

Research Article

Granular Computing Classification Algorithms Based on Distance Measures between Granules from the View of Set

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Granular computing classification algorithms are proposed based on distance measures between two granules from the view of set. Firstly, granules are represented as the forms of hyperdiamond, hypersphere, hypercube, and hyperbox. Secondly, the distance measure between two granules is defined from the view of set, and the union operator between two granules is formed to obtain the granule set including the granules with different granularity. Thirdly the threshold of granularity determines the union between two granules and is used to form the granular computing classification algorithms based on distance measures (DGrC). The benchmark datasets in UCI Machine Learning Repository are used to verify the performance of DGrC, and experimental results show that DGrC improved the testing accuracies.

1. Introduction

Granular computing (GrC) is computing method based on the partition of problem space and is widely used in pattern recognition, information system, and so forth. Zadeh identified three fundamental concepts of the human cognition process, namely, granulation, organization, and causation [1, 2]. Granulation is a process that decomposes a universe into parts. Conversely, organization is a process that integrates parts into a universe by introducing operation between two granules. Causation involves the association of causes and effects. Information granules based on sets, fuzzy sets or relations, and fuzzy relations are computed in [3]. In general, the fuzzy inclusion measure is induced by granule and union granule, such as the positive valuation functions of granules that are used to form the fuzzy inclusion measure [4–6]. But there are some problems; for example, the fuzzy inclusion measure between two atomic granules is zero no matter how far between two atomic granules is. These studies enable us to map the complexities of the world around us into simple theories.

GrC based algebraic system is a frame computing paradigm that regards the set of objects as granule, and the union operator and meet operator are the two keys of GrC. The union operator and meet operator are related to the shapes

of granule. There are granules with different shapes, such as hypersphere granules, hypercube granules, hyperdiamond granules, and hyperbox granules.

The present work uses distance measure between granules with the same shapes from the view of set. A granule is represented as a vector, and the distance between granules is defined by the centers of granules and the granularities, such as the half length of hyperdiamond diagonal, the radii of hypersphere, the half length of hypercube side, and the length of hyperbox diagonal. The granular computing classification algorithms based on distance measure (DGrC) are proposed.

The rest of this paper is presented as follows. Granular computing classification algorithm based on distance measure is described in Section 2. Section 3 demonstrates the comparative experimental results on two-class and multiclass problems. Section 4 summarizes the contribution of our work and presents future work plans.

2. Granular Computing Classification Algorithm Based on Distance Measure

For the dataset $S = \{(x_i, y_i) \mid i = 1, 2, \dots, n\}$ in N -dimensional space, we construct granular computing classification algorithms (GrC) in terms of the following steps. Firstly, the

single point in S is represented as the atomic granules which are indivisible. Secondly, the distance between two granules is proposed based on the view of set. Thirdly, the distance and granularity determine the union process jointly. Finally, the granule set is obtained and used to predict the class of unknown datum.

2.1. Representation of Granule and Granularity. In reality, the shapes of granules are irregular, the distance between two granules is not easily measured, the union granule, and the meet granule are related to the shapes of granules. In order to study granular computing, the granule is represented as regular shapes, such as hyperdiamond, hypersphere, hypercube, and hyperbox, especially diamond, sphere, cube, and box in 2-dimensional space. These four shape granules are represented as follows.

- (1) Hyperdiamond granule is represented as a vector including the hyperdiamond's center and the half of diagonal length.
- (2) Hypersphere granule is represented as a vector including the center and the radii of the hypersphere.
- (3) Hypercube granule is represented as a vector including the center and the half of side length of the hypercube.
- (4) Hyperbox granule is represented as a vector including vectors induced the beginning points and the end points.

Granularity is the size of granule, such as the half of diagonal length of hyperdiamond granule, the radii of hypersphere granule, the half of side of hypercube granule, and the maximal diagonal of hyperbox. The granularity of granule G is represented as $g_r(G)$.

For hyperdiamond granule

$$g_r(G) = r, \quad (1a)$$

where r is the half of diagonal length of hyperdiamond granule.

For hypersphere granule G

$$g_r(G) = r, \quad (1b)$$

where r is the radii of hypersphere.

For hypercube granule

$$g_r(G) = r, \quad (1c)$$

where r is the half of side of hypercube.

The granularity of hyperbox granules G is defined as the distance between the beginning point and the end point. For hyperbox granule $G = (\mathbf{x}, \mathbf{y})$, the granularity is the distance

$$g_r = \|\mathbf{x} - \mathbf{y}\|_2. \quad (1d)$$

In Figure 1, $G_1 = (0.1, 0.2, 0.5)$ is hyperdiamond granule in R^2 space, whose center is $(0.1, 0.2)$ and granularity is 0.5. $G_2 = (0.1, 0.2, 0.5)$ is hypersphere granule with center

$(0.1, 0.2)$ and granularity 0.5. $G_3 = (0.1, 0.2, 0.5)$ is hypercube granule with center $(0.1, 0.2)$ and granularity 0.5. $G_4 = [0.1 \ 0.2 \ 0.3 \ 0.5 \ 0.3606]$ is hyperbox granule with the beginning point $(0.1, 0.2)$, the end point $(0.3, 0.5)$, and the granularity 0.3606. These granules are shown in Figures 1(a), 1(b), 1(c), and 1(d). From Figure 1, we can see that different shape granules have different shapes even if they have the same forms of representations.

2.2. Distance Measure between Granules. The distance between granules refers to the minimal distance between two points which belong to different granules.

For two hyperdiamond granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the distance is

$$d(G_1, G_2) = \|C_1 - C_2\|_1 - r_1 - r_2, \quad (2)$$

where $C_1 = (x_1, x_2, \dots, x_N)$ and $C_2 = (y_1, y_2, \dots, y_N)$ are the centers of hyperdiamond granules G_1 and G_2 and r_1 and r_2 are granularities of hyperdiamond granules G_1 and G_2

$$\|C_1 - C_2\|_1 = |x_1 - y_1| + |x_2 - y_2| + \dots + |x_N - y_N|. \quad (3)$$

For two hypersphere granule $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the distance is

$$d(G_1, G_2) = \|C_1 - C_2\|_2 - r_1 - r_2, \quad (4)$$

where $C_1 = (x_1, x_2, \dots, x_N)$ and $C_2 = (y_1, y_2, \dots, y_N)$ are the centers of hypersphere granules G_1 and G_2 and r_1 and r_2 are granularities of hypersphere granules G_1 and G_2

$$\|C_1 - C_2\|_2 = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_N - y_N)^2}. \quad (5)$$

For hypercube granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the distance is

$$d(G_1, G_2) = \|C_1 - C_2\|_\infty - r_1 - r_2, \quad (6)$$

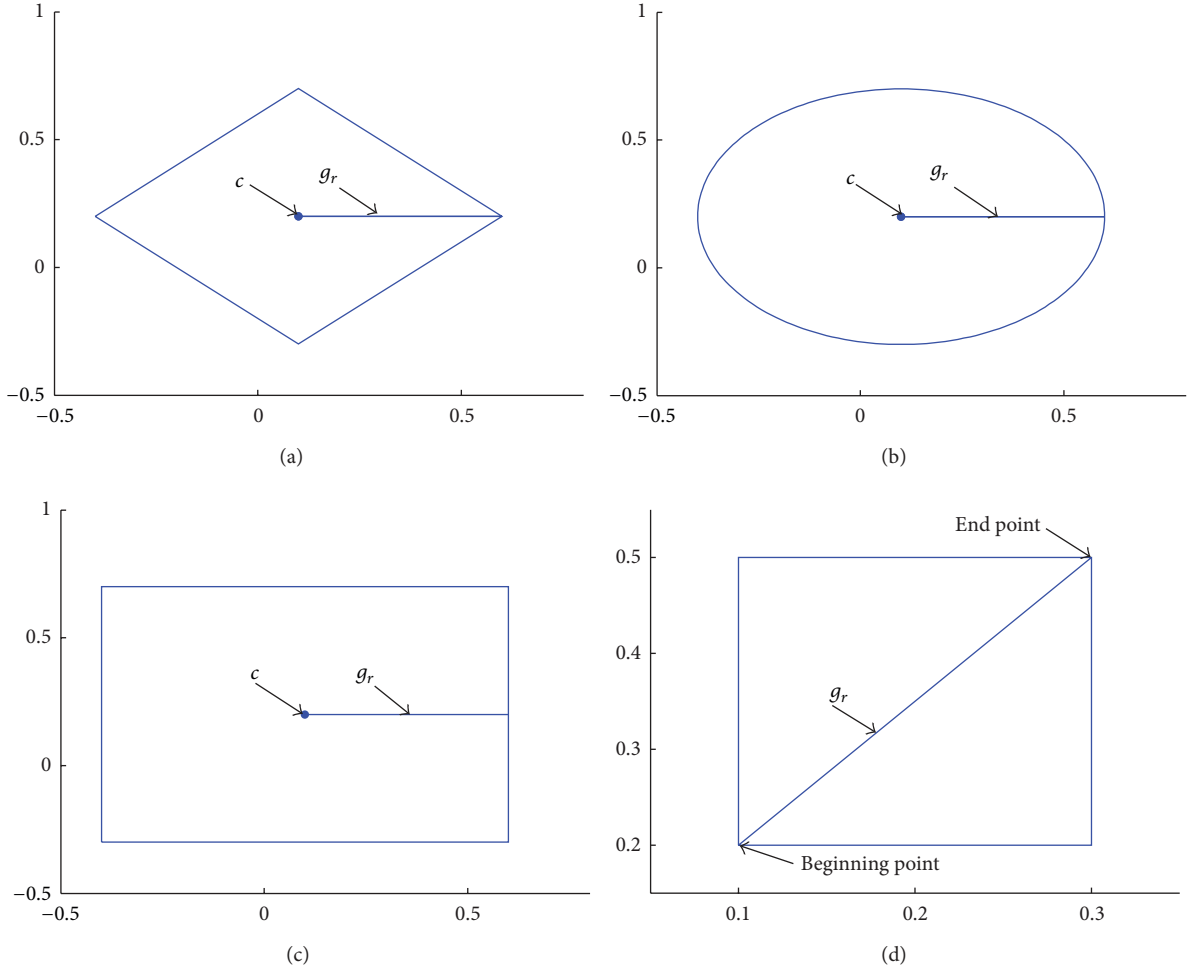
where $C_1 = (x_1, x_2, \dots, x_N)$ and $C_2 = (y_1, y_2, \dots, y_N)$ are the centers of hypercube granules G_1 and G_2 and r_1 and r_2 are granularities of hypercube granules G_1 and G_2

$$\|C_1 - C_2\|_\infty = \max\{|x_1 - y_1|, |x_2 - y_2|, \dots, |x_N - y_N|\}. \quad (7)$$

For two hyperbox $G_1 = (\mathbf{x}_1, \mathbf{y}_1)$ and $G_2 = (\mathbf{x}_2, \mathbf{y}_2)$, the distance is

$$\begin{aligned} d(G_1, G_2) &= |\max\{x_{11}, x_{21}\} - \min\{y_{11}, y_{21}\}| \times \dots \times |\max\{x_{1N}, x_{2N}\} \\ &\quad - \min\{y_{1N}, y_{2N}\}|, \end{aligned} \quad (8)$$

where $\mathbf{x}_1 = (x_{11}, x_{12}, \dots, x_{1N})$ and $\mathbf{x}_2 = (x_{21}, x_{22}, \dots, x_{2N})$ are the beginning points of hyperbox granules G_1 and G_2 and $\mathbf{y}_1 = (y_{11}, y_{12}, \dots, y_{1N})$ and $\mathbf{y}_2 = (y_{21}, y_{22}, \dots, y_{2N})$ are the end points of hyperbox granules G_1 and G_2 .

FIGURE 1: Granules with different shapes in R^2 space.

$\mathbf{x} = \mathbf{x}_1 \vee \mathbf{x}_2$, $\mathbf{y} = \mathbf{y}_1 \wedge \mathbf{y}_2$ are operators between two vectors and defined as

$$\begin{aligned} \mathbf{x}_1 \vee \mathbf{x}_2 &= (\max \{x_{11}, x_{21}\}, \max \{x_{12}, x_{22}\}, \dots, \max \{x_{1N}, x_{2N}\}), \\ \mathbf{y}_1 \wedge \mathbf{y}_2 &= (\min \{y_{11}, y_{21}\}, \min \{y_{12}, y_{22}\}, \dots, \min \{y_{1N}, y_{2N}\}). \end{aligned} \quad (9)$$

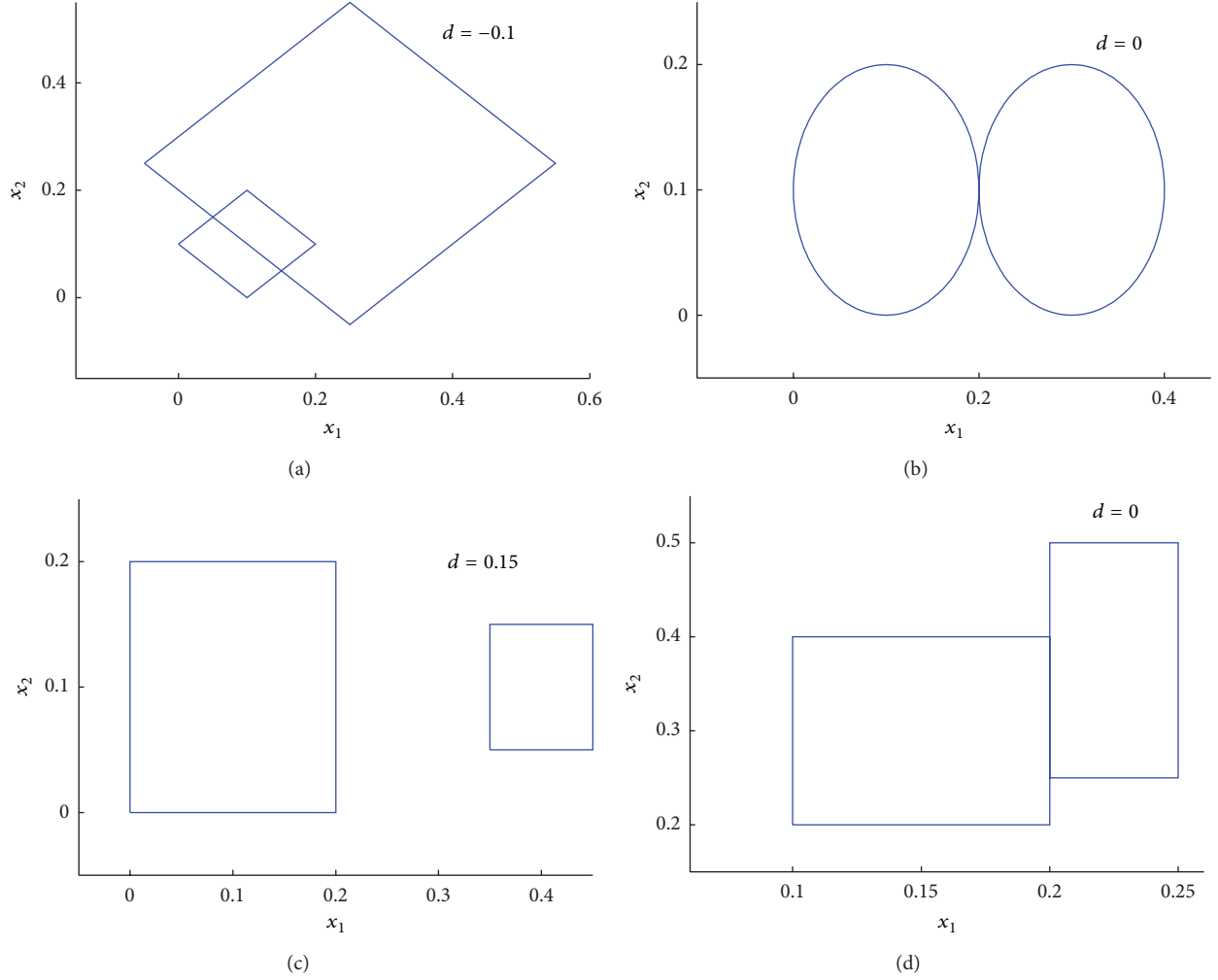
According to the distance between two granules mentioned above, the distance between two granules is the arbitrary real number. There is margin between two granules when $d > 0$, there is a same point between two granules when $d = 0$, and there is an overlap between two granules when $d < 0$. When $d > 0$, the greater d means the greater margin between two granules, and when $d < 0$, the greater d means the smaller overlap. Figure 2 shows the distance between two granules, including $d < 0$, $d = 0$, and $d > 0$.

2.3. Operators between Granules. Any points are regarded as atomic granules which are indivisible; the union process is the key to obtain the larger granules compared with atomic granules. Likewise, the whole space is a granule with the maximal granularity; the decomposition process is the key to divide the larger granules into smaller granules.

For two hyperdiamond granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the union hyperdiamond granule is

$$G = G_1 \vee G_2 = (C, R),$$

$$\begin{aligned} C &= \left(\frac{1}{2} (\max \{S(:, 1)\} - \min \{S(:, 1)\}), \right. \\ &\quad (\max \{S(:, 2)\} - \min \{S(:, 2)\}), \\ &\quad \dots, \left. \frac{1}{2} (\max \{S(:, N)\} - \min \{S(:, N)\}) \right), \\ R &= \frac{1}{2} S \| (id1, :) - S (id2, :) \|_1, \end{aligned} \quad (10)$$

FIGURE 2: Distances between two granules in R^2 space.

where $S = S_1 \cup S_2$, $S_1 = \{C_1 - r_1 e_i \mid i = 1, 2, \dots, N\}$, $S_2 = \{C_2 - r_2 e_i \mid i = 1, 2, \dots, N\}$, e_i is the vector whose i th component is 1, and the other components are 0,

$$id1 = \arg \max S(:, 1), \quad id2 = \arg \min S(:, 1). \quad (11)$$

For two hypersphere granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the union hypersphere granule is

$$G = G_1 \vee G_2 = [C, R] = \left[\frac{1}{2} (P + Q), \frac{1}{2} \|P - Q\| \right], \quad (12)$$

where $P = C_1 - r_1 (C_{12} / \|C_{12}\|)$, $Q = C_2 + r_2 (C_{12} / \|C_{12}\|)$, $C_{12} = C_2 - C_1$ the vector from C_1 to C_2 .

For two hypercube granules $G_1 = (C_1, r_1)$ and $G_2 = (C_2, r_2)$, the union hypercube granule is

$$G = G_1 \vee G_2 = (C, R),$$

$$C = \min \{C_1 - r_1 I, C_2 - r_2 I\} + R, \quad (13)$$

$$R = \max (C_1 \vee C_2 - C_1 \wedge C_2) + r_1 + r_2,$$

where I is the vector with the same length as vector C_1 , and all the components are 1.

For two hyperbox granules $G_1 = (\mathbf{x}_1, \mathbf{y}_1)$ and $G_2 = (\mathbf{x}_2, \mathbf{y}_2)$, the union hyperbox granule is

$$G = G_1 \vee G_2 = (\mathbf{x}_1 \wedge \mathbf{x}_2, \mathbf{y}_1 \vee \mathbf{y}_2). \quad (14)$$

We explain the union process between granules in Figure 3 for 2-dimensional space R^2 . Two granules $G_1 = [0.1 \ 0.2 \ 0.2]$ and $G_2 = [0.25 \ 0.25 \ 0.2]$ represent two hyperdiamond granules, hypersphere granules, or hypercube granules in 2-dimensional space, the union hyperdiamond granule is $[0.1750 \ 0.2250 \ 0.2791]$, the union hypersphere granule is $[0.1750 \ 0.2250 \ 0.38989]$, and the union hypercube granule is $[0.1750 \ 0.2250 \ 0.3]$. Suppose two hyperbox granules $G_1 = [0.1 \ 0.2 \ 0.2 \ 0.4]$ and $G_2 = [0.2 \ 0.25 \ 0.25 \ 0.5]$ in 2-dimensional space, the union hyperbox granule is $[0.1 \ 0.2 \ 0.25 \ 0.5]$. These union granules are shown in Figure 3.

2.4. Granular Computing Classification Algorithms Based on Distance between Granules. The granular computing classification algorithms include two algorithms, the first algorithm

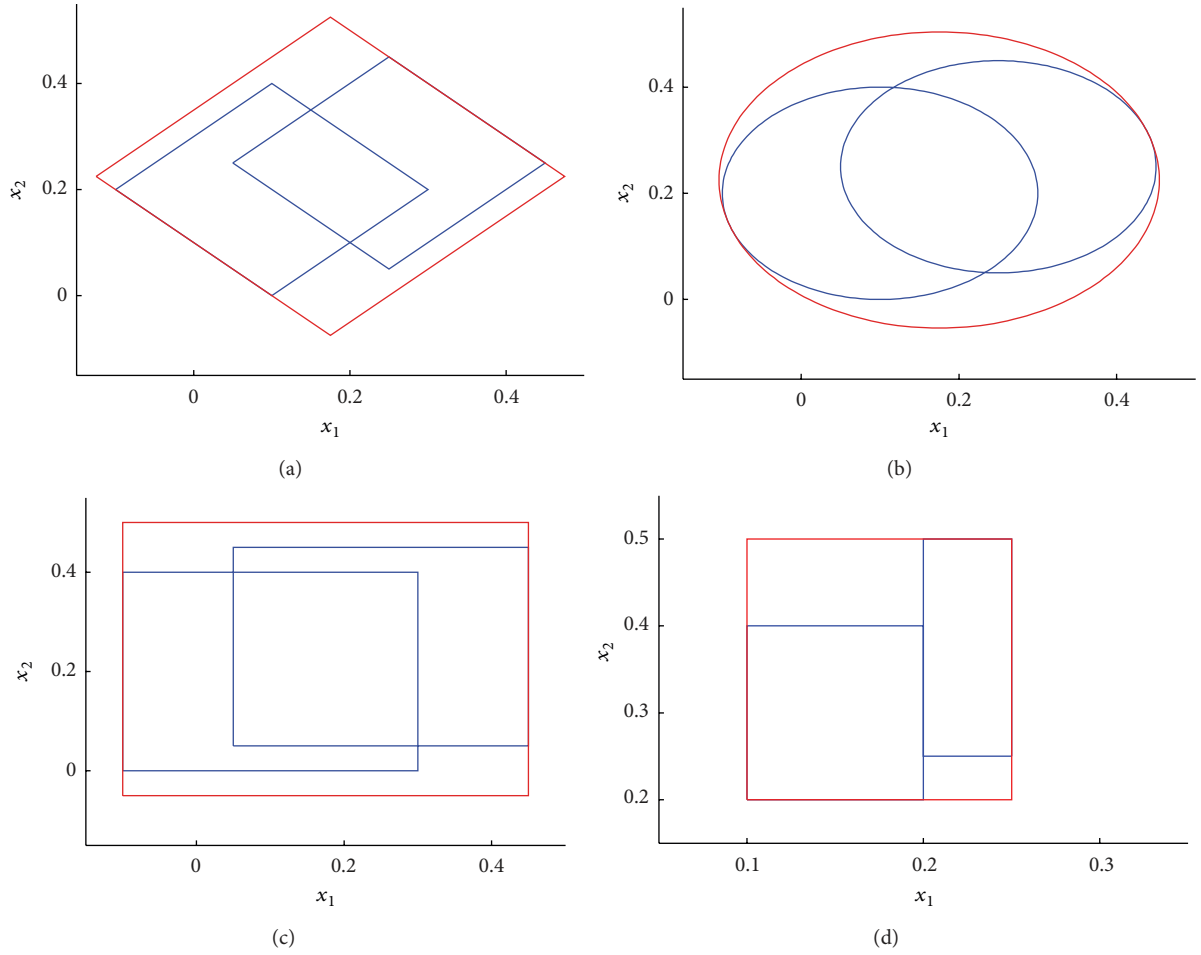


FIGURE 3: Unions between two granules. The union granules are represented as the red lines.

is the training algorithm and the second algorithm is the testing algorithm.

For training set TS, the training granular computing classification algorithms are proposed by the following steps. Firstly, the samples are used to form the atomic granule. Secondly, the threshold ρ of granularity is introduced to conditionally unite the atomic granules by the aforementioned union operator, and the granule set is composed of all the union granules. Thirdly, if all atomic granules are included in the granules of GS, the union process is terminated, otherwise, the second process is continued. The training algorithm is described as follows.

Suppose that the atomic granules with the same class labels induced by TS are g_1, g_2, g_3, g_4 , and g_5 . The training algorithm can be described as the following tree structure in Figure 4; leafs denote the atomic granules, root denotes GS including its child nodes G_2 and G_3 , G_1 is induced by union operation of child nodes g_1 and g_2 , G_2 is the union granule of G_1 and g_3 , and G_3 is the union granule of g_4 and g_5 . The whole process of obtaining GS is the bottle up process.

The threshold ρ , which is the cut of granularity induced by formulas (1a)–(1d) for the different shapes of granules, is selected in descending order. The larger ρ means the granule

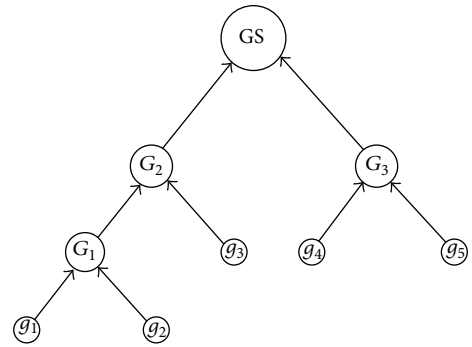


FIGURE 4: The training process of TS including 5 samples.

set induced by Algorithm 1 including the larger granules, conversely; the smaller ρ means the granule set induced by Algorithm 1 including the smaller granules. For the same training set, the smaller ρ means the induced granule set including more granules compared with the larger ρ .

The purpose of training algorithm is to obtain the granule set and the corresponding class lab, which are used to predict the class label of an unknown datum. The testing data

Input: Training set TS, threshold ρ of granularity, the class number n
Output: Granule set GS, the class label lab

(S1) initialize the granule set $GS = \emptyset$, lab = \emptyset
(S2) $i = 1$
(S3) select the samples with class i , and form set X
(S31) initialize the granule set $GSt = \emptyset$
(S32) $j = 1$
(S33) for the j th sample x_j in X , form the corresponding atomic granule G_j
(S34) $k = 1$
(S35) by formulas (2), (4), (6), and (8), compute the distance d_{jk} between the atomic granule G_j and the k th granule G_k in GSt
(S36) $k = k + 1$
(S37) find the minimal distance d_{jm}
(S38) form the union granules by formulas (10), (12), (13), and (14), and determine the granularity of the union granule by formulas (1a), (1b), (1c) and (1d) if the granularity of the union of G_j and G_m is less than or equal to ρ , the granule G_m is replace by the union, otherwise G_j is the new member of GSt .
(S39) remove x_j until X is empty.
(S4) $GS = GS \cup GSt$, lab = lab $\cup \{i\}$
(S5) if $i = n$, output GS and class lab, otherwise $i = i + 1$

ALGORITHM 1: Training algorithm.

including multiple data and their class labels are used to form the testing set, which is used to verify the performance of granular computing algorithms. If the prediction class labels of the testing data are same as the real class labels, the testing data are classified. Otherwise, the testing data are misclassified. The classification accuracy is one of the performances of granular classification algorithms. The testing algorithm is described as Algorithm 2.

3. Experiments

We evaluated the effectiveness of DGrC on both two-class and multiclass problems using Intel PIV PC with 2.8 GHz CPU and 2 GB memory, running Microsoft Windows XP Professional, and Matlab 7.0. We mainly analyze and discuss DGrCs with different shape granules from training accuracy (Tr (%)), testing accuracy (generalization ability) (Ts (%)), training time (Tr (s)), and testing time (Ts (s)).

3.1. Two-Class Problems. The spiral classification is a difficult problem to be classified and is used to evaluate the performance of classifiers. The training data are generated by the method proposed in [7]. The training set and the testing set in reference [8] are used to evaluate the performance of GrC.

The threshold ρ of granularity is from 0.2 to 0 with step 0.001; the maximal testing accuracy is the selection indicator of optimization algorithms. Performances of GrC with four kinds of shape are listed in Table 1. The training data and their granules were shown in Figure 5 in which the single points are the atomic granules. From the table, we saw that GrC with hypersphere granules achieved the optimization performance because of the minimal size of GS including 88 granules when $\rho = 0.094$, GrC with hypercube granules is poor because of maximal size of GS including 99 granules when $\rho = 0.079$, and GrC with hyperdiamond granules touched the

Input: inputs of unknown datum x , granule set GS, the class label lab
Output: class label of x

(S1) x is represented as granule g
(S2) for $i = 1: |GS|$
(S3) compute the distance d_i between g and g_i in GS
(S4) find the minimal distance d_m
(S5) find the corresponding class label of the g_m as the label of x

ALGORITHM 2: Testing algorithm.

best testing accuracy firstly. The training time and testing time are related to the size of granule set GS, so the granular computing classification algorithms with the minimal size of granule set are our pursuits in the same conditions for the maximal test accuracy.

3.2. Multiclass Problems. For multiclass problems, datasets listed in Table 2 are selected from the UCI Machine Learning Repository (<http://archive.ics.uci.edu/ml/>) to test DGrC. They are wall-following robot navigation data (sensor2, sensor4, and sensor24) which are divided into training data and testing data at random, optical recognition of handwritten digits (optdigits) including training data and testing data, pen-based recognition of handwritten digits (pendigits) including training data and testing data, letter recognition (letter) which is divided into training data and testing data, and shuttle including training data and testing data. These datasets are used to verify the performances of DGrC from the aspects of size, Tr (%), Ts (%), Tr (s), and Ts (s) (see Table 3).

TABLE 1: Performance of GrC with different shape granules.

Shapes	ρ	Size	Tr (%)	Ts (%)	Tr (s)	Ts (s)
Hyperdiamond	0.1	97	100	100	0.35938	0.015625
Hypersphere	0.094	88	100	100	0.3125	0.015625
Hypercube	0.079	99	100	100	1.4063	0.03125
Hyperbox	0.095	97	100	100	5.2656	0.0326

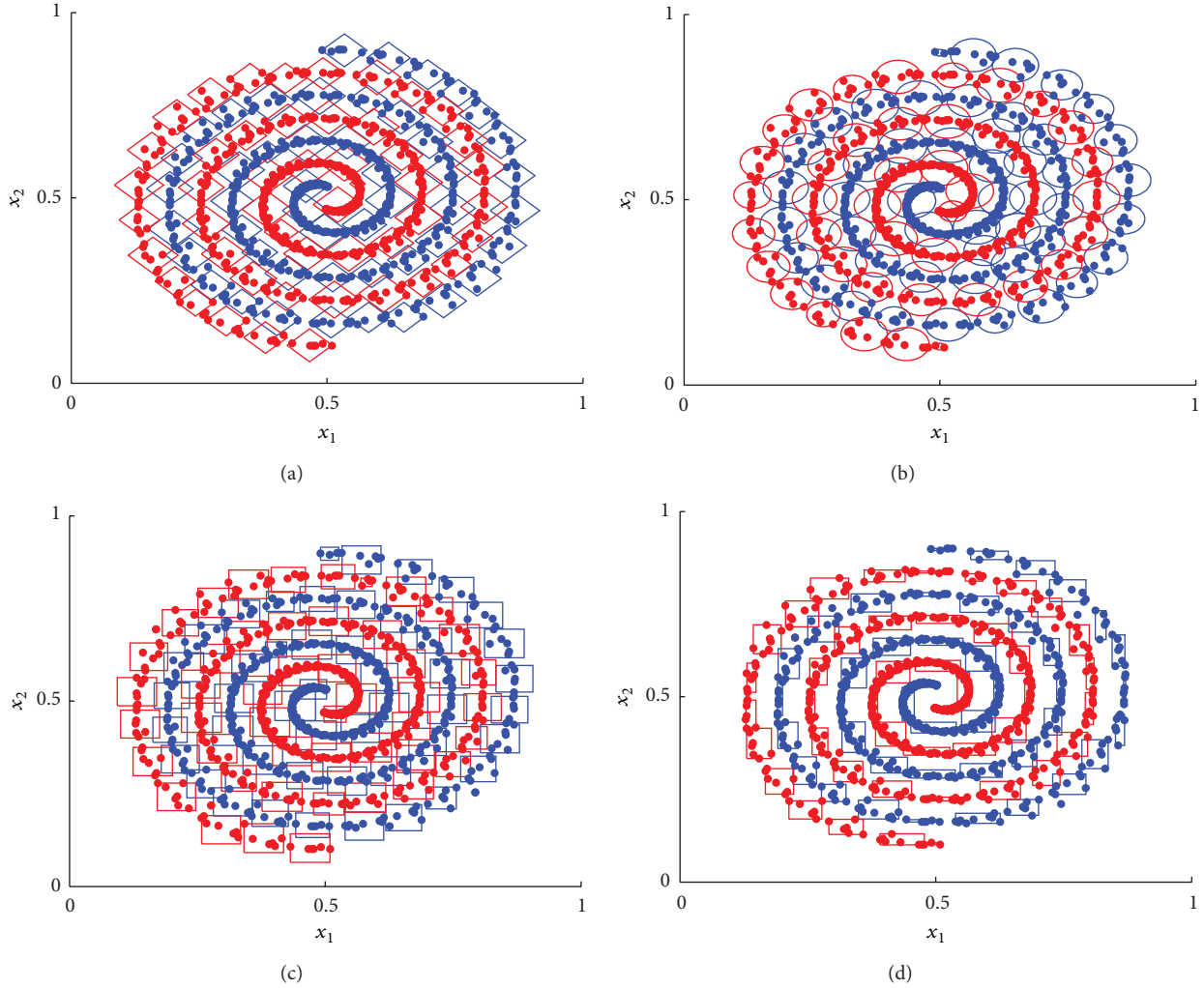


FIGURE 5: Spiral classification problem and GS (a) hypersphere granules, (b) hypercube granules, (c) hyperdiamond granules, and (d) hyperbox granules.

TABLE 2: Multiclass problems.

Data sets	Inputs	Outputs	Training size	Testing size
Sensor2	2	4	3636	1820
Sensor4	4	4	3638	1818
Sensor24	24	4	3636	1820
Optdigits	64	10	3823	1797
Pendigits	16	10	7494	3498
Letter	16	26	13333	6667
Shuttle	9	7	43500	14500

For the selected datasets, the optimal testing accuracies are 98.0769% (sensor2), 90.8691% (sensor4), 83.0220% (sensor24) 97.997% (optdigits), 97.799% (pendigits), 94.765% (letter), and 99.883% (shuttle) by KNN algorithms. We selected the optimal parameters that maximized the testing accuracy. DGrCs with 4 shapes are performed in the same environment, and the performance is listed in Table 3. From the table, we can see, for the optimal testing accuracies, that DGrC is better than KNN. (1) DGrC with hyperdiamond granules achieved the best testing accuracies 92.4092%, 87.8022%, and 99.9448%, which are highlighted by black fonts, for datasets sensor4, sensor24, and shuttle. (2) DGrC with hypersphere

TABLE 3: Performance of DGrC on multiclass problems.

Data sets	Shapes	ρ	Size	Tr (%)	Ts (%)	Tr (s)	Ts (s)
Sensor2	Hyperdiamond	0.005	948	99.67	98.3516	1.1875	0.2031
	Hypersphere	0.0115	338	99.3674	98.4615	1.2031	0.1406
	Hypercube	0.01	365	99.2299	98.1319	3.8125	0.0938
	Hyperbox	0.004	1560	99.945	98.297	1.2031	0.71875
Sensor4	Hyperdiamond	0.0195	974	99.5052	92.4092	1.1563	0.2344
	Hypersphere	0.0085	1387	99.9175	91.7492	0.9063	0.5938
	Hypercube	0.0255	466	98.3233	90.4290	2.1563	0.1563
	Hyperbox	0.00765	906	97.306	91.474	1.5469	0.8125
Sensor24	Hyperdiamond	0.4450	2154	99.6425	87.8022	9.2188	2.7969
	Hypersphere	0.4450	1700	98.3773	83.2418	4.7031	2.4063
	Hypercube	0.1750	2276	99.3674	75.6593	10.0625	3.7969
	Hyperbox	0.265	2711	99.67	83.132	7.5469	6.4531
Optdigits	Hyperdiamond	1.99	3685	99.9738	97.4958	18.5625	4.813
	Hypersphere	2	1005	100	98.1636	2.1094	2.4063
	Hypercube	0.15	3823	100	96.3272	6.5781	12.2344
	Hyperbox	2.1	2028	99.9738	98.0523	25.0625	7.1875
Pendigits	Hyperdiamond	0.62	2334	99.97856	97.5129	5.2500	2.3281
	Hypersphere	0.28	4041	100	97.9131	5.0000	7.5000
	Hypercube	0.25	2074	99.9733	97.5129	8.8594	4.2344
	Hyperbox	0.64	5801	99.9466	97.9417	2.7031	9.6563
Letter	Hyperdiamond	0.13	11993	100	94.6603	7.0156	18.2188
	Hypersphere	0.065	12685	100	94.7953	3.1719	28.0469
	Hypercube	0.08	7350	100	90.2955	9.2344	19.9063
	Hyperbox	0.5	10427	98.77	94.5853	6.9844	36.8438
Shuttle	Hyperdiamond	0.0028	3052	99.9931	99.9448	47	10.255
	Hypersphere	0.0015	3348	100	99.9448	36.3125	10.8594
	Hypercube	0.00006	5895	99.9977	99.9379	58.7969	31.1250
	Hyperbox	0.0025	2920	99.9885	99.9379	26.2688	28.7813

granules achieved the optimal testing accuracies 98.4615%, 98.1636%, 94.7953%, and 99.9448% for dataset sensor2, optdigits, letter, and shuttle. (3) DGrC with hyperbox granules achieved the optimal testing accuracy 97.9417% for pendigits.

4. Conclusions

The granular computing classification algorithms with different shape granules are proposed based on distance measures in the paper. Firstly, a training datum is represented as an atomic granule. Secondly, the distance measure between granules is form based on the centers and granularities of granules. Thirdly, the training process is constructed based on the union operator and the threshold of granularity jointly. Finally, the proposed granular computing classification algorithms are demonstrated by the dataset selected from references. DGrC is affected by the sequence of the training data the same as the other granular computing. For the future work, we will focus on the adaptive selection of threshold of granularities and apply the granular computing to image segmentations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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