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Governance of Swedish school mathematics — where and how did it happen? A study of different modes of governance in Swedish school mathematics, 1910-1980

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Abstract: The aim of this paper is to revise a standard narrative about governance of the Swedish school system in the period of 1910-1980. According to this narrative, the Swedish school system was centralized during this period. However, this narrative does not fit the history of Swedish mathematics education (years 1-9). The research questions are: where in the school system was change initiated and how was change enforced? On the basis of studies of syllabi, textbooks, teaching literature, teacher journals and reports from investigations and development projects, different modes of governance of school mathematics are identified. The main results are that textbook producers rather than national syllabi and exams were drivers of change in the period 1910-1960. Moreover, the centralized attempts to change school mathematics, prepared in the 1960s, were soon abandoned in the early 1970s. Thus, centralized governance of Swedish school mathematics, with the ambition to achieve change, was something that took effect relatively late and during a very short period of time.

Keywords: Governance; reform; mathematics education; Sweden; 20th century; primary schools; lower secondary schools.

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1. Introduction

This paper is about the governance of Swedish school mathematics (years 1-9). The research questions concern modes of governance: where in the school system was change initiated and how was change enforced? The aim is to revise a standard narrative about governance of Swedish schools. This standard narrative is further accounted for in the section *State of the art.*

The main material of the study is syllabi, textbooks, teaching literature, teacher journals and reports from investigations and development projects. Regarding the

reports, they have been used to collect descriptive facts about various phenomena, for instance how teachers perceived governance or how a reform was prepared. Syllabi, textbooks, teaching literature and journals have been analysed on the basis of three questions: what content were the students supposed to learn, how was the teaching supposed to be performed and what were the arguments for choosing certain contents or teaching methods.

The paper comprises eight sections, of which the introduction is the first. In the second section - *State of the art* - previous research and the contribution of this paper are discussed. In this section, the notions of governance are also specified. In the subsequent five sections, the important conclusions are laid out and motivated. In the first two of these five sections - *Passive centralized governance, 1910-1960* and *Attempts of active centralized governance, 1960-1980* - two modes of governance are studied. In the subsequent two sections, two ambitious attempts of centralized governance prepared in the 1960s are further studied: the New Math project and the IMU project. These two sections are followed by the section *Implementation in the* 1970s, in which the implementation of New Math and the IMU project are studied. In the final section, the main conclusions of the paper are presented.

Three types of school types are mentioned in this paper: *Folkskolan* (1-9), *Realskolan* (5-9) and *Grundskolan* (1-9). The first two were the main school types in the period 1910-1962. By far, *Folkskolan* had the most students; more or less all students went there for years 1-4, except for children whose parents could afford private schooling. *Realskolan* was a lower secondary school (4-9) that prepared for further theoretical studies or advanced vocational studies. Throughout the period 1910-1962, the number of students in *Realskolan* increased steadily. In 1962, *Grundskolan* was introduced and it replaced *Folkskolan* and *Realskolan* during a ten-year period. This is one of the greatest school reforms ever in Sweden and a significant feature was the integration of an education for all and educational programmes for further theoretical studies and advanced vocational studies. The introduction of *Grundskolan* was prepared during the 1950s. In a number of schools, models for teaching in integrated classes were tried and developed. This enterprise is also an example of centralized governance.

2. State of the art

In this paper, governance refers to a group of people trying to steer its own decisions or another group's decisions in a certain direction. Of particular interest is central governance, i.e. the central school authorities' attempts to govern groups of people in the school system, for instance teachers and textbook authors. A process where the central school authorities attempt to control more of the teachers' and textbook authors' decisions is denoted centralization. The reverse process is then denoted decentralization.

According to Lindensjö & Lundgren (2012, pp. 25-26), the central school authorities in Sweden have had three types of tools to govern schools: judicial, economical and ideological. Judicial governance refers to using laws and regulations; economical governance refers to locating resources to various ends; and ideological governance refers to specifying goals, contents and results. These types of governance are of course related; for instance, a syllabus is an example of judicial and ideological governance

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
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since it is a judicial document that aims to regulate teaching and learning, which to a great extent concern ideas. This paper mainly concerns ideological governance since school subjects to great extent concern ideas about what to teach and how to teach.

In Swedish academic literature about governance of the Swedish school system there is a standard narrative about central governance. The basic structure of the narrative is that during the period of 1900-1970, the state successively took greater control of the school system. In the 1970s, a highly centralized system was in operation, but it was being questioned and the state launched a number of investigations that resulted in plans for how the system could be decentralized. These plans were then realized in the early 1980s.

This narrative appears in three Swedish university textbooks on history of education or curriculum history (Larsson & Westberg, 2011, pp. 330-336; Richardson, 2010, pp. 95, 159; Lindensjö & Lundgren, 2012, pp. 29, 81-82, 93-96); this qualifies the use of the expression «standard narrative». But we also find this standard narrative in research papers and dissertations (e.g. Börjesson 2016, pp. 30-31, 77-98, 218-220; Oftedal Telhaug *et al.*, 2006, pp. 248-250, 255-256; Lundahl, 2006, pp. 254, 277-283).

According to Oftedal Telhaug *et al.* (2006, p. 245), centralized school governance was typical for Sweden, but also the other Nordic countries; it was part of the Nordic model. Oftedal Telhaug et al. (2006, p. 245) also maintain that this model was considered an ideal for school development in Western countries.

From the treatises mentioned above, we understand that centralization did not happen quickly and at the same time; it was more of a slow stepwise process where different parts of the school system were affected at different times.

However, exceptions from the narrative are not mentioned. As regards school subjects and ideological governance, the impression is that the governance of school subjects was centralized. For instance, the national syllabus issued in 1919 were firmer (Larsson & Westberg, 2011, p. 332); and even more so after the Second World War (Oftedal Telhaug et al., 2006, p. 255). Moreover, national standardized tests, developed by psychological experts rather than teachers, received a much more important role in the governance of the schools from the 1940s and onwards (Lundahl, 2006, p. 410). In the 1960s, another type of ideological centralization appeared as the state initiated development projects, whose aims were to change both the content and the methods of teaching (Lindensjö & Lundgren, 2012, pp. 66-68).

A weakness of this standard narrative about centralization is that it does not fit school mathematics, especially if we consider how change was initiated and enforced. In this paper it is shown that textbook producers rather than national syllabi and exams were drivers of change in the period 1910-1960. Moreover, the centralized attempts to change school mathematics, thoroughly prepared in the 1960s, were soon abandoned as the phase of implementation began in the early 1970s. My point here is that centralized governance of Swedish school mathematics, with the ambition to achieve change, was something that took effect relatively late and during a very short period of time. These results entail that we cannot understand changes in Swedish school mathematics in the period of 1910-1980 as a product of ideas implemented only through the central school authorities and national policy documents. If we want to understand these changes, more attention has to be given to producers of textbooks and thus other modes of governance. And of course, the

study presented in this paper raises questions about governance and change in other school subjects in Sweden.

This paper also contributes to research about the history of Swedish mathematics education. In particular, if we want to understand how and why the school subject has changed over time. Prytz (2009) and Prytz (2012) do address questions about power and influence, but they concern only people involved in textbook production and debates in teacher journals. The state's attempts to govern school mathematics are not part of the analysis. If we consider the more comprehensive treatises on the history of Swedish mathematics education (Prytz, 2007; Lundin, 2008; Hatami, 2007; Bjerneby Häll, 2002), issues concerning the intersection of governance and change in school mathematics are not studied. In these treatises the focus is on the content of syllabi, textbooks, teacher, journals and exams.

Obviously, the main contribution of this paper is to research about the Swedish school system. However, the Swedish case is relevant in an international perspective since centralization was not an isolated Swedish phenomenon. For instance, Green (1997, p. 107) in the major work *Education, Globalization and the Nation State*, considers the educational systems of France, Germany, Japan and Sweden as centralized by the 1980s. In fact, the Swedish system is seen as highly centralized (Green, 1997, p. 116). In contrast, he sees the educational systems of the UK and the USA as less centralized. However, Timar & Tyack (1999, pp. 15-23) describe how the educational system in the USA became increasingly centralized at state level in the period of 1900-1980. Timar & Tyack (1999, pp. 22) also observe that centralization was not an all-encompassing process and that some areas in the system remained untouched. In fact, they briefly mention mathematics education in California as an example of an untouched area. My findings suggest that mathematics education (1-9) in Sweden had a similar fate in the period of 1910-1980.

Whether or not mathematics education was an untouched area is of course significant in any analysis of the relation between modes of governance and outcomes, especially when outcomes in mathematics are a part of the analysis. Green (1997, pp. 108-129) presents such an analysis. Among other things, he considers results from international tests, mathematics included, in the period of 1981-1986. And, as mentioned above, he places the Swedish educational system in the group of centralized systems. Clearly, that analysis is weakened if mathematics education in Sweden was untouched by centralization in the period of 1910-1980. However, Green (1997, p. 116) does point out that «Sweden has given more discretion to schools concerning the curriculum». But on the basis of my findings I would like to rephrase and say: given lots of discretion to schools and textbook producers for a long time. Anyhow, Green (1997, pp. 128-129) concludes that more centralized systems perform better. Green (1997) is not alone in analysing the relation between modes of governance and outcomes. Hofman et al. (2010, p. 167-169) makes a similar type of analysis, but with more recent material from the 1990s and onwards. They reach the opposite conclusion: decentralized systems perform better. However, Hofman *et al.* (2010) also do not consider the existence of untouched areas. In the final section of this paper, I discuss the outcome of the international tests of 1964 and 1980 and my findings regarding mathematics education in Sweden being untouched by centralized governance in the period of 1910-1980.

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
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As far as I can see, the existence of areas untouched by central governance in the 20th century is not an explicit issue within research on history of mathematics education. This is true for the Swedish context; see above. The complete international context is difficult to evaluate. But from a recent overview, *Handbook on the History of Mathematics Education* (Karp & Schubring, 2014, pp. 197-323), we understand there were attempts to reform mathematics education in Italy, France, Germany, England and the USA in the 20th century, reforms that were initiated by national or regional school authorities and thus examples of central governance. We also understand that these reforms were met by resistance and far from always implemented in the way they were designed. However, the issue about centralization and to what extent mathematics education was affected is not explicitly addressed. Nor is the issue discussed in treatises dedicated to reforms of mathematics education in the 20th century. For instance, Phillips' (2015) extensive work on the New Math reform in the USA is focused on ideas, as is Rogers (2015) in a paper on communities of mathematics educators in England in the period of 1950-1980. Gispert & Schubring's (2011) paper on reform processes in France and Germany in the 20th century is focused on groups of people involved in reform, but not the role of central school authorities, and Smid (2012) applies a similar perspective on reform movements in the Netherlands in the 20th century. Thus, by this study I show how research in the history of mathematics education has the potential of making original and critical contributions to a more general history of education.

3. Passive centralized governance, 1910-1960

The main conclusion of this section is that centralized governance of school mathematics was passive up to about 1960, when *Grundskolan* was introduced. The basic argument is that national syllabi were not used as tools of change. Change was rather left to the teachers and, not least, the textbook producers. Moreover, it was a system able to change, which is also shown in this section. I also argue that textbook authors were an active part in that process.

During the period of 1900-1960, syllabi were not used as tools to change mathematics education. The mathematics syllabi of *Folkskolan* and *Realskolan* were comparatively short. And they did not, with the same level of detail as later syllabi, specify what the students should learn (Prytz, 2015, pp. 311-324). Another difference is that a new syllabus was quite a rare event before 1960. In the twentieth century, *Folkskolan* received new syllabi in 1900, 1919 and 1955; *Realskolan* received new syllabi in 1905, 1928, 1933 and 1955. In comparison, during its first 20 years *Grundskolan* received new syllabi more often: in 1962, 1969 and 1980.

If we consider the syllabi, there were few changes as regards the content of the teaching. The novelty in the new *Folkskolan* syllabus of 1919 was that percentage and equations became subtopics; the former introduced in year 6 and the latter in year 7. In the syllabus of 1955, the main novelty was that years 8 and 9 were added. These courses resembled the corresponding ones of *Realskolan*, but applications had a more prominent place. Another difference was that equations were introduced in year 6 rather than 7 (NP, 1900, pp. 12-13, 29; UP, 1920, pp. 58-60; SÖ, 1955, pp. 123-124). In the *Realskolan* syllabi of 1905, 1928 and 1933, the formulations about the content were

more or less the same. Another similarity was their briefness. They were, basically, lists of topics and subtopics with few details (Bergqvist-Nordfelt, 1910, pp. 204-205; Bergqvist & Wallin, 1928, pp. 226-227; Wallin & Grimlund, 1939, pp. 316-317).

In the syllabi of both *Folkskolan* and *Realskolan*, the sections on teaching methods were longer, except for the *Folkskolan* syllabi of 1900, and they were successively extended. The latter is as an example of more centralized governance. On the other hand, the basic ideas of the methodological guidelines were similar, both between the two school types and over time: not too much, not too fast, not too complicated, keep up the interest of the students, precision, training and repetition. For each subtopic and year, these guidelines were further specified, mainly in terms of what to teach. Beyond general formulations about the need for visualizations, there were no or few guidelines regarding how explanations and illustrations should be designed (NP, 1900, pp. 32-34, Bergqvist-Nordfelt, 1910, pp. 205-214; SÖ, 1935, pp. 129-142; SÖ, 1955, pp. 124-129).

Due to the stability of the syllabi, they cannot be considered a tool to enforce change. And due to their general character, they can neither be considered a tool to hinder change.

In the 1930s, another type of governing tool was added: the national textbook review. The first list of textbooks approved by the review board came into force in 1941 (Prytz, 2007, p. 127). Among the review board's aims during the period of 1935-1973 was to consider agreement with the syllabus (Johnsson Harrie, 2009, pp. 115-116). Hence, due to the stability of the syllabi, the textbook review can be regarded neither as a tool to enforce change nor as a tool to hinder change.

Still, this stability of the syllabi did not mean that there were no attempts to change school mathematics. Active in that respect were textbook producers. In a study by Prytz (2007) on geometry textbooks for *Folkskolan* and *Realskolan* in the period of 1905-1962, it is shown that content as well as methods changed over time. Here, the term 'method' also refers to how and in what order the content was represented.

For instance, in the *Folkskolan* textbooks the introductions and explanations of concepts followed the same routine throughout the period. The students should work with some experimental exercises before definitions and formulas or other propositions were explicitly stated. The design of the experimental exercises did change, however. In the beginning of the period, the students were guided through the experiments by written lines of thought. This changed in later textbooks; the students were then supposed to complete the experiments without the help of written lines of thoughts (Prytz, 2007, pp. 122-124).

These changes seem to have been deliberate in the sense that the authors followed a principle. According to the forewords, the authors based their work on ideas about visualization and student activity[1]. These ideas were also pivotal in teaching literature, used in teacher education, for instance, and were there comprehensively explained (Prytz, 2007, pp. 63-71).

In the same *Folkskolan* textbooks there are also examples of less extensive changes. In textbooks published after 1925, the authors included a bit more demanding exercises. I deem them more demanding by the fact that the solution

[1] The Swedish term used then was åskådning. Its meaning is described further down.

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
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required more than plugging in numerals in a formula and making computations. Still, the latter type of exercises was well represented in all textbooks throughout the period (Prytz, 2007, pp. 122-123).

There were also clear changes in the *Realskolan* textbooks. By the beginning of the century, the titles of the geometry textbooks for years 7-9 contained the term 'Euclid', placed in a very prominent manner on the front cover. As regards the content, they contained the same propositions and order of the propositions as traditional editions of Euclid's *Elements.*[2] Many of the proofs were also identical with Euclid's. These textbooks were already being challenged by new textbooks by the end of the nineteenth century. And most importantly, the new textbooks were used in more schools (220 versus 70) by the late 1920s (Prytz, 2007, p. 126). Observe that this was well before the establishment of the national textbook review and the first list of approved textbooks.

The new textbooks differed on important points. For example, the order of the propositions was altered and they were grouped thematically. This entailed a number of new theorems and proofs, even though several of them were quite similar to the theorems and proofs in Euclid's *Elements*. Some of the new theorems concerned straight lines, perpendiculars, foldings and symmetry. These theorems also had an important role since they were introduced early and were used to handle congruency. Thus, we can say that the new theorems replaced the traditional congruence theorems, which were central in Euclid's *Elements*. The traditional congruence theorems were, however, not excluded; they were introduced later (Prytz, 2007, pp. 158-160). It is important to note that these changes were motivated by educational arguments: the new theorems about straight lines, perpendiculars, foldings and symmetry should support learning. These arguments were ventilated in debates in a teacher journal (Prytz, 2007, p. 102).

Nonetheless, there were also striking similarities between old and new textbooks. The axiomatic-deductive method was applied in the design of the new textbooks; every theorem, except for the axioms, was proved on the basis of axioms, definitions or previously proved theorems. Hence, proofs continued to have a prominent position.

My thesis so far is that the change in mathematics education in the period of 1900-1960 was achieved not through syllabi, but rather through textbooks. This thesis is further confirmed when we consider the professional debate about geometry instruction in *Realskolan,* where textbooks were a key topic. A common denominator of the investigated articles is that they concerned ideals in teaching, something to strive for. This endeavour involved textbooks in some way.

In the period of 1910-1960, there were two comprehensive debates about the best way to teach geometry. The forum was *Elementa*, the teacher journal specialized in mathematics and science education in the secondary schools. The debates took place in the early 1920s and the late 1930s. To a great extent these debates concerned textbooks and the debaters were all textbook authors or editors of textbooks. And in both cases, authors or editors of less popular textbooks initiated the debates by claiming the popular textbooks were of inferior quality. For further details about these debates and the debaters, see Prytz (2009).

[2] Euclid & Heath (1956) is the reference point.

The journals for *Folkskolan*, which were not specialized on subjects, did not contain this type of extensive debate on textbooks. Still, textbooks were a part of the debate throughout the period 1910-1960. In some articles, the authors explicitly said that textbooks were important in teaching (e.g. FT article 4, 11, 29, 34). In others, the authors mentioned textbooks in a more critical tone, implying that textbooks might hinder the teaching in some way (e.g. FT article 2-4, 10-12, 15, 17-18, 26, 28-29, 34). But even more articles concerned textbooks in an indirect way. Several articles concerned arithmetic teaching, in particular which algorithms should be used and how they should be introduced, how different types of numbers (fractions for instance) should be introduced and explained, how symbols and expressions should be used or how many exercises the students should work with (FT article 1-4, 11-21, 24-29, 31-35). All these issues are related to textbook design, which textbook producers had influence over.

So far, I have claimed that change was achieved through textbooks rather than syllabi. But, who initiated change? My standpoint is that change, which we have examples of, was driven by the textbook producers. The arguments are the following. 1) As regards the authors of the geometry textbooks in *Realskolan*, their participation in the professional debate is an example of how they wanted to influence the teachers in some direction. 2) But generally, the producers of textbooks were operating on a market and in the period of 1900-1960, new textbooks were regularly published for *Folkskolan.* The same applies for *Realskolan*, except for the 1940s and early 1950s (Prytz, 2016, pp. 13-14)[3]. Thus, the producer of the newest textbooks had to make an active choice about change to attract buyers. And then it is reasonable to believe that the producers wanted to make the buyers appreciate the change.

An important aspect of this is that the textbook producers were dependent on the teachers as buyers of the textbooks. They had to be attentive to what the teachers wanted. Thus, they could not drive overly radical changes.

4. Attempts of active centralized governance, 1960-1980

By the introduction of *Grundskolan* in 1962, the state's ambition to govern mathematics education had changed. These changes concerned how the content of the courses was selected, but also the role of science in that process. Of course, science had played an important role in that process also before 1960, especially the science of mathematics. Even the arithmetic in elementary schools, i.e. numbers, operations, rules and algorithms, was a part of scientific mathematics. The novelty was that methods from the social sciences were also used to determine the content. Some years before the introduction of *Grundskolan*, a large-scale survey regarding the content of the school subjects Mathematics and Swedish was conducted (SB, 1960). The survey included people from university, schools, administration and industry, but also from the general population. The aim was to map the need for Mathematics and Swedish in different sectors of society.

Another change in governing policy concerned the format of the syllabus in mathematics. In comparison to previous syllabi, the first syllabus for *Grundskolan*

[3] A major exception was the arithmetic textbooks for Realskolan used in years 4-6. On the other hand, it might be that Folkskolan textbooks were used in Realskolan.

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
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contained directives regarding content and teaching methods with a higher degree of detail (Prytz, 2015, pp. 311-312, 319). This also meant a greater number of words. For instance, the syllabus of *Folkskolan* of 1919 comprised about 1,800 words; the first syllabus of *Grundskolan* comprised about 10,500 words (Prytz, 2015, pp. 311-312).

But the mathematical content also changed. Geometry became a less comprehensive topic, especially in years 7-9 and in comparison to *Realskolan*. In *Realskolan*, a more theoretical type of geometry, with emphasis on definitions, theorems and proofs, was taught in years 7-8. According to the first syllabus of *Grundskolan*, one should not strive for an «axiomatic structure» of the course (SÖ, 1962, p. 188); that is, definitions, theorems and proofs should have a less prominent position. This was probably a result of the survey mentioned above: the need for this type of geometry, outside the school system, was deemed low (SB, 1960, pp. 482-483). Furthermore, the relevance of this type of geometry within the school system was also questioned. The basic argument for including theoretical geometry in years 7-9 concerned transfer; the logic students acquired in geometry class was regarded beneficial when they learned not only other mathematical topics but also other school subjects. The final report regarding the survey concluded that there was no scientific support for that argument (SB, 1960, pp. 485-487).

However, apart from geometry, the first syllabus of *Grundskolan* brought few changes in content. The courses in years 1-6 were similar to corresponding courses in *Folkskolan*. The advanced courses in years 7-9 were similar to the courses in *Realskolan* (7-9), except for geometry. And, the basic courses in years 7-9 were similar to the courses in *Folkskolan* (7-9) (SÖ, 1962, pp. 164-170; UP, 1920, pp. 58-60; Wallin & Grimlund, 1939, pp. 316-317).

Thus, the new scientific way of selecting the content of the courses had little impact and the new syllabus of 1962, despite a higher level of details, did not bring many changes in mathematical content.

The recommendations in the syllabus of 1962 about teaching methods brought no significant changes either; they rather confirmed already prominent and established principles and gave them an even more prominent position. These principles concerned student activity and visualizations (SÖ, 1962, pp. 44-52). And throughout the first half of the twentieth century, they had a central position in discussions about mathematics teaching in teacher journals and methodological literature for both *Folkskolan* and *Realskolan*. The key concept in those discussions was *åskådning* (the German equivalent is *anschauung*), which involved visualizations, concrete materials and student activity; for further details see Prytz (2007, pp. 99-106). An important aspect of the debates in the first half of the twentieth century concerning *åskådning* is that the concept was not often called into question. Furthermore, we can also see that authors designed their textbooks (*Folkskolan* and *Realskolan*) according to principles about *åskådning*, which were expressed in teacher journals and methodological literature (Prytz, 2007, pp. 102, 122-123).

My summary of the state's attempt to govern school mathematics in connection to the introduction of *Grundskolan* in 1962 is the following. The state had developed tools, apart from national exams and a mandatory textbook review, for a more

centralized governance of the content and the teaching methods. However, the state did not make much use of these tools to change school mathematics in any significant way.

This summary is confirmed by the International Study of Achievement in Mathematics conducted in 1964. In the survey on the teachers' (7-9) perception of freedom in determining the content and teaching methods, the Swedish teachers came out on top; the mean value was 9.0 on a 10-point scale. The teachers were asked about freedom in relation to syllabus, textbooks and examinations (IEA, 1967, pp. 175-176). If the state had tried to achieve significant changes, it is likely that a greater number of teachers would have perceived it as a limitation of freedom. Note that the Swedish students (year 7) performed quite poorly in comparison with the other countries, coming in at last place together with the USA. In fact, in the second international study, conducted in 1980, the Swedish results were more or less unaltered (Murray & Liljefors, 1983, p. 44). Moreover, the report about the study of 1964 concluded that there was no support for the idea that great freedom for teachers results in high total scores (IEA, 1967, p. 176).

The state's ambition in governing school mathematics changed in connection to the planning and introduction of the curriculum of 1969. This time, the ambition was to achieve significant changes. In the 1960s, two major projects were initiated: the New Math project (NM project) and the IMU project. IMU stands for *Individualiserad MatematikUndervisning* (individualized mathematics teaching). The purpose of the NM project was to develop the content of the teaching and the teaching methods. The purpose of the IMU project was to facilitate efficient individualization, but also to develop how students were grouped and the teacher role.

Despite differences in purpose, the projects shared several characteristics. They lasted for several years and involved thousands of students and teachers. They involved scientific expertise and methods from social science were applied. And a central component in both projects was textbook development.

In the following two sections, the two projects are further described. My thesis is that both projects were attempts of far-reaching state governance. Far-reaching in the sense that the projects aimed at changing how the teachers communicated mathematics and how activities in the classroom were organized. The NM project in particular was expected to have this effect since the syllabus changed according to ideas central for New Math. A third section concerns the implementation of the reforms the two projects resulted in. It is shown that both reforms soon failed in important respects. From that I conclude that school mathematics to a great degree evaded centralized governance.

5. The New Math project

New Math was an international movement, with roots in the USA and France. The movement usurped a dominant position in discussions about school mathematics in the late 1950s in several countries, among those Sweden. Its aim was achieve comprehensive and radical changes in school mathematics from years 1 to 12. These changes should be founded on science; the scientific discipline of mathematics, of course, but also the fairly new scientific disciplines of psychology

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
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and education (cf. Kilpatrick, 2012, p. 563; Phillips, 2015, pp. 87-88). However, the changes were motivated not only by scientific progress; a basic argument was that the workforce needed a new type of school mathematics (cf. Kilpatrick, 2012, pp. 564-565; Phillips, 2015, pp. 26-27). Similar arguments were conveyed in Sweden (cf. Prytz, 2012, p. 410). Another element of this context was a more widespread and positive appreciation of science and rationality. At least in the USA, proponents of the New Math believed the new type of mathematics teaching should promote a general and rational mode of thinking (Phillips, 2015, p. 44-46). To my knowledge, though, this idea was not prominent in Sweden, at least not if we consider Swedish reports and articles about New Math

However, rationality in planning and organizing the society was definitely a characteristic of Sweden. The narrative about the centralization of school governance, mentioned above, concerns this. And the new mathematics syllabus of 1969, which was based on New Math, was prepared in this spirit. The preparations began in 1960, when the *Nordic Committee for the Modernization of Mathematics Teaching* (*Nordiska kommittén för matematikundervisningens modernisering*) was formed. Sweden, Norway, Denmark and Finland had decided to cooperate in the reformation of school mathematics. Apart from producing a new syllabus, a key element in the project was textbook development. About 30 authors were contracted to produce new textbooks that fitted a radically new syllabus. The textbooks were also tried in teaching, in some cases for up to three years. In total, the trials involved 1,310 school classes. By the end, the project received an even more scientific character as comparisons between experimental classes and control classes were made, but with much fewer classes (Prytz & Karlberg, 2016, p. 73). It is important to note that the results from the trials indicated that new types of textbooks were possible to use in teaching. For further details about the trials and their results, see Prytz & Karlberg (2016).

As regards both the content and teaching methods of school mathematics, a key component of New Math was set theory. It was not supposed to be a topic on its own, but should rather be a foundation for the other topics, such as arithmetic and algebra, from years 1 to 12. Set theory had received a similar role in the scientific discipline of mathematics in the first half of the twentieth century. The motivation for giving set theory this role in school mathematics already from year 1 came from psychology. The most renowned researchers engaged in the project were Jean Piaget (1896-1980) and Jerome Bruner (1915-2016). They saw similarities between mental structures and mathematical structures, already in young children, and argued that this should be utilized in teaching. A stronger focus on structures in the teaching should facilitate better understanding, which in turn should facilitate better learning (cf. Bjarnadottír, 2014, p. 451; Phillips, 2015, pp. 87-88, 94).

These ideas had an impact on the Swedish syllabus introduced in 1969. Concepts and notation from set theory were integrated in several topics and were to be used when new concepts in a subtopic were introduced and explained. Set theory was meant to be a bridge between and unite subtopics. Moreover, illustrations based on set theory were also recommended. Thus, the role of set theory concerned teaching methods - how teachers communicated with the students. Nonetheless, set theory was not the only innovation in this respect in the Swedish syllabus of 1969. The number line received a similar role. In some sense, set theory and the number

line were complementary; in passages in the syllabus where set theory was not included, the number line was used instead (SÖ, 1969b, pp. 4-26).

In relation to my claims about far-reaching governance, this role of set theory and the number line in the syllabus is important. It shows that the central school authorities, through the syllabus, wanted to change how teachers interacted with students about mathematics. Moreover, it was a radical change, both with respect to the methodological ideas and the range of governance. This had never been done before.

The content of the mathematics courses also changed. A new topic in all years was statistics and probability. New subtopics in geometry in years 7-9 were trigonometry and vectors. Another innovation was to introduce topics in earlier years. Concepts related to algebra, previously taught mainly in years 7-9, should now be introduced in year 1. Geometry, previously introduced in year 5, should also be taught from year 1. Statistics and probability should be introduced in year 2. Functions should be introduced in year 6 rather than in year 9 (SÖ, 1962, pp. 164-170; SÖ, 1969b, pp. 4-26).

If we consider the major Nordic report on New Math, the introduction of topics in earlier years seems to have had a methodological background. By focusing the teaching on algebraic structures and the concepts of relation, function and vector, the students would become more familiar with mathematics as a tool (NKMM, 1967, p. 173). Exactly what the latter meant was not explained in the report.

All these changes related to New Math were not enforced only through the syllabus. An important element in the governance was textbooks - the idea was to make the teachers switch to a new type of textbooks; recall that much of the NM project concerned development and trials of textbooks. However, the state was not supposed to produce the textbooks. This task was left to the publishing companies. Obviously, this meant that the control of textbook production was pivotal in the attempt to achieve a switch of textbooks and implement the reform.

And indeed the state had such a tool of governance: the national and mandatory textbook review. As mentioned above, the first list of approved textbooks was issued in 1941. Among the review board's aims during the period of 1935-1973 was to consider agreement with the syllabus (Johnsson Harrie, 2009, pp. 115-116). Consequently, in order to have a textbook accepted, the producers had to comply with the content as well as the methods of New Math, since both were clearly expressed in the syllabus.

In summary, in connection to the launch of the new mathematics syllabus of 1969, the state had planned for far-reaching governance. It was far-reaching in the sense that the proposed changes concerned how teachers and textbook authors should communicate the content with the students. Moreover, the state had the means to achieve these changes. A syllabus that described a new content as well as a new method was prepared and tested; it had been demonstrated what type of textbooks matched the new syllabus; and with a mandatory textbook review the state had the means to control the textbook production.

As regards teaching methods, the NM project was restricted to how teachers and textbook producers organized and communicated the content. But changes in

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

the teachers' role and the organization of the teaching were prepared elsewhere, more precisely within the IMU project.

6. The IMU project

IMU stands for *Individualiserad MatematikUndervisning* (*Individualized Mathematics teaching*) and the project mainly concerned years 7-9 (Löwbeer & Orring, 1968, bilaga 1; Larsson, 1973, p. 7). The purpose of the IMU project was linked to the introduction of *Grundskolan* and the need to handle more heterogeneous classes. Before *Grundskolan*, the school system was differentiated after year 4 in *Folkskolan*. By the introduction of *Grundskolan* in 1962, much of this differentiation disappeared. In years 1-6 all classes should be heterogeneous with respect to giftedness and plans for future education. In years 7-9, however, some differentiation remained. The students choose between advanced and basic courses in mathematics and English; students could also choose to study a second foreign language. All these choices were of course related to future studies. The IMU material was developed in line with the idea of heterogeneous classes and was supposed to work without a division in basic and advanced courses in mathematics in years 7-9 (Larsson, 1973, p. 7).

There were of course political arguments about democracy and levelling out differences between societal classes behind the creation of an undifferentiated school system and heterogeneous classes (Lundgren, 1989, pp. 105-112). But the IMU project was motivated from a psychological point of view as well. It was believed that students working at their own pace would learn better. Another belief was that if students were studying on the right level, they should become more motivated, which in turn should lead to better learning (Löwbeer & Orring, 1968, p. 78).

There was also a practical and economic motivation: there were not enough mathematics teachers in the 1960s. By having larger groups of students, the idea was to replace some teachers with assistants when using the IMU material, which would decrease the salary costs (Löwbeer & Orring, 1968, p. 79, Bilaga 6-7). Actually, the removal of advanced and basic courses was also about economy; the system was considered expensive since teachers often had to teach in relatively small classes (Löwbeer & Orring, 1968, Bilaga 6).

Nevertheless, the origin of the project was a teacher-initiated experiment, conducted in an experimental school in the early 1960s, in which the teachers tried to work without basic and advanced classes. The students who wanted a more advanced course were then given self-instructive textbook material - a correspondence course. In brief, the trials went well and gained interest on different levels in the school administration (Larsson, 1973, pp. 10-11).

Observe that correspondence education was by then well established in Sweden. Throughout the twentieth century, an increasing number of Swedes had received secondary education or advanced vocational education through correspondence courses (KU, 1975, pp. 59-60). The leading publishing company in this context was Hermods, which also became engaged in the IMU project.

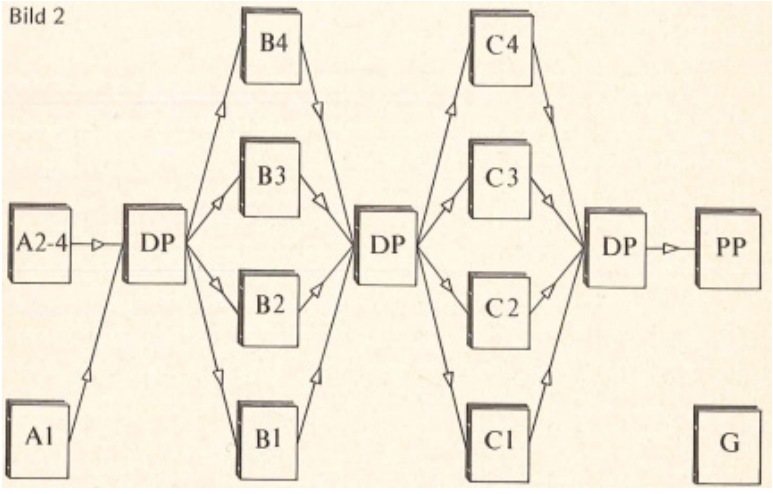
As an organized enterprise involving a greater number of people, the IMU project started in 1964 and ended in 1972 (Larsson, 1973, p. 3). It was a joint project between the central school authorities, the department of education and psychology at the teacher training college in Malmö and the publishing company Hermods. It comprised two parts: 1) development of a new type of individualized textbook material and 2) research about the effects of organizing the teaching on the basis of this material. In total, about 23,500 students took part in the project (Larsson, 1973, p. 12).

Regarding the goals of the IMU project, these varied as the project continued for several years (Larsson, 1973, p. 17). But in a later official report, the following goals were stated:

- To draw up and test self-instructional teaching material in mathematics
- To test suitable teaching methods for the use of this material
- To discover in what way the students should be grouped and the teachers used in order to obtain the maximum effect from the material and the method
- With the aid of the constructed material, to measure the effects of entirely individualised instruction (Håstad et al, 1968, pp. 1-2)

An innovative element of the IMU project was the development of a textbook system adapted to a situation where the needs of the students were more varied. For that reason, the system comprised a great number of booklets and the student should work with the booklet best suited to their ability. In order to match each student with the most suitable booklet, the system comprised a number of diagnostic tests. Another innovative idea was that the students should work by themselves; the material should be self-instructional.

Image 1. Structure of the IMU material, version 5



Source: (Larsson (1973), p. 9)

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

Image 1 shows the structure of one module. Three modules covered one year. The A booklets were common to all students, except for A1, which were intended for the low-performing students. DP stands for diagnostic tests and PP was a more comprehensive test. On the basis of the DPs, the students were matched with a suitable booklet: B or C. The G booklet contained practice exercises.

Apart from the DP diagnoses in Image 1, the students should perform smaller diagnoses in more or less every lesson. These were to be corrected by the students. In this way, the students should detect what they mastered and what exercises they could skip and thus move faster through the material (Håstad et al, 1968, p. 3). Clearly, the intention was not to keep the class together.

The new material also meant a new teacher role. The teachers were not supposed to provide whole-class teaching; they should instead spend more time on planning and providing individual teaching. But the plan was also to replace some teachers with administrative personnel to handle the correction of all diagnoses and the matching of students with the right booklet (Håstad et al, 1968, pp. 6-9). A key idea in that respect was to create bigger classes. And rather than having two teachers for every 60 students, there should be 1.5 teachers and one assistant.

Since the IMU material should be introduced in connection with the new syllabus of 1969, it was adapted to the coming syllabus. Thus, the IMU material included much from the NM project. Actually, one of the authors of the material was Matts Håstad, who was a key figure in the NM project (Larsson, 1973, p. 16; Prytz & Karlberg, 2016, p. 73).

The other two authors were Curt Öreberg, who had been the head teacher in mathematics at the experimental school where it all started, and Leif Svensson, who worked at Hermods and had developed the correspondence material used at the original experimental school (Larsson, 1973, p. 47).

In total, five versions of the IMU material were developed. Each version was modified on the basis of the trials. Another element in the development work was goal analysis, which followed a certain procedure. From the syllabus, a so-called *terminal behaviour* - a description of what the students should know by the end of year 9 - was determined. This was followed by an analysis of a possible content of the courses that could meet the *terminal behaviour*. The final step was to determine the content of the courses. A third element in the process was a higher degree of precision in the goals; they should be expressed in measurable and observable behaviours (Larsson, 1973, pp. 31-32). This was quite different from how goals had been expressed in previous syllabi, but also in the syllabus of 1969. Initially, however, this working process was not fully applied (Larsson, 1973, p. 30).

As regards the second part of the project, it comprised ten different research studies (Larsson, 1973, p. 103). They were designed to meet goals 3 and 4 mentioned above. My summary of these studies is based on Larsson (1973, pp. 103-150).

The studies concerned four elements of the teaching process:

- Individualization as a method

- The material
- The organization with different types of classes and personnel
- A new teacher role

And for the first three elements, the studies aimed at measuring effects on students and teachers. It is important to note that the experimental students followed a preliminary version of the coming syllabus of 1969. Because of that, it was considered impossible to make comparisons, as regards knowledge in mathematics, with students having traditional textbooks following the syllabus of 1962. The argument was that the textbooks were too different and therefore it was impossible to do a fair evaluation.

Regarding the effects on students, the studies showed that the experimental students were more positive about the IMU material than the control students were about their traditional textbooks. It was also shown that it was possible for many of the students to reach the goals in year 9; by the end of year 9, about 80 percent of the students had finished the course or were working with the very last booklet. Another result was that the material suited advanced and average-level students the best. Steps to adapt the material to less gifted students were taken, however, and a fourth low level was added in the fifth version of the material; see Image 1.

The analysis of effects on the teachers was about organization. Three types of organizations were most common: 80-90 students with 2 or 2.5 teachers and 1 assistant; 50-60 students with 1.5 teachers and 1 assistant; about 30 students with 1 teacher and no assistant. The effect was that the teachers in the two larger types of classes could spend more time on both conferences and teaching, while the teachers in the smallest types of classes had to spend more time on administration. Thus, the teacher role appears to have changed towards more individual contact with the students and more time for planning when using the IMU material. Observe that no comparisons with teachers using regular textbooks in regular-sized classes were made; this was clearly stated in the reports.

For the students, the choice of organization did not have clear effects on test results. Thus, reducing the number of teachers per student did not have negative effects on the results. Observe that no comparisons were made with students having regular textbooks and teaching.

In summary, in connection to the launch of the new syllabus of 1969, the state had planned for a new teacher role and a new organization of the teaching. This is another example of an attempt of far-reaching governance; by switching to a new type of textbook and new organization of teaching, the state aimed at changing the daily work of the teachers.

7. Implementation in the 1970s

The basic claim of this section is that the far-reaching reforms in school mathematics that were planned in the 1960s, more precisely those based on the NM project and the IMU project, were not enforced in the 1970s. The common denominator of the evidence presented below is that the central school authorities,

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

soon after the reform was launched, refrained from using or lost their tools of far-reaching governance.

It is important to note that the curriculum reform of 1969, which included the changes in school mathematics based on New Math and IMU, was introduced successively over a three-year period. Starting in 1970, just the students in years 1, 4 and 7 were affected by the reform (SÖ, 1969a, p. 5). This meant that the reform involved all students and teachers only in 1972.

However, by 1972 or 1973, the person in charge of mathematics (1-9) at the central school authorities in the period of 1972 -1977 (Sven-Erik Gode) seems to have given up on central parts of the reform. In an interview (*Nämnaren*, 1983, pp. 10-11), Gode explains that he from the beginning felt compelled to handle insufficiencies in the New Math. As a motivation for this, he mentions a survey to the teachers that indicated, for instance, that the syllabus was too comprehensive. Moreover, the results on the first national tests related to the new syllabus, conducted in 1973, were by Gode seen as a failure, at least in part; comparisons showed that students following the former syllabus had better skills in arithmetic. Gode also perceived the critique against the New Math as just. His policy was then to prioritize certain parts of the syllabus, which appears to have been arithmetical skills and less advanced geometry and algebra. Moreover, teachers should be allowed to choose the methods they found suitable to achieve established goals.

Observe that the question about the success and failure of New Math in Sweden is complex. If all test results in the 1970s are considered, we cannot dismiss New Math as a definite failure. This issue is analysed in a coming paper.

Anyhow, Gode's choice to let the teachers choose their methods was a deviation from the New Math. Recall that a basic element of the New Math concerned how teachers should organize and communicate the content with the students, i.e. teaching methods.

Gode's hesitant, if not negative, attitude towards the New Math, as expressed in 1983, was not an isolated phenomenon within the central school administration. In materials issued by the central school administration, as early as 1973, ideas central for New Math were peripheral.

In 1973, the mathematics syllabus was supplemented by *Basic Skills in Mathematics* (*Baskunskaper i matematik*), which was issued by the central school authorities. Actually, the whole document had a similar layout as the syllabus, aside from the colours of the cover, see Image 2. The purpose of *Basic Skills in Mathematics* (hereafter referred to as *Basic Skills*) and the preceding investigations, which began in 1971, was to handle low-performing students (SÖ, 1973, p. 2). This group comprised about 15 percent of the students (SÖ, 1973, p. 5). However, *Basic Skills* was not a separate course for low performers. The idea was to pinpoint knowledge that should be included in all regular teaching, but that should be prioritized in the teaching of low performers (SÖ, 1973, p. 5). Thus, *Basic Skills* concerned all students.

Image 2. Front covers

Basic Skills comprised two parts: the first was an analysis of the syllabus aiming to identify basic skills; the second comprised guidelines on how the teaching should be planned and executed. Indeed the book contained a great many references to set theory, but mainly in the first part where the syllabus was analysed (SÖ, 1973, pp. 5-37). Terminology from set theory was also used extensively in the syllabus. In the second part about teaching, set theory received little attention (SÖ, 1973, pp. 38-84).

In the second part, sets were mentioned, but then as a component in the expression «sets of tens» (*tiomängder*). It then concerned laboratory material and how students could form subsets of ten objects in a set containing more than ten objects (SÖ, 1973, pp. 58-59). A related expression was «ten base material» (*tiobasmaterial*), which was used for illustration of how to execute arithmetical computations (SÖ, 1973, pp. 59-68). However, sets of tens and tens-based material was a small part of New Math, at least as it was manifested in the syllabus of 1969.

Another difference between *Basic Skills* and the syllabus of 1969 was the use of set theory as a bridge between subtopics. In the syllabus, basic concepts from set theory were used in descriptions of what the teaching in arithmetic, geometry, algebra and function should concern and which illustration should be applied (SÖ, 1969b, pp. 6-12, 24, 26). Set theory did not have this function in *Basic Skills*. As regards algebra, this was impossible since algebra was more or less excluded; it was considered too difficult for low performers, especially in years 7-9 (SÖ, 1973, p. 43). However, geometry and functions were included and these subtopics were not connected with arithmetic via set theory (SÖ, 1973, pp. 13-35)

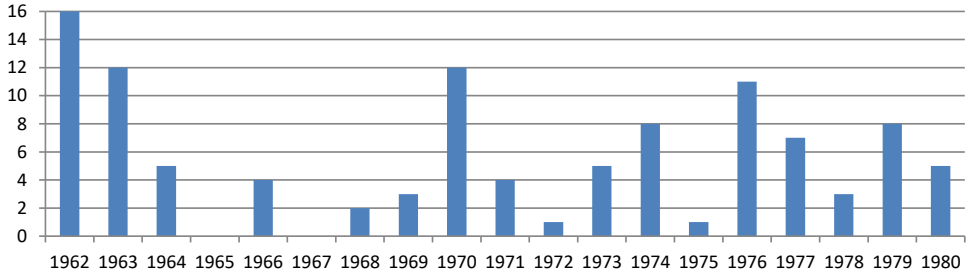
But not only did the central school authorities at an early stage weaken essential parts of the syllabus of 1969 related to the New Math. An important governmental tool disappeared equally fast. In 1974, the textbook review became mandatory for mathematics and many other subjects. The only exception was subjects related to social science and the humanities (Johnsson Harrie, 2009,

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

pp. 12-13). Recall that textbooks were essential in those parts of the reform that concerned New Math and IMU.

The publishing companies' reaction to the changes in policy and governing tools mentioned above seems to have come rather quick. In Diagram 1, below, we can see how the influx of brand-new textbooks took off again around 1974.

Diagram 1. Numbers of new series in Mathematics for *Grundskolan*, years 1-3, 4-6, 7-9



The diagram is based on a database about historical Swedish textbooks in mathematics, see Prytz (2016).

In Diagram 1, we see such peaks in connection to launches of new syllabi in 1962 and 1969, which were major policy changes. Indeed, the mathematics syllabus of 1962 brought few changes, but it was a new school type with new year spans. Hence, old textbooks could not be reused. My point is that the increase around 1974 supports my claims about a policy change in 1972 or 1973.

If we consider the content of a sample of textbook series published in the period of 1969-1979 (43 textbooks for years 1, 4 and 7 out of 63 series), we can see a difference between textbooks published before and after 1975. Before 1975, all textbooks included set theory, more or less extensively. From 1975 and onwards, a number of new textbooks series did not include set theory at all (see *Analysed textbooks*).

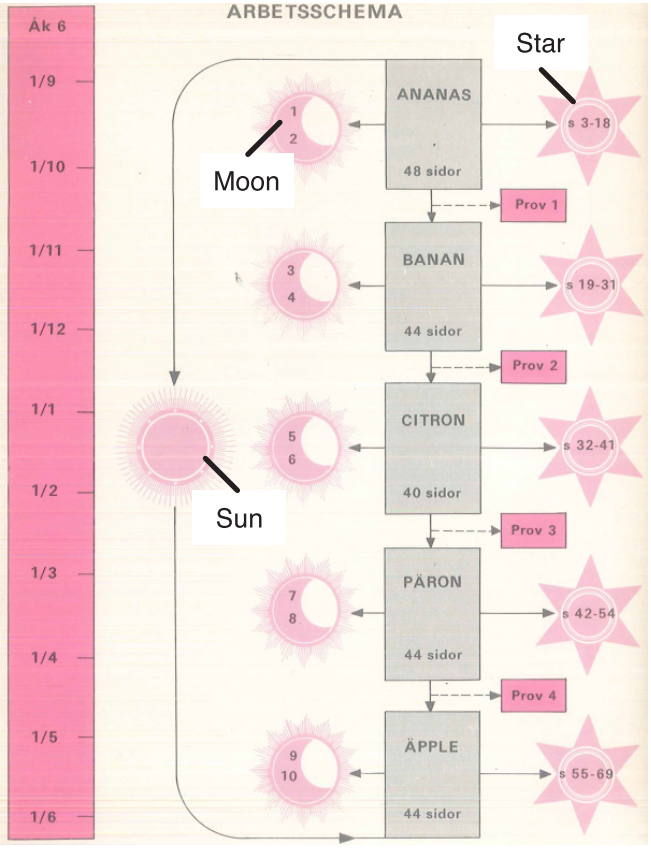
This influx of textbook series not based on set theory also indicates a demand for such products. It is not likely that profit-seeking publishing companies would produce these series without having knowledge of such demand. And we do know that the publishing companies by this time had good contacts with teachers (cf. Prytz, 2017). However, we do not know how great the demand for these series was. Both types of textbooks were printed throughout the second half of the 1970s.

The statistics mentioned above also show the importance of a mandatory textbook review in the implementation of the New Math reform. In the period of 1970-1974, when the review was mandatory, all investigated textbooks included set theory, the very basic element of the reform.

As regards the IMU material, already in 1968, before the project was finished, the central school authorities issued a report saying that the material should be used in schools (Löwbeer & Orring, 1968, p. 1). But for various administrational reasons this never happened (Larsson, 1973, pp. 24-27, 67-71).

Nonetheless, in 1970, the authors of the IMU material published the series *Hello Mathematics!* (*Hej Matematik!)*, which was based on the IMU material, and of course New Math. But there were differences in relation to the IMU material.

Image 3. Structure of *Hello Mathematics!*, year 6



Source: Håstad *et al.* (1973), p. 8

If we compare Images 1 and 3, the main difference in *Hello Mathematics!* was that all students should use the same booklets for longer periods of time. In Image 3, these booklets are entitled *Ananas* (*Pineapple*), *Banan* (*Banana*), *Citron* (*Lemon*), *Päron* (*Pear*) and Äpple (*Apple*). When a student had finished one of those booklets, they proceeded to the booklet *Stjärna* (*Star*), which contained extra exercises of varying degrees of difficulty. Hence, the diagnostic tests in the booklet *Måne* (*Moon*) should not be used to match the student with a new booklet; their purpose was to help the teacher keep track of the students (Håstad *et al.*, 1973, pp. 5-8). The structure of the series for the other years was similar.

Another difference concerned the role of the teacher. Originally in the IMU project, the teacher was not supposed to provide whole-class instruction. This had changed in *Hello Mathematics!*: the booklet *Sol* (*Sun*) should be used for whole-class teaching, which should be done in parallel with the work with the other booklets. And the idea was not to let every student work at their pace. Two models were recommended: keeping the class together or, on the basis of ability, dividing

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

the class into three groups. In the latter model, the groups should be instructed together (Håstad *et al*., 1973, pp. 9-12). The idea to have large classes was not mentioned.

In general, no textbooks series in the 1970s was based on the type of individualization the IMU project originally represented. In that respect, the IMU project had little impact. On the other hand, diagnoses and graded exercises were in various degrees included in several textbook series.

As a summary about New Math and IMU, both projects were examples of plans for far-reaching centralized governance. But this mode of governance was soon abandoned as the syllabus of 1969 was launched. Swedish school mathematics was governed this way only for brief period of time.

8. Conclusions

The questions of this paper concern modes of governance: where in the school system was change initiated and how was change enforced? In relation to previous research, see the section *State of the art*, the aim is to revise a standard narrative about governance of the Swedish school system in the period 1910-1980. A central element in that narrative is that governance was successively centralized up to about 1970 and remained so until the early 1980s; more precisely the state took greater control over the school system.

In some respects that was true for school mathematics. For instance, a mandatory national textbook review was established in the late 1930s; about the same time, national exams were introduced in *Folkskolan*, but they had existed much longer in *Realskolan*. After 1950, we can see further steps in this process as major development projects, more or less scientific, were carried out as preparations of major syllabus reforms. Moreover, the syllabi became much more detailed after 1960.

However, if we consider active attempts to achieve change as an element in governance, the standard narrative does not apply to school mathematics. In that perspective, centralization occurred fairly late and not before 1960. First of all, we can discard national exams as a main driver of change; they did not regulate teaching methods, for instance how to design explanations and exercises, nor the order and extent of topics, subtopics, concepts, explanations and exercises. Moreover, a textbook review alone could not drive change since the reviewers should check textbooks against the syllabus. And the syllabi before 1960 contained few changes and were kept on a general level. Thus, the textbook producers were given fairly great freedom regarding how to design explanations and exercises, but also in the order and extent of topics, subtopics, concepts, explanations and exercises.

My point here is that central governance of school mathematics before 1960 did not concern teaching methods, which is a central part of teaching. Instead, drivers of change as regards teaching methods were the textbook producers. In this paper, I have shown examples of deliberate changes in the textbooks and that textbooks were debated in the teacher journals.

However, the syllabus of 1962, despite a much greater level of detail than previous syllabi, brought only smaller changes. The most significant change

concerned geometry in years 7-9. The directives about teaching methods in the syllabus of 1962 were largely a confirmation of ideas expressed and accepted in teacher journals and teaching literature in the previous decades.

Only once in the period 1910-1980 did the state aim at achieving comprehensive change by means of a detailed national syllabus. This happened when the second syllabus of *Grundskolan* was issued in 1969. This syllabus was to a large extent influenced by the New Math. On the other hand, this mode of governance lasted only for a few years. The reform concerned all students first in 1972 and by 1973, we can already see how the central school authorities refrained from enforcing key elements of the reform. Moreover, in 1974, the national textbook review became non-mandatory. This appears to have opened up for textbooks not in line with New Math; it was only after 1974 that those types of textbooks were published. Thus, active centralized governance of school mathematics aimed at change, with a national syllabus as a key element, came late and was a brief event.

However, if we shift perspective away from central school authorities and national syllabi, we can discern continuity in how changes in Swedish school mathematics were initiated and enforced. Throughout the period 1910-1980, textbooks were a main tool in achieving change, but it was done differently in the periods of 1910-1960 and 1960-1980.

In 1910-1960, textbook authors introduced new ways of explaining mathematical concepts and new types of exercises and proofs. Textbooks were also a common topic in teacher journals, which indicates a central role in teaching.

The importance of textbooks as a tool for change did not, however, change in the 1960s. The launch of the syllabus of 1969 was preceded by two major development projects described in this paper: the New Math project and the IMU project. Both aimed at radical changes in school mathematics. In both these projects, development and trials of textbooks were the essential element. Moreover, the implementation of the syllabus of 1969 - the New Math syllabus - depended on teachers shifting to new textbooks.

The main difference in the 1960s and 1970s was that the central school authorities made a serious endeavour to seize control of the textbook production. Hence, the more rigorous forms of textbook development, in different ways inspired by social sciences, were part of that endeavour.

These results add to the greater standard narrative about how the state successively took greater control over the Swedish school system. If we assume this narrative was true for judicial and economic matters, my results suggest that the ideological governance developed differently, especially if we consider attempts to change a school subject. Of course, further studies about governance and other school subjects are needed.

In research about the efficiency of different modes of governance, scholars have reached different conclusions regarding centralization and decentralization. As mentioned in the section *State of the art*, Green (1997) suggests that the former is more efficient, while Hofman et al. (2010) suggests that the latter is more efficient. A problem with Green's (1997) analysis is that he assumes the Swedish system became highly centralized and at the same time he uses results

Espacio, Tiempo y Educación, v. 4, n. 2, julio-diciembre / july-december 2017, pp. 43-72.
e-ISSN: 1698-7802

from international studies on mathematic education, though not exclusively. The problematic aspect is that Swedish mathematics education was far from centralized. However, Green's (1997) conclusion is saved by the fact that Swedish students in year 7 performed quite poorly in the international studies of 1964 and 1980. My findings indicate that the absence of changes in school mathematics initiated by central school authorities in the period of 1910-1980 did not bring about good results in international studies on achievements in mathematics. Note that neither Green (1997) nor Hofman et al. (2010) pay much attention to governance of school subjects.

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(The journal of the Folkskolan teachers)

Article	Year	Nr	Pages	Title
1.	1921		293	Ett försvar för de allmänna bråken.
2.	1921		236-237	Räkneundervisningen enligt den nya undervisningsplanen
3.	1921		421	Räkneundervisningen
4.	1921		443-444	Räkneundervisningen i landsbygdsens skolformer
5.	1923		162	Matematiken i fortsättningsskolan
6.	1923		183-184	Räkning i fortsättningsskolan
7.	1923		694-695	Räkning i fortsättningsskolan
8.	1923		167	Räkning i fortsättningsskolan
9.	1929		836	Seminariernas matematikprov
10.	1930		916-917	Räkneundervisningen i folkskolan
11.	1931		332	En ny räknelära för folkskolan
12.	1940	14	9	Räkning i folkskolan
13.	1943	50	11-12	Förenklingar i räkneundervisningen
14.	1944	40	13	Räkneundervisningen i B 3-skolan
15.	1944	2	20	Räkneundervisning
16.	1945	21	15	Att göra bråk liknämniga
17.	1946	7	10	Huvudräkning
18.	1946	1	14	Aktiv huvudräkning
19.	1947	10	22	Nytt hjälpmedel vid räkneundervisningen
20.	1948	17	4-5	Räkneterminologin i folkskolan
21.	1948	16	8-11	Bör räkneundervisningen metodiskt likriktas?
22.	1952	45	11-12	Sjunkande räknefärdighet?
23.	1952	49	23-24	Räknefärdigheten. Diskussion om uppsalaundersökningen i mekanisk räkning
24.	1952	41	15-16	Räkneundervisningen
25.	1952	10	16-17	Kulramen i folkskolans räkneundervisning
26.	1952	49	25	Något om räkneundervisningen
27.	1953	41	15	Multiplikation utan ‘minne’
28.	1953	51-52	21-24	Undersökning om räknefärdigheten
29.	1953	41	8-11	Räknelära
30.	1954	7-8	20	Hur stiger den mekaniska räknefärdigheten under skolgången
31.	1954	11	23	Räkneundervisningen, Är enhetlighet i matematikundervisningen nödvändig?
32.	1954	16-17	37	Min metod. Skriftlig huvudräkning
33.	1954	22	22-23	Folkskolans matematikundervisning
34.	1954	5	8-10	Enhetlighet i matematikundervisningen
35.	1956	42	20-21	Begreppen i division

12. Analysed textbooks (chronological order)

The textbooks which do not contain set theory are marked by *.

Year 1 (13 books)

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Hultman, C., Ljung, B., & Kristiansson, M. (1972). *Ettans matematik*. Gävle: Skolförl. Gävle.

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Forssén, E.O., Olsén, A., & Frank, K. (1973). *Modern matematik*. Stockholm: Esselte studium.

* Unenge, J. (1975). *Matematik.* Lund: Corona.

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Year 4 (11 books)

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Lundgren, S., & Paulsson, K. (1976). *Mia: matematik i användning.* Stockholm: Esselte studium.

* Kilborn, W., Dahlström, K., & Johansson, B. (1977). *Min matematik.* Göteborg: Stegeland.

Håstad, M., Öreberg, C., & Svensson, L. (1977). *Hej matematik*. (new version) Malmö: LiberLäromedel.

Year 7 (19 books)

Kvist, U., & Larsson, J. (1969). *Högstadiets matematik, Lunda-systemet*. Lund: H. Ohlsson.

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