf{z}) = 2\mathcal{F}_{\mathrm{BM3D}}(\mathcal{B}_{k-1}(\mathbf{z})) - \mathcal{B}_{k-2}(\mathbf{z})$$
$$\mathcal{B}_0(\mathbf{z}) = \mathbf{z} \ , \quad \mathcal{B}_1(\mathbf{z}) = \mathcal{F}_{\mathrm{BM3D}}(\mathbf{z})$$

Experiment

Denoising performance assessment

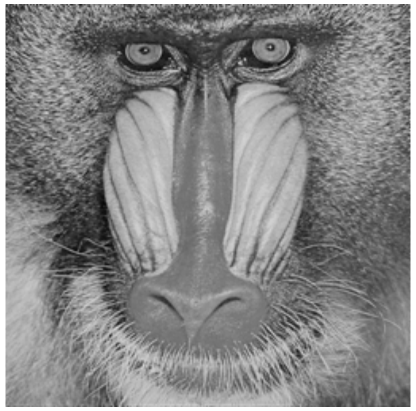
- **Comparison** — **BM3D, Global Image Denoising(GLIDE) [3]**
  - **GLIDE : Improving method by eigenvalue filtering**
- **Test images** — ***Bridge*, *Mandrill*, *Goldhill*, *Building***
- **Noise strength** — $\sigma \in \{10, 20, 30, 40, 50\}$
- **Measure** — **PSNR, SSIM**

Conditions

- **Intel Xeon E5-2690 2.9GHz CPU**
- **62.9 GB RAM**
- **12 core parallel computing**

Bridge

Mandrill



Goldhill

Building

[3] H. Talebi and P. Milanfar, “Global image denoising,” *IEEE Trans. Image Process.*, vol. 23, no. 2, pp. 755–768, Feb. 2014.

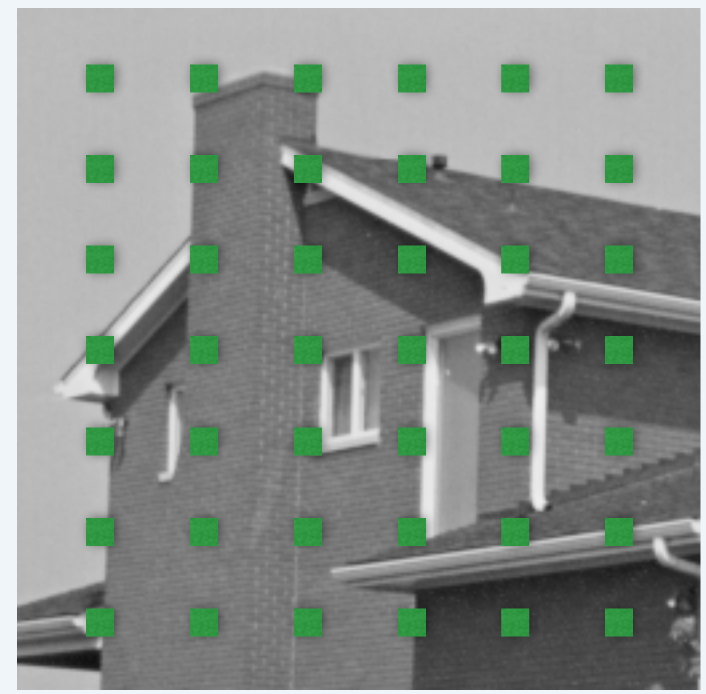
Experiment

Global Image Denoising (GLIDE)

estimate eigenvalue/eigenvector from a portion of a pre-filtered image

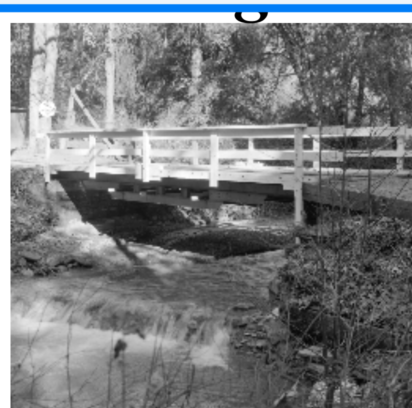
Pre-filtering

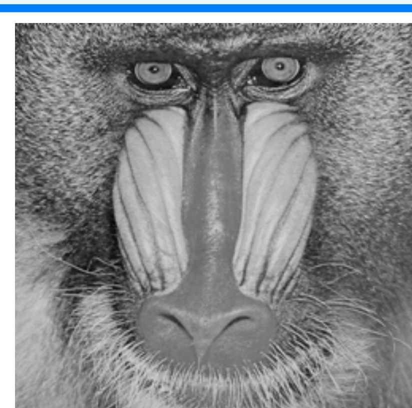
Sampling

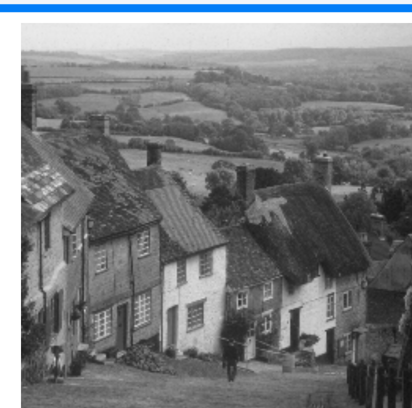


Advantage **Fast processing**

Disadvantage **Eigenvalue filtering can not be performed exactly**







[3] H. Talebi and P. Milanfar, "Global image denoising," *IEEE Trans. Image Process.*, vol. 23, no. 2, pp. 755–768, Feb. 2014.

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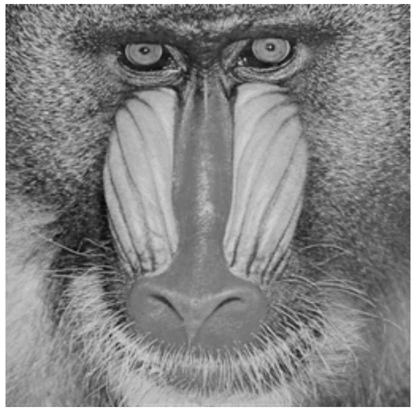
Mandrill



Goldhill

Building

[3] H. Talebi and P. Milanfar, “Global image denoising,” *IEEE Trans. Image Process.*, vol. 23, no. 2, pp. 755–768, Feb. 2014.

Performance Comparison

σ	Method	*Bridge*	*Mandrill*	*Goldhill*	*Building*
10	BM3D	29.84 / 0.911	30.56 / 0.905	31.80 / 0.880	**33.16** / **0.939**
	GLIDE	29.81 / **0.913**	30.54 / 0.904	31.72 / 0.881	32.91 / 0.938
	Proposed	**29.86** / **0.913**	**30.57** / **0.906**	**31.86** / **0.884**	**33.16** / **0.939**
20	BM3D	25.46 / 0.765	26.39 / 0.773	28.50 / 0.775	29.35 / 0.862
	GLIDE	25.62 / 0.784	26.55 / 0.788	28.57 / **0.785**	29.30 / 0.865
	Proposed	**24.66** / **0.789**	**26.56** / **0.791**	**28.59** / 0.784	**29.40** / **0.866**
30	BM3D	23.55 / 0.647	24.33 / 0.651	26.91 / 0.706	27.32 / 0.790
	GLIDE	23.68 / 0.678	24.57 / 0.686	26.71 / 0.711	27.26 / 0.792
	Proposed	**23.73** / **0.679**	**24.58** / **0.689**	**26.96** / **0.714**	**27.37** / **0.794**
40	BM3D	22.51 / 0.572	23.10 / 0.558	**25.84** / 0.654	25.89 / 0.722
	GLIDE	22.43 / 0.584	**23.23** / 0.573	25.70 / 0.640	25.87 / **0.729**
	Proposed	**22.55** / **0.586**	23.19 / **0.582**	25.83 / **0.655**	**25.90** / 0.724
50	BM3D	21.81 / 0.509	22.43 / 0.489	**25.04** / 0.610	24.93 / 0.663
	GLIDE	21.81 / **0.547**	**22.60** / 0.518	25.01 / **0.616**	24.85 / **0.680**
	Proposed	**21.93** / 0.540	22.59 / **0.525**	**25.04** / 0.615	**24.95** / 0.673

PSNR[dB] / SSIM

Original image

GLIDE

PSNR 22.43[dB]
SSIM 0.584

BM3D

PSNR 22.53[dB]
SSIM 0.571

Proposed

PSNR 22.71[dB]
SSIM 0.604

Bridge $\sigma = 40$

Visual Assessment

Original image

BM3D

BM3D	GLIDE	Proposed
22.53[dB] / 0.571	22.43[dB] / 0.584	22.71[dB] / 0.604

Visual Assessment

Original image

GLIDE

BM3D

22.53[dB] / 0.571

GLIDE

22.43[dB] / 0.584

Proposed

22.71[dB] / 0.604

Visual Assessment

Original image

Proposed

BM3D	GLIDE	Proposed
22.53[dB] / 0.571	22.43[dB] / 0.584	22.71[dB] / 0.604

Execution Time

Image size	BM3D	GLIDE	Proposed
256x256	0.8	115.4	51.8
512x512	3.1	Out of Memory	225.1
1024x1024	18.1	Out of Memory	946.4

[sec]

* Faster than GLIDE
* Can be executed in commodity computers

Conditions

Intel Xeon E5-2690 2.9GHz CPU
62.9 GB RAM
12 core parallel computing

Conclusion

- **Purpose**
  - **Improvement of denoising performance for BM3D**
- **Method**
  - **Eigenvalue filtering by CPA without matrix construction**
- **Result**
  - **Better denoising performance visually and numerically**
  - **Faster execution than GLIDE**
- **Future work**
  - **Improvement of MSE estimation**

Reference List

- **Eigenvalue filtering using CPA**

  M. Onuki, S. Ono, K. Shirai, and Y. Tanaka, "Non-local/local image filters using fast eigenvalue filtering," in *Proc. ICIP*, 2015.

- **BM3D**

  K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-D transform-domain collaborative filtering", *IEEE Trans. Image Process.*, vol. 16, no. 8, pp. 2080–2095, Aug. 2007.

- **Global image denoising**

  H. Talebi and P. Milanfar, "Global image denoising," *IEEE Trans. Image Process.*, vol. 23, no. 2, pp. 755–768, Feb. 2014.