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GOVERNMENT SPENDING IN A SIMPLE MODEL OF ENDOGENOUS GROWTH

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ABSTRACT

I extend existing models of endogenous economic growth to incorporate a government sector. Production involves private capital (broadly defined) and public services. There is constant returns to scale in the two factors, but diminishing returns to each separately. Public services are financed by a flat-rate income tax.

The economy's growth rate and saving rate initially rise with the ratio of productive government expenditures to GNP, g/y, but each rate eventually reaches a peak and subsequently declines. If the production function is Cobb-Douglas with an exponent α for public services, then the value $g/y = \alpha$ maximizes the growth rate, and also maximizes the utility attained by the representative consumer.

The distortion from the income tax implies that the decentralized equilibrium is not Pareto optimal; in particular, the growth and saving rates are too low from a social perspective. In a command optimum, growth and saving rates are higher, but $g/y = \alpha$ turns out still to be the best choice for the size of government. The command optimum can be sustained by picking the expenditure ratio, $g/y = \alpha$, and then financing this spending by lump-sum taxes.

If the share of productive spending, g/y, were chosen randomly, then the model would predict a non-monotonic relation between g/y and the economy's long-term growth and saving rates. However, for optimizing governments, the model predicts an inverse association between g/y and the rates of growth and saving.

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Recent models of economic growth can generate long-term growth without relying on exogenous changes in technology or population. Some of the models amount to theories of technological progress (Romer, 1986), and others to theories of population change (Becker and Barro, 1988). A general feature of these models is the presence of constant or increasing returns in the factors that can be accumulated (Romer, 1988, Lucas, 1988, Rebelo, 1987).

One strand of the literature on endogenous economic growth concerns models where private and social returns to investment diverge, so that decentralized choices lead to sub-optimal rates of saving and economic growth (Romer, 1986). In this setting private returns to scale may be diminishing, but social returns—which reflect spillovers of knowledge or other externalities—can be constant or increasing. Another line of research involves models without externalities, where the privately determined choices of saving and growth are Pareto optimal (Rebelo, 1987). These models rely on constant returns to private capital, broadly defined to encompass human and non-human capital.

The present analysis builds on both aspects of this literature by incorporating a public sector into a simple, constant-returns model of economic growth. Because of familiar externalities associated with public expenditures and taxes, the privately-determined values of saving and economic growth turn out to be sub-optimal. Hence there are interesting choices about government policies, as well as empirical predictions about the relations among the size of government, the saving rate, and the rate of economic growth.

1. Growth Models with Optimizing Households

As a background I begin with a brief sketch of the standard optimal growth model, due to Ramsey (1928), Cass (1965), and Koopmans (1965). The representative, infinite-lived household seeks to maximize overall utility, as given by

$$U = \int_0^\infty u(c)e^{-\rho t}dt \tag{1}$$

where c is consumption and $\rho>0$ is the constant rate of time preference. Population, which corresponds to the number of consumers, is constant. I use the iso-elastic utility function,

$$u(c) = [c^{1-\sigma}-1]/(1-\sigma) \tag{2}$$

where $\sigma>0$.

Each household-producer has access to the production function,

$$y = f(k) \tag{3}$$

where y is output per worker and k is capital per worker. I interpret $y = f(k)$ as output net of depreciation. The production function satisfies the usual properties, including positive net marginal product of capital ($f' > 0$) over some range of k, diminishing marginal productivity ($f'' < 0$), and the limiting conditions, $f'(0) = \infty$ and $f'(\infty) \leq 0$. There is no technological progress, in the sense that the function $f(\cdot)$ is time invariant. The economy is closed, and produced net output goes either for consumption or net investment. The number of workers equals the constant number of households, and each worker works one unit of time. That is, I abstract from the labor-leisure choice.

As is well known, the maximization of the representative household's overall utility in equation (1) implies that the growth rate of consumption at each point in time is given by

$$\dot{c}/c = (1/\sigma)\cdot(f'-\rho) \tag{4}$$

Since I omitted the two standard sources of exogenous growth—population change and technological progress—the model has steady-state levels of capital, k^*, and consumption, c^*. The value k^* is determined from equation (3) by the condition $f'(k^*) = \rho$. Since net investment is zero in the steady state, steady-state consumption is $c^* = f(k^*)$.

A recent strand of endogenous growth models, represented by Rebelo (1987), departs from the standard framework by replacing diminishing returns, $f'' < 0$, with constant returns. In the present setting, which has a single type of capital good, the modified production function with constant returns to capital is

$$y = Ak \tag{5}$$

where $A > 0$ is the constant net marginal product of capital.

The assumption of constant returns becomes more plausible when capital is viewed broadly to encompass human and non-human capital. Human investments include education and training, as well as expenses for having and raising children (Becker and Barro, 1988). In effect, a family can invest in human capital by improving the quality of its existing members or by deciding to have more members. In any event, while the returns to broadly-defined capital may not be precisely constant, it is hard to see why economists usually interpret capital narrowly to exclude labor input. It seems that labor services depend as much as capital services on prior investment decisions.

Of course, human and non-human capital need not be perfect substitutes in production. Therefore production may show roughly constant returns to scale in the two types of capital taken together, but diminishing returns in either input separately. The "Ak" production function shown in equation (5) can be modified to distinguish between two types of capital, and the model can be extended, along the lines of Lucas (1988) and Rebelo (1987), to allow for sectors that produce physical and human capital, respectively. In comparison with the "Ak" model, the main additional results involve transitional dynamics whereby an economy moves from an arbitrary starting ratio of physical to human capital to a steady-state ratio. This transition is particularly important in analyzing how countries that start from different initial conditions—possibly due to major disturbances such as World War II—might converge to similar levels of economic performance. However, for studying steady-state growth, the important element is constant returns to scale in the factors that can be accumulated—that is, the two types of capital taken together—and not the distinction between the factors.[1] Since my primary interest in this paper concerns long-term growth, I decided to use the simplest constant-returns-to-scale production function, as given by the "Ak" form in equation (5).

The analysis does assume that fixed factors are not important enough to cause significant departures from constant returns. Even with the existence of land and other natural resources, the results can go through as long as reproducible capital is a good substitute for these fixed factors. Alternatively, it may be that fixed factors become binding eventually, but not until the variable factors reach very high levels. Then the present analysis may be a good approximation over a wide range of accumulation where the returns to scale in the variable factors are nearly constant.

The concept of capital can include knowledge, accumulated through expenditures on research and development, as long as this knowledge is private property. Knowledge that is non-excludable or non-rival brings in issues of sub-optimal economic growth that have been studied by Romer (1986). My study of government as an element in economic growth turns out to parallel Romer's analysis in some respects.

The production function in equation (5) implies $f' = A$. Substituting into equation (4) then yields

$$(6) \qquad \gamma = \dot{c}/c = (1/\sigma)\cdot(A-\rho)$$

[1] Jones and Manuelli (1988) show that similar results can obtain if the marginal product of capital is diminishing but has a positive lower bound. In effect, diminishing returns can apply over a range of capital stocks, as long as roughly constant returns apply asymptotically.

where I use the symbol γ to denote a growth rate. I assume that the technology is sufficiently productive to ensure positive steady-state growth, but not so productive as to yield unbounded utility. The corresponding inequality conditions are

$$A > \rho > A(1-\sigma) \tag{7}$$

The first part implies $\gamma > 0$ in equation (6). The second part, which is satisfied automatically if $A>0$, $\rho>0$, and $\sigma\geq1$, guarantees that the attainable utility is bounded.

In this model the economy is always at a position of steady-state growth where all variables—c, k, and y—grow at the rate γ shown in equation (6). Given an initial capital stock, k(0), the levels of all variables are also determined. In particular, since net investment equals γk, the initial level of consumption is

$$c(0) = k(0)\cdot(A-\gamma) \tag{8}$$

The model is a theory of endogenous growth in that changes in the underlying parameters of technology and preferences map into differences in growth rates. From equation (6) the growth rate, γ, is higher if the economy is more productive (higher A), and lower if people are less patient (higher ρ) or less willing to substitute intertemporally (higher σ).

2. The Public Sector

The contribution of the present paper is to modify the above analysis by incorporating a public sector. Let g be the quantity of public services provided to each household-producer. I assume that these services are provided without user charges, and are not subject to congestion effects (which might arise for highways or some other public services). That is, the model abstracts from externalities associated with the use of public services.

I consider initially the role of public services as an input to private production. It is this productive role that creates a potentially positive linkage between government and growth. I assume now that production exhibits constant returns to scale in k and g together, but diminishing returns in k separately. That is, even with a broad concept of private capital, production exhibits decreasing returns to private inputs if the (complementary) government inputs do not expand in a parallel manner. In a recent empirical study, Aschauer (1988) argues that the services from government infrastructure are particularly important in this context.

I assume the Cobb-Douglas form of production function,

$$y = f(k,g) = Ak^{1-\alpha}g^{\alpha} \tag{9}$$

where $0 < \alpha < 1$. In equation (9) k is the representative producer's quantity of capital, which would correspond to the per capita amount of aggregate capital. I assume that g can be measured correspondingly by the per capita quantity of government purchases of goods and services. A number of problems can arise here. First, the flow of public services need not correspond to government purchases, especially when the government owns capital and the national accounts omit an imputed rental income on public capital in the measure of current purchases. This issue is important for empirical implementation of the model. But conceptually, it is satisfactory to think of the government as doing no production and owning no capital. Then the government just buys a flow of output (including services of highways, sewers, battleships, etc.) from the private sector. These purchased services, which the government makes available to households, correspond to the input that matters for private production in equation (9). As long as the government and the private sector have the same production functions, the results would be the same if the government buys private inputs and does its own production, instead of purchasing only final output from the private sector, as I assume.

A second issue arises if the public services are non-rival for the users (as is true for the space program). Then it is the total of government purchases, rather than the amount per capita, that matters for each individual. As is well known at least since Samuelson (1954), this element is important for determining the desirable scale of governmental activity. However, the present analysis can be modified to include this aspect of publicness without changing the nature of the subsequent results.

Government expenditure is financed contemporaneously by a flat-rate income tax,

$$g = T = \tau y = \tau A k^{1-\alpha} g^{\alpha} \tag{10}$$

where T is government revenue and τ is the tax rate. I have normalized the number of households to unity, so that g corresponds to aggregate expenditures and T to aggregate revenues.

The production function in equation (9) implies that the marginal product of capital is now

$$(11) \qquad f_k = A(1-\alpha)(g/k)^{\alpha}$$

Note that f_k is calculated by varying k in equation (9), while holding fixed g. That is, the representative producer assumes that changes in his quantity of capital and output do not lead to any changes in his amount of public services.

It is convenient to substitute $g = \tau y$ in equation (9) and simplify to get

$$(12) \qquad y = k \cdot A^{1/(1-\alpha)} \cdot \tau^{\alpha/(1-\alpha)}$$

Therefore, for a given expenditure ratio, τ, y is proportional to k, as in the "Ak" production function in equation (5). An increase in τ means an increase in the relative amount of public input, and therefore an upward shift in the coefficient that connects y to k.

The ratio of the two productive inputs is

$$(13) \qquad g/k = (g/y) \cdot (y/k) = \tau \cdot (y/k) = (A\tau)^{1/(1-\alpha)}$$

where the value for y/k comes from equation (12). Substituting from equation (13) for g/k into equation (11) leads to

$$(14) \qquad f_k = (1-\alpha)\cdot A^{1/(1-\alpha)}\cdot \tau^{\alpha/(1-\alpha)}$$

Therefore, an increase in the expenditure ratio, τ, implies an upward shift in the marginal product of capital, f_k.

Private optimization still leads to a path of consumption that satisfies equation (4), except that f' is replaced by the private marginal return to capital. With the presence of a flat-rate income tax at rate τ, this return is $(1-\tau)f_k$. Therefore, substituting for f_k from equation (14), the growth rate of consumption is now

$$(15) \qquad \gamma = \dot{c}/c = (1/\sigma)\cdot[(1-\alpha)\cdot A^{1/(1-\alpha)}\cdot(1-\tau)\cdot\tau^{\alpha/(1-\alpha)} - \rho]$$

As long as τ is constant—that is, the government sets g and T to grow at the same rate as y—the growth rate γ is constant. Hence the dynamics is the same as that for the "Ak" model analyzed before. Consumption starts at some value c(0) and then grows at the constant rate γ. Similarly, k and y begin at initial values k(0) and y(0) and then grow at the constant rate γ. The economy has no transitional dynamics, and is always in a position of steady-state growth where all quantities grow at the rate γ shown in equation (15).

Given a starting amount of capital, k(0), the levels of all variables are again determined. In particular, the initial quantity of consumption is

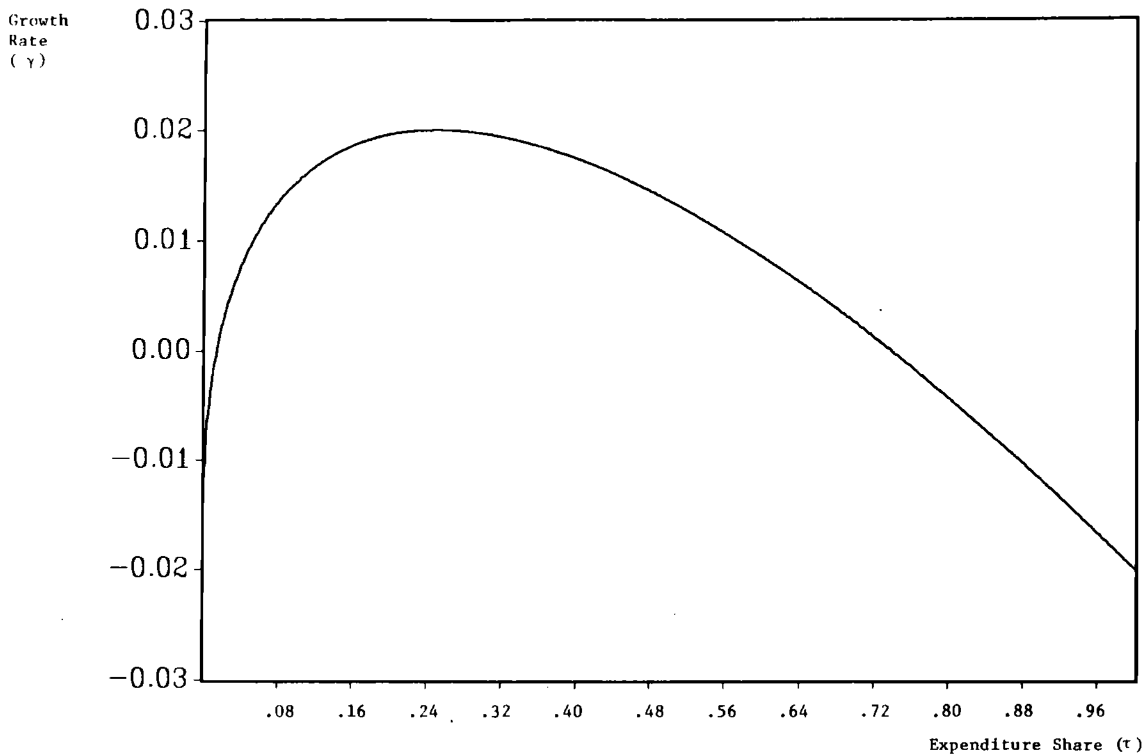
$$(16) \qquad c(0) = k(0)\cdot[(1-\tau)\cdot A^{1/(1-\alpha)}\cdot\tau^{\alpha/(1-\alpha)} - \gamma]$$

where γ is given in equation (15). The first term inside the brackets of equation (16) corresponds to $y(0) - g(0)$, and the second term to initial investment, $\dot{k}(0)$.

Figure 1 (originally drawn on a menu) shows the relation between the growth rate, γ, and the expenditure- and tax-rate, τ. A higher expenditure ratio raises f_k in equation (14) and thereby raises γ in equation (15). However, a higher tax rate means that people retain a smaller fraction, $1-\tau$, of their before-tax income, which tends to reduce γ. At low values of τ, the first force dominates and creates a net positive effect of τ on γ. (The Cobb-Douglas technology shown in equation (9) implies that anarchy is very unproductive.) However, for high enough τ, the second force dominates and therefore leads to a net negative effect of τ on γ. As τ approaches 0 or 1, the growth rate from equation (15) approaches the same negative value, $-\rho/\sigma$.

The growth rate, γ, is positive for a range of τ if the economy is sufficiently productive relative to the rate of time preference. The condition for a range with positive growth (which generalizes the condition $A > \rho$ from the "Ak" model) is $A^{1/(1-\alpha)}\cdot(1-\alpha)^2\cdot\alpha^{\alpha/(1-\alpha)} > \rho$. Also, as before, I assume that the economy is not so productive to allow the attained utility to become unbounded—the condition here is $\rho > A^{1/(1-\alpha)}\cdot(1-\sigma)(1-\alpha)^2\cdot\alpha^{\alpha/(1-\alpha)}$. As in the "Ak" model, the latter condition must hold if $A>0$, $\rho>0$, and $\sigma\geq1$.

Equation (15) shows that maximizing γ is equivalent to maximizing the expression, $(1-\tau)\cdot\tau^{\alpha/(1-\alpha)}$. The solution is $\tau = \alpha$. Roughly speaking, to maximize the growth rate, the government's sets its share of GNP, $\tau = g/y$, to equal the share it would get if public services were a competitively supplied input of production. Note that the value of τ that maximizes γ depends only on the production parameter, α, and not on the preference parameters, ρ and σ. (The independence from ρ and σ follows for any constant-returns



<u>Figure 1</u>
<u>The Growth Rate and the Size of Government</u>

NOTE: The curve shows the growth rate, γ, from equation (15). The parameter values are $\sigma = 1$, $\alpha = .25$, $\rho = .02$, $A^{1/\alpha} = .113$. These values imply that the maximum value of γ is .02.

production function, and does not depend on the Cobb-Douglas form.)

The (net) saving rate is given by

$$s = \dot{k}/y = (\dot{k}/k)\cdot(k/y) = \gamma\cdot A^{-1/(1-\alpha)}\cdot\tau^{-\alpha/(1-\alpha)} \tag{17}$$

where k/y comes from equation (12), and γ is given in equation (15). The curve in Figure 2 is a graph of s versus τ. Because k/y declines with τ, the saving rate peaks before the growth rate. That is, a value $\tau < \alpha$ would maximize s. Recall that saving and investment are broader concepts than usual in this model since they encompass accumulations of human capital (including additions to the stock of population). On the other hand, net product, net investment, and net saving are reduced by depreciation of stocks of human capital.

Presumably, there is no reason for the government to wish to maximize γ or s, *per se*. For a benevolent government, the appropriate objective in this model is to maximize the utility attained by the representative household. Because the economy is always in a position of steady-state growth, it is straightforward to compute the attained utility as a function of τ, as long as τ is constant over time.[2] With γ constant, the integral in equation (1) can be simplified to yield (aside from a constant),

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[2]An optimizing, benevolent government would choose a constant value of τ in this model. If the government is not benevolent, then its objective may also entail a constant value of τ, although not the value that maximizes the utility attained by the representative household. For example, this result obtains in a model, considered below, where the government maximizes an expression that relates to the present value of its net receipts.

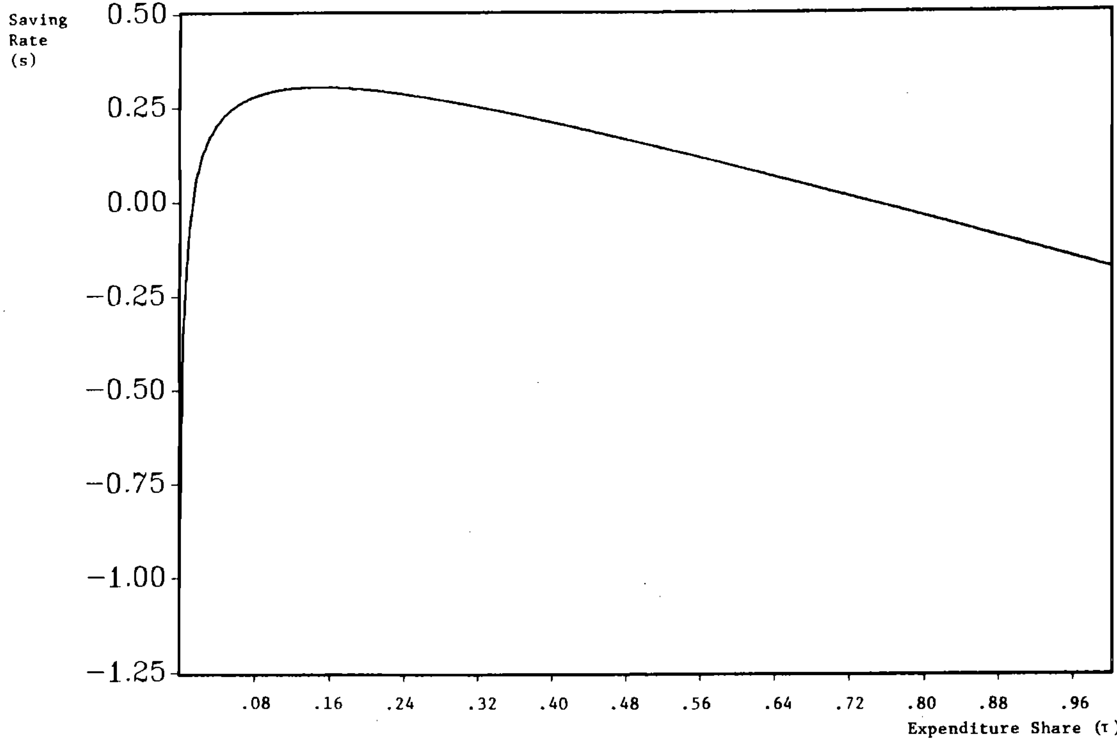

Figure 2
The Saving Rate and the Size of Government

NOTE: The curve shows the saving rate, s, from equation (17). Parameter values are indicated in Figure 1.

$$U = \frac{[c(0)]^{1-\sigma}}{(1-\sigma)[\rho - \gamma(1-\sigma)]} \tag{18}$$

The condition that utility be bounded, mentioned before, ensures that $\rho > \gamma(1-\sigma)$.

Equations (15) and (16) determine γ and c(0), respectively, as functions of τ. Hence, these formulas can be used to determine the value of τ that maximizes U in equation (18). To see the nature of the results, it is useful to note that equations (15) and (16) imply that c(0) can be written as a function of γ (with τ not appearing separately),

$$c(0) = [k(0)/(1-\alpha)] \cdot [\rho + \gamma \cdot (\sigma+\alpha-1)] \tag{19}$$

Substituting into equation (18) yields a relation between U and γ,

$$U = \frac{[\rho + \gamma(\sigma+\alpha-1)]^{1-\sigma}}{(1-\sigma)[\rho - \gamma(1-\sigma)]} \tag{20}$$

It can then be shown that the effect of γ on U in equation (20) is positive for all values of $\sigma>0$ and $0<\alpha<1$, as long as utility is bounded, which ensures $\rho > \gamma(1-\sigma)$. (This result follows although an increase in γ need not raise c(0) in equation (19).) Therefore the maximization of U corresponds to the maximization of γ. It follows that $\tau = \alpha$ is the tax rate that delivers the maximum of attained utility.[3]

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[3]This calculation holds fixed the initial capital stock, k(0). Therefore, I do not allow k(0)—that is, prior investment decisions—to respond to "once-and-for-all" changes in τ at date 0. As long as the policymaker is restricted to a flat-rate income tax—so that pure capital levies are

3. A Planning Problem for the Government

The result $\tau = \alpha$ is the solution to a second-best policy problem. Because of familiar externalities implied by public expenditures and taxation, the decentralized choices of saving turn out to generate outcomes that are not Pareto optimal. In fact, the departures from Pareto optimality are analogous to those in Romer's (1986) growth model, which relied on the public-goods nature of privately-created knowledge.

In the present model the easiest way to assess the external effects is to compare the decentralized outcomes with those from a planning problem. Suppose that the government chooses a constant expenditure ratio, τ, and can then dictate each household's choices for consumption over time.[4] Given the value of τ, the government picks the consumption path to maximize the representative household's attained utility, where the expression for utility is again given in equations (1) and (2). The resulting condition for the planned growth rate of consumption turns out to be

$$\text{(21)} \qquad \gamma_p = \dot{c}/c = (1/\sigma)\cdot[A^{1/(1-\alpha)}\cdot(1-\tau)\cdot\tau^{\alpha/(1-\alpha)} - \rho]$$

In equation (15), the expression within the brackets and to the left of the

precluded—the optimal tax rate, $\tau = \alpha$, is time-consistent in this model. It turns out, as shown below, that the solution is also time consistent if the policymaker can use a consumption tax, which amounts to a lump-sum tax in the present model.

[4]See n. 2 above on the constancy of τ.

minus sign was the private marginal return on capital, $(1-\tau)f_k$. In contrast, the corresponding term in equation (21) is the social marginal return on capital, given that the expenditure ratio, τ, is assumed constant. Equation (12) shows that the marginal effect of k on y, for fixed τ, equals $A^{1/(1-\alpha)} \cdot \tau^{\alpha/(1-\alpha)}$. However, to maintain τ, an increase of y by 1 unit requires an increase in g by τ units. Since the increase in g comes out of current output, the effect of k on y is adjusted by the factor, $(1-\tau)$, to calculate the net social return on capital in equation (21). Hence, the difference between the private choice in equation (15) and the planning solution in equation (21) is the presence of the term, $1-\alpha$, in the former.

Figure 3 shows how τ affects the planning growth rate, γ_p, and the decentralized growth rate, γ. It is clear from a comparison of equations (21) and (15) that γ_p exceeds γ for all values of τ. That is, the decentralized choices involve too little growth. (The insufficiency of growth corresponds to too low a saving rate—see the comparison of saving rates in Figure 4.) Moreover, since equation (21) differs from equation (15) only by the absence of the term, $1-\alpha$, it follows that the shape of the graph of γ_p versus τ is the same as that of γ versus τ. In particular, the maximum of γ_p also occurs at the tax rate, $\tau = \alpha$. (The result $\tau = \alpha$ is the solution to almost all problems in this paper.)

It is straightforward to show, following the procedure used for the decentralized case, that the planner who desires to maximize the utility attained by the representative household would choose the value of τ that maximizes the growth rate, γ_p. But this growth-maximizing expenditure share is again the value α. Hence the government selects the expenditure share, $\tau = \alpha$, in two circumstances: first, if it uses an income tax to finance

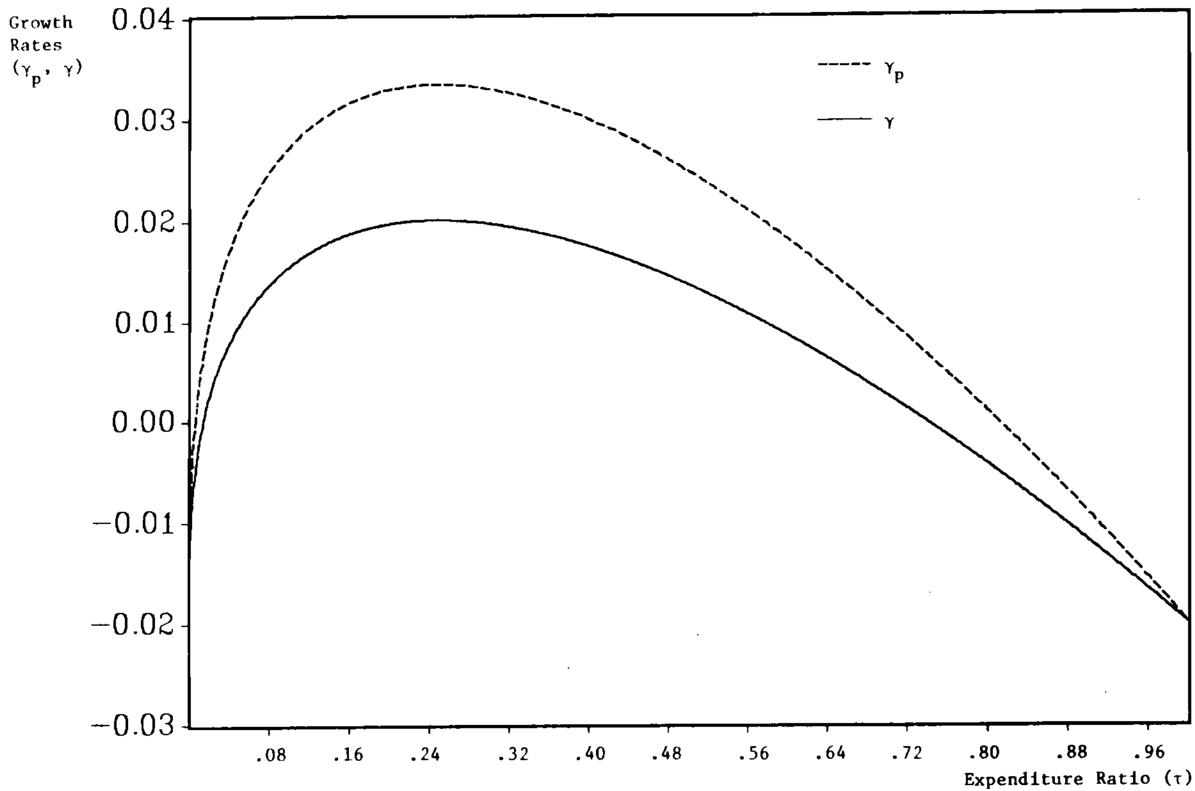


Figure 3

The Planning Growth Rate (γ_p) and the Decentralized Growth Rate (γ)

NOTE: γ_p comes from equation (21) and γ from equation (15). Parameter values are given in Figure 1.

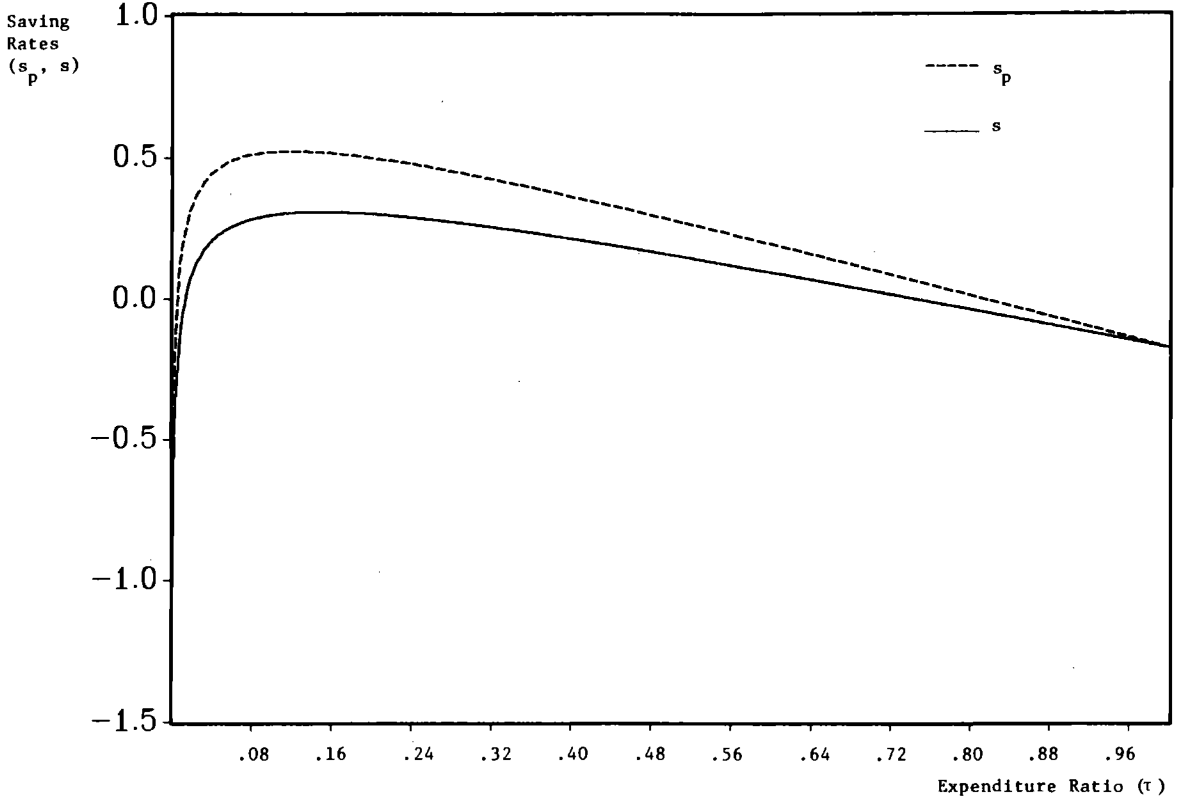


Figure 4

The Planning Saving Rate (s_p) and the Decentralized Saving Rate (s)

NOTE: $s_p = \gamma_p \cdot (k/y)$, where γ_p is given in equation (21) and k/y in equation (12). s is given in equation (17). Parameter values are as in Figure 1.

spending in an environment of decentralized households, and second, if it has the power to dictate each household's saving behavior. The growth rate, saving rate, and level of attained utility are all lower in the first environment than the second. But the optimal share of government in GNP is the same.

It is natural to consider whether the command optimum can be implemented by replacing the income tax with a lump-sum tax in an environment of decentralized households. (In this model, which lacks a labor-leisure choice, a consumption tax would be equivalent to a lump-sum tax.) With lump-sum taxes, the private marginal return on capital is f_k, rather than $(1-\tau)f_k$. Therefore, instead of equation (15), optimizing individual households would choose the growth rate of consumption,

$$(22) \quad \gamma_L = \dot{c}/c = (1/\sigma)\cdot[(1-\alpha)\cdot A^{1/(1-\alpha)}\cdot\tau^{\alpha/(1-\alpha)} - \rho]$$

Thus, γ_L differs from γ by the absence of the term, $1-\tau$, inside the brackets.

Figure 5 graphs γ_L, along with γ and γ_p, as a function of τ. As is apparent from equation (22), γ_L is monotonically increasing in τ. That is because a higher τ means a higher expenditure share, which shifts upward the marginal product, f_k. With a lump-sum tax, households respond to the higher f_k by choosing a higher growth rate for consumption (and a higher saving rate—see Figure 6).

A comparison of equations (21) and (22) indicates that γ_p contains the term, $1-\alpha$, where γ_L contains the term, $1-\tau$. At the point, $\tau = \alpha$, which was already shown to correspond to the command optimum, the two terms coincide. Note from Figures 5 and 6 that the growth rates, γ_p and γ_L, and saving rates,

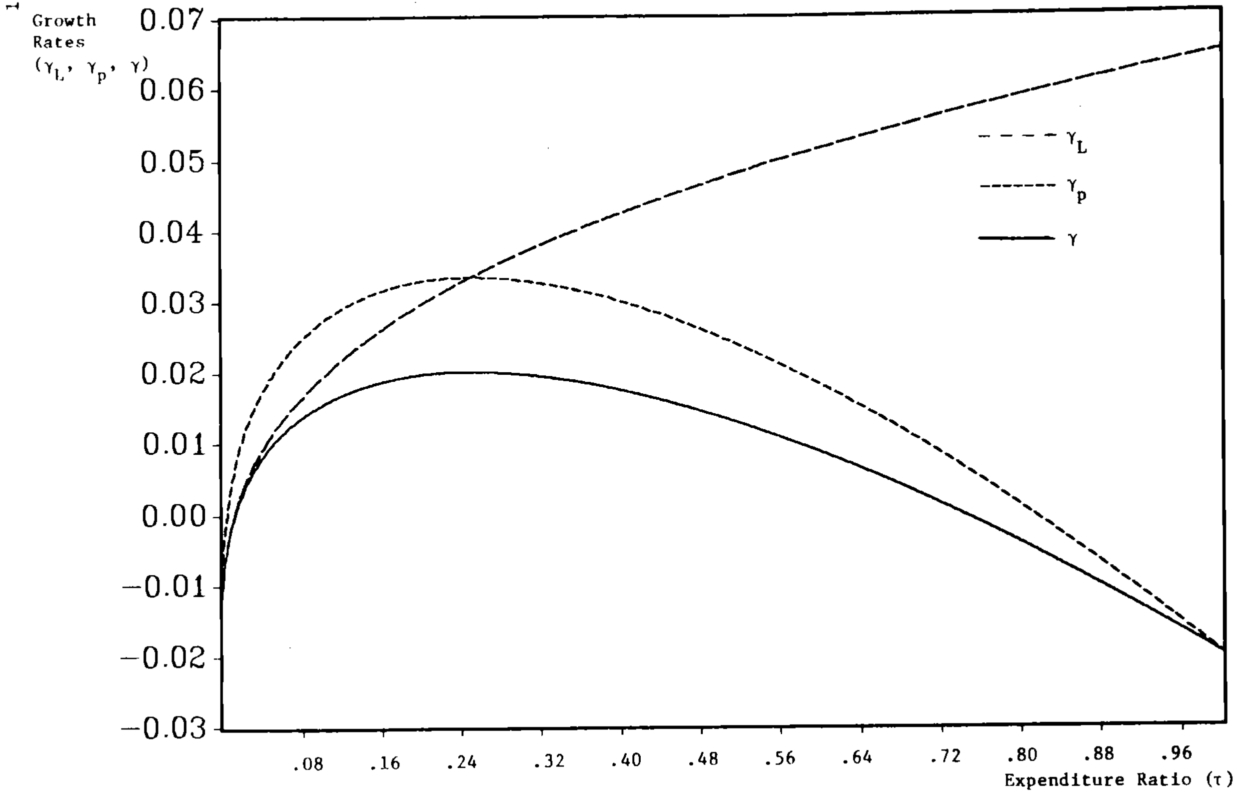


Figure 5

The Growth Rate in Three Environments

NOTE: γ_L is from equation (22), γ_p from equation (21), and γ from equation (15). Parameter values are given in Figure 1.

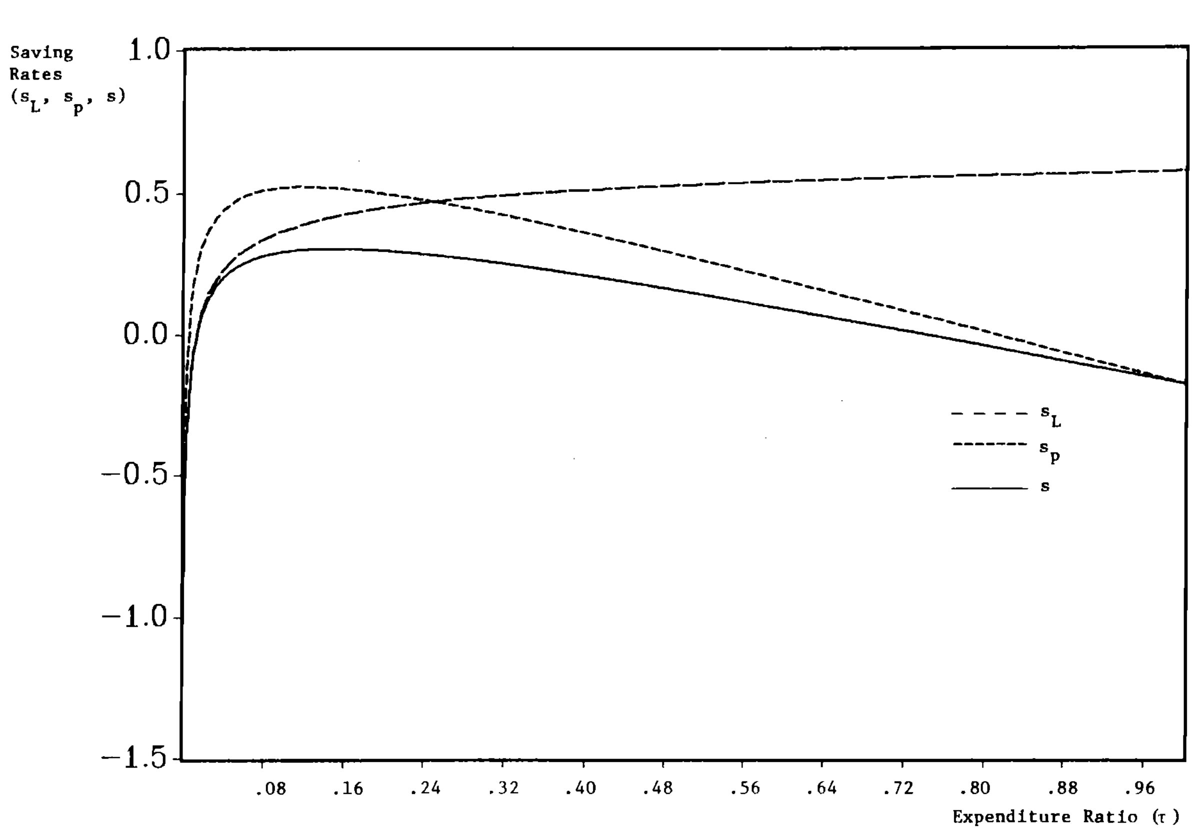


Figure 6
The Saving Rate in Three Environments

NOTE: $s_L = \gamma_L \cdot k/y$ and $s_p = \gamma_p \cdot k/y$, where γ_L is from equation (22), γ_p from equation (21), and k/y from equation (12). s is from equation (17). Parameter values are given in Figure 1.

s_p and s_L, are equal at this point. These results mean that lump-sum taxation supports the command optimum if the expenditure share is set at the optimal value, $\tau = \alpha$.

If the expenditure share is set non-optimally, $\tau \neq \alpha$, then the planning solution for consumption—contingent on this incorrect choice of τ—does not coincide with the solution under lump-sum taxation. This result indicates that the income tax is not the only distortion in the model. I am uncertain whether the other distortion is economically interesting, but I will now explain what it is.

An individual producer computes the marginal product, f_k, while holding constant the quantity of public services, g, that he receives from the government. This assumption is appropriate for some types of public services, and I continue to assume that it is right in the present context. (In some other cases the expansion of someone's own production or property automatically gets that person more public services; an example might be police protection.) But, if the government sets a given expenditure ratio, τ, then an increase in national product by one unit induces the government to raise the aggregate of its public services by τ units. Thus, when an individual producer decides to raise his individual k and y, he is indirectly causing the government to increase its aggregate spending. The effect on that individual's public services, which entered into his production function, would be negligible, and can therefore be ignored. But it is nevertheless true, with τ fixed, that an individual's decision that raises national product by 1 unit causes the total of government purchases to expand by τ units. The effects depend on whether the size of the government is optimal. If so—namely, at the point $\tau = \alpha$—a marginal change in government expenditures is just worth its cost. Hence there is no distortion and the lump-sum tax result replicates the planning optimum, as noted before. But suppose that the government is too large, $\tau > \alpha$. Then the induced expansion of government expenditures constitutes a negative externality. On this count, each individual has too much of an incentive to expand individual output; in particular, in this model, each individual has too much incentive to save. Hence, if $\tau > \alpha$, $\gamma_L > \gamma_p$ in Figure 5, and $s_L > s_p$ in Figure 6. Analogously, the incentive to expand individual output is too low when the government is too small, $\tau < \alpha$. Hence, $\gamma_L < \gamma_p$ and $s_L < s_p$ apply in this range.

Figures 5 and 6 also allow a comparison between lump-sum taxes (which could be consumption taxes in this model) and income taxes. At the point $\tau = \alpha$, the lump-sum tax generates the command optimum and is therefore superior to the income tax. For $\tau < \alpha$, the lump-sum tax comes closer than the income tax to the command optimum; therefore, the lump-sum tax would also be preferred here. However, for $\tau > \alpha$ the comparison becomes ambiguous, because the lump-sum-tax choices, γ_L and s_L, are too large, while the income-tax choices, γ and s, are too small. For very large governments (that is, τ well above α), the outcome under income taxes can be superior to that under lump-sum taxes. The reason is that the income tax is an imperfect way to get individual producers to internalize the distortion described above. With $\tau > \alpha$, people have too great an incentive to expand output by an additional unit because the government is thereby induced to increase its expenditures by τ units. If government spending were worthless, then the way to internalize this distortion would be to tax the individual's income at rate τ. As τ gets well above its ideal value, α, the social return from more government spending diminishes—that is, it becomes more nearly accurate that government spending is worthless at the margin. Therefore, the income tax becomes more nearly the right way to offset the negative externality, and the value γ in Figure 5 gets steadily closer to the value γ_p. Similarly, in Figure 6, s and s_p converge as τ approaches 1.

4. Government Consumption Services

Suppose that the government's expenditures also finance some services that do not enter into households' production functions. For example, the government might provide services that appear directly in households' utility functions. I assume that total spending per household is g+h, where the quantity h represents the government's consumption services. The utility function for each household is now

$$(23) \qquad u(c,h) = \frac{(c^{1-\beta}\cdot h^{\beta})^{1-\sigma} - 1}{(1-\sigma)}$$

where $0 < \beta < 1$. The household's overall utility is still given by equation (1), except that u(c,h) replaces u(c) in the integral.

I still assume a flat-rate income tax, so that the government's budget constraint is

$$(24) \qquad T = (\tau_g+\tau_h)\cdot y$$

where $\tau_g = g/y$ is the government's expenditure ratio for productive services, and $\tau_h = h/y$ is the ratio for consumption services.

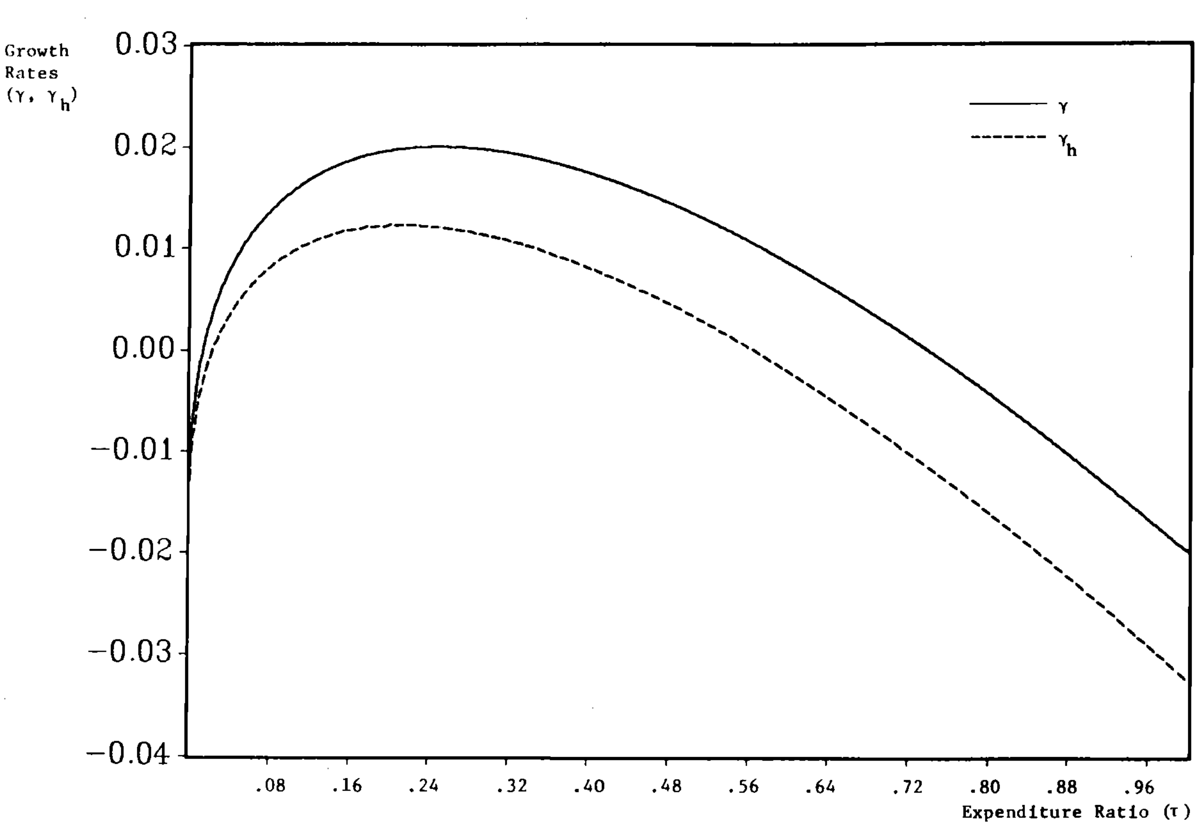
Households' decentralized choices for consumption and saving now lead to the growth rate

$$(25) \qquad \gamma_h = (1/\sigma)\cdot[(1-\alpha)\cdot A^{1/(1-\alpha)}\cdot(1-\tau_g-\tau_h)\cdot(\tau_g)^{\alpha/(1-\alpha)} - \rho]$$

This expression modifies equation (15) in a straightforward manner: $(1-\tau_g-\tau_h)$ replaces $(1-\tau)$, and $(\tau_g)^{\alpha/(1-\alpha)}$ replaces $\tau^{\alpha/(1-\alpha)}$. The tax rate τ_h amounts to an existing distortion that reduces private choices of saving and growth rates. The dotted curve in Figure 7 shows the relation between γ_h and the share of productive government spending, τ_g, taking account of the positive value of τ_h. The growth rate lies uniformly below the value γ, shown by the solid curve, that would have been chosen if $\tau_h = 0$. Figure 8 shows the corresponding saving rates, s_h and s.

For a given τ_h, it is easy to show that the value of τ_g that maximizes γ_h in equation (25) is $\alpha(1-\tau_h)$. In other words, the growth-maximizing share of productive government spending is smaller if the government is also using the income tax to finance other types of spending. Moreover, if the value of τ_h were set arbitrarily (that is, not in order to maximize utility), and if the presence of h in households' utility were neglected, then the choice $\tau_g = \alpha(1-\tau_h)$ also turns out to maximize the utility attained by the representative household.

However, in most cases it would be uninteresting to treat the choices of τ_g and τ_h in this asymmetric manner. Presumably, if it is appropriate to think of τ_g as chosen from a utility-maximizing criterion, then it would be similarly appropriate for τ_h. Suppose then that each household's utility function is given by equation (23), and that τ_g and τ_h are set to maximize



<u>Figure 7</u>

<u>Growth when the Government also Provides Consumption Services</u>

NOTES: γ is from equation (15) and γ_h from equation (25). The graph of γ_h assumes τ_h = .15. Other parameter values are from Figure 1.

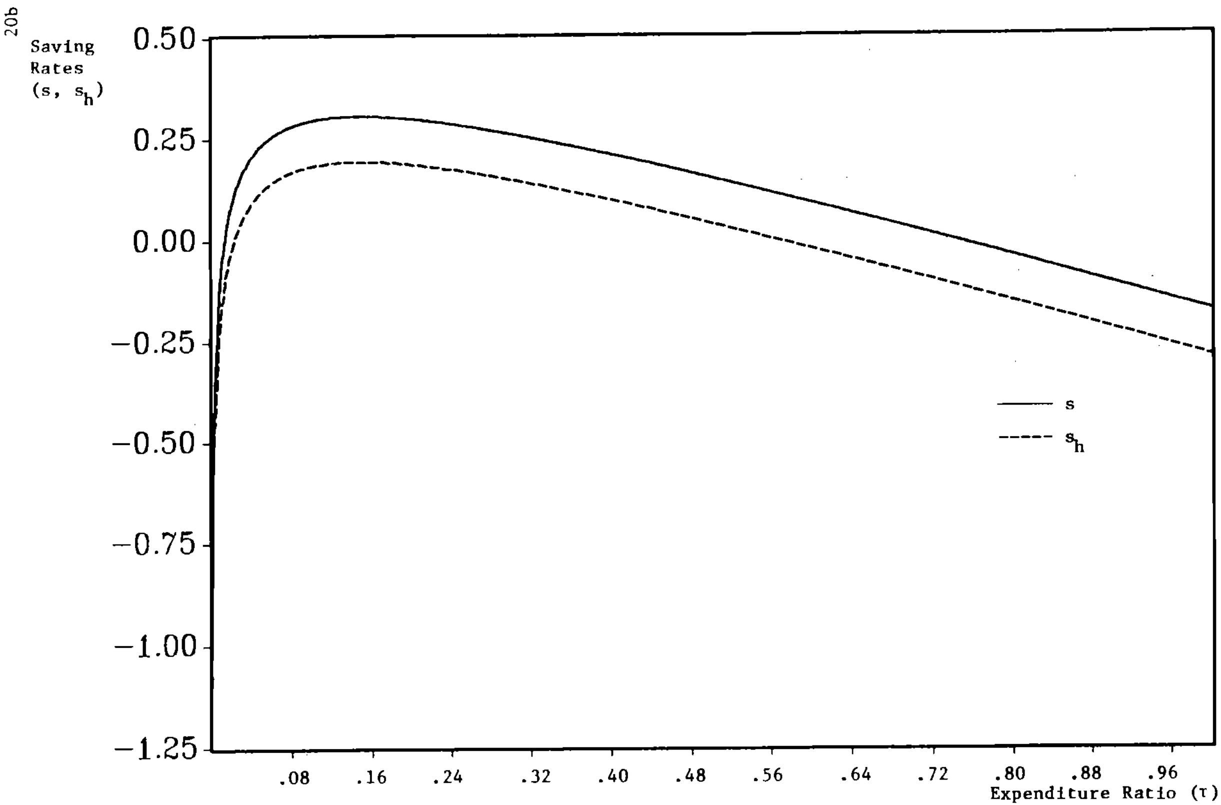


Figure 8

The Saving Rate when the Government also Provides Consumption Services

NOTE: s is from equation (17). $s_h = \gamma_h \cdot k/y$, where γ_h is from equation (25), and k/y from equation (12). $\tau_h = .15$. Other parameter values are in Figure 1.

the overall utility attained by the representative household in the form of equation (1). The effects of the tax rates on γ_h are shown in equation (25). As before, it is possible to determine the initial level of consumption, c(0), and thereby calculate the entire path of consumption as $c(t) = c(0)\cdot e^{\gamma_h t}$. The path of the government's consumption services is given by $h(t) = \tau_h\cdot y(t) = \tau_h\cdot y(0)e^{\gamma_h t}$. Using these results, it is feasible to relate the attained utility, U, to the tax rates, τ_g and τ_h. There are then two first-order conditions corresponding to the maximization of U. Combining these conditions leads to the familiar result: $\tau_g = \alpha$. That is, as long as τ_h is chosen optimally, the optimal ratio for productive government expenditures, τ_g, is the same as before. In particular, the choice depends again only on the productivity parameter, α, and not on aspects of preferences (including now the parameter β, which determines households' preferences for private consumption, c, versus government consumption services, h).

5. Self-Interested Government

Thus far, I assumed that the government was benevolent and therefore sought to maximize the utility attained by the representative household. I now consider the alternative that the government is run by an agent who has no electoral constraints and seeks to maximize his own utility.

Return to the setting where all government expenditures, g, serve as productive inputs for private producers. The government still uses a flat-rate income tax, but instead of automatically balancing the budget, the government can earn the net revenue,

$$(26) \qquad c_g = (\tau-\epsilon)y$$

where the expenditure ratio, $\epsilon = g/y$, can differ from the income-tax rate, τ. The government agent uses his net revenue to purchase the quantity of consumer goods, c_g. (The results would not change if the agent were allowed to hold capital, and perhaps owned a nonzero quantity of capital at time 0.) The agent receives utility from consumption in the same manner as any household—that is, the flow of utils is $u(c_g)$ from equation (2), and the overall attained utility, U, is given by the integral in equation (1). In particular, the government agent has the same discount rate, ρ, as each household.

Assuming constant values for τ and ϵ (which will be optimal for the government), the privately determined growth rate is now

$$(27) \qquad \gamma_\epsilon = (1/\sigma)\cdot[(1-\alpha)\cdot A^{1/(1-\alpha)}\cdot(1-\tau)\cdot\epsilon^{\alpha/(1-\alpha)} - \rho]$$

This result is a straightforward modification of equation (15) when $\epsilon \neq \tau$. The government agent's consumption is $c_g(t) = (\tau-\epsilon)\cdot y(0)e^{\gamma_\epsilon t}$. Therefore, using the same procedure as before, it is possible to write the agent's attained utility as a function of τ and ϵ. The two first-order conditions for maximization of utility then lead to the results

$$(28) \qquad \tau > \epsilon = \alpha$$

The optimal expenditure rate, ϵ, equals α, as in previous models. This choice is basically one of efficient production, which means that the self-interested government chooses the same value as the benevolent government. Basically, the government agent sets $\epsilon = \alpha$ in order to maximize the tax base that he has to work with. Then he is also in the position to set $\tau > \epsilon$ in order to secure the net flow of revenue, c_g.

The results in this section parallel those in the preceding one. In effect, the government agent's consumption, c_g, plays the same role that the government's consumption services, h, played in the previous model. In both cases the presence of these consumption flows does not upset the conditions for productive efficiency, which imply that the government's productive expenditures are the fraction α of total output. However, the ratio of government revenues to output exceeds α in both cases; in one case to provide consumption to the government agent, and in the other to provide government consumption services to each household.

6. Some Empirical Implications

The theory has implications for relations between the size of government and the rates of growth and saving. Since the analysis applies to steady-state growth paths, the natural empirical application would be to differences in average performance across countries over long periods of time.

As is usual in empirical investigations, the hypothesized effects of government policy are easier to assess if the government's actions can be treated as exogenous. That is, the results are simple if governments randomize their actions and thereby generate useful experimental data. In this case, variations in the share of productive government expenditures in

GNP, g/y, affect growth and saving rates, γ and s, as shown by the curves in Figures 7 and 8, respectively. Countries could be arrayed along the horizontal axes by the size of g/y, and the responses of γ and s would be non-monotonic, as shown in the figures.

An increase in the share of non-productive government expenditures, say h/y in the model of section 4, leads to the types of shifts shown by the movements from the solid to the dashed curves in Figures 7 and 8. For a given value of g/y, an increase in h/y lowers the growth and saving rates. These effects arise because a higher h/y has no direct effect on private-sector productivity, but does lead to a higher income-tax rate. Since individuals retain a smaller fraction of their returns from investment, they have less incentive to invest, and the economy tends to grow at a lower rate.

The predictions are similar for any other differences across countries that imply that private investors get to retain a smaller fraction of their returns from investment. For example, if g/y is held fixed, an increase in the average marginal tax rate—resulting, say, from a difference in the tax system—would tend to lower the growth and saving rates. Similarly, any other source of reduction in property rights would tend to reduce the growth and saving rates.

Aside from problems of measuring public services and the rates of growth and saving, the empirical implementation of the model is complicated by the endogeneity of the government. Within the theoretical model, the government sets the share of productive expenditures, g/y, to equal α. Therefore, instead of being arrayed along the horizontal axes in Figures 7 and 8, each government would operate at the same point, $g/y = \alpha$. Within this framework of optimizing governments, cross-sectional variations in g/y arise only if α differs from country to country.

The parameter α, which measures the productivity of public services relative to private services, could vary across countries for a number of reasons. These include geography, the share of agricultural production, urban density, and so on. For present purposes it is unnecessary to predict how any specific element would affect α, and therefore g/y for an optimizing government. As long as the variations in α are independent of the overall level of productivity (measured by the coefficient, $A^{1/(1-\alpha)}$, which connects y to k for a given τ in equation (12)), the model predicts how the induced variations in g/y will correlate with differences in γ and s. Using equations (15) and (17), substituting $\tau = \alpha$, and holding constant $A^{1/(1-\alpha)}$, the model implies that a rise in α, and hence in g/y, will reduce γ and s. Therefore, the theory predicts that these types of endogenous variations in productive expenditures will be inversely correlated with γ and s. For variations in other forms of government expenditures, the negative relation with γ and s follows straightforwardly.

I am planning an empirical study of cross-country relationships among the size of government, growth, and saving. But since I have no results at this point, it may be worth summarizing some findings in the literature. Kormendi and Meguire (1985) studied 47 countries in the post-World War II period, using data on total government "consumption" expenditures and other variables from *International Financial Statistics*. This measure of government spending, used by many authors because of the availability of the data, excludes public investment and transfers from overall public expenditures. Although the category is called consumption, it does not necessarily follow that these public services enter mainly into utility functions, rather than production functions. Using data for each country averaged over roughly 20-year periods, Kormendi and Meguire found (p. 147) no relation between average growth rates of real GDP and average growth rates of the share of government consumption spending in GDP. They matched up the growth of GDP with the change in g/y because they were thinking of a framework where, with diminishing returns, a different level of g/y would have only a transitory effect on economic growth. However, in my model, it would be more appropriate to examine the relation between the growth rate of GDP and the level of g/y.

Grier and Tullock (1987) extended the Kormendi-Meguire form of analysis to 115 countries, using data on total government consumption and other variables from Summers and Heston (1984). This extension was a pooled cross-section, time-series analysis, using data averaged over 5-year intervals. They found a significantly negative relation between the growth of GDP and the growth of the government share of GDP, although most of the relation derived from the 24 OECD countries (see Tables 1 and 2).

Landau (1983) studied 104 countries on a cross-sectional basis, using an earlier form of the Summers-Heston data. He found (Table 1) significantly negative relations between the growth rate of real GDP per capita and the level of government consumption expenditures as a ratio to GDP.[5] This

[5]The regressions held constant the average level of real per capita GDP for each country. Therefore, the results should not be affected by "Wagner's Law," which is the proposition that g/y rises with the level of per capita income (presumably for reasons related to utility functions, rather than production functions).

formulation is easier to interpret in the context of my model. However, Landau's regressions held constant a measure of investment in education, which would be one component of an economy's broadly defined saving. Since the negative effect of more government on growth in my model works through a reduced incentive to save, the interpretations are very different if saving rates are held constant.

Ram (1986) looked at 115 countries using the Summers-Heston data. He reported (Table 1) positive effects of the growth of government consumption spending on the growth of GDP. However, there are two problems with the results. First, they hold constant the ratio of investment (private plus public) to GDP, which eliminates the channel in my model for the negative effect of government on growth. Second, the results amount to a positive coefficient in a regression of the growth rate of GDP on the growth rate of government consumption expenditures. This regression would pick up the reverse effect of income on government consumption; that is, it probably amounts to a demand function for public services.

The literature contains a number of additional empirical studies, with results similar to those reported above. In my empirical work, I plan to consider these results in more detail, as well as carrying out new research along the lines suggested by my theoretical analysis.

References

Aschauer, D.A., "Is Public Expenditure Productive?" unpublished, Federal Reserve Bank of Chicago, March 1988.

Becker, G.S. and R.J. Barro, "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 103, February 1988, 1-25.

Cass, D., "Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, 32, July 1965, 233-240.

Grier, K.B. and G. Tullock, "An Empirical Analysis of Cross-National Economic Growth, 1950-1980," unpublished, California Institute of Technology, December 1987.

Jones, L.E. and R.E. Manuelli, "A Model of Optimal Equilibrium Growth," unpublished, Northwestern University, April 1988.

Koopmans, T.C., "On the Concept of Optimal Economic Growth," in *The Econometric Approach to Development Planning*, North Holland, Amsterdam, 1965.

Kormendi, R.C. and P.G. Meguire, "Macroeconomic Determinants of Growth," *Journal of Monetary Economics*, 16, September 1985, 141-163.

Landau, D., "Government Expenditure and Economic Growth: a Cross-Country Study," *Southern Economic Journal*, 49, January 1983, 783-792.

Lucas, R.E., "On the Mechanics of Economic Development," forthcoming, *Journal of Monetary Economics*, 1988.

Ram, R., "Government Size and Economic Growth: a New Framework and Some Evidence from Cross-Section and Time-Series Data," *American Economic Review*, 76, March 1986, 191-203.

Ramsey, F.P., "A Mathematical Theory of Saving," *Economic Journal*, 38, December 1928, 543-559.

Rebelo, S., "Long Run Policy Analysis and Long Run Growth," unpublished, University of Rochester, November 1987.

Romer, P.M., "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94, October 1986, 1002-1037.

-----------, "Capital Accumulation in the Theory of Long Run Growth," February 1988, forthcoming in R.J. Barro, ed., *Modern Business Cycle Theory*, Harvard University Press.

Samuelson, P.A., "The Pure Theory of Public Expenditure," *Review of Economics & Statistics*, 36, November 1954, 387-389.

Summers, R. and A. Heston, "Improved International Comparisons of Real Product and its Composition: 1950-80," *Review of Income and Wealth*, 30, June 1984, 207-262.