GPS satellite and receiver instrumental biases estimation using least squares method for accurate ionosphere modelling

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The positional accuracy of the Global Positioning System (GPS) is limited due to several error sources. The major error is ionosphere. By augmenting the GPS, the Category I (CAT I) Precision Approach (PA) requirements can be achieved. The Space-Based Augmentation System (SBAS) in India is known as GPS Aided Geo Augmented Navigation (GAGAN). One of the prominent errors in GAGAN that limits the positional accuracy is instrumental biases. Calibration of these biases is particularly important in achieving the CAT I PA landings. In this paper, a new algorithm is proposed to estimate the instrumental biases by modelling the TEC using 4th order polynomial. The algorithm uses values corresponding to a single station for one month period and the results confirm the validity of the algorithm. The experimental results indicate that the estimation precision of the satellite-plus-receiver instrumental bias is of the order of ± 0.17 nsec. The observed mean bias error is of the order -3.638 nsec and -4.71 nsec for satellite 1 and 31 respectively. It is found that results are consistent over the period.

1. Introduction

Global Positioning System (GPS) is a satellite-based positioning system based on the radio ranging technique. The accuracy of the standalone GPS system is limited by several errors such as ionospheric error, tropospheric error, clock error, multipath error, ephemeris error, etc. Because of these errors, GPS navigation signals alone are not adequate to support CAT I Precision Approach (PA) landings. Therefore, GPS augmentation system is required to provide users with orbit, clock, and ionosphere corrections. The first space-based augmentation system (SBAS) was initiated by USA for providing coverage of the Continental United States (CONUS) region. This augmentation system is called Wide Area Augmentation System (WAAS). The European Geo-stationary Navigation Overlay System (EGNOS) is being implemented by the European Space Agency since 1996 for the European countries and the MTSAT Satellite Augmentation System (MSAS) is being implemented by Japan. The Canadian WAAS (CWAAS) is also at an advanced stage of its implementation and is expected to be ready by 2006. Countries such as Brazil, Mexico and China are also developing their own SBAS. As in USA, the Airports Authority of India (AAI) has decided to implement an indigenous satellite-based regional GPS augmentation system, known as GAGAN as a part of the civil aviation requirements in India Ramalingam (2002). The GAGAN system for this purpose will be implemented jointly by the Indian Space Research Organization (ISRO) and AAI. GAGAN architecture consists of:

- Indian Reference Stations (INRESs)
- Indian Mission Control Center (INMCC)
- Indian Navigation Land Uplink Station (INLUS)

Keywords. Instrumental biases; Global Positioning System; GPS Aided Geo Augmented Navigation; Space-Based Augmentation System.

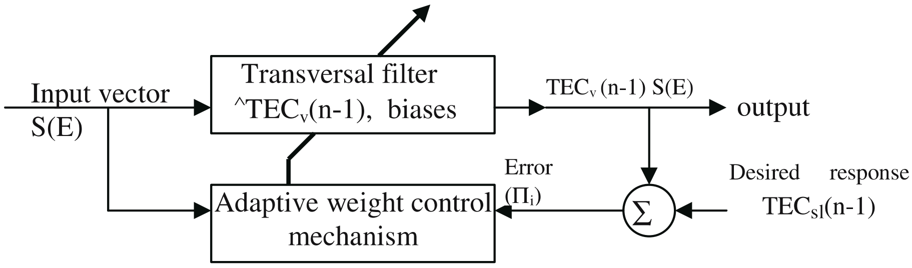

Figure 1. Block diagram of adaptive filter algorithm.

- Geostationary Earth Orbit (GEO) payload and
- User GNSS receivers.

Initial studies on the placement of 20 TEC stations for the Indian region are carried out by Sarma *et al* (2000). Based on this, 20 TEC stations are placed at surveyed locations over widely separated geographical areas in India. The networks of INRESs and 20 TEC stations receive and monitor the GPS signals for estimating the clock, ephemeris and ionospheric error corrections. Data from these stations are transmitted to the INMMC, where the validity of the signals from each satellite is assessed and corrections are computed. The INMMC also develops the ephemeris and clock information of the Geostationary Earth Orbiting Satellites (GEOs). All these data are packed into a GAGAN message and is sent to the INLUS. The INLUS uplinks this message on 6455.2 MHz to the GEOS that broadcasts GPS like signals to the GNSS users. INMCC is collocated with the INLUS at Bangalore. One 40 kg navigation payload with EIRP of 33.5 dBW is planned in the Indian Ocean Region (IOR) between the orbital location 48° to 100°E longitude through GSAT4 to meet the objectives of GAGAN (Suryanarayan Rao and Pal).

Ionospheric delay is one of the prominent errors in the GAGAN that limits positional accuracy. The ionospheric delay corrections are broadcast as vertical delay estimates at specified Ionospheric Grid Points (IGPs) in the predefined global IGP grid to suitably modify single frequency GPS receivers (WAAS 1997). The predefined global IGP grid consists of 1808 IGPS. For providing the ionospheric error corrections over the GAGAN service region, 60 IGPS are identified (Sarma *et al* 2000). However, the estimation of the IGP delay, which is a function of TEC, is limited by instrumental biases. The instrumental bias is the difference between the propagation paths of L1 and L2 signals and is due to the circuitry in the GPS satellite and receiver hardware. Even though the bias errors are of the order of ±10 nsec it will become critical in SBAS (Brain *et al* 1999). Calibration of hardware biases is particularly important in augmented GPS systems where vertical accuracy of 4.5 m is required for PA landings. If the differential delay parameters are not calibrated, they propagate into the differential correction through the ionospheric models (Brian *et al* 1999). The differential delays are environmentally dependent and hence time-varying. In the case of hardware calibration, it will be difficult for the master station of the GAGAN located at Bangalore, India to continually monitor all the geographically distributed 20 TEC stations. Therefore, software calibration is the solution and is described in this paper.

2. Estimation of satellite and receiver biases

A new algorithm based on Coco *et al* (1991) is proposed to estimate the instrumental biases. In this algorithm, the combined satellite and receiver differential delays are estimated using the least-squares method. This algorithm is an approximation of the steepest descent algorithm, which uses an instantaneous estimate of the gradient vector of a mean square error. The vertical TEC at each ionospheric pierce point (IPP) is represented as the 4th order polynomial model in this algorithm. The inputs to the algorithm are azimuth, elevation angle of each satellite tracked, slant factor, slant TEC, IPP latitude and longitude. The slant TEC measurement (TEC_{sl}) due to GPS signal at TEC station is the sum of the observed slant TEC, satellite differential delay (b_S) and receiver differential delay (b_R). The differential delay can be modeled as the sum of a receiver bias, a satellite transmitter bias, and a constant times the line-of sight ionospheric total electron content (TEC) (Coco *et al* 1991). The following three assumptions are made in implementing this algorithm:

- The slant and vertical TECs are related by a constant obliquity factor.
- Satellite-plus-receiver (SPR) differential delays are assumed to be constant over several hours.

Table 1. $\overline{\sigma}$ *of the mean SPR instrumental biases for 13 GPS satellites for one month period.*

SV PRN	31	27	25	24	21	16	15	13	10	8	6	4	1
	SPR value (nsec)												
Mean	−4.71	−2.96	5.41	−3.86	1.53	5.02	4.25	0.66	−6.4	−3.88	0.66	−3.63	−3.63
$\overline{\sigma}$	2.91	0.79	1.27	0.91	1.70	1.04	1.24	1.09	0.6	0.77	1.14	0.81	0.81

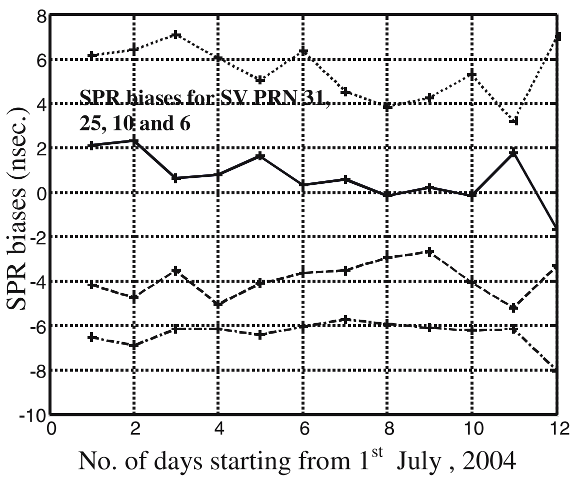

Figure 2. SPR instrumental biases for 4 satellites observed from Hyderabad (17.431°N, 78.453°E).

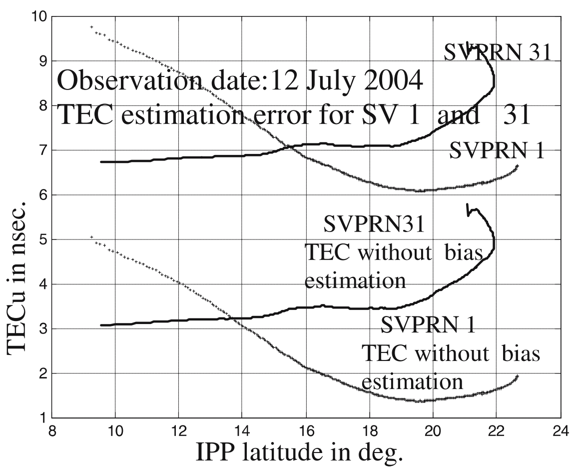

Figure 3. Comparison of TEC estimation after modelling of instrumental biases.

- The TEC, at the IPP is represented by 4th degree polynomial as follows (Lao-Sheng Lin):

$$\begin{aligned}\mathrm{TEC}_v(\phi_m, \lambda_{cr}) = {} & a_0 + a_1\phi_m + a_2\lambda_{cr} + a_3\phi_m^2 \\ & + a_4\lambda_{cr}^2 + a_5\phi_m\lambda_{cr} + a_6\phi_m^3 \\ & + a_7\lambda_{cr}^3 + a_8\phi_m^2\lambda_{cr} + a_9\phi_m\lambda_{cr}^2 \\ & + a_{10}\phi_m^4 + a_{11}\lambda_{cr}^4 + a_{12}\phi_m^3\lambda_{cr} \\ & + a_{13}\phi_m^2\lambda_{cr}^2 + a_{14}\phi_m\lambda_{cr}^3, \qquad (1)\end{aligned}$$

where $a_0, a_1, \ldots, a_{14}$ are the unknown ionosphere model coefficients. ϕ_m and λ_{cr} are the IPP latitude and longitudes in geomagnetic coordinate system.

3. Modelling of instrumental biases

The biases can be modeled in terms of vertical TEC as (Ma and Maruyama 2003):

$$S(E)_{ji} \times \mathrm{TEC}_{v\,i} + (b_{Sj} + b_{Rj}) = \mathrm{TEC}_{sl\,ji} \qquad (2)$$

where, $\mathrm{TEC}_{sl\,ji}$ = measured slant TEC from the receiver j to the satellite i, E = elevation angle from the receiver j to the tracked satellite i, $S(E)_{ji}$ = slant factor, TEC_{vi} = vertical TEC at the ionospheric pierce point due to the satellite $i, b_{Si} + b_{Rj}$ = satellite-plus-receiver (SPR) differential delay of the satellite i and receiver j.

The algorithm is a linear adaptive filtering algorithm and it consists of two basic processes:

- a filtering process, which involves computing the output of a linear filter in response to an input signal and generating an estimation error by comparing this output with a desired response and
- an adaptive process, which involves the automatic adjustment of the parameters of the filter in accordance with the estimation error.

The combination of these two processes working together constitutes a feedback loop (see figure 1). The figure shows a transversal filter, around which the least mean square algorithm is built; this component is responsible for performing the filtering process. The second component is a mechanism for performing the adaptive weight control process on the tap weights of the transversal filter. The detailed structure of the transversal filter consists of 3 basic weight elements, namely, a unit delay element, a multiplier and an adder. The number of delay elements used in the filter determines the finite duration of its impulse response. The role of each multiplier in the filter is to multiply the tap input by a filter coefficient referred to as a tap weight. The combined role of the adders in the filter is to sum the individual multiplier outputs and produce an overall filter output.

The physical phenomenon is characterized by the two set of variables $\mathrm{TEC}_{sl}(i)$ and $S(E)(i)$. The variable $\mathrm{TEC}_{sl}(i)$ is observed at time i in response to the subset of variables $S(E)(i)$, $S(E)(i-1), \ldots, S(E)(i-M+1)$, applied as inputs. The $\mathrm{TEC}_{sl}(i)$ is a function of the inputs $S(E)(i)$, $S(E)(i-1), S(E)(i-1), \ldots, S(E)(i-M+1)$. This functional relationship is modeled as (Ma and Maruyama 2003),

$$\mathrm{TEC}_{sl}(i) = \sum_{k=0}^{M-1} a_k b_k (S(E)(i-k)) + \Pi(i), \quad (3)$$

$$\prod(i) = \mathrm{TEC}_{sl}(i) - \mathrm{TEC}_{v}(n-1) S(E)(i), \quad (4)$$

$$\mathrm{TEC}_{\bar{V}}(n+1) = \mathrm{TEC}_{\bar{V}}(n) + 2\mu \Pi_n S(\bar{E})(n), \quad (5)$$

where, a_0, a_{M-1}, and b_k are unknown parameters of the model, $\mathrm{TEC}_{\bar{V}}(n+1)$ is the tap weight vector adoption, μ is the step size parameter and Π_i represents the measurement error.

4. Data acquisition and results

As a part of the GAGAN setup, 20 dual frequency GPS receivers (Novtel Model No. GSV 4004 A) are located at various places spread all over India. However, in this work, three receivers located at Hyderabad (17.431°N, 78.453°E), Bangalore (12.95°N, 77.68°E) and Visakhapatnam (17.72°N, 83.22°E) are considered. Several days of navigation and observation data in RINEX format were collected and analyzed. The navigation data file consists of 38 parameters. However, in our analysis only 23 parameters are used. Navigation data are available for every two hours. In between data are generated using standard formulae. Observation data file consists of C/A, P1 and P2 pseudoranges and L1 and L2 phases for all the visible satellites. From this information satellite position, elevation and azimuth angle of satellite, IPP local time, IPP latitude, longitude, geomagnetic latitude, geomagnetic longitude, slant factor, ionospheric time delay and slant TEC for all the visible satellites are estimated.

For one month data (1 to 31st July 2004), the biases are estimated. Using the satellite elevation and azimuth information, for each satellite the IPP latitude and longitude are estimated. A mesh grid with a square grid spacing of $5° \times 5°$ in latitude and longitude is assumed at an altitude of 350 km above the earth surface. In each 5° square grid, the number of IPPs available are determined. The instrumental biases are assumed to be constant over several hours in a particular mesh grid of 5° square grid size. The differential delay $(b_{Si} + b_{Rj})$ and the 15 coefficients of the polynomial for all the 29 satellites that were visible were estimated for a particular 5° square grid $(17.431 \pm 2.25°, 78.4530 \pm 2.25°)$. In this particular grid the IPPs are due to 13 visible SVs (PRNs 1, 4, 6, 8, 10, 13, 15, 16, 21, 24, 25, 27, 31).

The mean standard deviation $(\overline{\sigma})$ of instrumental biases for 13 satellites are presented in table 1. The $\overline{\sigma}$ values indicate the day-to-day variability of the SPR differential delay estimates. Figure 2 shows the SPR instrumental biases for the SV PRN 31, 25, 10 and 6. The biases observed are positive values (2 to 7 nsec) for SVs 6 and 31 and negative values (−3 to −8 nsec) for SVs 10 and 31. Figure 3 compares the TEC estimation for a SV PRN 1 and 31 (12 July 2004) after modelling of instrumental biases. The bias error estimated is −3.638 nsec and −4.71 nsec for satellites 1 and 31 respectively. From the results, it is found that the SPR differential delays of 13 satellites are varying from −6.4060 to 5.4117 nsec. The results indicate that day-to-day variation of SPR differential delay is small and it is less than 1 nsec. The average value of the σ of the SPR differential delay estimate is 1.17 nsec, which represents an error estimate of the SPR differential delays.

5. Conclusions

A new algorithm is proposed to estimate the instrumental biases by modelling the ionospheric TEC using 4th order polynomial. This algorithm is an approximation of the steepest descent algorithm, which uses an instantaneous estimate of the gradient vector of a cost function. The estimate of the gradient is based on sample values of the tap-input vector and an error signal. The algorithm can be used to calibrate the dual frequency GPS receivers for precise TEC measurement even when the receiver internal hardware calibration is not available. The experimental results indicate that the estimation precision of the satellite and receiver differential delay is of the order of ±0.17 nsec. It is found that the error in the TEC estimation for the SV PRN 1 and 31 are −3.638 nsec and −4.71 nsec respectively. It is also found that the results are consistent over the period and the method is accurate and faster for real-time applications like GAGAN systems.

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