Governmental Transfers Can Reduce a Moral Hazard Problem

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Abstract

An altruistic agent who may aid a person with a low income may cause that person to exert little effort to increase his income. Such behavior generates a Dilemma, in which welfare is lower than when no one is altruistic. We show how governmental transfers, which do not allow for reallocation from a person who saves much to one who saves little, reduces the effect, and can lead to an outcome which is Pareto-superior to the outcome under a Nash equilibrium with no government taxation and transfers.

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JEL classification: D13; D64; D91

1 Introduction

Members of a family are often altruistic to other members. Think of parents caring about their children. Or think of a husband and wife who support their elderly parents. Though some family members may be altruistic, others may be selfish. A child may want to obtain a large transfer from his parents, even if that impoverishes his parents, and even if the transfer comes at the expense of reduced transfers to his brothers and sisters. One mother-in-law may care nothing about the welfare of the other mother-in-law and would be perfectly happy to see a married couple give greater support to her and less to the other in law.

Such attitudes make the donor's generosity a common pool, leading one person, say recipient 1, to behave in a way which increases the transfers the donor (say parents) makes to recipient 1, recognizing that the donor will reduce transfers to recipient 2.[1] Faced with such motivation, the donor may favor policies which reduce the incentives of the recipient to behave in ways which induce transfers.

The donor may benefit by committing to make a small transfer. For example, a parent may tell a child that even if the child has no income the parent will give no more than $10,000. The child will then have a marginal incentive to work. But an individual may find it difficult to commit to his future actions; in contrast, policy set by government, can effectively make that commitment. Suppose the government enacts a law requiring the donor to pay $10,000 in taxes and making a transfer payment of $5,000 to each of the two recipients. That has two effects. First the donor's wealth declines by $10,000 and so he will be less willing to make any transfer to either recipient. Second, the donor cannot take some of the $5,000 government transferred to one recipient and give it to the other. Therefore, each recipient has less incentive to reduce his own wealth with the aim of increasing the private transfer he gets.

For intuition, suppose that the donor will transfer a fixed total amount, say $10,000 to the recipients. He will allocate that amount between the two recipients so that in period 2 the marginal utility of consumption to the recipients is equal. Therefore, for a given transfer, the less recipient 1 saves in period 1, the higher his marginal utility of consumption in period 2, and the less the donor will transfer to recipient 2 and the more the donor will transfer to recipient 1. Each recipient therefore has an incentive to save little in period 1. This race between the two recipients is avoided if government taxes the donor $10,000, and gives each recipient $5,000; the transfers a recipient gets in period 2 is then independent of the recipient's savings decision in period 1.

One application of this reasoning is that a potential donor may favor a compulsory government transfer program. For example, a person may favor a social security system that transfers money to his elderly parents and parents-in-law. This reasoning differs from the justifications for social security commonly found in the literature. Our model does not have social security tax future

[1] McGarry and Schoeni (1995) find that parents indeed give greater financial assistance to their children with low incomes than to their children with high incomes.

recipients of the government transfer. Instead, the model has social security tax current potential donors who would otherwise or in addition make transfers to current recipients.

2 Literature

Consideration of how altruism can lead to a moral hazard problem is examined under the rubric of the Good Samaritan Dilemma (see Bruce and Waldman 1990). That in turn builds on literature which supposes that donors are motivated by altruistic concern over the well-being of the recipients of charity (Hochman and Rodgers 1969; Warr 1982; Roberts 1984). A solution to the Good Samaritan Dilemma is to have altruists commit not to make transfers; in our analysis, in contrast, a solution is to have government commit to tax potential donors and use the proceeds to make transfers to needy recipients.

We shall consider two potential recipients within a family, with each believing that if he is poorer than the other, that the donor will reallocate transfers from the less poor recipient to the poorer one. That analysis relates to the Rotten Kid Theorem. Becker (1974) claims that if all potential recipients get transfers from an altruist, then the potential recipients, even if selfish, have an interest in maximizing the joint income of donors and recipients. Bergrstom (1989), however, shows that the result fails if utility is non-transferable. We assume such non-transferability.

Social security systems have been justified, or explained, on two main grounds. One is paternalism (Diamond, 1977). The other justification is to alleviate the Good Samaritan's Paradox described above, but without considering reallocation of transfers across recipients (Buchanan, 1977; Coate, 1995).

3 Assumptions

The population consists of identical families, each consisting of one donor and two potential recipients. Each recipient is selfish, his utility increasing only with his own consumption. A recipient lives for two periods, endowed with wealth w in period 1. In period 1 recipient i ($i = 1$ or $i = 2$) allocates wealth between consumption and saving. The savings of potential recipient i is S_i. In period 2 a recipient's consumption equals his savings, plus a transfer from the donor, plus a transfer from government.

The donor has wealth w in period 2. His utility increases with his consumption and with the consumption of each recipient. A donor can spend his endowment on his own consumption, on a transfer to each recipient, and on taxes which finance governmental transfers to the recipients. The donor's transfer to recipient i is called d_i. In period 2 the donor pays a tax of T, with government transferring t_i to recipient i, with $t_1 + t_2 = T$. We emphasize symmetric behavior, where $t_1 = t_2 = T/2$, but we shall show that similar results hold when only one person gets a governmental transfer. Thus, in period 2 recipient i consumes

$S_i + d_i + t_i$.[2]

The timeline follows:

1. The donor determines the tax he will pay in period 2, and the governmental transfers in period 2
2. Each recipient determines how much of his endowment to save
3. Government makes transfers to recipients
4. A donor makes transfers to recipients in his family
5. Utilities are realized

A recipient's utility from consumption in the two periods is

$$\ln(w - S_i) + \beta \ln\left(S_i + d_i + \frac{T}{2}\right), \quad i = 1, 2, \tag{1}$$

where β is a parameter indicating the weight on consumption in period 2.

In period 2 a donor consumes $w - d_1 - d_2 - T$; his utility from such consumption is $\ln(w - d_1 - d_2 - T)$. A donor's utility also increases with the utility of each recipient:

$$\begin{aligned} \ln(w - d_1 - d_2 - T) &+ \alpha\left[\ln(w - S_1) + \beta \ln\left(S_1 + d_1 + \frac{T}{2}\right)\right] \\ &+ \alpha\left[\ln(w - S_2) + \beta \ln\left(S_2 + d_2 + \frac{T}{2}\right)\right], \end{aligned} \tag{2}$$

where α is the weight a donor places on the utilities of recipients in his family.

The governmental transfer T is determined, for example, by voting; no one person's vote is decisive so that no individual donor or recipient can alter government policy: a donor must pay T, even if he alone wants to pay more or less than that. A donor can increase consumption of a recipient in his family by making private transfers. A recipient can affect the amount of transfer he gets from the donor in his family by choosing his savings.

4 Transfers and savings with no commitment

A donor cannot commit to the amount he will transfer. Recipients simultaneously choose their savings, S_1 and S_2. After that, a donor makes private transfers, d_1 and d_2 to maximize his objective function (2). His decision is influenced by the choices of S_1 and S_2. Hence, recipients will choose S_1 and S_2 taking into account the donor's behavior.

Therefore, to analyze this game, we first consider the donor's problem; after that we analyze the recipient's choice.

[2] We can think of N identical families, each consisting of a donor and of two recipients. The government's total tax revenue is NT, which is divided among $2N$ recipients in the population.

4.1 Donor's choice of private transfer

A donor maximizes the objective function (2) with respect to d_1 and d_2, subject to the constraints $d_1 \geq 0$ and $d_2 \geq 0$. The donor's private transfers d_1 and d_2 depend on the sizes of recipients' savings S_1 and S_2, and on the governmental transfer T.

If both S_1 and S_2 are sufficiently small to satisfy

$$S_i \leq \left(\frac{\alpha\beta}{1+\alpha\beta}\right) w + \left(\frac{\alpha\beta}{1+\alpha\beta}\right) S_j - \frac{1}{2}\left(\frac{1+2\alpha\beta}{1+\alpha\beta}\right) T, \quad i, j = 1, 2, \quad i \neq j, \tag{3}$$

then

$$d_i = \frac{\alpha\beta w + \alpha\beta S_j - (1+\alpha\beta) S_i}{1+2\alpha\beta} - \frac{1}{2} T \geq 0, \quad i, j = 1, 2, \quad i \neq j. \tag{4}$$

If S_i is too large to satisfy (3) yet S_j is sufficiently small to satisfy

$$S_j \leq \alpha\beta w - \frac{1}{2}(1+2\alpha\beta) T, \tag{5}$$

then

$$d_i = 0, \tag{6}$$

and

$$d_j = \frac{\alpha\beta w - S_j - \left(\frac{1}{2} + \alpha\beta\right) T}{1+\alpha\beta} \geq 0. \tag{7}$$

Lastly, if both S_1 and S_2 are too large to satisfy (5), then $d_1 = d_2 = 0$. See Appendix 1 for the derivations. Figure 1 shows, for a numerical example, combinations of (S_1, S_2) satisfying each of the cases.

Let the private transfer from a donor to recipient i as a function of the recipient's savings S_i, the other recipient's savings S_j, and the governmental transfer T, be $d(S_i, S_j, T)$.

In the absence of transfers, a recipient's utility in period 2 increases with his savings in period 1. A donor therefore is more willing to give the recipient a transfer the less the recipient saved. As we will see later, if pre-commitment for the private transfer is impossible, a recipient has an incentive to save little and rely on a private transfer from the donor.

4.2 Savings by recipient

A recipient expects to receive a larger transfer from the donor the less the recipient saves. He also knows that the recipient who saves less than the other will get a larger transfer from the donor. Specifically, recipient i chooses S_i to maximize $\ln(w - S_i) + \beta \ln\left(S_i + d_1 + \frac{T}{2}\right)$ recognizing that d_i is a function of S_1 and of S_2.

Figures 2, 3, and 4 show recipient 1's utility (1). Recipient 1's best response S_1 to recipient 2's choice of S_2 is also shown.

There are three candidates for recipient 1's best response. From (3) and (5), the interval of (S_1, S_2) in which recipient 1 saves so much that he gets no private transfer is

$$S_1 \geq \text{Min}\left[\left(\frac{\alpha\beta}{1+\alpha\beta}\right)w + \left(\frac{\alpha\beta}{1+\alpha\beta}\right)S_2 - \frac{1}{2}\left(\frac{1+2\alpha\beta}{1+\alpha\beta}\right)T, \alpha\beta w - \frac{1}{2}(1+2\alpha\beta)T\right]. \tag{8}$$

Recipient 1's optimal, or utility-maximizing, savings limited to this interval is

$$\frac{\beta w - \frac{1}{2}T}{1+\beta}. \tag{9}$$

If recipient 1 chooses a value of S_1 that does not satisfy (8), he will get a private transfer. If recipient 2 saves little, recipient 1's optimal savings in this interval are

$$\left(\frac{\beta-1}{1+\beta}\right)w - \left(\frac{1}{1+\beta}\right)S_2. \tag{10}$$

In contrast, if recipient 2 saves much, recipient 1's utility-maximizing savings are:

$$\frac{(\beta-1)w + \frac{1}{2}T}{1+\beta}. \tag{11}$$

Appendix 2 gives details about recipient 1's savings and the derivations.

Recipient 1 chooses between (9), (10) or (11), to give him the highest utility; we call his choice of savings S_1. The larger is S_2, the greater is recipient 1's incentive to save little, and to rely on a private transfer from the donor, as we can see in Figures 2, 3 and 4.[3] An increase in S_2 reduces the donor's gain from making a transfer to recipient 2, and increases the donor's incentive to make a transfer to recipient 1. Therefore, the donor transfers more to recipient 1 when recipient 1 chooses (10) or (11), making that choice more attractive to recipient 1. In contrast, if recipient 2 saves little, recipient 1 will get only a small transfer from the donor, even if the recipient chooses (10) or (11). Recipient 1 will then avoid saving little.

5 An equilibrium with little savings

5.1 Private transfers and savings in equilibrium

The game between the two recipients can have three different equilibria. An important equilibrium has each recipient save little, so that each gets a private transfer from the donor. Figure 3 depicts an example in which this outcome is a Nash equilibrium. From (10), savings in this equilibrium are

$$S_i = \left(\frac{\beta-1}{2+\beta}\right)w, \quad i = 1, 2. \tag{12}$$

[3]In some cases, as in Figure 2, recipient 1 may save so much as to induce the donor to make no private transfers.

From (4) and (12), private transfers are:

$$d_i = \left[\frac{1-\beta+\alpha\beta^2+2\alpha\beta}{(1+2\alpha\beta)(2+\beta)}\right] w - \frac{1}{2}T, \quad i=1,2, \tag{13}$$

and from (1), the corresponding maximized utilities are

$$\ln\left(\frac{3}{2+\beta}\right) w + \beta \ln\left[\frac{3\alpha\beta^2}{(1+2\alpha\beta)(2+\beta)}\right] w. \tag{14}$$

To consider the conditions under which (12) is a Nash equilibrium, we examine whether recipient 1 gains by changing his savings from (12), given that recipient 2 chooses (12).

Suppose that recipient 1 *slightly* increases his savings from (12), thereby improving the intertemporal allocation of his consumption. The increased savings reduce the donor's benefit from making a transfer. The reduced transfer dominates the improved intertemporal allocation, so that recipient 1's utility declines.

When, however, recipient 1's savings exceed some critical value, the donor reduces his transfer to zero. Further increased savings by recipient 1 do not further reduce the private transfer, so that increased savings improves intertemporal allocation of consumption, and so benefit the recipient. Recipient 1's utility-maximizing savings in the interval (S_1, S_2) where S_2 equals $\left(\frac{\beta-1}{2+\beta}\right) w$ yet S_1 is too large to satisfy (3) (with $i=1$ and $j=2$) are $\frac{\beta w - \frac{1}{2}T}{1+\beta}$. Recipient 1's maximized utility in this interval is:

$$\ln\left(\frac{1}{1+\beta}\right)\left(w+\frac{1}{2}T\right) + \beta \ln\left(\frac{\beta}{1+\beta}\right)\left(w+\frac{1}{2}T\right). \tag{15}$$

If, however, the maximized utility with no private transfer (15) is less than the maximized utility with a private transfer (14), recipient 1 does not deviate from (12). In the Nash equilibrium a person's savings are given by (12) and the private transfers are given by (13).

If the governmental transfer is zero, whether (14) is larger than (15), and whether in a Nash equilibrium each recipient saves in the amount given by (12) depends on how much the donor places on the recipient's utility (α), and on how much recipients value consumption in period 2 (β).

Proposition 1
Let α be the weight the donor places on the recipient's utility, and let β be the intertemporal discount factor. If α is sufficiently large and β is sufficiently small, in the Nash equilibrium each recipient saves $\left(\frac{\beta-1}{2+\beta}\right) w$ and each gets a private transfer.

Proof See Appendix 3.

In Figure 5, Area I corresponds to the set of (α, β) which Proposition 1 indicates. Equations which show the set of (α, β) are complicated, as shown in Appendix 3.

Large α means that the donor is more eager to make a private transfer. Hence, for a wider range of other parameters will the recipient enjoy higher utility by saving little, and receiving a private transfer. Let $\bar{d}$ denote the private transfer from the donor to recipient i for the case discussed in Proposition 1, where $T = 0$ and α is large. This $\bar{d}$ equals (13) in which $T = 0$.

5.2 Governmental transfer

We consider whether a governmental transfer, which encourages recipients to save, can be efficient.

Proposition 2
Let the private transfer from the donor to each recipient when there is no governmental transfer be $\bar{d}$. There exists a positive governmental transfer smaller than $2\bar{d}$ which generates a Nash equilibrium in which each recipient saves $\frac{\beta w - \frac{1}{2}T}{1+\beta}$ and the donor makes no private transfer. Both the donor and the recipients are better off than in the equilibrium with no governmental transfer.

Proof See Appendix 4.

To understand this effect, suppose government imposes a tax on the donor, and makes a transfer to each recipient, equal to the amount of the private transfer that a recipient gets in the equilibrium when government makes no transfer. That is, $T = 2\bar{d}$.

As we discussed, an increase in the governmental transfer reduces the donor's benefit from making a private transfer. Therefore, even a recipient who saves little, in the amount $\left(\frac{\beta-1}{2+\beta}\right) w$, gets no private transfer. The amount the recipient gets is the same as it was when government made no transfer. Hence, his utility is also the same as it was, (14).

A recipient can enjoy higher utility by saving more than $\left(\frac{\beta-1}{2+\beta}\right) w$. Increased savings by recipient i do not further reduce the private transfer, so that increased savings increases his utility by improving intertemporal allocation of consumptions.

Therefore, his utility (15) when saving $\frac{\beta w - \frac{1}{2}T}{1+\beta}$ is strictly higher than his utility when he saves only $\left(\frac{\beta-1}{2+\beta}\right) w$. Even when T is to somewhat smaller than $2\bar{d}$, the recipient gains by increasing his saving. Therefore, when $0 < T < 2\bar{d}$, it is a stable Nash equilibrium for each recipient to save $\frac{\beta w - \frac{1}{2}T}{1+\beta}$.

Such a governmental transfer benefits the donor and the recipients. The recipient's utility in the new Nash equilibrium is (15), which is higher than in the equilibrium with no governmental transfer (14). Also, the donor's utility from his own consumption is higher: in the new equilibrium, $T < 2\bar{d}$, and

$d_1 + d_2 = 0$, his total payments, in the new Nash equilibrium, T plus private transfer $d_1 + d_2$, are smaller than transfers in the initial equilibrium, $2\bar{d}$.

Conversely, a Nash equilibrium with no governmental transfer is inefficient. As we saw in Proposition 1, Pareto inefficiency tends to occur when the donor makes large private transfers. Recipients depend on the private transfers from the donor, and the competition between the two recipients to attract private transfers makes matters worse.

A governmental transfer, however, need not always increase welfare. One situation to consider has a governmental transfer crowd out a private transfer. An increase in the governmental transfer reduces the donor's wealth, while recipients become richer. Hence, the donor gains less from private transfers to recipients. If the recipients' savings, S_1 and S_2, are sufficiently small to satisfy (3), the donor will make a positive transfer to each recipient, yet the donor will reduce private transfers d_1 and d_2 by the amount that offsets the increase in governmental transfer, as we can see in (4). Therefore, the sum of private and governmental transfers, $d_i + T/2$, is unchanged, and each recipient's utility (1) is unaffected by an increase in the governmental transfer. [4] Hence, if recipient j chooses $\left(\frac{\beta-1}{2+\beta}\right) w$ as in (12), the utility-maximizing choice for recipient i in the range of S_i in (3), that is, when the recipient gets a private transfer, is also $\left(\frac{\beta-1}{2+\beta}\right) w$, and his utility is (14). This utility is independent of T.

Another situation to consider has a recipient get no private transfer, and so gains from an increase in the governmental transfer. Suppose recipient i saves so much that the donor makes no private transfer to the recipient ($d_i = 0$). An increase in the governmental transfer increases recipient i's utility (1), as the sum of private and governmental transfers to him, $d_i + T/2 = 0 + T/2$, increases. Therefore, recipient i's utility when he gets no private transfer (15) also increases.[5]

To summarize, an increase in the governmental transfer can increase a recipient's incentive to save. Put differently, when the government transfer is large, the total amount of transfers (the sum of private and governmental transfers) to a recipient is less sensitive to a change in his savings. To see this, recall that the private transfer $d(S_i, S_j, T)$ decreases with the recipient's savings, whereas the governmental transfer is independent of savings. When the governmental transfer is large, the sensitive private transfer is very small, and so changes in the total transfer will not induce recipients to reduce savings.

[4] We can easily check this by substituting d_i in (1) using (4).

[5] If saving by recipient i is too large to satisfy (3) or (5), then $d_i = d(S_i, S_j, T) = 0$, as we saw in section 4.1 and Appendix 1. We can check that if $d_i = 0$, recipient i's utility (1) increases with T. The saving which maximizes (1) when $d_i = 0$ is $S_i = \frac{\beta w - \frac{1}{2}T}{1+\beta}$, which declines with T. Recipient i gets a governmental transfer in period 2. Therefore, as T increases he will smooth out his intertemporal consumption path by saving less in period 1.

6 Governmental transfer to only one recipient

One might think that governmental transfers are efficient only because an individual realizes that his increased savings will not reduce the governmental transfer to him, so that he has greater incentive to save. But there is an important added effect—a donor can reallocate his private transfer from one recipient to another, but cannot reallocate a governmental transfer. To highlight this effect, we consider a governmental transfer to only one of the two recipients in each family, and show how it affects the behavior of both recipients. Let the governmental transfer to recipient 2 be T. Recipient 1 gets no governmental transfer. For mnemonic purposes, call the recipient who gets a governmental transfer Recipient R_G; call the recipient who gets no governmental transfer Recipient R_{NG}.

Recipient R_{NG}'s best response can take one of three forms, which differ from those we derived in section 4. If recipient R_{NG} gets no private transfer, he saves

$$\frac{\beta w}{1+\beta}. \tag{16}$$

If a recipient saves little, he will get a private transfer. If recipient R_G saves little, recipient R_{NG}'s utility-maximizing savings given that he gets a private transfer are the same as (10). In contrast, if recipient R_G saves much, recipient R_{NG}'s utility-maximizing savings given that he gets a private transfer are

$$\frac{(\beta-1)w+T}{1+\beta}. \tag{17}$$

So Recipient R_{NG} will choose between the values of expressions (16), (16), and (17) which yield him the highest utility; call his savings S_{NG}

Recipient R_G's best response is not the same as recipient R_{NG}'s best response. If recipient R_G gets no private transfer, he saves

$$\frac{\beta w - T}{1+\beta}. \tag{18}$$

If recipient R_{NG} saves little, recipient R_G's utility maximizing savings given that he gets a private transfer are:

$$\left(\frac{\beta-1}{1+\beta}\right) w - \left(\frac{1}{1+\beta}\right) S_{NG}. \tag{19}$$

If recipient R_{NG} saves much, recipient R_G's utility-maximizing savings given that he gets a private transfer are:

$$\left(\frac{\beta-1}{1+\beta}\right) w. \tag{20}$$

Appendix 2 gives the details about the derivations. Each recipient will choose between the different levels of savings to give him the highest utility.

Suppose the weight (α) the donor places on the recipient's utility is large, and that the intertemporal discount factor (β) is small. As Proposition 1 indicates, if the governmental transfer is zero, the Nash equilibrium has recipients save little, in the amount $\left(\frac{\beta-1}{2+\beta}\right)w$ and rely on a private transfer from the donor in the amount $\bar{d}$. Now suppose that government imposes a tax on the donor, and makes a transfer to recipient R_G, equal to the amount that a recipient gets in the equilibrium when government makes no transfer. That is, $T = \bar{d}$. Figure 6 shows recipients' utilities (1) and the recipients' best responses when such a governmental transfer is introduced.

If both recipients save little, in the amount $\left(\frac{\beta-1}{2+\beta}\right)w$, each recipient's utility is the same as when government made no transfer. Recipient R_{NG} gets a private transfer of $d_{NG} = \bar{d}$ that is the same as he got when government made no transfer. Recipient R_G gets a governmental transfer $T = \bar{d}$, but gets no private transfer, or $d_G = 0$. Thus, recipient R_G gets the same total transfer as when government made no transfer.

Recipient R_G, however, can enjoy higher utility by saving more than $\left(\frac{\beta-1}{2+\beta}\right)w$. When government made no transfer, if he saved more than $\left(\frac{\beta-1}{2+\beta}\right)w$, the donor reduced the transfer to him to be less than $\bar{d}$. Now, however, recipient R_G gets $\bar{d}$ not from the donor but from the government. An increase in recipient R_G's savings does not reduce the governmental transfer but improves his intertemporal consumption allocation. Therefore, recipient R_G increases his savings to $\frac{\beta w-T}{1+\beta}$.

Recipient R_{NG} will also save more, though not as much as recipient R_G will. With the government transfer to recipient R_G, recipient R_{NG} will save $\frac{(\beta-1)w+T}{1+\beta}$. An increase in recipient R_{NG}'s savings reduces the private transfer he gets from the donor. However, the private transfer is less sensitive to a change in recipient R_{NG}'s savings than when government makes no transfer to recipient R_G. When government makes no transfer and one recipient saves little, an increase in the other recipient's savings induced the donor to reduce the transfer to the high-saver and increase the transfer to the low saver. When, however, government makes a transfer only to recipient R_G, and recipient R_G saves much, recipient R_G becomes richer. An increase in recipient R_{NG}'s savings still induces the donor to reduce his transfer recipient R_{NG}, but the reduction is smaller than when government made no transfer. The less sensitive change in a private transfer makes the recipient less eager to rely on it.

Thus, a governmental transfer to only one recipient benefits both recipients. The donor's total payments, $T = \bar{d}$ plus the private transfer to recipient R_{NG}, which is smaller than $\bar{d}$, are smaller than total payments in the initial equilibrium $2\bar{d}$. Therefore, the governmental transfer also benefits the donor. In short, the government transfer yields a Pareto-superior outcome.

7 Conclusion

Most work on altruism does not consider the interactions between potential recipients—either only one recipient is considered, or else each recipient is atomistic, so that strategic interactions are absent. We do consider such interactions, showing how they matter. A potential recipient recognizes that if he is more needy than other recipients, the donor will give him a larger transfer. Such behavior can lead to a race to the bottom, in which each recipient has an incentive to make himself more needy. Some governmental policies can mitigate the problem. In particular, transfers by government which are not under the control of the donor, and which are insensitive to the needs of recipients, can eliminate the race to the bottom. The governmental transfer has two effects on the behavior of the recipients. First, if the governmental transfer made to any one person not affected by his behavior (as largely holds for social security) then the recipient gets a greater benefit from increasing his saving. Second, the governmental transfer is not reallocated as a recipient saves more. The constraint on re-allocation induces increased savings, and so increases efficiency. This second effect appears even if some recipients get no governmental transfer—person A will save more if person B gets a governmental transfer. So the purpose of governmental assistance to a person is not only to benefit that person, but also to address the moral hazard problem arising with people not given governmental assistance.

Though we spoke of social security and of savings, the line of reasoning can apply to other areas. Thus, a similar analysis can apply when a potential recipient can choose a level of effort which determines his income. And the mechanism we highlight need not be the only one in operation. It can make a difference only at the margin, or explain why donor do not strongly oppose government taxes and spending.

8 Appendix 1: Donor's choice of private transfer

The values of d_1 and d_2 that maximize (2) satisfy:

$$\frac{-1}{w-d_1-d_2-T}+\frac{\alpha\beta}{S_1+d_1+t_1}\leq 0, \tag{21}$$

$$d_1\geq 0, \tag{22}$$

$$\left(\frac{-1}{w-d_1-d_2-T}+\frac{\alpha\beta}{S_1+d_1+t_1}\right)d_1=0, \tag{23}$$

$$\frac{-1}{w-d_1-d_2-T}+\frac{\alpha\beta}{S_2+d_2+t_2}\leq 0, \tag{24}$$

$$d_2\geq 0, \tag{25}$$

and

$$\left(\frac{-1}{w-d_1-d_2-T}+\frac{\alpha\beta}{S_2+d_2+t_2}\right)d_2=0, \tag{26}$$

where t_i is the governmental transfer to recipient i, and $t_1+t_2=T$.

If both d_1 and d_2 are positive, from (23) and (26) we can see that both (21) and (24) hold as an equality, and solving them for d_1 and d_2 yields:

$$d_i=\left(\frac{\alpha\beta}{1+2\alpha\beta}\right)(w-T+S_j+t_j)-\left(\frac{1+\alpha\beta}{1+2\alpha\beta}\right)(S_i+t_i)\geq 0,\quad i,j=1,2,\ \ i\neq j. \tag{27}$$

From (27), the conditions under which d_1 and d_2 are positive are

$$S_i\leq\left(\frac{\alpha\beta}{1+\alpha\beta}\right)(w-T+S_j+t_j)-t_i,\quad i,j=1,2,\quad i\neq j. \tag{28}$$

When $t_1=t_2=\frac{1}{2}T$, expression (27) is written as (4), and (28) is written as (3).

If d_2 is positive yet $d_1=0$, from (26) we can see that (24) hold as an equality. Solving (24) as an equality and $d_1=0$ yields:

$$d_2=\left(\frac{\alpha\beta}{1+\alpha\beta}\right)(w-T)-\left(\frac{1}{1+\alpha\beta}\right)(S_2+t_2)\geq 0. \tag{29}$$

The condition under which (29) is positive is obtained as:

$$S_2\leq\alpha\beta(w-T)-t_2. \tag{30}$$

Substituting d_2 and d_1 in (21) using (29) and $d_1=0$ and rearranging it yields the condition under which $d_1=0$. This condition is the inverse of the condition (28) (with $i=1$ and $j=2$). When $t_1=t_2=\frac{1}{2}T$, (29) is written as (7) (with $j=2$), and (30) is written as (5) (with $j=2$).

Lastly, if $d_1=0$ and $d_2=0$, from (23) and (26) we can see that (21) and (24) hold. Rearranging them yields:

$$S_i\geq\alpha\beta(w-T)-t_i,\qquad i=1,2. \tag{31}$$

9 Appendix 2: Recipient 1's savings as a function of recipient 2's savings

From (28) and (31), the interval of (S_1, S_2) in which recipient 1 gets no transfer is:

$$S_1 \geq \text{Min}\left[\left(\frac{\alpha\beta}{1+\alpha\beta}\right)(w - T + S_2 + t_2) - t_1, \alpha\beta(w-T) - t_1\right]. \tag{32}$$

When $t_1 = t_2 = \frac{1}{2}T$, (32) is written as (8).

9.1 Appendix 2.1: Recipient 1's optimal savings limited to the interval (32)

Differentiating (1) with respect to S_1 with $d_1 = 0$ yields:

$$\frac{-1}{w - S_1} + \beta\left(\frac{1}{S_1 + t_1}\right). \tag{33}$$

If S_1 in the interval (32) makes (33) zero, this S_1 is recipient 1's optimal savings in this interval. From (33), this S_1 is calculated as:

$$S_1 = \frac{\beta w - t_1}{1+\beta}. \tag{34}$$

When $t_1 = \frac{1}{2}T$, this becomes (9). When $t_1 = 0$ and $t_2 = T$, (34) becomes (16), and (34) for recipient 2 becomes (18). This value minus $\alpha\beta(w-T) - t_1$ yields:

$$\frac{\beta(1 - \alpha(1+\beta))w + \beta(t_1 + \alpha(1+\beta)T)}{1+\beta}. \tag{35}$$

As $0 < \beta < 1$ and $0 < \alpha < 1/2$, this is positive. Savings $\frac{\beta w - t_1}{1+\beta}$ is higher than the border of (32) and thus it is the optimal choice.

9.2 Appendix 2.2: Recipient 1's optimal savings limited to the interval of S_1 too small to satisfy (32)

The interval in which S_1 is smaller than the right side of (32) is divided into two intervals. In one of the intervals, S_1 is smaller than the border of (28) (with $i = 1$ and $j = 2$), but larger than the border of (28) (with $i = 2$ and $j = 1$), so that the donor makes positive transfers to both of recipient 1 and 2 as in (27). That is,

$$S_1 \in \left[-(w - T + t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2), \left(\frac{\alpha\beta}{1+\alpha\beta}\right)(w - T + S_2 + t_2) - t_1\right]. \tag{36}$$

In the other interval, S_1 is smaller than the border of (28) (with $i = 2$ and $j = 1$) and the border of (31) (with $i = 1$). When (S_1, S_2) is in this interval, the donor makes a transfer only to recipient 1. This interval is:

$$S_1 \leq \text{Min}\left[-(w - T + t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2), \alpha\beta(w - T) - t_1\right]. \tag{37}$$

We first consider recipient 1's utility-maximizing savings limited to the interval (36). Differentiating (1) with respect to S_1 with d_1 that is equal to (27) yields:

$$\frac{-1}{w - S_1} + \beta\left(\frac{1}{S_1 + d(S_1, S_2, T) + t_1}\right)\left[1 - \left(\frac{1+\alpha\beta}{1+2\alpha\beta}\right)\right]. \tag{38}$$

If S_1 in the interval (36) makes (38) zero, this S_1 is recipient 1's optimal savings in this interval. From (38) and (27) (with $i = 1$), this S_1 is calculated as (10). If (10) is larger than the upper limit in the interval (36), recipient 1's optimal S_1 limited to this interval is the upper limit of this interval. The value of (10) exceeds the upper limit if:

$$S_2 \leq \left(\frac{\beta - 1 - 2\alpha\beta}{1 + 2\alpha\beta + \alpha\beta^2}\right) w + \left[\frac{\alpha\beta(1+\beta)}{1 + 2\alpha\beta + \alpha\beta^2}\right](T - t_2) + \left[\frac{(1+\alpha\beta)(1+\beta)}{1 + 2\alpha\beta + \alpha\beta^2}\right] t_1. \tag{39}$$

In contrast, if (10) is smaller than the lower limit in the interval (36), recipient 1's optimal S_1 in this interval is the lower limit of this interval. The value of (10) is lower than the lower limit if:

$$S_2 \geq \left(\frac{2\alpha\beta^2}{1 + \beta + 2\alpha\beta + \alpha\beta^2}\right) w - \left[\frac{\alpha\beta(1+\beta)}{1 + \beta + 2\alpha\beta + \alpha\beta^2}\right](T - t_1) - \left[\frac{(1+\beta)(1+\alpha\beta)}{1 + \beta + 2\alpha\beta + \alpha\beta^2}\right] t_2. \tag{40}$$

Next, we consider recipient 1's optimal savings limited to the interval (37). Differentiating (1) with respect to S_1 when d_1 equals the value of (29) for recipient 1 yields:

$$\frac{-1}{w - S_1} + \beta\left(\frac{1}{S_1 + d(S_1, S_2, T) + t_1}\right)\left[1 - \left(\frac{1}{1+\alpha\beta}\right)\right]. \tag{41}$$

If S_1 in the interval (37) makes the value of (41) zero, this S_1 is recipient 1's optimal savings in this interval. From (41) and (29) for recipient 1, this S_1 is

$$S_1 = \frac{(\beta - 1)w + T - t_1}{1 + \beta}. \tag{42}$$

If (42) exceeds $\alpha\beta(w - T) - t_1$, that is, if:

$$\frac{(\beta - 1)w + T - t_1}{1 + \beta} \geq \alpha\beta(w - T) - t_1, \tag{43}$$

then recipient 1's optimal savings limited to this interval is the upper limit of this interval. If (42) is smaller than $\alpha\beta(w - T) - t_1$ but higher than $-(w - T +$

$t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2)$, the last value is recipient 1's optimal savings limited to this interval. The value of (42) exceeds $-(w - T + t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2)$ if:

$$S_2 \leq \left[\frac{2\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right] w - \left[\frac{\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right](T - t_1) - t_2. \tag{44}$$

The right-hand side of (40) is necessarily larger than that of (39), and the right-hand side of (44) is necessarily larger than that of (40). Therefore, to summarize, recipient 1's optimal savings limited to the interval of S_1 too small to satisfy (32) are:
if $S_2 \leq \left(\frac{\beta-1-2\alpha\beta}{1+2\alpha\beta+\alpha\beta^2}\right) w + \left[\frac{\alpha\beta(1+\beta)}{1+2\alpha\beta+\alpha\beta^2}\right](T - t_2) + \left[\frac{(1+\alpha\beta)(1+\beta)}{1+2\alpha\beta+\alpha\beta^2}\right] t_1$,

$$S_1 = \left(\frac{\alpha\beta}{1+\alpha\beta}\right)(w - T + S_2 + t_2) - t_1, \tag{45}$$

if $\left(\frac{\beta-1-2\alpha\beta}{1+2\alpha\beta+\alpha\beta^2}\right) w + \left[\frac{\alpha\beta(1+\beta)}{1+2\alpha\beta+\alpha\beta^2}\right](T - t_2) + \left[\frac{(1+\alpha\beta)(1+\beta)}{1+2\alpha\beta+\alpha\beta^2}\right] t_1 \leq S_2 \leq \left(\frac{2\alpha\beta^2}{1+\beta+2\alpha\beta+\alpha\beta^2}\right) w - \left[\frac{\alpha\beta(1+\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right](T - t_1) - \left[\frac{(1+\beta)(1+\alpha\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right] t_2$, S_1 is (10),
if (43) holds and if $\left(\frac{2\alpha\beta^2}{1+\beta+2\alpha\beta+\alpha\beta^2}\right) w - \left[\frac{\alpha\beta(1+\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right](T - t_1) - \left[\frac{(1+\beta)(1+\alpha\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right] t_2 \leq S_2$,

$$S_1 = \text{Min}\left[-(w - T + t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2), \alpha\beta(w - T) - t_1\right]. \tag{46}$$

if (43) does not hold and if $\left(\frac{2\alpha\beta^2}{1+\beta+2\alpha\beta+\alpha\beta^2}\right) w - \left[\frac{\alpha\beta(1+\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right](T - t_1) - \left[\frac{(1+\beta)(1+\alpha\beta)}{1+\beta+2\alpha\beta+\alpha\beta^2}\right] t_2 \leq S_2 \leq \left[\frac{2\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right] w - \left[\frac{\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right](T - t_1) - t_2$,

$$S_1 = -(w - T + t_1) + \left(\frac{1+\alpha\beta}{\alpha\beta}\right)(S_2 + t_2), \tag{47}$$

if (43) does not hold and if $\left[\frac{2\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right] w - \left[\frac{\alpha\beta^2}{(1+\alpha\beta)(1+\beta)}\right](T - t_1) - t_2 \leq S_2$, then S_1 is (42).

When $t_1 = t_2 = \frac{1}{2}T$, (42) becomes (11). When $t_1 = 0$ and $t_2 = T$, (42) becomes (17), and (42) for recipient 2 becomes (20).

10 Appendix 3: Proof of Proposition 1

The value of (14) minus the value of (15) is

$$(1+\beta)\ln\left[\frac{3(1+\beta)}{2+\beta}\right] + \beta\ln\left(\frac{\alpha\beta}{1+2\alpha\beta}\right). \tag{48}$$

If (48) is positive, the Nash equilibrium has each recipient save little, as in (12). When $\beta = 0$ and $\alpha = 1/2$, the value of (48) is positive. Differentiating (48) with respect to α yields:

$$\frac{\beta}{\alpha(1+2\alpha\beta)}. \tag{49}$$

This is positive, and thus large α generates the Nash equilibrium in which each recipient saves little. Differentiating (48) with respect to β yields:

$$\frac{3+\beta+2\alpha\beta}{(2+\beta)(1+2\alpha\beta)} + \ln\left[\frac{3(1+\beta)}{2+\beta}\right]\left(\frac{\alpha\beta}{1+2\alpha\beta}\right). \tag{50}$$

This is negative when β is small. Hence, the smaller β is, the more likely will the Nash equilibrium have each recipient save little.

11 Appendix 4: Proof of Proposition 2

We show that if $t_1 = t_2 = \bar{d}$ ($T = 2\bar{d}$) and if recipient 2 saves in the amount $\frac{\beta w - t_1}{1+\beta}$, as in (34), recipient 1 will save the same amount.

From Appendix 2.2, if recipient 2 saves $\frac{\beta w - \frac{1}{2}T}{1+\beta}$ and if (43) holds, the optimal S_1 limited to the interval of $S_1 \leq \alpha\beta(w-T) - t_1$ is $S_1 = \alpha\beta(w-T) - t_1$. In other words, recipient 1's utility increases with S_1 in this interval. Hence, he can enjoy higher utility by choosing S_1 in the interval of (32). From Appendix 2.1, recipient 1 will choose (34).

Therefore, it suffices to show that (43) holds when $t_1 = t_2 = \bar{d}$. From (13), $\bar{d}$ is calculated as:

$$\bar{d} = \left[\frac{1-\beta+\alpha\beta^2+2\alpha\beta}{(1+2\alpha\beta)(2+\beta)}\right] w. \tag{51}$$

Using $t_1 = t_2 = \bar{d}$, $T = 2\bar{d}$ and the equation above, $\frac{(\beta-1)w+T-t_1}{1+\beta} - [\alpha\beta(w-T) - t_1]$ becomes :

$$\begin{aligned}
&\left[\frac{\beta-1-\alpha\beta-\alpha\beta^2}{1+\beta}\right] w + \left[\frac{\beta+2+2\alpha\beta+2\alpha\beta^2}{1+\beta}\right]\bar{d} \\
&= \left[\frac{\beta-1-\alpha\beta-\alpha\beta^2}{1+\beta}\right] w + \left[\frac{\beta+2+2\alpha\beta+2\alpha\beta^2}{1+\beta}\right]\left[\frac{1-\beta+\alpha\beta^2+2\alpha\beta}{(1+2\alpha\beta)(2+\beta)}\right] w \\
&= \left[\frac{3\alpha\beta^2}{(1+\beta)(2+\beta)(1+2\alpha\beta)}\right] w > 0
\end{aligned} \tag{52}$$

Hence, (43) holds.

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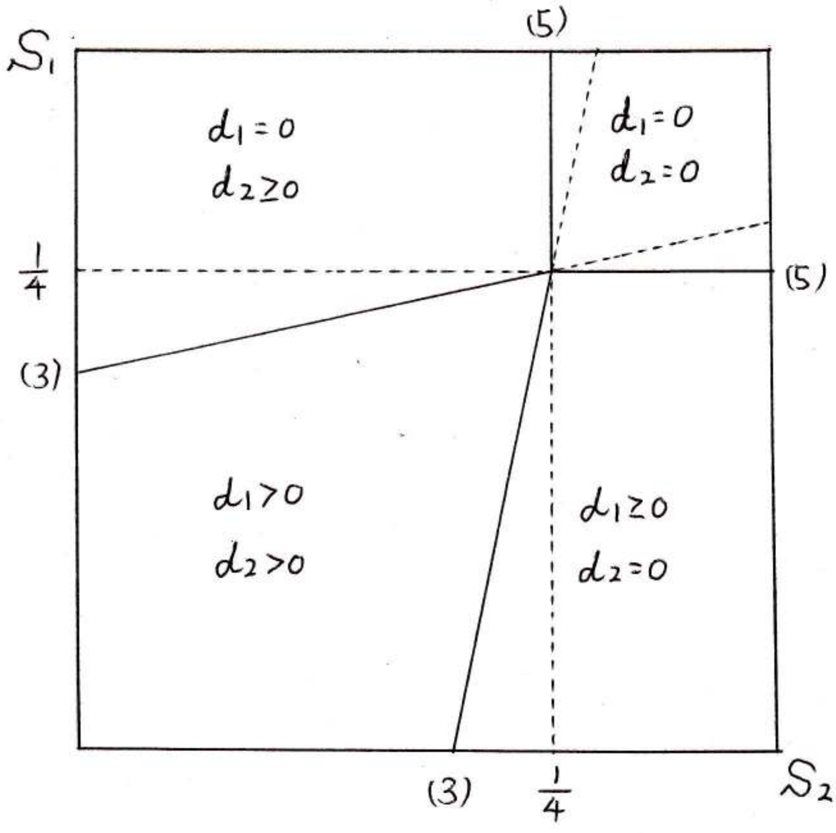

Figure 1

Private transfer (d_1 and d_2) as a function of recipients' savings (S_1 and S_2)

$\alpha = 0.5$, $\beta = 0.5$ and $T = 0$.

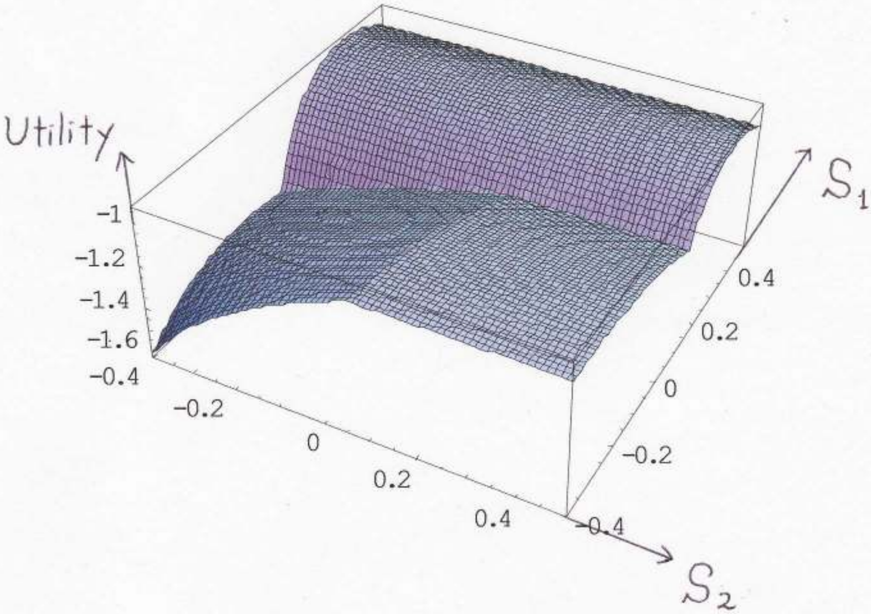

Figure 2(i)
Recipient 1's utility as a function of savings (S_1 and S_2) when government makes no transfer.

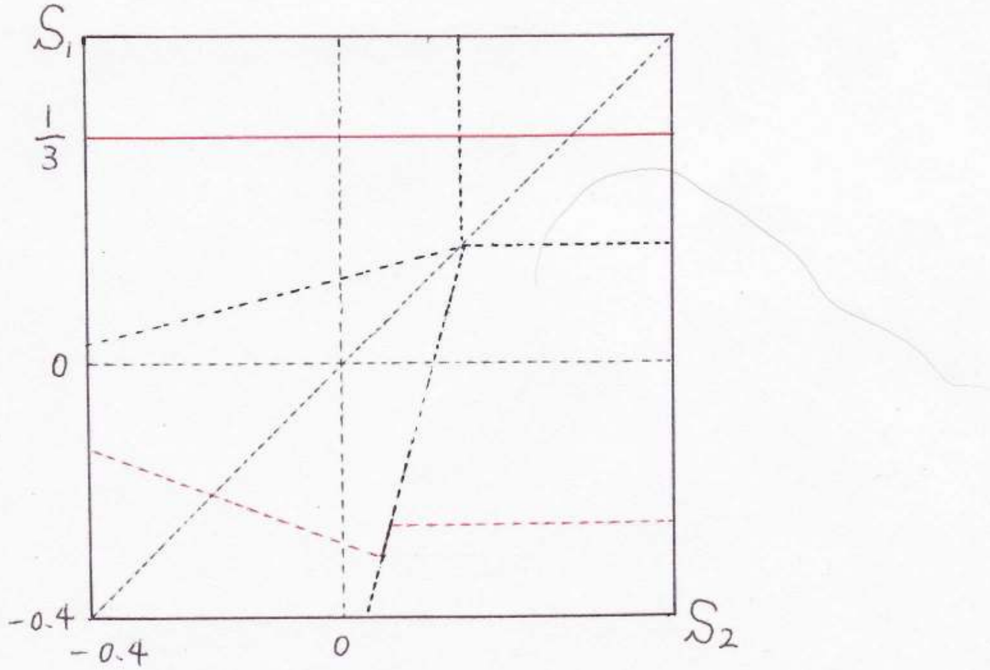

Figure 2(ii)
Recipient 1's savings (S_1) as a function of the other's savings (S_2) when government makes no transfer

.25 and $\beta = 0.5$.

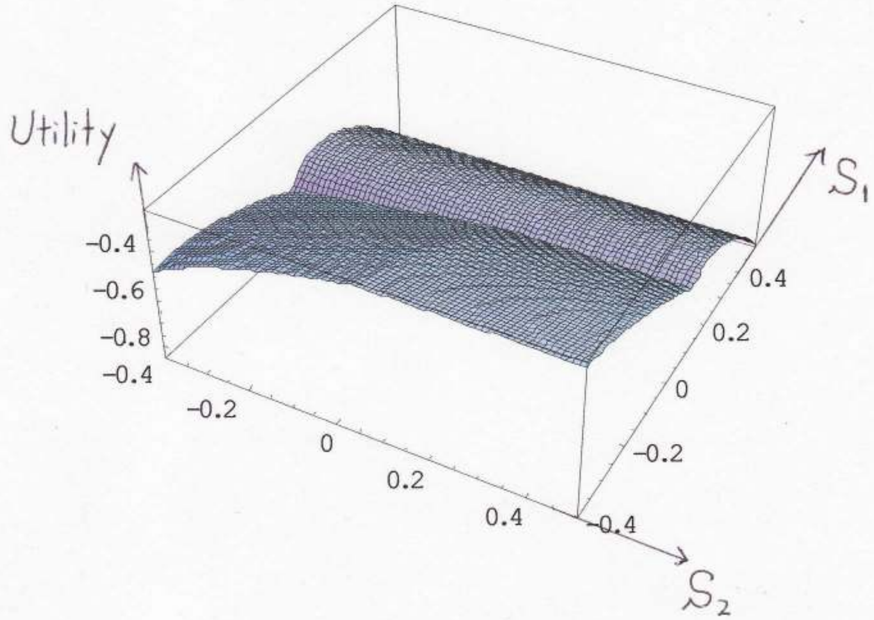

Figure 3(i)
Recipient 1's utility as a function of savings (S_1 and S_2) when government makes no transfer.

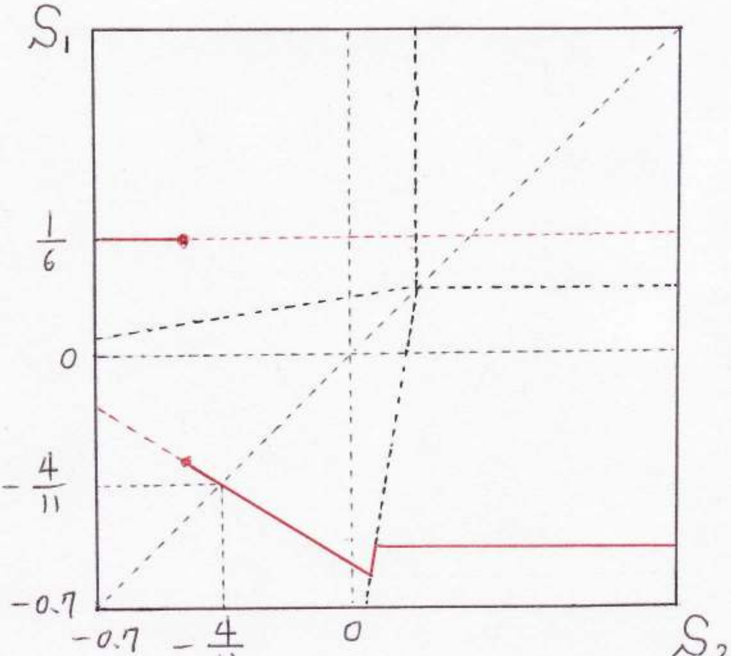

Figure 3(ii)
Recipient 1's savings (S_1) as a function of the other's savings (S_2) when government makes no transfer

$\alpha = 0.5$ and $\beta = 0.2$.

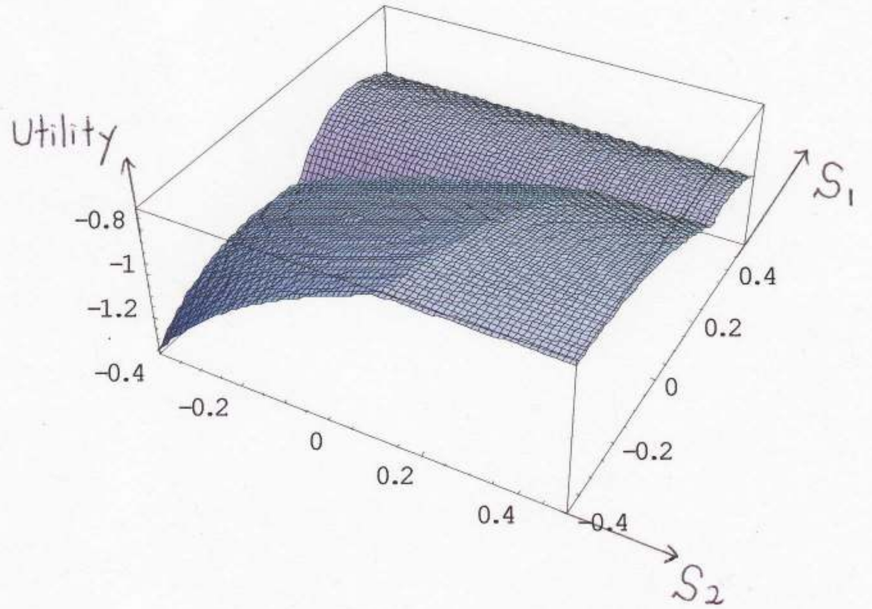

Figure 4(i)
Recipient 1's utility as a function of savings (S_1 and S_2) when government makes no transfer.

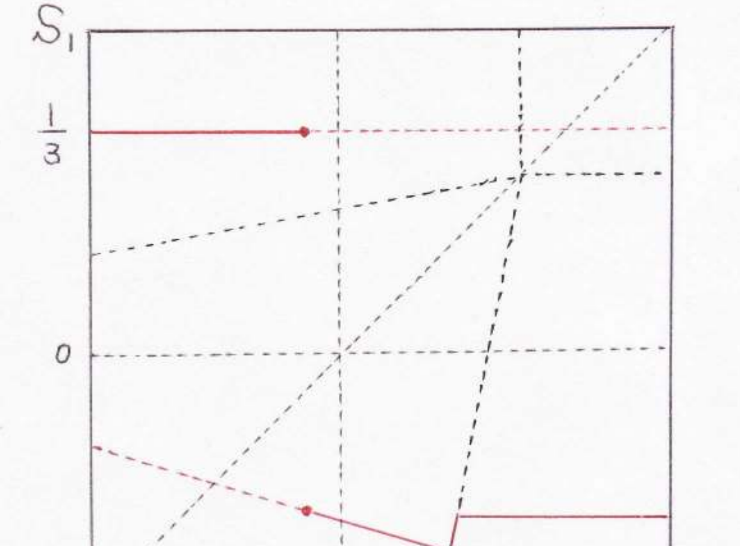

Figure 4(ii)
Recipient 1's savings (S_1) as a function of the other's savings (S_2) when government makes no transfer

.5 and $\beta = 0.5$.

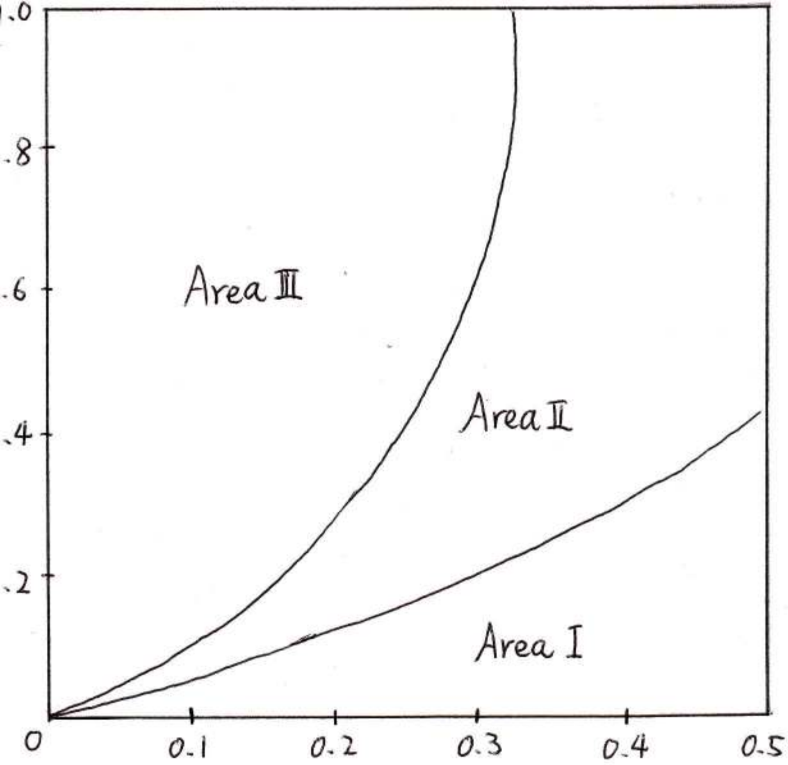

Figure 5

Types of equilibria depend on donor's altruism (α) and recipient's discount factor (β)

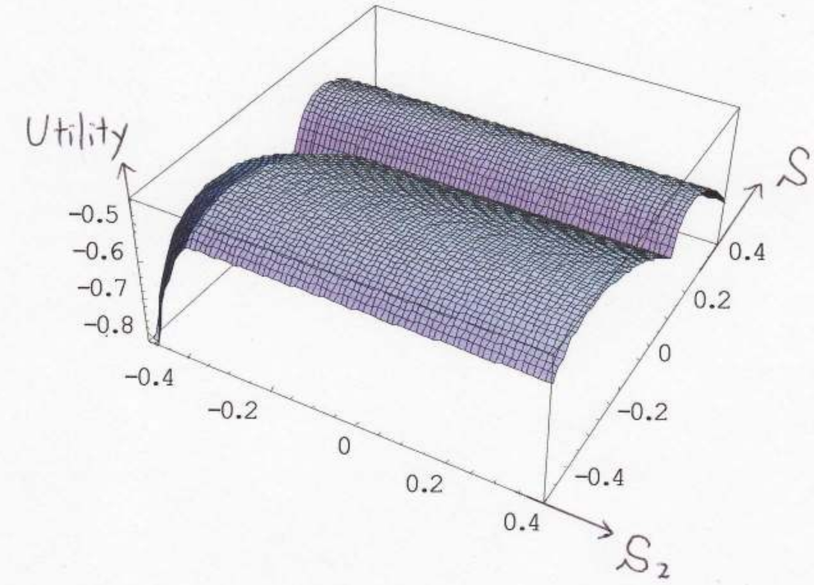

Figure 6(i)

Recipient 1's utility as a function of savings (S_1 and S_2) when government makes a transfer to only recipient 2.

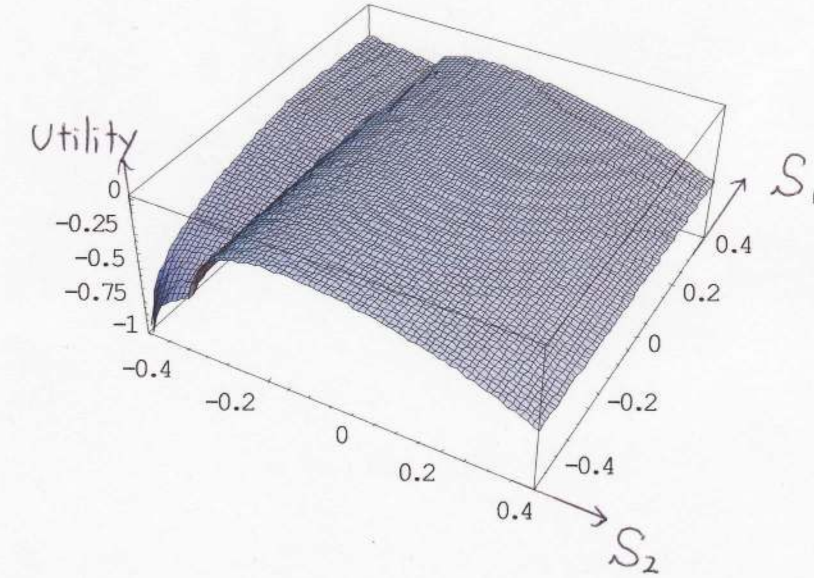

Figure 6(ii)

Recipient 2's utility as a function of savings (S_1 and S_2) when government makes a transfer to only recipient 2.

$\alpha = 0.25$ and $\beta = 0.5$.

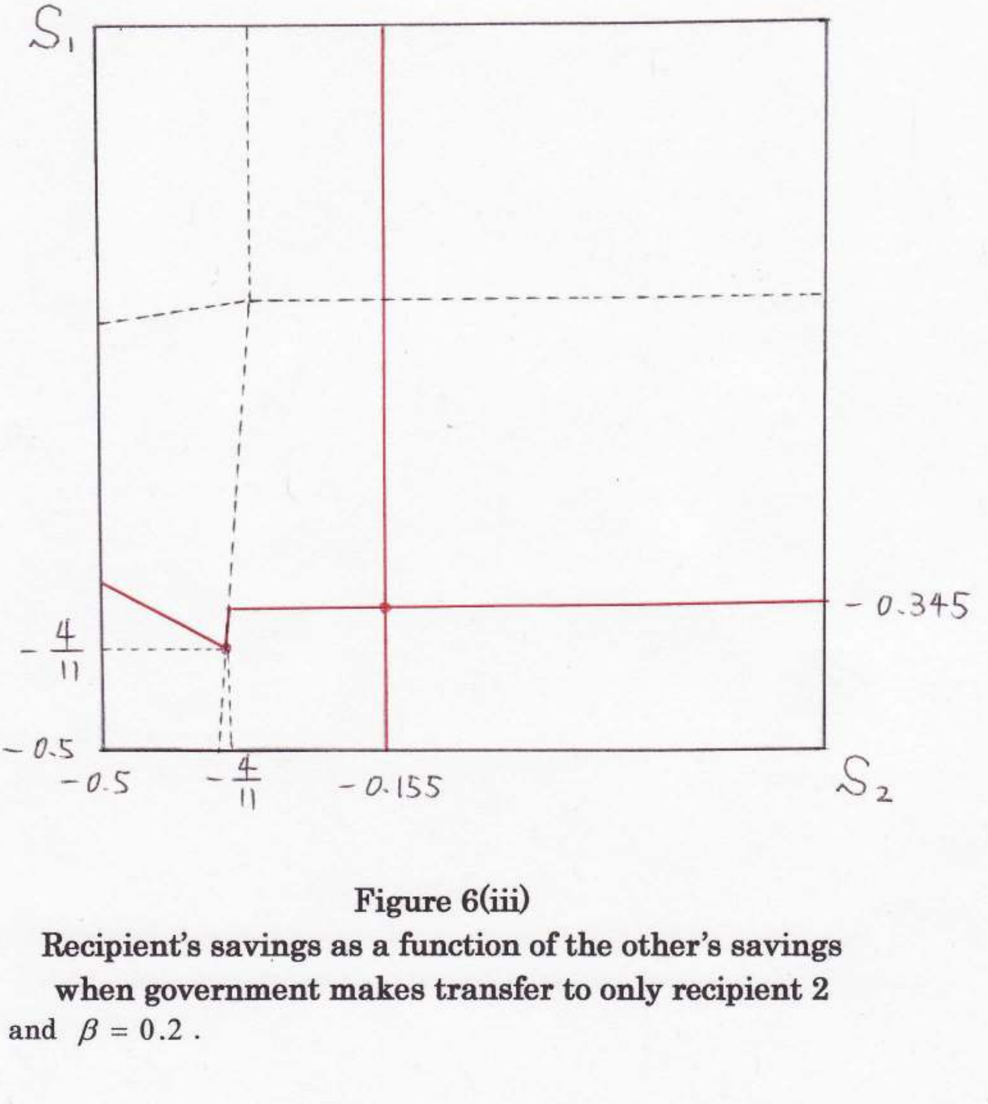

Figure 6(iii)

Recipient's savings as a function of the other's savings when government makes transfer to only recipient 2

$\alpha = 0.5$ and $\beta = 0.2$.