Gradual Product Replacement, Intangible-Asset Prices and Schumpeterian Growth

by

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Abstract

The paper develops a general-equilibrium model of scale-invariant Schumpeterian (R&D-based) growth. New higher-quality products are discovered through stochastic and sequential R&D races in each industry. The market share of an R&D race winner increases gradually and is governed by an exponential deterministic process. The introduction of gradual (as opposed to instantaneous) product replacement sheds more light on the effects of the rate of technology diffusion on long-run growth and on long-run dynamics of intangible asset prices. An economy with faster product diffusion rates experiences higher long-run innovation rates, faster transitional growth, and is populated by younger firms. As the typical firm becomes older, the earnings yield (i.e., the inverse of the price earnings (P/E) ratio) increases and expected earnings growth declines. Younger firms have lower earnings, lower market shares, but higher P/E ratios and higher expected earnings growth associated with their higher potential market growth.

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1. Introduction

Models of Schumpeterian (R&D-based) growth have routinely assumed that each newly discovered product captures its potential market instantaneously.[1] The assumption of instantaneous product replacement, which is made for analytical convenience and simplicity of exposition in variety-accumulation and quality-improvement models or economic growth, does not enjoy empirical support: The transmission of new technology and its adoption is a gradual and lengthy process. A trip to any Blockbuster store, where the space allocated to DVD movies is gradually expanding at the expense of the space for VHS videos constitutes one of numerous examples of gradual (as opposed to instantaneous) product replacement process.[2] One implication of instantaneous product adoption is that long-run (per-capita) asset prices of technology leaders are independent of the firm's age and constant over time. For example, in quality-ladder Schumpeterian growth models the instant a new technology leader is born, all potential customers buy the new product and the flow of monopoly profits grows at a constant growth rate which is equal to the rate of growth of population. When a new leader emerges the incumbent's market share goes to zero instantaneously. There is no difference between young and old firms.

The main innovation of the present paper is to analyze the effects of gradual product replacement on Schumpeterian growth and on intangible asset prices. The focus on intangible asset prices is motivated by the relative lack of understanding how total productivity growth and general equilibrium forces shape the behavior of financial markets and the market valuation of intangible assets. The second half of the 1990s is a case in point. During the period 1995-1999 the U.S. economy experienced 4.8 percent annual output growth.[3] Financial markets enjoyed unprecedented growth during the second half of the 1990s. At the microeconomic level, several popular books described the characteristics of high-technology markets and offered rules that would identify the winners of technological competition.[4] At the macroeconomic level, the evolution of asset prices raised concerns regarding the long-run sustainability of economic growth and the rationality of consumers and investors. Alan Greenspan, the chairman of the Federal Reserve Board, expressed these concerns in December of 1996: "How do we know when

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[1] Schumpeterian growth is a type of growth generated through the endogenous introduction of new products and/or processes which is based on Schumpeter's (1942) description of endogenous technological progress. See Jones (1995) Aghion and Howitt (1998), Segerstrom (1998), Young (1998), Dinopoulos and Thompson (1998) among many others for models of Schumpeterian growth. Jones (1999) and more recently Dinopoulos and Sener (2004) provide overviews of recent developments in Schumpeterian growth theory.

[2] Klepper and Simmons (1997) report long-diffusion periods for a variety of new product and process innovations: For example it took more than 10 years before the balloon cord technology of making tires achieved a 90 percent market share and more than 18 years before the fabric tire technology was replaced by the balloon-cord and high-pressure cord technologies.

[3] See Oliner and Sichel (2000, Table 1).

[4] See Kelly (1998), Varian and Shapiro (1998) and Moore et al (1999) among others. For instance, according to Moore at al. (1999) if one had invested $10,000 in Cisco Systems in 1990, the investment would have been worth over $3.5 million by 1999. And a $10,000 investment in Yahoo in 1996 would have been worth more than $300,000 three years later. The book describes the dynamics of high-tech markets and offers rules of thumb to investors to identify "gorilla candidates", that is companies that would dominate their markets and earn exceptional monopoly profits for an extended period of time.

irrational exuberance has unduly escalated asset values?…And how do we factor that assessment into monetary policy?"[5] Indeed, based on the belief that assets were overvalued, the Federal Reserve Bank engaged in a policy of gradually raising interest rates in an attempt to slowly let the air out of the bubble.

The above-mentioned developments raise several novel questions: What is the relationship between sustainable (i.e., long-run) economic growth and the market valuation of technology leaders? What are the general-equilibrium forces that determine the evolution of high-tech markets? How do stock market values depend on economic factors such as population growth, total factor productivity growth, the expected lifetime of a typical firm, and the rate of technological diffusion?

A small but growing body of literature has examined the role of technology in shaping the evolution of asset prices and has provided valuable insights on some of the above-mentioned questions. Hall (2001) has estimated the quantity of intangible capital from the market value of businesses emphasizing the role of cash-flow and intangible assets in explaining the evolution of the stock market. Hobijin and Jovanovic (2001) have analyzed the effects of information technology on stock prices using a vintage-capital model. Helpman and Trajtenberg (1998a, 1998b) and Petsas (2003) among others have examined the impact of gradual diffusion of general purpose technologies on economic growth and cycles.[6] Laitner and Stolyarov (2003, 2004) have developed models of physical and intangible capital to examine the evolution of Tobin's q and the measurement of total factor productivity. These studies have examined the role of gradual technology diffusion on the aggregate stock-market performance.

The present paper complements the above-mentioned studies by developing a dynamic general equilibrium model of an economy populated by technology leaders and experiencing sustained Schumpeterian (R&D-based) growth. The model is used to analyze the long-run behavior of intangible assets under the assumption that there is a gradual (as opposed to instantaneous) adoption of new higher-quality products within each high technology market. The economy is populated by identical rational consumers and firms. The former engage in utility maximization over an infinite time horizon. The latter maximize expected discounted profits. In the model, new higher-quality products are discovered through sequential R&D races. The firm that discovers the state-of-the-art quality product becomes the new leader and replaces the product produced by an incumbent monopolist. We focus on balanced-growth equilibrium dynamics for tractability purposes and also assume that all markets are efficient. Consequently, at each instant in time asset prices do not reflect informational asymmetries, heterogeneous tastes or speculative bubbles.[7] Therefore our results should be interpreted as long-run relations between economic fundamentals and the behavior of intangible asset prices.

Following one of the main insights of Moore et al. (1999, chapter 2), we assume that the winner of an R&D race replaces the incumbent firm gradually. The market share of the leader increases gradually and is governed by an exogenous and deterministic dynamic process that

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[5] Shiller (2000) provides more details on the concept of irrational exuberance.

[6] Helpman and Tranjteberg (1998a, 1998b) introduce gradual diffusion of a general purpose technology by assuming that its adoption depends on the production of complementary inputs. The model is used to generate cycles in per-capita output growth and the stock-market valuation of innovating firms. Petsas (2003) analyzes the stock-market growth effects of a gradual adoption of a general purpose technology. In his model, unlike the present one, the general purpose technology is adopted across a continuum of structurally identical industries.

[7] Economies populated by rational consumers engaged in maximizing behavior over an infinite horizon, as in the present model, cannot generate financial speculative bubbles. See Blanchard and Fischer (1989, chapter 5) for more details on this issue in the contest of the neoclassical growth model.

mimics the exponential cumulative distribution function. In contrast, other Schumpeterian growth models have routinely assumed an instantaneous product-replacement mechanism. We abstract from market-structure considerations, which are described in detail in Moore et al (1999), in order to increase the tractability of the model. Therefore, the model does not deal with the question of which early participant in a high-tech market is more likely to dominate. We model the selection process as a stochastic R&D race, with the winner of a race destined to dominate its market, earn temporary monopoly profits, and eventually be replaced by the winner of the next R&D race.

The introduction of a gradual product replacement mechanism in a standard model of Schumpeterian growth allows us to shed more light on the long-run dynamics of intangible asset prices. These prices incorporate simultaneously the marginal cost of creating the intangible assets and the expected discounted earnings of the technology leader. The model has a unique balanced growth equilibrium in which all per-capita variables grow at constant rates. In the steady-state equilibrium, firms maximize expected discounted profits, rational consumers maximize their discounted lifetime utility and all markets clear instantaneously. Economies with faster product diffusion rates experience higher innovation rates and growth, and are populated by younger firms. The total rate of return on an intangible asset is constant in the steady-state equilibrium and equals the earnings yield rate (which is the inverse of the price earnings –P/E – ratio) plus the expected rate of capital gains. The latter is equal to the growth rate of the market, measured by the rate of growth of population, plus the rate of growth of the firm's market share. Unlike models of Schumpeterian growth with instantaneous product replacement, the present model gives rise to nontrivial dynamics for asset prices in the steady state. More specifically, as the age of the quality leader increases, the dividend yield increases and capital gains decrease. Therefore, younger firms have lower earnings, lower market shares, but higher P/E ratios and higher expected capital gains associated with their potential market growth.

The paper is organized as follows. Section 2 describes the behavior of consumers and firms. Section 3 describes the role of the economy's stock market and its valuation of monopoly profits. Section 4 completes the description of the model by focusing on the labor market. Section 5 derives the steady-state equilibrium of the economy and section 6 analyzes the steady-state dynamics of asset prices. Finally section 7 offers a summary of the main results and suggestions for further research.

2. The Model

2.1 Consumers

The economy is populated by identical households located on a unit interval. Each household is modeled as a dynastic family whose size grows at an exogenous growth rate $g_N > 0$. The representative family maximizes the following expected utility over an infinite time horizon

$$U = \int_0^\infty e^{-(\rho - g_N)t} \log u(t) dt \tag{1}$$

where ρ is the constant subjective discount rate, $g_N > 0$ is the constant growth rate of population, and subutility $\log u(t)$ is defined as follows:

$$\log u(t) = \int_0^1 \log\left[\sum_j \lambda^j Z(j,\omega,t)\right] d\omega \qquad (2)$$

Parameter $\lambda > 1$ is the quality improvement between two consecutive final consumption goods and measures the size of innovations; $Z(j,\omega,t)$ denotes the per-capita quantity consumed of a good that has experienced j innovations (quality improvements) in industry $\omega \in (0,1)$ at time t. Each consumer maximizes (2) subject to a static budget constraint.[8] The solution to this static problem yields

$$Z(t) = \frac{c(t)}{p(t)} \qquad (3)$$

where $p(t)$ is the price level of a particular good and $c(t)$ is the per-capita consumption expenditure at time t. The aggregate demand for a typical good is given by

$$Q(t) = \frac{c(t)N(t)}{p(t)} \qquad (4)$$

where $N(t)$ is the level of the economy's population at time t. Substituting (3) into (2) and the resulting expression into (1), one can solve the intertemporal consumer optimization problem. The solution yields the familiar differential equation

$$\dot{c}(t)/c(t) = r(t) - \rho \qquad (5)$$

where $r(t)$ is the market interest rate.

2.2 Producers

There is a continuum of structurally identical industries producing final consumption goods. Each industry $\omega \in (0,1)$ is characterized by two activities: manufacturing of final goods and R&D investment to discover higher quality products. The latter are discovered through endogenous, stochastic and industry-specific sequential R&D races. Each activity utilizes only labor, which is the only factor of production. Denote with $N(t)$ the endowment of labor at time t. We assume that $N(t)$ grows at constant rate of growth $g_N = \dot{N}(t)/N(t)$. At each instant in time there are three distinct types of firms in a typical industry: a quality leader producing the state-of-the-art quality product, say j; followers producing a product $j - 1$ with quality one step below the leader's product; and challengers who engage in R&D to discover the next state-of-the-art quality product $j + 1$.

[8] See Segerstrom (1998) and Dinopoulos and Segerstrom (1999) for more details on the consumer's maximization problem.

Since all industries are assumed to be structurally identical, one can omit argument ω from functions and variables in order to simplify the notation. Challenger i, who invests $R_i(t)$ resources in R&D discovers the next higher-quality product with probability $I_i(t)dt$, where dt is an infinitesimal interval of time and $I_i(t)$ is given by

$$I_i(t) = \frac{R_i(t)}{X(t)}. \tag{6}$$

The $X(t)$ term in (6) captures the difficulty of R&D and will be explained shortly. We assume that the returns to R&D investment are independently distributed across firms, industries, and over time. As a result, the probability that one firm (challenger) will discover the next quality product is given by

$$I(t) = \sum_i I_i(t) = \frac{R(t)}{X(t)} \tag{7}$$

where $R(t) = \sum_i R_i(t)$ is the aggregate R&D investment in a typical industry. Equation (7) defines the intensity of the Poisson process that governs the arrival of innovation in a typical industry. Since time is continuous, the probability of two products discovered instantaneously is zero. Following the standard practice of quality-ladders growth models, we will refer to *I(t)* as the rate of innovation. This specification implies instantaneous constant returns to R&D at the aggregate level.

The scale-effects property is removed by following the approach proposed in Jones (1995), Segerstrom (1998) and Dinopoulos and Segerstrom (1999). We assume that R&D starts off being equally difficult in all industries $X(\omega,0)=1$ for all $\omega \in (0,1)$ and the level of R&D difficulty evolves according to

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu I(\omega,t) \tag{8}$$

where $\mu > 0$ is a constant. Equation (8) implies that the level of aggregate R&D difficulty is an increasing function of cumulative R&D investment and the level of "knowledge" in a particular industry. It also implies –as we will establish later- that the long-run rate of innovation and per capita growth is proportional to the constant rate of population growth and therefore policies do not have a permanent growth effect.[9]

[9] Jones (1999) and Dinopoulos and Thompson (1999) and Dinopoulos and Sener (2004) provide more details for the implications of the scale-effects property for early R&D-based growth models and describe recent attempts to develop models of growth without scale effects. In a previous version of the paper we analyzed a different specification of (8) which states that the level of R&D difficulty is proportional to the level of population, i.e., $X(\omega,t) = kN(t)$ and results in permanent growth effects. The main results of the analysis are not affected by the choice of either specification, although the one adopted in the present paper allows us to derive closed form steady-state solutions.

Denote with α_Q and α_R the constant unit-labor requirements for producing manufacturing output and R&D services in a typical industry. Total costs for these activities are therefore $w\alpha_Q Q(t)$ and $w\alpha_R R(t)$ respectively, where w is the wage of labor. Denote with *j* the state-of-the-art quality step (i.e., the quality level is λ^j) produced by a quality leader in a typical industry and with τ the time of discovery of product *j*. We assume that when a new technology *j* is discovered the old *j* - 1 technology becomes common knowledge. Hence the technology of producing all products with quality lower than *j* is public knowledge. This means that all followers in this industry produce the *j* – 1 product, charge a price equal to unit labor costs (due to free entry), and have zero profits.

The profit flow of the quality leader can be written as

$$\pi(t,\tau) = (p(t) - \alpha_Q w)\frac{c(t)N(t)}{p(t)} s(t,\tau) \qquad (9)$$

where $p(t)$ is the price charged and $s(t,\tau)$ is the market share of the leader at time t. Argument τ denotes the time of the technological leader's birth. In other words, at time t the leader's age is $t-\tau$; each leader charges a price $p(t)$; she/he faces unit manufacturing costs $\alpha_Q w$; and enjoys a market share $s(t,\tau)$ of total output demanded $Q(t) = c(t)N(t)/p(t)$.

2.3 Gradual Product Replacement

The rest of the literature assumes that at the time of discovery, τ, the leader's market share jumps from zero to unity instantaneously. This assumption is justified by the notion that consumers are fully informed about the quality of the new good and that they switch instantaneously to the product with the lowest quality adjusted price. In other words, if a quality leader offers the slightly lower price than $\lambda\alpha_q w$ consumers migrate immediately to the newly introduced product and the old product becomes obsolete. Here we depart from this unrealistic assumption by supposing that $s(t,\tau)$ evolves gradually over time. In other words, we assume that there is a fraction of consumers in the economy who cannot immediately recognize the quality of the newly introduced product, but they do so gradually as its market share increases. We would like to mimic an S-curve diffusion process, provide tractability, and generalize the Schumpeterian approach to economic growth. In order to meet our goals, we choose to model the evolution of $s(t,\tau)$ as follows: At each instant in time there are two types of products in each industry: the state-of-the-art-quality product and generic ones that can be produced by all other firms.

Following the rest of the Schumpeterian growth literature, we assume that every newly discovered good is protected by a perfectly enforceable patent which expires when the next higher-quality product is discovered. We assume that the technology of how to produce all generic products, whose patents have expired, is public knowledge and perfect competition prevails in their production. This unrealistic assumption is made for tractability considerations.[10]

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[10] Dinopoulos and Segerstrom (1999), Grossman and Lai (2004) and Dinopoulos at al. (2005) among others have used the assumption that the technology of products with expired patents is common knowledge and perfect competition prevails in their production.

As a result, the only generic produced (by the old quality leader or other followers) will be the one-quality-step-behind product j - 1. This assumption prevents the old quality leader from earning positive economic profits by exploiting those consumers who are not fully informed about the quality-adjusted price of the newly introduced product.[11]

When the state-of-the art quality product is introduced in the market, the product instantaneously captures a small market share $s_0 > 0$, and consumers start migrating gradually from the generic product to the state-of-the-art quality product. In the presence of population growth, $dN = [\partial N(t)/\partial t]dt$ is the number of consumers born between t and $t + dt$. When a challenger discovers a new product, he charges a limit price which equals the size of the quality improvement times the marginal cost of the follower (i.e., $p(t) = \lambda w a_Q$). In principle, fully informed consumers are indifferent between the two products and the typical assumption used in the literature is that all consumers $dN + N(t)$ switch instantaneously to the state-of-the art quality product. We replace this assumption by the following process: At the time of discovery, the new quality leader producing product j engages in limit pricing and its market share jumps instantaneously to $s_0 > 0$. This initial market share is the same in all industries and does not vary over time. It will be treated as a parameter. One could think of $s_0 = s(\tau, \tau)$ as the exogenous fraction of fully informed consumers in the economy who based their product-choice decision on quality adjusted prices. The market share of generic product $j - 1$ becomes $1 - s_0 = 1 - s(\tau, \tau)$, and the demand for product j – 2 jumps down to zero. Over time, the market share of each quality leader $s(t, \tau)$ increases gradually until the discovery of the next higher quality product, and the market share of the lower-quality (generic) product $1 - s(t, \tau)$ declines. New consumers, dN, are uniformly distributed between the two products. We assume that $s(t, \tau)dN$ buy the new product and $(1 - s(t, \tau))dN$ consumers buy the generic product.

We assume that the leader's market share evolves over time according to the following exponential deterministic process

$$s(t, \tau) = 1 - (1 - s_0)e^{-\delta(t-\tau)} \tag{10}$$

where $s(\tau, \tau) = s_0$ is the initial market share and $\delta > 0$ is a parameter that captures the speed of product adoption. Notice that as $t \to \infty$ the market share approaches unity. Earlier quality-ladders growth models assumed that $\delta \to \infty$ which implies instantaneous product replacement.

A few remarks about the proposed product replacement mechanism are in order. Equation (10) represents a very rough approximation of the S-curve diffusion process. It partitions the diffusion process in two parts: The business literature on technology adoption assumes that a small segment of the market consists of early adopters who are well informed about the quality of the new product and quickly switch to the newest products which have the lowest quality-adjusted price.[12] We assume that this segment of population adopts the new product

[11] We would like to thank an anonymous referee for helping us clarify this point.
[12] Implicit in the business literature is the idea that these early adopters get extra utility from being first. See Moore et al. (1999) for a description of the technology adoption model and its implications for the valuation of high-tech leaders. This extra utility is not a feature of our model. Indeed in our model (and all other models of Schumpeterian growth) consumers are indifferent toward adopting the new technology because the new quality leader engages in limit pricing and charges a price that exactly captures the extra utility from the new innovation. After the initial

instantaneously. The assumption of instantaneous product adoption by early adopters is captured in (10) by the initial market share s_0. Starting at the initial market share, equation (10) states that the market share of the leader is a concave function of time and therefore it captures the concave portion of an S-curve.[13] In other words, the consumer migration process is exogenous by assumption, and therefore it can not be affected (with the exception of its expected length) by other parameters. This strong assumption renders the analysis tractable and allows us to derive closed-form general- equilibrium solutions. One could make the diffusion process endogenous by assuming that the diffusion parameter is a decreasing function of the product's price (i.e., $\delta = \lambda\delta_0 / p$, where δ_0 is a positive constant): The higher is the product's price relative to the quality increment, the slower is the rate of market-share growth. This specification introduces a profit-maximizing price/age profile that is increasing in the firm's age. Young firms with low market shares would charge a lower price and enjoy lower price-cost margin in an effort to accelerate the growth of their market; and older firms experience lower market share growth and higher prices. However this extension would substantially decrease the tractability of the model.[14]

Equation (10) implies that

$$\dot{s}(t,\tau) = \delta(1 - s(t,\tau)). \tag{11}$$

That is, the rate at which market share increases depends only on the potential (untapped) market share. It also implies the leader's customers, $m = s(t,\tau)N(t)$, grow at the rate

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jump in market share, the rest of the consumers migrate to the new product gradually based on word of mouth, advertising, network-based externalities, or replacement dynamics of durable goods. The factors that determine the gradual migration process are not modeled in the present paper in order to maintain the model's analytical tractability.

[13] In other words, one can think of equation (10) as a truncated S-curve diffusion process in which the segment from time zero to the inflection point occurs simultaneously. This specification provides an alternative to the traditional way of modeling adoption of technology through a logistic (S-curve) deterministic function. Klepper and Simmons (1997, Table 7) provide annual observations for market shares associated with the adoption of balloon-cord and straight-side-rim technologies in automobile tire market. We used a non-linear regression analysis to estimate and compare an exponential diffusion process (described by (10)) to a logistic diffusion process. In the case of balloon cord tires the estimated coefficients are s_0 = -0.032 (standard error: 0.028) and δ = 0.226 (standard error: 0.010). The regression yielded an R squared of 0.98 and a DW statistic of 1.91 The logistic diffusion model yielded an s_0 = 0.12, a diffusion coefficient δ = 0.50 an R squared of 0.97 and a DW statistic of 0.69. Consequently, in the case of balloon cord tires the exponential diffusion model did slightly better than the logistic diffusion model. In the straight side rims case the opposite was true.

[14] In addition to its tractability, the proposed diffusion process can also be justified as follows: Assume that within each dynastic family the adoption of the new quality product occurs instantaneously (there is instantaneous transmission of information). Suppose that a fraction s_0 of population consists of informed families. Each dynastic family in the remaining population adopts the new product with an exogenous instantaneous probability δdt. Then equation (10) is the cumulative exponential distribution which shows the probability that a family adopts the new product in the time period $(t-\tau)$. The assumption that there is a continuum of identical dynastic families allow us to invoke the law of large numbers and claim that the fraction of population adopting the new product evolves deterministically over time and its evolution is governed by an exponential function given by (10).

$$\frac{\dot{m}}{m} = \frac{\partial sN / \partial t}{sN} = g_N + \delta\left(\frac{1}{s(t,\tau)} - 1\right) \tag{12}$$

At the time of discovery t, the leader starts with a low market share, s_0, and enjoys a high percentage growth of sales that converges to the rate of population growth as $s \to 1$. As long as s is less than unity, the growth rate of sales exceeds the rate of population growth.

3. Stock-Market Valuation of Intangible Asset Prices

There is a stock market in the economy that channels consumer savings to firms engaged in R&D. At each instant in time each challenger issues a flow of shares promising to pay the flow of monopoly profits if the firm wins the R&D race and zero otherwise. Therefore, at each instant in time there are two types of stocks in this economy. Stocks issued by quality leaders producing the state-of-the-art quality product in each industry and stocks issued by challengers engaged in R&D to replace the incumbent firm in each industry. The stock market value of the industry leader, $V(t,\tau)$, is the price of the leader's intangible assets and is based on a comparison between the riskless rate of return and the expected return of holding the stock of a technology leader. Since the financial risk is industry specific and there is a continuum of industries, consumers can hold a completely diversified portfolio of stocks and earn the riskless rate of return, which is equal to the market interest rate $r(t)$. In equilibrium, shareholders need to be indifferent between the two.

$$\pi(t,\tau)dt + (1 - I(t)dt)dV(t,\tau) - I(t)V(t,\tau)dt = r(t)V(t,\tau)dt \tag{13}$$

where $\pi(t,\tau)$ is the flow of monopoly profits defined in (9) and $I(t)$ is the intensity of the Poisson process that governs the arrival of innovations in each industry.

A stockholder holding the quality leader's stock over an infinitesimal interval *dt* receives dividends equal to the firm's current profits. With probability 1- $I(t)dt$, no challenger discovers the next higher quality product, the incumbent retains leadership and stockholders receive capital gains. With probability $I(t)dt$, a new leader emerges and the stock of the incumbent becomes worthless. The expected returns from holding the incumbent's stock must equal the return on bonds of equivalent value. Dividing by *dt*, and taking the limit as *dt* approaches zero yields

$$V(t,\tau) = \frac{\pi(t,\tau)}{r(t) + I(t) - \dot{V}(t,\tau)/V(t,\tau)} \tag{14}$$

where $\frac{\dot{V}(t,\tau)}{V(t,\tau)} = \frac{dV(t,\tau)}{dt}\frac{1}{V(t,\tau)} = \frac{\partial V(t,\tau)}{\partial t}\frac{1}{V(t,\tau)}$. Equation (14) states that the market valuation of intangible assets equals the expected flow of monopoly profits discounted appropriately. The discount factor equals the market interest rate plus the probability of default minus the growth rate of these intangible assets, which reflects the market growth rate.

Equation (14) is the main channel through which the parameters of the model affect the evolution of intangible asset prices. This equation arises in all quality ladder models of

Schumpeterian growth and one can ask whether or not (14) is empirically relevant. Yardani and Quintana (2002) report a strong correlation between the ratio of expected operating earnings to the price index for the S&P 500 companies, using 12 month ahead consensus earnings estimates, which corresponds to the ratio $\pi(t, \tau) / V(t, \tau)$ using our notation, and the 10-year Treasury bond yield (i.e., $r(t)$). The average spread between the two is 29 basis points for the period 1974- July 2002. This strong correlation was used by the US Federal Reserve Bank to assess the extent of overvaluation of stock prices in 1997. Yardani and Quintana refer to equation $\pi(t, \tau) / V(t, \tau) = r(t)$ as the "Fed's Stock Valuation Model". According to these authors equation (14) corresponds closely to a new improved version of the Fed's Stock Valuation Model because it incorporates the risk of default and accounts for earnings growth. The authors use information on earnings forecasts, long-term Treasury yields, corporate bond yields, and growth of earnings forecasts to estimate the right-hand-side of equation (14) and calculate the "fair value" of an intangible asset.[15]

The empirical relevance of (14) does not mean that all predictions of the model enjoy empirical support. Rather the above discussion and evidence serve as a motivation to proceed with the analysis. Notice that all variables in (14) are endogenous and are related to the model's parameters (i.e., economic fundamentals) in a complex fashion. We proceed by solving the dynamic general-equilibrium model in order to unravel the long-run relationship between the economic fundamentals and the behavior of intangible-asset prices. The long-run behavior of intangible assets will hopefully shed light to the question of whether asset prices have "escalated" due to "irrational exuberance", or whether the economic fundamentals are consistent with relatively high asset prices.

Maximizing the flow of profits $\pi(t, \tau)$ with respect to price and taking into account that goods are perfect substitutes adjusted for quality, one obtains the standard limit-price equilibrium condition

$$p = \lambda w \alpha_Q \tag{15}$$

In other words, Betrand competition between the technology leader and the followers generates a price for the state-of-the-art quality product which is λ times the marginal costs of the generic producers $w\alpha_Q$. The latter charge a price equal to $w\alpha_Q$, earn zero economic profits and have stock market values of zero.

Each challenger maximizes the expected discounted profits of engaging in R&D:

$$V(t,t)I_j(t)dt - w\alpha_R R_j(t)dt$$

where $V(t,t)$ is the stock market valuation of monopoly profits at the time of discovery (i.e., the market value of a start up company manufacturing the state-of-the-art quality product with a market share s_0), $I_j(t)dt$ is the instantaneous probability of discovering the next higher-quality product given by (6), and $w\alpha_R R_j(t)dt$ is the total labor costs of producing $R_j(t)$ R&D services.

[15] According to Yardani and Quitana (2002) the simple version of (14) has worked well historically because the long-term growth component in the denominator of (14) has been offset on average by the risk variable in the corporate bond market!

We assume that there is free-entry into each R&D race, which implies that the market value of a start up company $V_0(t)$ is given by

$$V_0(t) \equiv V(t,t) = w\alpha_R X(t) . \tag{16}$$

4. Resource Conditions

We assume that workers can move instantaneously between activities within each industry and across industries. Since prices and wages are flexible the aggregate demand for labor equals its supply at each instant in time and full employment of labor prevails. In other words, at each instant in time, the given labor force, $N(t)$, is split between three uses: manufacturing by quality leaders, manufacturing by followers, and R & D for quality improvements. The economy's full employment condition for labor is

$$N(t) = \left(\frac{1}{\lambda}\right)\int_0^1 \frac{c(t)N(t)}{w} s(t,\tau_\omega)d\omega + \int_0^1 \frac{c(t)N(t)}{w}(1-s(t,\tau_\omega))d\omega + \alpha_R \int_0^1 R_\omega(t)d\omega \tag{17}$$

where the limit–pricing condition (15) has been used.

Consider the demand for labor by a quality leader in industry ω. This firm hires $a_Q Q(t)s(t,\tau_\omega)$ workers, where Q(t) is the industry demand for output (see (4)), $s(t,\tau_\omega)$ is its market share and α_Q is the unit-labor requirement in manufacturing. Substituting $Q(t)$ from (4), using the limit-pricing condition(15), and integrating across all industries yields the first term on the right-hand-side (RHS) of (17). The second term on the RHS is the demand for manufacturing labor employed by the quality followers. It is derived in the same way as the first term with two modifications. These firms enjoy a market share equal to 1 - $s(t, \tau_\omega)$ and charge a price $p = \alpha_Q w$. The third term on the RHS of (17) equals the aggregate demand for workers engaged in R&D, where $R_\omega(t)$ is the amount of R&D services in industry ω and α_Q is the constant unit-labor requirement in R&D. Equation (17) completes the description of the model's equations.

5. Steady-State Equilibrium

The model has a unique steady-state equilibrium in which several variables of interest are constant over time or grow at constant rates. In what follows, it is convenient to analyze the steady state behavior of four variables: $v_0 \equiv V(t,t)/N(t)$ is the per-capita stock-market value of a quality leader with market share s_0, $x = x(t)/N(t)$ is the per-capita difficulty of R&D, c is per capita consumption, and I is the aggregate innovation rate which is the intensity of the Poisson process that governs the arrival of innovations in each industry. We also choose the wage of labor as a numeraire: $w = 1$. In the steady-state equilibrium, both I and c are constant over time and therefore (5) implies $r(t)=\rho$. The solution to equation (14) is then

$$V(t,\tau) = \int_t^\infty \left\{\left[\frac{(\lambda-1)c(v)N(v)}{\lambda}\right]\left[1-(1-s_0)e^{-\delta(v-\tau)}\right]\right\} e^{-(\rho+I)(v-t)}dv \ , \tag{18}$$

Performing the integration, evaluating the above expression at $t = \tau$, and dividing by $N(t)$ yields

$$v_0 = V(t,t)/N(t) = c\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{1}{\varphi}-\frac{(1-s_0)}{\delta+\varphi}\right\} = c\left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{\delta+\varphi s_0}{\varphi(\delta+\varphi)}\right\}, \tag{19}$$

where $\varphi = \rho + I - g_N$ is the effective discount rate.

Expression (19) gives the per-capita value of a manufacturing start up in terms of discounted current and expected future earnings. Rewriting the R&D condition (16) and using (6) yields another expression that relates v_0 to the per-capita marginal costs of R&D.

$$v_0 = V(t,t)/N(t) = \alpha_R x \tag{20}$$

The full-employment of labor condition can be written as

$$N(t) = \left(\frac{cN(t)}{\lambda}\right)\int_0^1 s(t,\tau_\omega)d\omega + cN(t)\int_0^1 (1-s(t,\tau_\omega))d\omega + \alpha_R R(t).$$

This condition is equivalent to

$$1 = c + \bar{s}\left(\frac{1}{\lambda}-1\right)c + \alpha_R \frac{R(t)}{N(t)},$$

where $\bar{s} = 1-(1-s_0)I/(\delta+I) = (\delta+s_0 I)/(\delta+I)$ denotes the average steady-state market share. In other words, $\bar{s} = \int_0^1 s(t,\tau_\omega)d\omega = \int_0^\infty [1-(1-s_0)^{-\delta z}]Ie^{-Iz}dz$.

Substitute $R(t) = X(t)I(t)$ (see equation (7)) and the expression for $\bar{s}$ into the full-employment condition to obtain

$$1 = \left[1-\left(\frac{\lambda-1}{\lambda}\right)\left(\frac{\delta+s_0 I}{\delta+I}\right)\right]c + \alpha_R I x. \tag{21}$$

This equation is the resource condition and provides another equation in v_0, I, x, and c. These four endogenous variables are determined by the system of four equations (8), (19), (20), and (21).

The unique steady-state equilibrium is associated with constant growth in each consumer's utility over time. Because goods are perfect substitutes, each consumer is indifferent between consuming the state-of-the-art quality product and the product offered by followers. Therefore one can substitute for consumer demand $c(t)/\lambda$ into static utility (2) and obtain the following expression for consumer's instantaneous utility at time t

$$\log u(t) = \log c(t) - \log \lambda + It \log \lambda \tag{22}$$

Differentiating this expression with respect to time and noticing that per-capita consumption expenditure is constant in the steady-state equilibrium yields the standard expression for long-run growth

$$g_u \equiv \frac{\dot{u}(t)}{u(t)} = I \ln \lambda \, . \tag{23}$$

According to (23), long-run Schumpeterian growth is the product of two terms, the rate of innovation I and the logarithm of the size of quality increments. This is a standard result in quality-ladders models of economic growth. Every time an innovation occurs, the instantaneous utility jumps by $\ln \lambda$. During a time interval dt, the expected number of innovations in a particular industry is given by the intensity of the Poisson process I(t). Therefore if the economy had only one industry, the instantaneous expected growth of instantaneous utility would be given by the right-hand-side of (23). Since there is a continuum of structurally identical industries in the economy, the law of large numbers implies that the aggregate growth rate is deterministic and given by the above expression.

Next consider the determination of the long-run rate of innovation. From (8), the steady-state value of $x = X/N$ is given by

$$\frac{\dot{x}}{x} = \frac{\dot{X}}{X} - \frac{\dot{N}}{N} = \mu I - g_N \, . \tag{24}$$

Since x is constant in the steady-state equilibrium, $\dot{x} = 0$ and equation (24) implies

$$I = \frac{g_N}{\mu} \, . \tag{25}$$

Equation (25) states that the long-run rate of innovation and growth is proportional to the exogenous rate of population growth and inversely proportional to the technological parameter that determines the marginal contribution of the rate of innovation to the growth rate of R&D difficulty. Both of these parameters are invariant to standard policy changes, and therefore long-run growth is in effect exogenous.

Substituting (25) into the definition of φ yields an exogenous effective discount rate

$$\varphi = \rho + \frac{g_N}{\mu} - g_N \tag{26}$$

The system of equations (19), (20), and (21) can be written as

$$v_0 = \left(\frac{\lambda - 1}{\lambda}\right) \left\{ \frac{\delta + \varphi s_0}{\varphi(\delta + \varphi)} \right\} c \tag{27}$$

$$v_0 = \alpha_R x \tag{28}$$

$$1 = \left[1 - \left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{\delta + s_0(g_N/\mu)}{\delta + (g_N/\mu)}\right)\right]c + \alpha_R\left(\frac{g_N}{\mu}\right)x \tag{29}$$

This system is linear in the endogenous variables and can yield the following closed-form solutions for the per-capita steady-state values of the stock value of a manufacturing start up v_0^*, R&D difficulty x* and consumption expenditure c*.

$$v_0^* = \frac{1}{A} \tag{30}$$

$$x^* = \frac{1}{a_R A} \tag{31}$$

$$c^* = \frac{1}{A\left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{\delta + s_0\varphi}{\varphi(\delta + \varphi)}\right)}, \tag{32}$$

where A depends on virtually all parameters of the model and is given by

$$A = \left\{\frac{\lambda}{\lambda - 1} - \left(\frac{\delta + s_0(g_N/\mu)}{\delta + (g_N/\mu)}\right)\right\}\left(\frac{\varphi(\delta + \varphi)}{\delta + s_0\varphi}\right) + \frac{g_N}{\mu}. \tag{33}$$

Parameter A is always positive because the first term in curly brackets exceeds unity and the second term is less than unity. The following proposition summarizes the steady-state properties of the model.

Proposition 1: T*here exists a unique steady-state equilibrium with the following characteristics:*

a. *The rate of innovation I, per-capita difficulty of R&D x, per-capita market value of a manufacturing start-up v_0, per-capita consumption expenditure c, wage of labor w, and per-capita R&D investment R(t)/N(t) are all constant over time.*
b. *The steady-state scale-invariant Schumpeterian growth g_U is exogenous, bounded, constant over time, and directly proportional to the growth rate of population g_N.*

Any policy that affects the steady-state value of per-capita R&D difficulty *x* results in transitional (temporary) changes in the innovation rate *I*. For instance, consider a parameter change that increases the steady-state value of *x,* say an increase in the magnitude of innovation, λ. Equation (24) implies that, during the transition, the rate of growth of *x* will be positive (i.e., $\dot{x} > 0$) and this can happen only if *I(t)* temporarily exceeds it long-run value g_N/μ. Consequently, the rate of innovation has to rise temporarily. A similar reasoning implies that a permanent decline in *x* is associated with a temporary decline in the rate of innovation. Of course, the effect of changes in x on transitional Schumpeterian growth depend not only on changes in the rate of innovation *I* but also on changes in the growth rate of per-capita consumption expenditure *c* (see equation (22)). These changes cannot be examined without analyzing the transitional dynamics of the model which are complicated (due to the introduction of gradual product replacement) and beyond the scope of this paper. Differentiating equations (30), (31) and taking into account (33) yields the following propositions:

Proposition 2: *The long-run per-capita R&D difficulty x, and the transitional rate of innovation I(t):*

a. *increase in the size of innovations λ, in the initial market share s_0, and in the rate of technology adoption δ;*
b. *decrease in the subjective discount rate ρ, and in the R&D unit-labor requirement α_R*

Proposition 3: *The long-run price of intangible asset associated with a manufacturing start up v_0:*

a. *increases in the size of innovations λ, in the initial market share s_0, and in the rate of technology adoption δ;*
b. *decreases in the subjective discount rate ρ, and is independent of the R&D unit-labor requirement α_R.*

Propositions 2 and 3 reveal the general equilibrium links among intangible-asset prices, total factor productivity growth, and the degree of new product diffusion. Economies with faster product-replacement rates (i.e., higher δ and/or higher s_0) enjoy higher long-run per-capita asset prices and higher transitional innovation rates. The same holds for economies experiencing larger innovations (higher λ). Economies with higher subjective discount rates (higher r(t) = ρ) that discount the future more end up with lower per-capita intangible-asset prices.

6. Long-run Intangible-Asset-Price Dynamics.

The steady-state equilibrium has several novel properties. In each industry new products are discovered through R&D races, old products are replaced gradually by higher quality ones, and firms are born and die. Temporary monopoly profits fuel innovation in a highly uncertain environment. The arrival of innovations in each industry is governed by a Poisson process with intensity equal to the rate of innovation. The duration of temporary monopoly power associated with each newly discovered product is random and exponentially distributed. R&D resources and manufacturing output measured in units of labor increase exponentially over time at the rate of population growth although the rate of growth of per-capita utility remains constant over time. Consequently the model captures the essence of Schumpeter's (1942) process of creative destruction.

Financial markets play a pivotal role in this economy: They channel consumer savings into firms engaged in R&D investment, diversify industry-specific risk associated with the introduction of new products and determine intangible-asset prices. The latter are associated with the valuation of monopoly profits earned by firms producing the state-of-the-art quality product in each industry and distributed back to stockholders as dividends. Like other Schumpeterian growth models, all agents have rational expectations, each consumer maximizes her discounted lifetime utility, firms maximize expected discounted profits, all prices are flexible and all markets clear instantaneously. However, unlike other Schumpeterian growth models, the introduction of a gradual (as opposed to an instantaneous) product replacement mechanism gives rise to nontrivial long-run dynamics for intangible asset prices. We explore these properties by focusing on the steady-state valuation of monopoly profits (i.e., intangible asset prices) and on the price earnings (P/E) ratio. In order to facilitate the economic intuition or the results, we would like to introduce additional notation. Denote with $T = t - \tau$ the age of a typical quality leader; and using equation (10), denote with $\zeta(s_0, \delta, T) = (1 - s_0)e^{-\delta T}$ the potential increase in market share of a quality leader, whose age is T.

Performing the integration of equation (18) and substituting (32) yields a general-equilibrium expression for the steady-state value of an intangible asset:

$$
\begin{aligned}
V(T,t) &= \frac{(\lambda-1)}{\lambda}\left[\frac{1}{\varphi} - \frac{\zeta(T)}{\delta+\varphi}\right] c^* N(t) \\
&= \left[\frac{1}{\varphi} - \frac{\zeta(T)}{\delta+\varphi}\right]\left[\frac{1}{\varphi} - \frac{\zeta(0)}{\delta+\varphi}\right]^{-1} \frac{N(t)}{A},
\end{aligned}
\tag{34}
$$

where parameter A is given by equation (33) and depends on virtually all exogenous parameters of the model. It is obvious that the steady-state value of a typical intangible asset increases over time as the size of the economy (measured by the level of population) expands; and the per-capita value of an intangible asset ($v(T) = V(T,t)/N(t)$) is an increasing and concave function of the asset's age T. In addition, an increase in the magnitude of innovations λ decreases the value of A and shifts the age profile of a typical asset upward. The dependence of the age profile on other parameters of interest is ambiguous.

Next we want to analyze the long-run properties of a popular ratio used routinely by financial analysts, the price-earnings (P/E) ratio of an intangible asset. Dividing equation (34) by the earnings of a quality leader $\pi^* = \left[(\lambda-1)(1-\zeta(T))c^* N(t)\right]/\lambda$ and using (10) yields a general-equilibrium expression for the P/E ratio of a typical firm:

$$
\frac{P}{E} = \frac{\left[1 - \frac{\varphi}{(\delta+\varphi)}\zeta(T)\right]}{\varphi\left[1-\zeta(T)\right]}. \tag{35}
$$

Consider now the shape of the age profile of the P/E ratio, and its dependence on the other parameters of the model. The expression in the numerator of (35) is a weighted average of current and expected future market share. Earnings, in the denominator, depend on current market share only. Notice that as the age of the firm T increases both of these terms increase, but the current market share in the denominator increases faster because $\varphi/(\delta+\varphi) < 1$. Consequently the P/E ratio of a manufacturing start up is high and declines monotonically overtime. The upper limit of the P/E ratio is obtained by evaluating (35) at T = 0 and yields

$(P/E)(0) = [\delta + s_0\,\varphi]/[\,s_0(\delta + \varphi\,)\varphi]$. As the firm's age approaches infinity its P/E ratio approaches (P/E) (∞) = $1/\varphi$, the inverse of the effective discount rate. Moreover, the potential increase in market share is a monotonic and decreasing function of time and this means that younger firms will have higher P/E ratios than older firms. Differentiating equation (35) reveals the dependence of the P/E age profile on the model's parameters. These properties are summarized in the following proposition:

Proposition 4: *The long-run price-earnings (P/E) ratio:*

a. *is a monotonic, decreasing, convex and bounded function of a firm's age T.*
b. *For any given T, the P/E ratio decreases in the subjective discount rate ρ and in the initial market share s_0;*
c. *The dependence of the P/E on the rate of growth of population g_N and the diffusion parameter δ is ambiguous.*

The age profile of the P/E ratio inherits the properties of the potential increase in market share which is a decreasing and convex function of time. For any given T, a higher discount rate means price decreases, hence the P/E ratio decreases. For any given T, a higher initial market share means a smaller potential increase in market share, hence the P/E ratio declines. The economic intuition for part (c) of proposition 4 requires more discussion. The P/E ratio is discounted future earnings divided by current earnings. Hence an increase in the effective discount rate φ decreases the P/E age profile by reducing discounted future earnings. However, an increase in the rate of growth of population has an ambiguous effect on the effective discount rate: On the one hand, a higher population growth rate generates the traditional "market growth effect", that is a faster increase in the rate of market expansion, measured by the population level, reduces the effective discount rate; on the other hand, since the long-run risk of default $I = g_N / \mu$ is proportional to the rate of population growth, an increase in g_N increases the risk of default, that is the "creative destruction" effect and increases the effective discount rate. Iff $\mu < 1$, then the creative destruction effect dominates and a higher level of population growth decreases the P/E age profile. The economic intuition for the ambiguous effect of the market-share growth parameter δ on the P/E ratio is that an increase in δ decreases the potential increase in market share but increases the speed at which that remaining market share is acquired. An increase in δ reduces the potential increase in market share $\zeta(T;\delta)$ in the same way as an increase in the asset's age, and therefore it generates a downward shift in the P/E age profile for a given weight $\varphi/(\delta+\varphi)$. However, an increase in δ reduces the weight attached to the potential market share in the numerator of equation (35) which reflects the speed at which the remaining market share is acquired and generates an upward shift in the age profile rendering the overall effect ambiguous.

Further interpretation of the P/E ratio is aided by looking at the composition of stock returns over time. Differentiating equation (34) with respect to time, substituting into (14), and using the definitions of the flow of profits and $V(t,T) = v(\mathrm{T})N(t)$, one can derive the following no-market arbitrage expression which holds in the steady-state equilibrium:

$$\frac{(\lambda-1)}{\lambda}\frac{[1-\zeta(T)]}{v(T)} + g_N + \delta\left[\frac{(\delta+\varphi)}{\varphi[1-\zeta(T)]} - 1\right]^{-1} - I = \rho\,, \qquad (36)$$

The first term in the left-hand-side of equation (36) is the per-dollar dividend $\pi(t, T)/V(t, T)$, offered by a quality leader at time t, which equals the inverse of the intangible asset's P/E ratio.[16] The next three terms reflect expected capital gains due to population growth, expected capital gains due to increased market share, and expected capital losses due to the possible loss of leadership to challengers. The right- hand-side of equation (36) is the personal discount rate, which equals the market interest rate. Equation (36) states that the expected rate of return to a stock issued by a quality leader is equal to the personal discount rate at each instant in time (see discussion associated with equations (13) and (15)). As the age of a typical quality leader increases the per-dollar dividend increases, but the per dollar expected capital gains declines by the same amount to ensure that the no market arbitrage condition holds at each instance in time. Similar considerations apply to the price sales ratio of a typical quality leader.

In models of Schumpeterian growth with instantaneous product replacement the left-hand side of equation (36) is independent of the age of a quality leader. In those models, the dividend rate and the capital-gains rate are constant over time in the steady-state equilibrium. In the present model, however, these terms vary with the age of the quality leader although their sum is constant over time. More specifically, a quality leader starts with a small market share, a small rate of profits, and a small dividend rate. This firm will enjoy a high rate of capital gains and high P/E ratio. In other words, new firms with low present market shares and high potential market share have all the characteristics of firms populating the new economy. Over time, the quality leader enjoys a higher market share and a higher flow of profits, the dividend component of the expected rate of return increases, and the P/E ratio declines. Equation (36) implies that the capital-gains component decreases over time to ensure that the total expected rate of return remains the same. In other words, loosely speaking, the stock-market value of a quality leader starts in a "growth and equity" category and ends up in the "growth and income" category where the firm is bigger, older, has a relatively low P/E ratio, and its dividend rate is the largest component of its expected return.

7. Concluding Remarks

The present paper developed a dynamic-general equilibrium model of Schumpeterian growth without scale effects and with gradual (as opposed to instantaneous) product replacement. Gradual product replacement is modeled by assuming that a firm that discovers the state-of-the-art quality product starts with an exogenous market share that grows gradually overtime following an exponential deterministic process. The gradual evolution of the market share introduces nontrivial asset price dynamics that are absent from previous models of Schumpeterian growth.

The parameters of the gradual product replacement mechanism-the initial market share and the rate of market-share acquisition- affect the long-run values of intangible asset prices, per-capita consumption expenditure, and have an impact on temporary acceleration of the rate of innovation and growth. Intangible assets are associated with the flow of temporary monopoly profits generated by the endogenous introduction of higher-quality products. In addition, unlike other growth models, the present one generates interesting long-run age profiles of price-earning and price-sales ratios, dividend rates and intangible-asset appreciation rates. The total expected rate of return associated with an intangible asset is constant over time and equals the personal

[16] The per-dollar dividend equals the earnings yield in the present model.

discount rate. This rate of return can be decomposed into the dividend rate (which equals the inverse of the P/E ratio in the present model) and the expected capital gains rate (which reflects the expected capital gains due to population growth, the expected capital gains due to increased market share, and the expected capital losses due to the possible loss of leadership to challengers). Young firms start with high P/E ratios, low dividend rates, low market shares, and high expected rates of appreciation. Over time, as the market share of a quality leader increases, its P/E ratio falls, its dividend rate rises, and the capital gains component of its total rate of return declines. In other words, the introduction of a gradual product replacement mechanism allows the standard quality-ladders growth model to capture some of the basic features of the "new" economy.

Of course, the model's properties and predictions depend on several restrictive assumptions, which simplified the analysis at the expense of empirical validity. Introducing gradual adoption of a general purpose technology would make the model a more realistic, but would not allow the existence of steady-state equilibrium.[17] Introduction of physical capital accumulation would permit to study the differences between tangible and intangible prices and the evolution of Tobin's q ratio. This type of inquiry would complement the prominent work of Laitner and Stolyarov (2003, 2004). Using alternative specifications for the evolution of R&D difficulty over time would create a richer and more complex picture of the relationship between long-run growth and intangible asset prices. The model could also be enriched by incorporating the role of venture capital and entrepreneurship following the pioneering work of King and Levine (1993a, 1993b).

Finally, the assumption of an exogenous initial market share and an exogenous rate of diffusion mask several fascinating aspects related to the evolution of market structure: Network externalities, heterogeneous consumers and firms, S-curve diffusion dynamics, industrial shake outs, R&D strategies associated with first-mover advantages, incumbent firm's responses to market-share erosion, and asymmetric information considerations are important elements that could in principle be incorporated into the model and increase its empirical relevance. However, we suspect that these important additions will increase the complexity and tractability of the model and might obscure the intuition of some findings. These issues represent fruitful avenues of future research.

[17] Petsas (2003) has developed a model of scale-invariant Schumpeterian growth with gradual diffusion of a general-purpose technology across a continuum of industries.

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