Ann. Data. Sci. (2015) 2(4):439–451
DOI 10.1007/s40745-015-0052-x

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Goal-Programming-Based Procedure for Calculating Positive Multipliers Under a Multiple Criteria Data Envelopment Analysis Framework: An Application to UEFA EURO 2012

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Received: 25 November 2015 / Revised: 25 November 2015 / Accepted: 27 November 2015 / Published online: 14 December 2015

Abstract One of the motivations for the arise of the multiple criteria data envelopment analysis (MCDEA) model was the need to yield more reasonable input-output multipliers than those derived from standard data envelopment analysis (DEA), without using priori information. The problem of unreasonable multipliers occurs when some production units are efficient in standard DEA simply because the optimization problem allows those units to select few inputs/outputs to attach positive multipliers, discarding all of the others. Notwithstanding, MCDEA may fail in providing multipliers schemes free of non-null values. Therefore, in this paper, we propose an alternative procedure, based on goal programming, for calculating positive multipliers within a MCDEA framework. This procedure is applied to a previously reported problem, concerning the performance evaluation of national teams that participated in the 2012 UEFA European Football Championship (UEFA EURO 2012), where the MCDEA model has not succeeded in providing strictly positive multipliers schemes. The results derived by the proposed procedure indicate that, to assure non-null multipliers, it is necessary a mild detachment of non-dominated solutions.

Keywords Multiple criteria data envelopment analysis · Multipliers schemes · Goal programming · Sports · football · UEFA EURO

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1 Introduction

Data envelopment analysis (DEA) [1] is a non-parametric method based on mathematical programming for measuring relative efficiency of production units, referred as decision-making units (DMUs).

DEA calculates relative efficiency for each DMU as the weighted sum of its outputs divided by the weighted sum of its inputs, on a bounded ratio scale.

Notwithstanding, DEA presents some widely reported inconveniences, amongst which we may highlight [2]:

(i) lack of discrimination among efficient DMUs, which occurs when the number of evaluated DMUs is small in comparison with the total number of variables (inputs and outputs) used in the evaluation; and
(ii) inadequacy of weighting coefficients (hereinafter, called multipliers), derived from multipliers DEA optimization problem, which could be unreasonable, for example, in situations where large multipliers are attached to variables of less importance and small (or even null) multipliers to important variables.

Problems (i) and (ii) are intertwined because an excessive number of variables compared to the number of evaluated DMUs allows each DMU to select few variables to which they attach positive multipliers, discarding all the others, to maximize its own relative efficiency.

To address these two problems, Li and Reeves [3] developed a multiple objective linear programming approach, called multiple criteria data envelopment analysis (MCDEA). However, there are cases in which MCDEA may not succeed in providing multipliers schemes free of non-null values (see, e.g., [4]).

For that reason, the objective of this paper is to propose an alternative procedure, based on goal programming [5], for calculating strictly positive multipliers within a MCDEA framework.

The proposed procedure is applied to a problem previously reported by Rubem and Brandão [4], concerning performance evaluation of national teams that participated in 2012 UEFA European Football Championship (UEFA EURO 2012).

The remainder of this paper is organized as follows: in Sect. 2, we address multipliers flexibility in DEA, and review main approaches used to avoid the inconvenience this flexibility may generate. Section 3 describes the MCDEA model used in this analysis and UEFA EURO 2012 performance evaluation to be further investigated. In Sect. 4, we present the goal programming procedure here developed to search for non-null multipliers and apply it to the aforementioned MCDEA-based performance evaluation problem. Finally, the last section presents some conclusions and suggestions for future work.

2 Multipliers Flexibility in DEA

Flexibility of multipliers has been regarded as one of the major advantages of DEA. Such characteristic allows, for instance, the identification of inefficient DMUs that have low performance, even when selecting their best feasible multipliers [2].

Nevertheless, total flexibility of multipliers derived from DEA's optimization problem may bring an inconvenience: important factors could be neglected in the analysis. This happens when multipliers of some inputs/outputs are null or contradict a priori knowledge [6], as mentioned in Sect. 1.

This inconvenience of DEA is addressed in the literature, mainly, by multipliers restrictions methods. The main purpose of these methods is to establish bounds within which multipliers may vary, preserving some flexibility [2]. The works of Allen et al. [6] and Pedraja-Chaparro et al. [7] provide a comprehensive revision on the incorporation of value judgements for restricting flexibility of DEA multipliers, some of which are mentioned hereinafter.

Dyson and Thanassoulis [8] developed an approach, further generalized by [9], which imposes direct numerical bounds on multipliers. These bounds depend on context and information provided by an expert. Such bounds shall be defined after analysing the multipliers derived from the original DEA problem. Thus, a standard DEA model must be run to determine multipliers ranges for each input and output, because the method in [8] is highly dependent on those ranges. Only after the analysis of multipliers for all inputs and outputs and all DMUs, restrictions could be introduced. In case of unfeasible solutions, restrictions shall be relaxed until unfeasibility disappears.

Charnes et al. [10] proposed a 'cone-ratio' DEA model in an attempt to restrict multiplier flexibility directly in multipliers space. For that, relative ordination of inputs and outputs is incorporated in the analysis, to allow better multipliers and, thus, efficiency results become more consistent with a priori knowledge.

Thompson et al. [11] developed the concept of 'assurance region', introducing homogeneous linear restrictions for multipliers in their domain. Such approach allows successive increment of an assurance region, until efficiency levels meet expert's preferences.

Wong and Beasley [12] provided a multiplier restriction method by setting bounds on proportions of individual inputs (or outputs) to total input (or output).

All aforementioned methods require priori information and involve subjective value judgment [2,3]. Such methods present some disadvantages, related to subjectivity [2]:

(i) value judgements or priori information can be incorrect or biased, or the ideas may not be consistent with reality; and
(ii) there may be a lack of consensus among experts or decision-makers, and this can slow down or adversely affect analysis.

In fact, preserving the DEA's spirit, in the sense of not including priori information, may be desirable. Within this context, Li and Reeves [3] departed from a multiple objective perspective to introduce MCDEA, where one of the purposes was to yield multipliers that are more reasonable, without requiring priori information.

However, the literature reports cases in which MCDEA fails in providing strictly positive multipliers schemes. This condition may sometimes lead to situations where all DMUs neglect the same input/output in the analysis (see, e.g., [4]).

To deal with such situations, and based on possible non-uniqueness of multipliers schemes [2], in this paper, we propose the usage of goal programming [5], in search for possible positive multipliers that correspond to MCDEA's non-dominated solutions.

3 MCDEA

MCDEA models [3] incorporate two additional objective functions to standard DEA multipliers optimization problems. In most cases, no single optimal solution satisfies all objectives simultaneously, requiring a set of non-dominated solutions. For further details on multiple objective linear programming see, for example, [13].

In (1), we present the output-oriented MCDEA model proposed by Rubem and Brandão [4], which differs from the input-oriented formulation of Li and Reeves [3]. In addition, distinctively from [3], where MCDEA model was based on CCR [1], the formulation shown in (1) is based on BCC [14], as done by [15], although still combined to input orientation.

$$\begin{aligned}
&\text{Min } d_o (\text{or Min } h_o = \sum\nolimits_{i=1}^{r} v_i x_{io} + v_*) \\
&\text{Min } M \\
&\text{Min } \sum\nolimits_{k=1}^{n} d_k \\
&\text{s.t.} \sum\nolimits_{j=1}^{s} u_j y_{jo} = 1 \\
&\sum\nolimits_{j=1}^{s} u_j y_{jk} - \sum\nolimits_{i=1}^{r} v_i x_{ik} + d_k - v_* = 0, \forall k \\
&M - d_k \geq 0, \forall k \\
&u_j, v_i, d_k \geq 0, \forall j, i \\
&v_* \in \mathcal{R}
\end{aligned} \tag{1}$$

In (1), the first objective function is the same of standard output-oriented BCC models. Nevertheless, traditionally, in MCDEA a deviation variable $d_o = (h_o - 1)$ is used in place of h_o, which originally denotes the inverse relative efficiency (i.e., $h_o = 1/Eff_o$) of the DMU under evaluation (DMU$_o$); v_i and u_j are multipliers of inputs and outputs, respectively; x_{io} and y_{jo} are inputs and outputs of DMU$_o$, respectively; and v_* represents a scale factor. DMU$_o$ is efficient if, and only if, $d_o = 0$(or $h_o = 1$).

The second objective function comprises minimization of the maximum deviation (minimax), and the third objective function represents minimization of the sum of deviations (minisum).

Variable M denotes maximum value of deviations d_k $(k = 1, \ldots, n)$, as ensured by third constraint.

It is noteworthy that, although using BCC's variable returns-to-scale assumption, negative efficiency values are avoided, due to output orientation. As reported in previous works [15–17], negative efficiency values, although implicit, usually appear when combining BCC's assumption to advanced input-oriented DEA models.

Under MCDEA's framework, a DMU is minimax efficient if, and only if, value d_o corresponding to the solution that minimizes second objective function is null. Analogously, a DMU is minisum efficient if, and only if, value d_o corresponding to the solution that minimizes third objective function is null [18,19]. Thus, when a DMU is minimax or minisum efficient, it must be necessarily efficient in standard DEA, because, by definition, both minimax and minisum efficiencies require $d_o = 0$ [3].

MCDEA's additional objectives, used to measure relative efficiency, tend to restrict efficiency results obtained by DMUs, reducing as well flexibility of multipliers in the optimization process [18,19]. However, there are cases in which MCDEA may not succeed in providing strictly positive multipliers, and this may not seem reasonable. This is the case of MCDEA analysis performed by Rubem and Brandão [4].

3.1 Illustrative Example of MCDEA-Based Analysis

Hereinafter, we reproduce the MCDEA-based performance evaluation of UEFA EURO 2012 national teams, presented by Rubem and Brandão [4], which resulted in null multipliers for the same input of all DMUs.

DMUs are sixteen national football teams engaged in the championship, and the evaluation was based on market's expectation and favouritism, to measure to which extent these two factors determine actual performance of teams.

For that purpose, two proxies were used as inputs: sum of players' market value and total points in FIFA's ranking, whose calculation was based on data available at www.financefootball.wordpress.com/ and www.fifa.com/, respectively; while the output was final tournament ranking, retrieved from www.uefa.com; which had to be converted into a cardinal scale. This conversion followed MACBETH-based [20] procedure proposed by [21].

Table 1 displays input and output data, originally reported by Rubem and Brandão [4], and reproduced here for ease of reference.

It is noteworthy that Rubem and Brandão [4] opted for using BCC's assumption because it seemed unreasonable to assume input-output proportionality and, besides that, the output variable is bounded. In addition, as teams seek to improve performance rather than reduce market value and FIFA scores, the authors used output orientation.

Results exhibited in Table 2 were originally presented by Rubem and Brandão [4], and refer to the application of the MCDEA model in (1) to the normalized data of Table 1. This normalization avoids distortions arising from different input-output ranges and allows consistent analysis of multipliers.

Figure 1, reproduced from [4], shows the weights space decomposed into indifference regions for each DMU, and, thus, the corresponding area in the indifference region for each non-dominated solution.

This area gives a clear idea of possible weights combinations for each objective function, allowing a stability evaluation for each non-dominated solution [19]. In this sense, large indifference regions indicate that the evaluation does not alter with moderate changes in weights of objective functions. Based on that, Rubem and Brandão [4] selected for each DMU the non-dominated solution that exhibited the largest indifference region. These solutions may be directly identified in Fig. 1, and their respective efficiency scores are in bold at Table 2.

With these MCDEA solutions, discrimination was improved, as shown in Rubem and Brandão [4]. Indeed, the number of efficient teams reduced from six (Spain, Italy, Czech Republic, Poland, Greece an Ireland) in standard DEA (solutions 1 in Table 2) to two (Italy and Greece) in their MCDEA approach (solutions in bold).

Nonetheless, the MCDEA-based analysis performed by Rubem and Brandão [4] did not succeed in providing better multipliers schemes, as shown in Table 3, which exhibits multipliers for the MCDEA non-dominated solution of each DMU corresponding to the largest indifference region. Table 3 was also extracted from [4].

Table 1 Data for UEFA EURO 2012 efficiency evaluation [4]

DMU	Input 1: market value (million €)	Input 2: points in FIFA ranking	Final tournament ranking	Output: M-MACBETH ranking
Spain	625	1456	1	100
Germany	475	1288	3	30
England	415	1145	5	17
Portugal	350	996	4	30
France	345	964	8	17
Netherlands	320	1234	15	10
Italy	310	977	2	54
Russia	165	975	9	10
Croatia	155	1053	10	10
Sweden	130	910	13	10
Ukraine	110	572	12	10
Czech Republic	105	771	6	17
Poland	95	518	14	10
Denmark	90	1019	11	10
Greece	85	953	7	17
Ireland	70	907	16	10

Table 2 MCDEA efficiency scores for each non-dominated solution [4]

DMU	Solution 1	Solution 2	Solution 3	DMU	Solution 1	Solution 2	Solution 3
Spain	1	**0.945180**		Croatia	**0.350741**	0.318826	
Germany	0.384146	**0.369772**		Sweden	0.424683	0.360825	**0.409836**
England	0.245192	**0.238541**		Ukraine	0.738533	0.403329	**0.473684**
Portugal	0.304365	0.284085	**0.280631**	Czech Republic	1	0.706464	**0.837897**
France	0.322025	0.287594	**0.284492**	Poland	1	0.442416	**0.536353**
Netherlands	0.180309	**0.179712**		Denmark	**0.561097**	0.457184	
Italy	**1**	1		Greece	**1**	0.804054	
Russia	**0.331614**	0.304642		Ireland	1	0.527638	**0.688073**

Actually, when comparing the results from Table 3 to the multipliers derived from standard DEA, except for Spain, the number of null multipliers for all DMUs remained unaltered or even increased [4].

Thus, using the criterion proposed by those authors, that is, choosing a preferable non-dominated solution based on its stability, the second input (points in FIFA ranking) was always disregarded by DMUs.

In the next section, we propose an alternative procedure to search for strictly positive multipliers, which shall preferably generate one of the MCDEA's non-dominated solutions. The proposed procedure will be applied to the same data analysed by Rubem and Brandão [4], in an attempt to derive non-null multipliers for the input 'points in

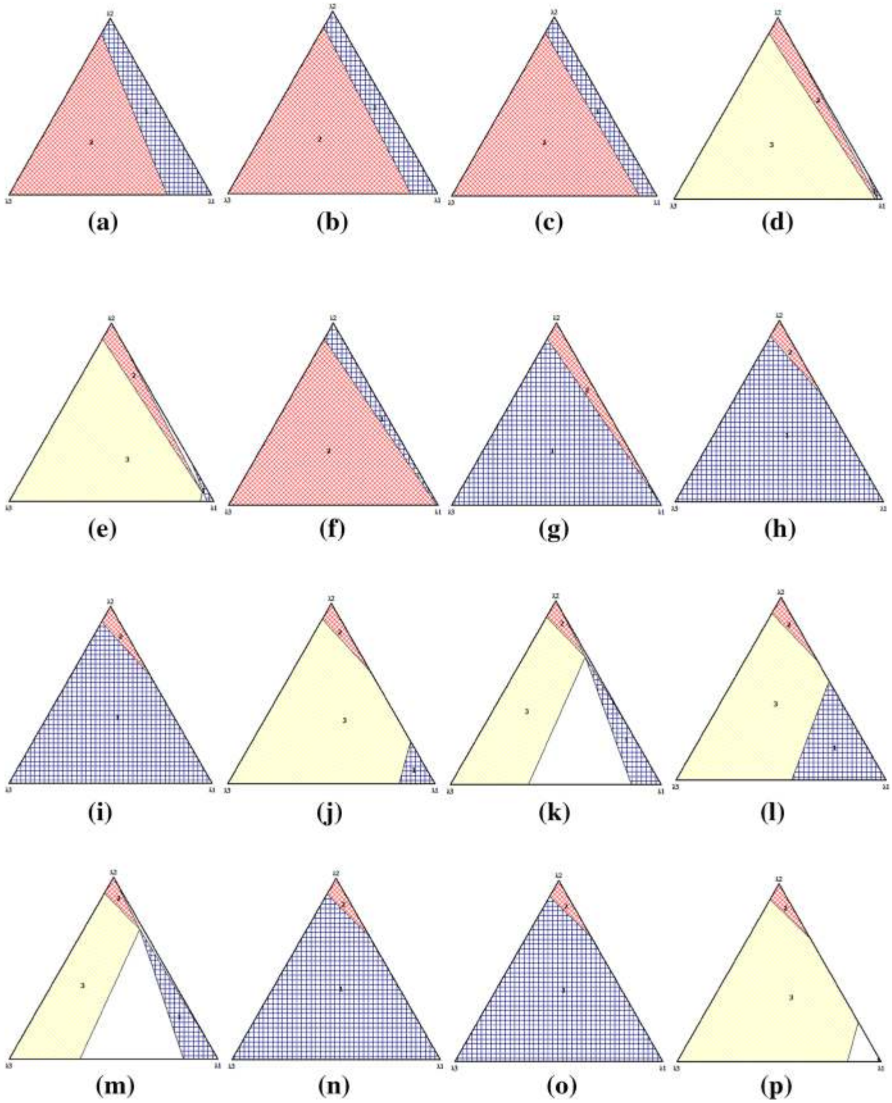

Fig. 1 **a** Spain, **b** Germany, **c** England, **d** Portugal, **e** France, **f** Netherlands, **g** Italy, **h** Russia, **i** Croatia, **j** Sweden, **k** Ukraine, **l** Czech Republic, **m** Poland, **n** Denmark, **o** Greece, **p** Ireland [4]

FIFA ranking', restricted, if possible, to MCDEA non-dominated solutions selected for DMUs in that work.

4 Goal Programming Procedure for Non-null Multipliers

In this paper, we propose the use of a goal programming procedure, in an attempt to find strictly positive multipliers, which shall preferably generate one of the MCDEA's non-dominated solutions.

Table 3 Multipliers for MCDEA non-dominated solution corresponding to largest indifference regions [4]

DMU	v_1	v_2	u_1	*Eff*	DMU	v_1	v_2	u_1	*Eff*
Spain	1.027778	0	1	0.945180	Croatia	10.27778	0	10	0.350740
Germany	3.425926	0	3.333333	0.369762	Sweden	10.27778	0	10	0.409836
England	6.045752	0	5.882353	0.238541	Ukraine	10.27778	0	10	0.473684
Portugal	6.045752	0	5.882353	0.280631	Czech Republic	6.045752	0	5.882353	0.837897
France	6.045752	0	5.882353	0.284492	Poland	10.27778	0	10	0.536353
Netherlands	10.27778	0	10	0.179712	Denmark	10.27778	0	10	0.561097
Italy	1.903292	0	1.851852	1	Greece	6.045752	0	5.882353	1
Russia	10.27778	0	10	0.331614	Ireland	10.27778	0	10	0.688073

Goal Programming (GP) was originated in the work Charnes and Cooper [5], with later developments attributed to [23–25], among others. Examples of GP applications to various areas can be seen in [26].

GP is a type of multiple objective programming, which seeks a solution that best meets aspiration levels predefined by decision maker.

In GP's basic approach, objectives of multiple objective programming problems are turned into goals, by defining a numerical aspiration level for each objective. Then, the model searches for a solution that minimizes the achievement function, which is generally represented by the weighted sum of undesirable deviations between aspiration levels and results actually achieved.

However, the proposal presented hereinafter is based on multiple goal approach [27,28], where each goal is associated with an achievement function that minimizes undesirable deviations.

$$
\begin{aligned}
&\text{Min } n_1 + p_1 \\
&\text{Min } n_2 + p_2 \\
&\text{Min } n_3 + p_3 \\
&\text{Min } n_4 \\
&\text{s.t. } \sum_{j=1}^{s} u_j y_{jo} = 1 \\
&\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} + d_k = 0, \forall k \\
&M - d_k \geq 0, \forall k \\
&V - v_i \leq 0, \forall i \\
&d_o + n_1 - p_1 = f_1^* \\
&M + n_2 - p_2 = f_2^* \\
&\sum_{k=1}^{n} d_k + n_4 - p_4 = f_3^* \\
&V + n_4 - p_4 = \varepsilon \\
&u_j,\ v_i, d_k,\ n_1,\ n_2,\ n_3,\ n_4,\ p_1,\ p_2,\ p_3, p_4 \geq 0, \forall j, i, k \\
&v_* \in \mathcal{R}
\end{aligned} \qquad (2)
$$

In this sense, the procedure presented in (2) seeks non-null inputs multipliers, by defining aspiration levels for all three MCDEA objectives as the corresponding values of non-dominated solutions, and for the inputs multipliers a minimum value, set in advance.

Thus, the proposed procedure comprises two stages. On the first stage, we shall run the MCDEA model and select which non-dominated solution is preferable. If multipliers derived are non-null or reasonable, the second stage is avoided; otherwise, we shall solve the GP problem described in (2).

In (2), n_1, n_2, n_3, n_4, p_1, p_2 and p_3 are undesirable deviations to goals of the four objective functions; f_1^*, f_2^*andf_3^* are the values of the corresponding MCDEA's objective functions associated with preferable non-dominated solution derived and used as aspiration levels in the corresponding goals; and ε is a non-null value used as aspiration level in the goal for inputs multipliers (in the application hereafter, $\varepsilon = 0.01$).

4.1 Application

Hereinafter, the procedure proposed in (2) is applied to the MCDEA-based performance evaluation of the national teams engaged in UEFA EUO 2012, previously analysed by Rubem and Brandão [4], and reproduced in Sect. 3.1.

To verify the possibility of obtaining non-null inputs multipliers to the non-dominated solutions selected by those authors, the GP problem in (2) is applied to each DMU (national football team).

In this application, we seek to match the values obtained for the three MCDEA objective functions (f_1^*, f_2^* and f_3^*) in the preferred non-dominated solutions, exhibited in Table 4, and match or surpass the predefined aspiration level (ε, herein set as 0.01) for the inputs multipliers.

For the model implementation, we resorted to iMOLPe, and used the weighted-sum method, assigning equal weights to each achievement function. Such assignment of equal weights implies that all four achievement functions have the same relative importance. Table 5 presents non-dominated solutions, which resulted in non-null multipliers, derived from the GP problem in (2), as previously described.

From Table 5, we note that using the proposed procedure it was not possible to meet all goals simultaneously for any of the DMUs; otherwise, all achievement functions would be null, and we would find non-dominated solutions with non-null multipliers.

However, for DMUs Portugal, France, Italy, Sweden, Czech Republic, Greece and Ireland, it was possible to meet two goals, namely, the first (standard DEA objective) and the fourth (non-null inputs multipliers objective).

In addition, for all DMUs, the proposed procedure helped us finding at least one strictly positive inputs multipliers scheme, even though such solutions do not correspond exactly to a non-dominated MCDEA solution.

With regard to the multipliers schemes for the input ‘points in FIFA ranking’, it was always attached the minimum value admissible (i.e., 0.01).

Thus, the GP procedure proposed herein, for finding non-null inputs multipliers schemes, reached its purpose, despite of mild deviations from the non-dominated MCDEA solutions, displayed at Table 4.

Table 4 MCDEA non-dominated solutions corresponding to largest indifference regions

DMU	Solution	Objective function values			DMU	Solution	Objective function values		
		f_1^*	f_2^*	f_3^*			f_1^*	f_2^*	f_3^*
Spain	2 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	0.058	0.547	3.316	Croatia	1 ($\lambda_1 = \lambda_3 = 1;\ \lambda_2 = 0$)	1.851	5.427	33.164
Germany	2 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	1.704	1.809	11.055	Sweden	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	1.440	5.427	33.164
England	2 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	3.192	3.192	19.501	Ukraine	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	1.111	5.427	33.164
Portugal	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	2.564	3.192	19.509	Czech Republic	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	0.193	3.192	19.501
France	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	2.515	3.192	19.509	Poland	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	0.864	5.427	33.164
Netherlands	2 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	4.564	5.427	33.164	Denmark	1 ($\lambda_1 = \lambda_3 = 1;\ \lambda_2 = 0$)	0.782	5.427	33.164
Italy	1 ($\lambda_1 = \lambda_3 = 1;\ \lambda_2 = 0$)	0	1.005	6.142	Greece	1 ($\lambda_1 = \lambda_3 = 1;\ \lambda_2 = 0$)	0	3.192	19.509
Russia	1 ($\lambda_1 = \lambda_3 = 1;\ \lambda_2 = 0$)	2.016	5.427	33.164	Ireland	3 ($\lambda_1 = \lambda_2 = 0;\ \lambda_3 = 1$)	0.453	5.427	33.164

Table 5 Non-dominated GP solutions derived using procedure in (2) applied to UEFA EURO 2012 data, which resulted in non-null multipliers and maximum number of goals met

DMU	MCDEA solution	GP solution	Achievement functions				Multipliers		
			a_1	a_2	a_3	a_4	v_1	v_2	u_1
Spain	2	1	0.003059	0.001077	0.001548	0	1.02732	0.01	1
Germany	2	4	0.002015	0.001077	0.001552	0	3.425468	0.01	3.33333
England	2	3	0.001077	0.001077	0.0015447	0	6.045293	0.01	5.88235
Portugal	3	1	**0**	**0.0008107941**	**0.004369794**	**0**	**6.04371**	**0.01**	**5.882353**
France	3	1	**0**	**0.001222808**	**0.002643408**	**0**	**6.04557**	**0.01**	**5.882353**
Netherlands	2	4	0.001758	0.001077	0.001552	0	10.27732	0.01	10
Italy	1	1	**0**	**0.001077**	**0.0015475**	**0**	1.902834	0.01	1.85185
Russia	1	4	0.000092	0.001077	0.001552	0	10.27732	0.01	10
Croatia	1	3	0.000636	0.001077	0.001552	0	10.27732	0.01	10
Sweden	3	1	**0**	**0.000885**	**0.003584**	**0**	**10.27618**	**0.01**	**10**
Ukraine	3	3	0.002634	0.001077	0.001552	0	10.27732	0.01	10
C. Republic	3	1	**0**	**0.0004290822**	**0.008423211**	**0**	**6.041437**	**0.01**	**5.882353**
Poland	3	1	0.002994	0.001077	0.001552	0	10.27732	0.01	10
Denmark	1	1	0.000450	0.001077	0.001552	0	10.27732	0.01	10
Greece	1	1	**0**	**0.001077**	**0.001544**	**0**	**6.045293**	**0.01**	**5.88235**
Ireland	3	1	**0**	**0.000945**	**0.002954**	**0**	**10.27653**	**0.01**	**10**

Nevertheless, the problem of presenting a value for ε shall be further investigated, for example, by means of a sensibility analysis.

A comparative advantage of the proposed procedure is that it allows more transparency than methods of multipliers restrictions. In other words, when using the GP-based procedure, the decision-maker may identify how far the solution with the "ideal" multipliers is from the non-dominated solution.

5 Conclusions

In this paper, we proposed an alternative procedure, based on goal programming, for calculating strictly positive inputs multipliers within a MCDEA framework.

The proposal was applied to a problem previously examined by [4] on performance evaluation of national teams engaged in UEFA EURO 2012, in which MCDEA model did not succeed in providing non-null inputs multipliers.

The results derived from the application of the proposed procedure revealed that, to assure non-null multipliers, a mild detachment of non-dominated solutions was necessary.

In future works, we intend to use other goal programming approaches for calculating alternative multipliers schemes (e.g., lexicographical ordering of achievement functions or maximizing the lesser of unwanted deviations). Another possibility would be to turn one or more of the goals into objective functions (e.g., maximizing standard DEA efficiency could cease to be a goal and become the objective function).

Acknowledgments We acknowledge the financial support of CNPq (Brazilian Ministry of Science and Technology). An earlier version of this paper was presented at the International Conference on Information Technology and Quantitative Management (ITQM 2015), Rio, Brazil, July 21–24, 2015 (see Procedia Computer Science, Volume 55, 2015, Pages 186–195).

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