Going Home Through an Unknown Street

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Abstract

We present a new strategy for searching for a goal in a street. The strategy works in two phases. First it follows an angular bisector, then it uses circular arcs based only on one side of the street. A competitive factor of 1.514 is achieved which is remarkably close to the lower bound of $\sqrt{2}$.

Secondly, we assume that the location of the goal is known to the robot. We prove a lower bound of $\sqrt{2}$ on the competitive ratio of any deterministic strategy for searching in streets with known destination.

Keywords: simple polygons; path planning; navigation; competitive analysis; on-line algorithms; computational geometry.

1 Introduction

Many problems in computer science cannot be solved optimally simply because of the *lack of complete information*. Nevertheless, we want to obtain solutions for these problems which are as good as possible, compared to the perfect solution which is obtained when complete information is available.

A *strategy* for solving a class of problems is an algorithm which, for any possible situation and whenever a new piece of information becomes available, gives instructions on how to proceed such that the problem is finally solved.

Generally, solutions are measured according to their consumption of resources, e. g. time, space, energy, path length, or the like. Following the concept of Sleator and Tarjan [24] we call a strategy *competitive with factor c* if any solution produced by this strategy is guaranteed to consume at most c times as much resources as the perfect solution does.

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In this paper, we consider a fundamental 2-dimensional navigation problem. An autonomous system (robot) with a vision system can move freely inside a room with opaque walls, its aim is to find a goal t. Whenever the goal becomes visible the robots goes there and its task is accomplished.

Our aim is to describe a strategy for searching this goal. The length of the path travelled by our strategy will be compared to the perfect solution, i. e. the length of the shortest path within the room from the start position to the goal, which is not known in advance.

In Section 2 we recall the main facts about searching in streets, a subclass of simple polygons. Section 3 explains how different strategies can be combined into one and how our new strategy operates. In a first phase, it uses *angular bisectors* to reach a certain intermediate position, from there it walks on *circular arcs* to view the goal. Its competitive factor of 1.514 is remarkably close to the known lower bound of $\sqrt{2} \approx 1.414$. Finally, in Section 4 we show that $\sqrt{2}$ remains a lower bound for searching in orthogonal streets even if the location of the goal is known in advance.

Competitive on-line searching has also been investigated in many other settings such as searching in other classes of simple polygons [6, 7, 11, 18, 20], among rectangles [2, 3, 4, 5, 21, 22], convex polygons [12], and on the real line [1, 8, 9].

2 Searching for a goal in a street

In our model the room is a simple polygon P in the plane, the robot is just a point moving inside the polygon, and the start position s and the goal t are two of P's vertices. Two points are mutually *visible* (see each other) if the connecting line segment is contained within P. As usual, two sets of points are said to be *mutually weakly visible* if each point of one set can see at least one point of the other set.

It is easy to see that for general simple polygons a constant competitive strategy can not exist. Therefore, it is an interesting question what type of polygons admit competitive searching, and the street polygons defined by Klein [13] form such a class.

Definition 1 A simple polygon P in the plane with two distinguished vertices s and t is called a *street* if the two boundary chains from s to t are weakly mutually visible, for an example see Figure 1.

Definition 2 A strategy is *c-competitive* for searching a goal in a street if its path never is longer than c times the length of the shortest path from s to t.

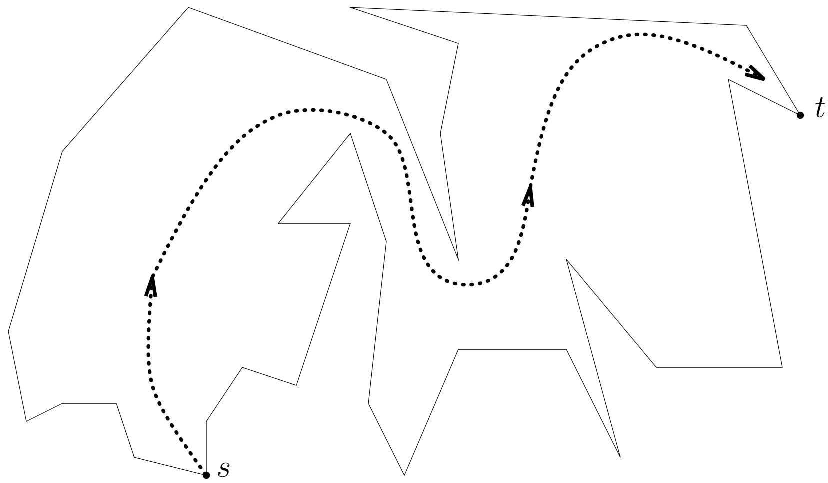

Figure 1: A street polygon.

In [13], the first strategy to search in streets called *local absolute detour* (*lad*) was described and proven to have a competitive factor of at most 5.72. The analysis of *lad* was later improved by Icking to ≈ 4.44 [10].

A number of other strategies have been presented since by Kleinberg [14] and López-Ortiz and Schuierer [16, 17, 19]. The currently best known competitive factor is ≈ 1.73 [19].

In the rest of this section we summarize other important facts about searching in streets which also stem from [13].

The only important case for all strategies is how they behave in the so-called *funnel* polygons. A funnel is a special street consisting of two chains of reflex[1] vertices with a common start point s, see Figure 2 for an example. The two reflex chains end in vertices t_l and t_r, respectively, and the line segment $t_l\, t_r$ closes the polygon. The goal t is either t_l or t_r, and the strategy will know which case applies latest when the line segment $t_l\, t_r$ is reached. For analyzing a strategy, both cases have to be considered and the worse of them determines the competitive factor.

A strategy which achieves a competitive factor c in each funnel can easily be extended to a c-competitive strategy for searching in streets, using the so-called high level strategy in [13].

There is a *lower bound* of $\sqrt{2} \approx 1.414$ for the competitive factor of any strategy for searching streets. To see this, consider a right-angled, isosceles triangle with s at the right angle and t_l and t_r at the other vertices.

It remains the open question if there is an *optimal competitive strategy*, i. e. a strategy whose competitive factor matches such a lower bound.

[1] A vertex of a polygon is called *reflex* if its internal angle exceeds 180°, as opposed to convex.

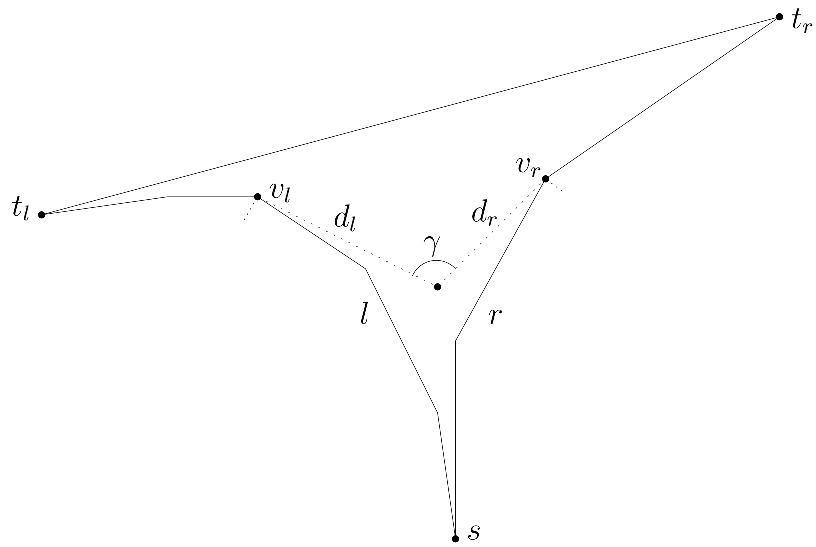

Figure 2: A funnel polygon.

3 A strategy in two phases

3.1 Putting strategies together

As mentioned above, we only consider funnel polygons. While a strategy proceeds, we always denote the most advanced visible point on the left chain with v_l and the corresponding vertex on the right chain with v_r. The visible boundary's length from s to the actual v_l is called l, and r from s to v_r. Let d_l and d_r denote the distances from the actual position to v_l and v_r, resp., cf. Figure 2.

We define the *opening angle* to be the angle between the directions from the actual position to v_l and to v_r, see γ in Figure 2.

As already proposed in [13], all reasonable strategies select a direction within the opening angle, in other words they always aim at something in between v_l and v_r. Other directions obviously produce unnecessary detours. As a consequence, the opening angle is always *strictly increasing*, it starts as the angle between the two edges adjacent to s and reaches, but never exceeds, 180° when finally the goal can be seen. By this property, the *opening angle* is predetermined as the natural choice for parameterizing the strategies.

Definition 3 We say a strategy *holds a competitive factor c up to an angle γ* if for each funnel we have for the strategy's path length, w, from s to the position with opening angle γ

$$w \leq c \cdot \min(l - d_l, r - d_r) \ .$$

Note that the condition of Definition 3 is somewhat stronger than that for competitiveness; in particular, holding a factor up to 180°, that is, when the goal is finally seen, implies the same overall competitive factor, but not the converse. In the following lemma we see how this definition can be applied.

Lemma 4 *Suppose we have a strategy A which holds a competitive factor c_A up to an angle γ_A.*

(i) If we have a second strategy B which holds factor c_B up to an angle γ_B for any funnel with initial opening angle $\geq \gamma_A$ then there is a combined strategy which holds the competitive factor $\max(c_A, c_B)$ up to γ_B.

(ii) Similarly, if we have a strategy B which is c_B-competitive for all funnels with initial opening angle $\geq \gamma_A$ then there is a $\max(c_A, c_B)$-competitive strategy.

Proof. We assume a funnel with opening angle $< \gamma_A$, otherwise the claims are obviously true. We give a proof only for the first one, it is nearly identical for the second.

The combined strategy works as follows. We use strategy A until the opening angle equals γ_A at a position p, see Figure 3. Let l_1 and r_1 be the lengths of the visible left and right chains at this point, and let d_{l_1} and d_{r_1} be the distances to their endpoints v_{l_1} and v_{r_1}.

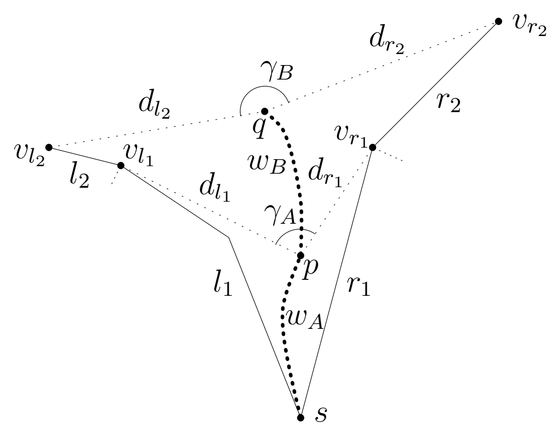

Figure 3: Combining two strategies.

From p we proceed with strategy B as if we have a funnel starting at p with visible edges $p\,v_l$ and $p\,v_r$ and which continues with the (yet invisible) edges of the original funnel. At some point q the angle γ_B is reached. Here, we have endpoints v_{l_2} and v_{r_2} of the visible chains and corresponding distances d_{l_2} and d_{r_2}. The length of the left boundary from v_{l_1} to v_{l_2} is called l_2 and analogously for r_2.

Let w_A be the path length generated by strategy A from s to p and w_B the path length from p to q.

From Definition 3 we know that $w_A \leq c_A \cdot \min(l_1 - d_{l_1}, r_1 - d_{r_1})$ as well as $w_B \leq c_B \cdot \min(d_{l_1} + l_2 - d_{l_2}, d_{r_1} + r_2 - d_{r_2})$. If we add the two inequalities and use instead of c_A and c_B their maximum, we get

$$
\begin{aligned}
& w_A + w_B \\
& \leq \max(c_A, c_B) \cdot \Big(\min(l_1 - d_{l_1}, r_1 - d_{r_1}) + \min(d_{l_1} + l_2 - d_{l_2}, d_{r_1} + r_2 - d_{r_2}) \Big) \\
& \leq \max(c_A, c_B) \cdot \min(l_1 + l_2 - d_{l_2}, r_1 + r_2 - d_{r_2})
\end{aligned}
$$

which is what we were looking for. □

3.2 Walking on angular bisectors

A very natural idea for a strategy is the following [15, 23].

> Strategy AB (angular bisector):
> Choose the *angular bisector* of the initial opening angle and walk straight in that direction. After some distance, but at least whenever one of v_l and v_r changes, determine the new angular bisector of the actual opening angle and continue.

As we will see in the analysis, how often we choose a new direction is not so important as long as we do this at least at any change of the most advanced visible vertices. Let us analyze the first step of this strategy.

Lemma 5 *As long as v_l and v_r do not change, strategy AB holds the competitive factor $1/\cos\frac{\gamma}{2}$ up to an opening angle γ, for any $\gamma < 180°$.*

Proof. Let γ_0 be the initial opening angle. Since v_l and v_r do not change during this step, we have a quadrilateral as shown in Figure 4. Let w be the length of the angular bisector segment from s to the point p where angle γ is reached. The remaining angles of the quadrilateral on the right and the left are called ρ and λ, we have $\gamma = \gamma_0 + \rho + \lambda$.

First, we concentrate on the quantities on the right, r being the length of the right side and d_r the distance from p to v_r. We rotate segment $v_r\,p$ about v_r onto the right side and obtain an isosceles triangle with angle ρ. The adjacent triangle has edge lengths w and $r - d_r$ and an angle $\frac{\gamma_0}{2}$ in between. The other two angles in this triangle are $\phi = \frac{\pi+\rho}{2}$ and $\frac{\pi-\gamma_0-\rho}{2}$.

Using the law of sines for this triangle we get

$$\frac{w}{r - d_r} = \frac{\sin\frac{\pi+\rho}{2}}{\sin\frac{\pi-\gamma_0-\rho}{2}} = \frac{\cos\frac{\rho}{2}}{\cos\frac{\gamma_0+\rho}{2}} \leq \frac{1}{\cos\frac{\gamma_0+\rho}{2}} \leq \frac{1}{\cos\frac{\gamma_0+\rho+\lambda}{2}} = \frac{1}{\cos\frac{\gamma}{2}}.$$

With symmetric arguments we get $w \leq 1/\cos\frac{\gamma}{2} \cdot (l - d_l)$, and the claim follows. □

Combining several steps of strategy AB we get the same behavior, as the next lemma shows.

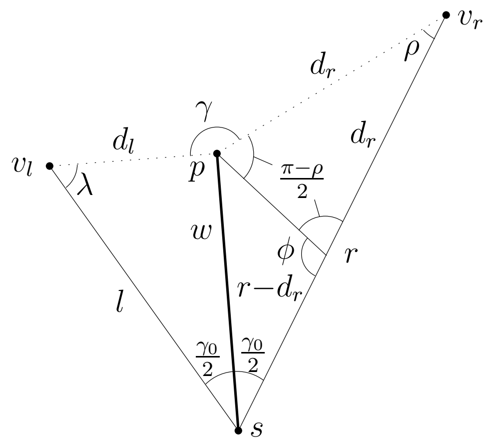

Figure 4: One step along the angular bisector.

Lemma 6 *Strategy AB holds the competitive factor* $1/\cos\frac{\gamma}{2}$ *up to an opening angle* γ*, for any* $\gamma < 180°$*.*

Proof. First let us observe what happens when one of the most advanced visible points, say v_l, changes at an opening angle γ. At this point the distance d_l from the actual position to the new v_l increases by exactly the same amount as the length, l, of the visible chain, but in Definition 3 only their difference is taken into account. Thus, if a competitive factor is held up to all angles strictly smaller than γ then this property also remains true for γ itself.

It remains to handle the direction changes, here we proceed by induction on the number of line segments in our path. The induction base holds because of Lemma 5, and for concluding from n segments to $n+1$ we apply Lemma 4 (i) to the $(n+1)$st step. The claim follows from the fact that $1/\cos\frac{\gamma}{2}$ is increasing with γ. □

Unfortunately, if γ approaches 180°, then $1/\cos\frac{\gamma}{2}$ tends to infinity, so we have not proven that AB is overall competitive. On the other hand, the above analysis is based on Definition 3 which is a much stronger requirement than competitiveness alone; indeed, it can be shown that strategy AB is at most 3-competitive [15]. Moreover, if we modify it just a little bit, the competitive ratio can be significantly improved: Proceed as described until the opening angle equals 120°, then, without taking care of new vertices becoming visible, walk straight along the angular bisector until the goal becomes visible. It can be shown that this is a 2-competitive strategy, the simple proof is omitted here because of the better bound in Section 3.3.

It is also interesting to note that all funnels with initial opening angle $\gamma < 90°$ are completely settled, in a sense: If we have a c-competitive strategy C for the funnels starting with $\gamma \geq 90°$ then C can be combined with strategy AB to search all funnels with the same factor c because AB holds the competitive factor $\sqrt{2}$

up to a right angle and $\sqrt{2}$ is a lower bound for c anyway. The same result was also obtained for the strategy *clad* by López-Ortiz and Schuierer [19].

3.3 Walking on circular arcs

Now let us assume that we are given a funnel with initial opening angle $\gamma \geq 90°$. We describe a non-symmetric strategy which depends only on the right side of the street.

> Strategy *CA-θ* (circular arcs with constant angle θ):[2]
> Walk along the curve with the property that each point p of the curve sees the visible part of the right side at a constant angle θ.

In other words, at a point p of the path the line to the most advanced visible point v_r and the line to the least advanced but still visible point (point s in most cases) intersect at an angle θ.

It is a well-known fact that such a curve consists of circular arcs. We switch from one circle to another whenever the visible chain changes, see Figure 5 for an example.

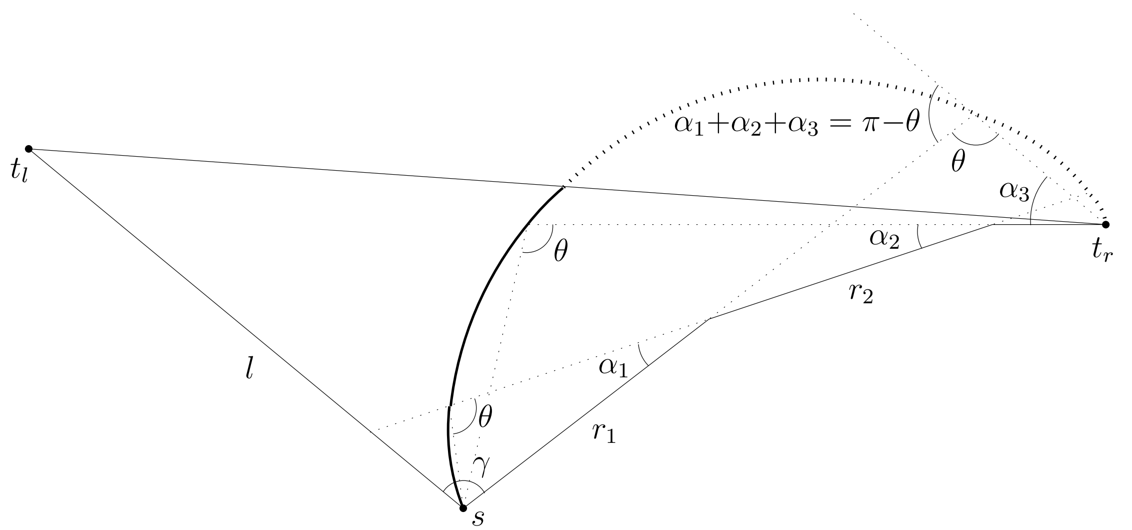

Figure 5: Circular arcs with constant angle θ to the right side.

Lemma 7 *Let $\gamma = 1.698 \approx 97.3°$ and $\theta = 1.631 \approx 93.5°$. Then strategy CA-$\theta$ is 1.514-competitive for all funnels starting with an opening angle $\geq \gamma$.*

[2] The strategy *CA-$\pi/2$* and its analysis are already contained in [23].

Proof. Two cases have to be distinguished.
Case 1: $t = t_r$, the goal is on the right side.
We estimate the length of a completed path of circular arcs which goes from s to t, each point p of the path sees the visible part of the right side at a constant angle θ. We neglect the fact that the real path is shorter because the goal t will actually be visible at some time before being reached.

From elementary geometry we obtain that the length of one circular arc equals the distance between the two spanning vertices (the most and the least advanced visible points) times $\alpha/\sin\theta$, where α is the angle of rotation, see Figure 5. The mentioned distance is not greater than the length of the boundary between the two points. The complete path length, w, is the sum over all such arcs.

$$w \leq \sum_i \frac{\alpha_i}{\sin\theta}(r_{i_x} + \cdots + r_{i_y})$$

Here, the α_i are the subsequent angles of rotation, and $r_{i_x}, \ldots, r_{i_y}$ are the lengths of the edges which are visible from a particular arc. After reordering the sum by collecting the terms in r_1, r_2, etc., we obtain the form

$$w \leq \frac{1}{\sin\theta} \sum_j r_j(\alpha_{j_x} + \cdots + \alpha_{j_y}) .$$

A particular edge r_j is visible from a certain interval on the path of circular arcs. We observe a moving angle θ whose sides always pass through the least and the most advanced visible points. From the first position where r_j is visible to the last one the angle θ is rotated by exactly $\pi - \theta$. This is because initially the right side of the angle lies on r_j while finally the left side does.

This means that for any j the angles $\alpha_{j_x}, \ldots, \alpha_{j_y}$ add up to exactly $\pi - \theta$, see edge r_1 and the angles α_1, α_2, and α_3 in Figure 5. As a consequence, length w is not greater than $(\pi - \theta)/\sin\theta$ times the length of the right boundary. For the given value of θ this factor is nearly 1.514.

Case 2: $t = t_l$, the goal is on the left side.
In this case we try to simplify the path while maintaining or increasing its length. If the right side has only two edges, r_1 and r_2, and corresponding angles of rotation α_1 and α_2, see Figure 6, we can argue as follows.

The strategy's path consists of two circular arcs and a line segment from point q, where the goal is discovered, to t. The first arc measures $1/\sin\theta \cdot r_1\alpha_1$ while the second is shorter than $1/\sin\theta \cdot (r_1 + r_2)\alpha_2$. For the sum we have

$$\begin{aligned} \frac{1}{\sin\theta}\Big(r_1\alpha_1 + (r_1 + r_2)\alpha_2\Big) &= \frac{1}{\sin\theta}\Big(r_1(\alpha_1 + \alpha_2) + a\frac{\sin(\alpha_1 + \alpha_2)}{\sin\alpha_2}\alpha_2\Big) \\ &\leq \frac{1}{\sin\theta}(r_1 + a)(\alpha_1 + \alpha_2). \end{aligned}$$

Here, a is the prolongation of edge r_1 to the line $t\,t_r$. The last inequality is based on $\alpha_2/\sin\alpha_2 \leq (\alpha_1 + \alpha_2)/\sin(\alpha_1 + \alpha_2)$.

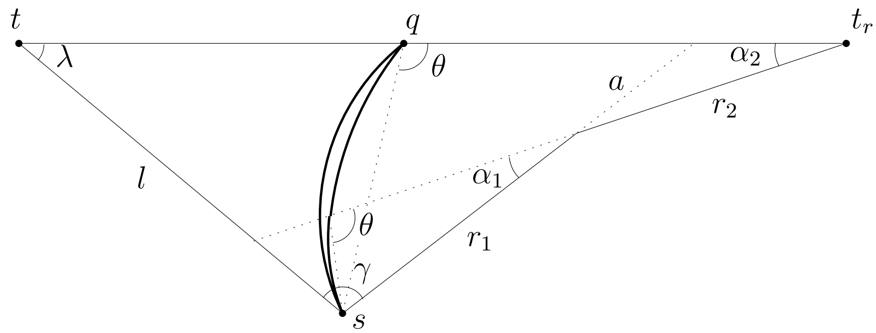

Figure 6: Estimating the detour to the left side.

We have bounded the length of the two arcs by the length of one arc with angle $\alpha_1 + \alpha_2$ over the edge prolongation $r_1 + a$ without changing the endpoints s and q. The new arc is just the path of our strategy if the right side would consist of r_1 and its prolongation a.

If there are more than two edges on the right side, this procedure can be repeated for all subsequent edges until only one arc remains. Note that the fact $\theta < \gamma$ implies that s remains visible all the time until t is seen.

By standard trigonometry we get for the length of the arc

$$w = \frac{(\pi - \gamma - \lambda)\sin\lambda}{\sin\theta \sin(\gamma + \lambda)} l$$

and $z = \sin(\theta - \lambda)/\sin\theta \cdot l$ for the line segment $q\,t$. Here, l denotes the length of segment $s\,t$ which is certainly not longer than the left side of the funnel, and λ is the angle between $s\,t$ and $q\,t$, angle λ may vary from 0 to $\pi - \gamma$.

Altogether, we have a competitive factor of

$$\sup_\lambda \frac{w + z}{l} = \frac{1}{\sin\theta} \cdot \sup_\lambda \Big(\frac{\sin\lambda}{\sin(\gamma + \lambda)} (\pi - \gamma - \lambda) + \sin(\theta - \lambda) \Big).$$

The angles γ and θ are constants, and the supremum can easily be determined by numerical methods. For the given values of γ and θ we obtain a factor of nearly 1.514 just as in Case 1. □

3.4 The main result

It remains to put strategies AB and CA-θ together.

Theorem 8 *Using strategy AB (angular bisector) up to an angle of* $\gamma = 1.698$ *and continuing with strategy CA-*θ *(circular arcs with constant angle* $\theta = 1.631$*) is* 1.514*-competitive for searching a goal in a street.*

Proof. The values for γ and θ in Lemma 7 have been chosen not only such that the factors for both cases of the previous proof are the same but also such that strategy AB holds the same factor 1.514 up to γ, which can be verified by calculating $1/\cos\frac{\gamma}{2}$.

By applying Lemma 4 (ii) we obtain this factor also for the combined strategy. □

4 Known Destination Search

In the previous sections we have always assumed that the position of the goal is not known to the robot in the beginning. A natural question to ask is if there is a strategy with a better competitive ratio if the location of the goal is known. Clearly, the polygon shown in Figure 7a[3] no longer provides a lower bound since the robot knows the position of the goal and can move directly to it; however, by connecting a number of these polygons, it is still possible to show a lower bound of $\sqrt{2}$ on the competitive ratio of any strategy to search in streets even if the position of the goal is known in advance.

We construct a family $\mathcal{F}$ of streets such that, for all $n \geq 0$ and for all on-line strategies S, there is a street P in $\mathcal{F}$ such that the competitive ratio of S in P is $\sqrt{2} - O(1/\sqrt{n})$.[4]

We call the polygon of Figure 7a an *eared-rectangle*. Eared-rectangles can be connected to create larger polygons. This is shown in Figure 7b. In the

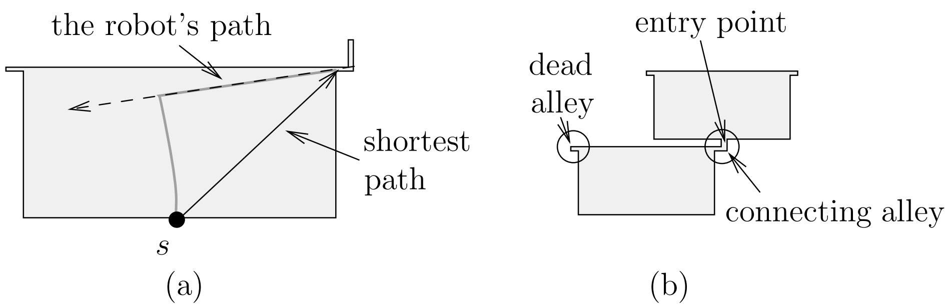

Figure 7: (a) An eared-rectangle. (b) Connecting two eared-rectangles

construction of Figure 7 each eared-rectangle has a *connecting alley* and a *dead-alley*. The *entry point* is the point where the robot enters an eared-rectangle.

If the robot is located inside an eared-rectangle and wants to decide which of the two alleys is dead and which is connecting, then it has to move up to a point in the eared-rectangle from which one of the alleys is completely visible. By

[3]For now, the paths drawn in the figure should be disregarded.

[4]A preliminary version of the proof has appeared in [16].

making the alleys very narrow, we can force the robot to move arbitrarily close to the horizontal line that connects the alleys before it can decide which alley is dead and which is connecting.

Assume we are given a strategy S to search in an orthogonal street with known destination. In the beginning the robot is located in an eared-rectangle of width 2 units and height 1 unit. The goal is located directly above s at a distance of n units.

We present the strategy of an *adversary* to S that constructs a polygon consisting of at most $n^2/2$ connected eared-rectangles in which the path traversed by the robot using S is at least $\sqrt{2} - O(1/\sqrt{n})$ times longer than the shortest path from s to t.

The adversary's strategy is as follows. If the robot moves into the left half of the eared-rectangle in order to find out which alley is connecting, then the adversary opens the right alley and connects a new eared-rectangle to it and vice versa. If the robot travels in the middle of the eared-rectangle, then the adversary opens an arbitrary alley. In this way the length of the path generated by S in one eared-rectangle has a length of $2 - \varepsilon$ where ε depends on the width of the alleys whereas the shortest path has a length of $\sqrt{2}$ (see Figure 7b).

The adversary puts one eared-rectangle on top of the other until the top edge of the current eared-rectangle has the same height as t. In this case the next eared-rectangle is rotated by 90° and placed on the side of the current eared-rectangle that is closer to t. We denote the entry point of this rotated eared-rectangle by s_2 (see Figure 8a).

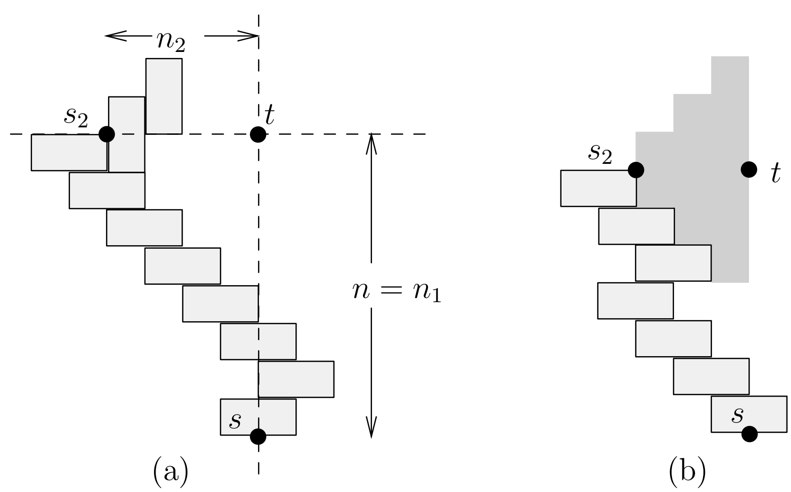

Figure 8: (a) Constructing a new polygon with eared-rectangles. (b) The situation at s_2 is analogous to the situation at s.

First of all we note that at s_2 the situation is exactly analogous to the situation at s just rotated by 90°. This is due to the fact that the shaded region in Figure 8b does not contain any eared-rectangles and the goal t is again on an axis-parallel line through s_2. Hence, the adversary can apply the same strategy recursively now starting at s_2. Since the distance of s_2 to t is at least one less than the distance of s to t, this construction ends after at most $k \le n$ iterations. We denote the starting point of the ith iteration s_i, for $1 \le i \le k$, where $s_1 = s$.

We now analyse the distance traveled by the robot. As we observed above, the length of the path generated by S in one eared-rectangle is at least $2-\varepsilon$ units whereas the length of the shortest path is $\sqrt{2}$ units. This is true for all eared-rectangles except for the last eared-rectangle of an iteration whose top edge has the same height as t. In this case the action of the adversary does not depend on S, but the adversary always rotates the new eared-rectangle and opens the alley that is closer to t. We assume that S is given this knowledge in advance and, hence, S is able to choose the shortest path in the last eared-rectangle of an iteration. Note that if the distance of s_i to t is n_i, then the adversary places n_i eared-rectangles on top of each other until the horizontal or vertical line through t is reached. Hence, the distance traveled by the robot in the ith iteration is $(n_i - 1)(2-\varepsilon) + \sqrt{2}$ whereas the length of the shortest path is $n_i\sqrt{2}$, and the competitive ratio of S is at least

$$\begin{aligned} \frac{\sum_{i=1}^{k}(n_i - 1)(2-\varepsilon) + \sqrt{2}}{\sqrt{2}\sum_{i=1}^{k} n_i} &= \left(\sqrt{2} - \frac{\varepsilon}{\sqrt{2}}\right) - \frac{(2-\sqrt{2}-\varepsilon)k}{\sqrt{2}\sum_{i=1}^{k} n_i} \\ &\ge \left(\sqrt{2} - \frac{\varepsilon}{\sqrt{2}}\right) - \frac{k}{\sum_{i=1}^{k} n_i} \qquad (1) \end{aligned}$$

with $1 \le n_k < n_{k-1} < \cdots < n_2 < n_1 = n$. The Strategy S can choose the numbers n_i in order to minimize Expression 1. It is minimized if $\sum_{i=1}^{k} n_i$ is as small as possible, that is, if $n_k = 1$, $n_{k-1} = 2$, and so on. Therefore, Expression 1 is bounded by

$$\left(\sqrt{2} - \frac{\varepsilon}{\sqrt{2}}\right) - \frac{k}{\sum_{i=1}^{k-1} i + n} = \left(\sqrt{2} - \frac{\varepsilon}{\sqrt{2}}\right) - \frac{k}{(k-1)k/2 + n}.$$

This is minimized for $k = \sqrt{n}$, and the competitive ratio of S is at least

$$\sqrt{2} - \frac{\varepsilon}{\sqrt{2}} - \frac{1}{\sqrt{n}},$$

for $n \ge 4$. By choosing the alleys so small that $\varepsilon = 1/\sqrt{n}$, the claim follows. Since n can be arbitrarily large, we have shown the following result.

Theorem 9 *If S is a deterministic strategy to search in streets with known location of the goal, then the competitive ratio of S is at least $\sqrt{2}$.*

5 Conclusions

We have considered two problems in this paper. First, we have presented two strategies for a robot to search in streets. In the strategy *AB* the robot follows the bisector of its current opening angle. A minor modification of this strategy has a competitive factor of two. In particular, the strategy *AB* can be shown to hold a competitive factor of $\sqrt{2}$ up to an angle of $\pi/2$ which is optimal. The second strategy *CA*-θ is an asymmetric strategy where the robot walks along circular arcs with the property that each point p on an arc sees the visible part of the right side at a constant angle θ. Strategy *CA*-θ is 1.514-competitive if $\theta = 1.631$. Since strategy *AB* holds a competitive factor of 1.514 up to an angle of 1.631, combining strategies *AB* and *CA*-θ yields a strategy with an overall competitive factor of 1.514 which is remarkably close to the lower bound of $\sqrt{2}$.

Secondly, we provide a lower bound of $\sqrt{2} - O(1/\sqrt{n})$ for the competitive ratio of any deterministic strategy that a robot may use to search in a rectilinear street if the coordinates of the target are given in advance to the robot. This implies that knowledge of the location of the target does not provide any advantage even for searching in rectilinear streets.

The major open problem is, of course, to design and analyse a strategy for searching in arbitrarily oriented streets that achieves an optimal competitive factor of $\sqrt{2}$. The bounds obtained in this paper suggest that such a strategy exists.

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