Granger Causality and Dynamic Structural Systems

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Objective

Relate Granger causality to a notion of structural causality

- Granger (G) causality

(Granger, 1969 and Granger and Newbold, 1986)

- Structural causality

(White and Chalak, 2007 and White and Kennedy, 2008)

Outline

1. Define G non-causality and structural non-causality
2. Relation between (retrospective, weak) G non-causality and structural non-causality
3. Testing (retrospective) weak G non-causality
4. Testing (retrospective) conditional exogeneity and structural non-causality
5. Applications
6. Conclusions

1. Definitions of G non-causality and structural non-causality

Granger Causality

- Let $\mathbb{N}_0 := \{0, 1, 2, 3...\}$ and $\mathbb{N} := \{1, 2, 3...\}$.
- subscript$_t$ denotes a variable at time t.
- superscript^t denotes a variable's "t-history", (e.g., $X^t = \{X_0, X_1, ..., X_t\}$)

Definition: *Granger non-causality*
Let $\{D_t, S_t, Y_t\}$ be a sequence of random vectors. Suppose that

$$Y_{t+1} \perp D^t \mid Y^t, S^t \text{ for all } t \in \mathbb{N}_0$$

then we say D **does not** G**-cause** Y **with respect to** S.
Otherwise, we say D G**-causes** Y **with respect to** S.

Data Generating Process (DGP)

Assumption A.1(a) (White and Kennedy, 2008) Let V_0, W_0, D_0, Y_0 be random vectors and let $\{Z_t\}$ be a stochastic process. $\{V_t, W_t, D_t, Y_t\}$ is generated by the structural equations

$$\begin{aligned} V_{t+1} &= b_{0,t+1}(V^t, Z^t) \\ W_{t+1} &= b_{1,t+1}(W^t, V^t, Z^t) \\ D_{t+1} &= b_{2,t+1}(D^t, W^t, V^t, Z^t) \\ \boxed{Y_{t+1} \;=\; q_{t+1}(Y^t, D^t, V^t, Z^t)} & \qquad t = 0, 1, 2\ldots \end{aligned}$$

- Cause of interest: D^t. Response of interest: Y_{t+1}.
- $\{D_t, Y_t, W_t\}$ observable; some components of $\{Z_t, V_t\}$ unobservable
- Covariates: $X_t := \{W_t$, observable components of Z_t and $V_t\}$
- Unobservables: U_t
- $b_{0,t+1}, b_{1,t+1}, b_{2,t+1}, q_{t+1}$ unknown functions.

Alternative Data Generating Process (DGP)

Assumption A.1(a) (White and Kennedy, 2008) Let V_0, W_0, D_0, Y_0 be random vectors and let $\{Z_t\}$ be a stochastic process. $\{V_t, W_t, D_t, Y_t\}$ is generated by the structural equations

$$\begin{aligned} V_{t+1} &= b_{0,t+1}(V^t, Z^{t+1}) \\ W_{t+1} &= b_{1,t+1}(W^t, V^{t+1}, Z^{t+1}) \\ D_{t+1} &= b_{2,t+1}(D^t, W^{t+1}, V^{t+1}, Z^{t+1}) \\ \boxed{Y_{t+1} \;=\; q_{t+1}(Y^t, D^{t+1}, V^{t+1}, Z^{t+1})} & \qquad t = 0, 1, 2\ldots \end{aligned}$$

Structural Causality

- Implicit dynamic representation of the DGP:

$$\boxed{Y_{t+1} = r_{t+1}(Y_0, D^t, V^t, Z^t)} \quad t = 0, 1, 2, \ldots$$

- **Definition:** *Structural non-causality*

Suppose for given t and all y_0, v^t, and z^t, the function

$$d^t \rightarrow r_{t+1}(y_0, d^t, v^t, z^t)$$

is constant in d^t. Then we say D^t **does not structurally cause** Y_{t+1} and write $D^t \not\Rightarrow_{\mathcal{S}} Y_{t+1}$. Otherwise, we say D^t **structurally causes** Y_{t+1} and write $D^t \Rightarrow_{\mathcal{S}} Y_{t+1}$.

- **Example:** $Y_{t+1} = \beta_0 + Y_0\beta_1 + D^{t\prime}\beta_2 + V^{t\prime}\beta_3 + Z^{t\prime}\beta_4$

  $\beta_2 = 0$: structural non-causality
  $\beta_2 \neq 0$: structural causality

2. Relation between G non-causality and structural non-causality

Weak Granger Causality

- **Definition**: *Weak G non-causality*

Let $\{D_t, S_t, Y_t\}$ be a sequence of random vectors. Suppose that

$$Y_{t+1} \perp D^t \mid Y_0, S^t \text{ for all } t \in \mathbb{N}_0$$

then we say D does not **weakly** G-cause Y with respect to S. Otherwise, we say D **weakly** G-causes Y with respect to S.

Note: G non-causality says $Y_{t+1} \perp D^t \mid Y^t, S^t$ for all $t \in \mathbb{N}_0$

Conditional Exogeneity

- **Assumption A.2 (a)**

$$D^t \perp U^t | \; Y_0, X^t, \; t = 0, 1, 2, \dots \;.$$

We say D^t is **conditionally exogenous with respect to U^t given (Y_0, X^t)**, $t = 0, 1, 2, \dots$. For brevity, we just say D^t is **conditionally exogenous**.

Structural Non-causality and (Weak) G Non-causality

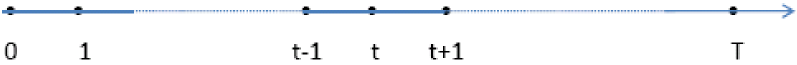
- **Proposition 1**

Suppose Assumption A.1(a) holds and that $D^t \not\Rightarrow_{\mathcal{S}} Y_{t+1}$ for all $t \in \mathbb{N}_0$. If Assumption A.2(a) also holds, then D does not (weakly) $G-$cause Y with respect to X.

- Structural non-causality and conditional exogeneity imply (weak) G non-causality

Retrospective Weak Granger Causality

- **Time line**



- **Definition**: *Retrospective weak G non-causality*

Let $\{D_t, S_t, Y_t\}$ be a sequence of random variables. For a given $T \in \mathbb{N}$, suppose that

$$Y_{t+1} \perp D^t | Y_0, S^T \text{ for all } 0 \leq t \leq T - 1$$

Then we say D does not **retrospectively** weakly G-cause Y with respect to S. Otherwise, we say D **retrospectively** weakly G-causes Y with respect to S.

Retrospective Conditional Exogeneity

- **Assumption A.2 (b)**

$$D^t \perp U^t | \ Y_0, X^T, \ t = 0, 1, 2, \ldots \ .$$

We say D^t is **retrospectively** conditionally exogenous with respect to U^t given (Y_0, X^T), $t = 0, 1, 2, \ldots$. For brevity, we just say D^t is **retrospectively** conditionally exogenous.

Structural Non-causality and Retrospective (Weak) G Non-causality

- **Proposition 2**

Suppose Assumption A.1(a) holds and that $D^t \not\Rightarrow_{\mathcal{S}} Y_{t+1}$ for all $t \in \mathbb{N}_0$. If Assumption A.2(b) also holds, then for the given T, D does not retrospectively (weakly) G cause Y with respect to X.

- Structural non-causality and retrospective conditional exogeneity imply retrospective (weak) G non-causality

Some Converse Results

- **Assumption A.3(a)** there exist measurable sets B_Y, B_0, B_D, and B_X such that:

(i)

$$P[Y_{t+1} \in B_Y, Y_0 \in B_0, D^t \in B_D, X^t \in B_X] > 0$$

(ii)

$$P[D^t \in B_D | Y_0 \in B_0, X^t \in B_X] < 1; \text{ and}$$

(iii) with

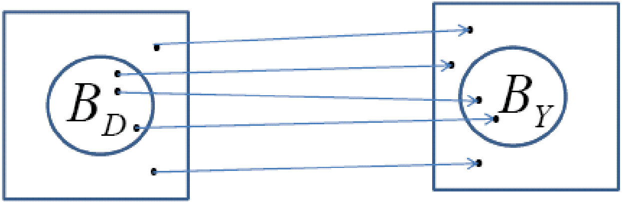
$$B_U(d^t, y_0, x^t) \equiv \text{ supp}(U^t \mid D^t = d^t, Y_0 = y_0, X^t = x^t),$$

for all $d^t \notin B_D$, $y_0 \in B_0$, and $x^t \in B_X$, and all $u^t \in B_U(d^t, y_0, x^t)$

$$r_{t+1}(y_0, d^t, v^t, z^t) \notin B_Y.$$

Some Converse Results (Cont'd)

- Intuition of A.3(a)



- Example of A.3(a):

$Y_{t+1} = D_t + U_t,\ D_t \sim N(0,1)\ ,\ U_t \sim \text{Uniform}(0,1)\ ;$
$B_D = (-\infty, 0) \cup (1, \infty)$ and $B_Y = (-\infty, 0) \cup (2, \infty)$.
$d_t \notin B_D$ means $d_t \in [0,1]$. For all $d_t \in [0,1]$ and $u_t \in (0,1)$,
$y_{t+1} = d_t + u_t \in (0,2)$, which is not contained in B_Y.

Some Converse Results (Cont'd)

- **Definition**: *Strong Causality*

Suppose A.1(a) and A.3(a) hold. Then we say that D^t **strongly causes** Y_{t+1}. Otherwise, we say that D^t **does not strongly cause** Y_{t+1}.

- **Proposition 3**

If D^t strongly causes Y_{t+1} for all t, then D weakly G-causes Y with respect to X.

Some Converse Results (Cont'd)

- Similarly, we can define Retrospective Strong Causality by replacing X^t with X^T in A.3(a).

- **Proposition 4**

If D^t retrospectively strongly causes Y_{t+1}, then D retrospectively weakly G-causes Y with respect to X.

Summary of the relation between (retrospective) weak G causality and structural causality

- Under (retrospective) conditional exogeneity, structural non-causality implies (retrospective) weak G non-causality.

Conversely,

- (Retrospective) strong causality implies (retrospective) weak G causality.

3. Testing (retrospective) weak G non-causality

Testing (Retrospective) Weak G Non-causality

- Weak G non-causality:

$$Y_{t+1} \perp D^t | Y_0, X^t$$

- (Retrospective) weak G non-causality:

$$Y_{t+1} \perp D^t | Y_0, X^T$$

Testing (Retrospective) Weak G Non-causality (Cont'd)

- **Proposition 5**

(a) Under some conditional stationarity and memory assumptions, then

$$Y_{t+1} \perp D^t \mid Y_0, X^t \Leftrightarrow Y_{t+1} \perp D_t \mid Y_t, X_{t-\tau}^t$$

Notation: $X_{t-\tau}^t := (X_{t-\tau}, X_{t-\tau+1}, ..., X_t)$

(b) Under some conditional stationarity and memory assumptions, then

$$Y_{t+1} \perp D^t \mid Y_0, X^T \Leftrightarrow Y_{t+1} \perp D_t \mid Y_t, X_{t-\tau}^{t+\tau}$$

Notation: $X_{t-\tau}^{t+\tau} := (X_{t-\tau}, X_{t-\tau+1}, ..., X_t, X_{t+1}, X_{t+2,}..., X_{t+\tau})$

Flexible Parametric Tests of Conditional Independence

$$\text{Test}: \mathcal{Y} \perp \mathcal{D} \mid \mathcal{S}$$

- CI test Regression 1: testing conditional mean independence with linear conditional expectations

$$E(\mathcal{Y} \mid \mathcal{D}, \mathcal{S}) = \alpha + \mathcal{D}'\beta_0 + \mathcal{S}'\beta_1.$$

- CI test Regression 2: testing conditional mean independence with flexible conditional expectations

$$E(\mathcal{Y} \mid \mathcal{D}, \mathcal{S}) = \alpha + \mathcal{D}'\beta_0 + \mathcal{S}'\beta_1 + \sum_{j=1}^{q} \psi(\mathcal{S}'\gamma_j)\beta_{j+1}$$

- CI test Regression 3: testing conditional independence using non-linear transformations
  $\mathcal{Y} \perp \mathcal{D} \mid \mathcal{S} \Rightarrow \psi_y(\mathcal{Y}) \perp \psi_d(\mathcal{D}) \mid \mathcal{S}$

$$E(\psi_y(\mathcal{Y}) \mid \psi_d(\mathcal{D}), \mathcal{S}) = \alpha + \psi_d(\mathcal{D})'\beta_0 + \mathcal{S}'\beta_1 + \sum_{j=1}^{q} \psi(\mathcal{S}'\gamma_j)\beta_{j+1}.$$

4. Testing (retrospective) conditional exogeneity

(Retrospective) Conditional Exogeneity

- Conditional exogeneity:

$$U^t \perp D^t | Y_0, X^t$$

- Retrospective conditional exogeneity:

$$U^t \perp D^t | Y_0, X^T$$

Challenge: U_t is unobservable.

Resolution: Observe additional proxies for U_t, say $\tilde{W}_t$ – can use $\tilde{W}_t$ to test (retrospective) conditional exogeneity.

Testing Conditional Exogeneity

- **Assumption A.6** (a) $\tilde{W}_0$ is an observable random variable and $\{\tilde{U}_t\}$ is an unobservable stochastic process such that (i) $\{\tilde{W}_t\}$ is generated by the structural equations

$$\tilde{W}_{t+1} = b_{3,t+1}(\tilde{W}^t, X^t, U^t, \tilde{U}^t), \quad t = 0, 1, ...,$$

  where $b_{3,t+1}$ is an unknown measurable function; and (ii)

$$D^t \perp (\tilde{U}^t, \tilde{W}_0) \mid Y_0, U^t, X^t, \quad t = 1, 2, \dots .$$

- **Assumption A.7** (a) $(\tilde{W}_{t+1}, \tilde{W}_t) \perp (\tilde{W}_0, Y_0) \mid X^t$ for all $t = 1, 2, \dots$.

Testing Conditional Exogeneity (Cont'd)

- **Proposition 6**

Suppose Assumptions A.1(a), A.6(a), and A.7(a) hold. Then $D^t \perp U^t \mid Y_0, X^t$ for all $t \in \mathbb{N}$ implies $\tilde{W}_{t+1} \perp D^t \mid \tilde{W}_0, X^t$ for all $t \in \mathbb{N}_0$.

- **Proposition 7**

Under some conditional stationarity and memory assumptions,

$$\begin{aligned} \tilde{W}_{t+1} \quad &\perp \quad D^t \mid \tilde{W}_0, X^t \text{ for all } t \in \mathbb{N}_0 \\ &\Leftrightarrow \quad \tilde{W}_{t+1} \perp D_t \mid \tilde{W}_t, X_{t-\tau}^t \text{ for all } t \in \mathbb{N}_0 \end{aligned}$$

- Similarly, test Retrospective Conditional Exogeneity by replacing $X^t, X_{t-\tau}^t$ with $X^T, X_{t-\tau}^{t+\tau}$.

A Pure Test of Structural Non-causality

Reject structural non-causality if

- the (retrospective) (weak) G non-causality test *rejects;* and
- the (retrospective) conditional exogeneity test *fails to reject*.

Significance Level and Power of the Structural Non-causality Test

- Test levels
  α_1 : conditional exogeneity test
  α_2 : G non-causality test
  α : structural non-causality test
- Test powers
  π_1 : conditional exogeneity test
  π_2 : G non-causality test
  π : structural non-causality test

Proposition 8

$$\max\left\{0, \min\left\{(\alpha_2 - \alpha_1), (\pi_2 - \pi_1), (\alpha_2 - \pi_1)\right\}\right\} \leq$$
$$\alpha \leq \max\{\min\{1 - \alpha_1, \alpha_2\}, \min\{1 - \pi_1, \pi_2\}, \min\{1 - \pi_1, \alpha_2\}\}$$

$$\pi_2 - \alpha_1 \leq \pi \leq \min\left\{1 - \alpha_1, \pi_2\right\}.$$

Significance Level and Power of the Structural Non-causality Test (Cont'd)

- Test levels

  $\alpha_{1T} \longrightarrow \alpha_1$: conditional exogeneity test

  $\alpha_{2T} \longrightarrow \alpha_2$: G non-causality test

  α_T : structural non-causality test

- Test powers

  $\pi_{1T} \longrightarrow 1$: conditional exogeneity test

  $\pi_{2T} \longrightarrow 1$: G non-causality test

  π_T : structural non-causality test

Proposition 9

$$0 \leq \liminf \alpha_T \leq \limsup \alpha_T \leq \min\{1 - \alpha_1, \alpha_2\} \quad \text{and}$$

$$\pi_T \rightarrow 1 - \alpha_1.$$

5. Applications

Applications

- Crude oil prices and gasoline prices (White and Kennedy, 2008)
- Monetary policy and industrial production (Angrist and Kuersteiner, 2004)
- Economic announcements and stock returns (Flannery and Protopapadakis, 2002)

Crude Oil Prices and Gasoline Prices

- Y_t : the natural logarithm of the spot price for US Gulf Coast conventional gasoline
- D_t : the natural logarithm of the Cushing OK WTI spot crude oil price
- U_t : all unobservable drivers of gasoline prices
- Structure:

$$Y_t = r_t(Y_0, D^t, U^t), \quad t = 0, 1, ..., T.$$

- Note: "Contemporaneous" effects allowed.
- Sample period: January 1987-December 1997

Crude Oil Prices and Gasoline Prices (Cont'd)

- $X_t = W_t$:
  (1) natural logarithm of Texas Initial and Continuing Unemployment Claims
  (2) Houston temperature
  (3) winter dummy for January, February, and March
  (4) summer dummy for June, July, and August
  (5) natural logarithm of U.S. Bureau of Labor Statistics Electricity price index
  (6) 10-Year Treasury Note Constant Maturity Rate
  (7) 3-Month T-Bill Secondary Market Rate
  (8) Index of the Foreign Exchange Value of the Dollar
- $\tilde{W}_t$: natural logarithm of the U.S. Bureau of Labor Statistics Natural Gas Price Index.

Crude Oil Prices and Gasoline Prices (Cont'd)

- Test retrospective conditional exogeneity by testing

$$D_t \perp \tilde{W}_t \mid \tilde{W}_{t-1}, X_{t-\tau}^{t+\tau} \tag{1}$$

- Test retrospective weak G non-causality by testing

$$Y_t \perp D_t \mid Y_{t-1}, X_{t-\tau}^{t+\tau} \tag{2}$$

- Results:

1. Fail to reject (1) using CI test Regressions 1, 2, 3 for almost all the choices of τ $(\tau = 0, 1, ..., 5)$ and q $(q = 1, 2, ..., 5)$.
2. Reject (2) using CI test Regressions 1, 2, 3 for almost all the choices of τ $(\tau = 0, 1, ..., 5)$ and q $(q = 1, 2, ..., 5)$.

Crude Oil Prices and Gasoline Prices (Cont'd)

- Conclusions: reject the hypothesis of structural non-causality from crude oil prices to gasoline prices.
- Similar conclusions using non-retrospective conditional exogeneity and weak G non-causality tests.
- Conclusions not surprising – But they critically support subsequent inferences about effect magnitudes.

Conclusions

This paper

- Links G non-causality and a notion of structural non-causality
- Provides explicit guidance as to how to choose S so G non-causality gives structural insight
- Extends G non-causality to new *weak* and *retrospective weak* versions
- Provides new tests of (retrospective) weak G non-causality, (retrospective) conditional exogeneity, and structural non-causality