

GRANULATION, MAGNETO-HYDRODYNAMIC WAVES, AND THE HEATING OF THE SOLAR CORONA

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Summary

In an electrically conducting liquid situated in a magnetic field any motion gives rise to magneto-hydrodynamic waves. Since the granulation is considered to constitute a turbulence in the photosphere, it must produce magneto-hydrodynamic waves, which are transmitted upwards to the chromosphere and the corona. The energy of the waves is estimated to the order of one per cent of the energy radiated by the Sun. It is shown that the waves are damped mainly in the inner corona where their energy is converted into heat. It is possible that the very high temperature found in the corona is produced through this magneto-hydrodynamic heating.

I. *Magneto-hydrodynamic waves in Sun.*—If a magnetic field H_0 is given in an electrically conducting liquid, any motion of the liquid gives rise to magneto-hydrodynamic waves † which travel parallel to the magnetic field with the velocity

$$V = H_0 \sqrt{(\mu/4\pi\rho)} \quad (I.1)$$

(ρ = mass density, μ = permeability). Any state of motion is transferred along the magnetic lines of force through the waves. These are characterized by a mechanical (hydrodynamic) motion, a change in the magnetic field and a set of electric currents.

If the electrical conductivity is finite, the waves are damped. A sine wave with the frequency ω travelling in the direction of the z -axis, taken parallel to H_0 , can be characterized through its induced magnetic field H' :

$$H' = H_1 e^{-\alpha z} e^{i\omega(t-z/V)}, \quad (I.2)$$

where H_1 is a constant, and the damping exponent is

$$\alpha = \frac{\sqrt{\pi}}{\mu^{5/2}} \frac{c^2 \omega^2 \rho^{3/2}}{\sigma H_0^3}. \quad (I.3)$$

Here σ is the conductivity (in e.s.u.) and c the velocity of light. ‡

As the Sun has a good electric conductivity (and a low viscosity) and possesses a general magnetic field, the conditions for generation of magneto-hydrodynamic waves exist. According to a recent theory § the sunspots are due to such waves which are produced by convection in the solar core whence they are transmitted along the magnetic lines of force. After having travelled a few decades of years through the Sun they reach the solar surface, where the magnetic fields associated with the waves give rise to sunspots.

* Originally submitted in different form on 1946 July 15.

† H. Alfvén, *Ark. Mat. Astr. Fys.*, **29B**, No. 2, 1942.

‡ The formula can easily be derived from the general equations of the waves, for example (1)–(6) in reference †.

§ H. Alfvén, *Ark. Mat. Astr. Fys.*, **29A**, No. 12, 1943; C. Walén, *Ark. Mat. Astr. Fys.*, **30A**, No. 15, 1944, and **31B**, No. 3, 1944; H. Alfvén, *M.N.*, **105**, 3, 1945; and **105**, 382, 1945.

It seems worth while to investigate whether magneto-hydrodynamic waves could be produced elsewhere in the Sun. As any motion changing the shape of a magnetic line of force produces a magneto-hydrodynamic oscillation of the line, we must expect that the turbulence of the photosphere which we observe as *granulation* gives rise to magneto-hydrodynamic waves. The scope of this paper is to study the possible effects of these waves.

The velocity of the magneto-hydrodynamic waves in the photosphere where the granulation takes place can according to (1) be calculated if we know the general magnetic field H_0 and the density ρ . For the former the value $H_p = 25$ gauss at the poles (and half of that at the equator) has generally been used, founded upon Hale's measurements of the Zeeman effect. According to a recent report, Thiesen has found the value $H_p = 53 \pm 12$ gauss, as a result of Zeeman effect measurements by a new method.* The displacement of the sunspot zone indicates that the dipole moment of the Sun is likely to be within the limits 1.5×10^{33} and 6.2×10^{33} gauss cm.³, corresponding to $H_p = 9$ and $H_p = 37$ gauss.† We adopt as a reasonable value $H_p = 40$ gauss (equatorial value $H_e = 20$ gauss, average for the whole surface ~ 30 gauss).

The density of the photosphere is estimated as 10^{-7} to 10^{-8} g. cm.⁻³. Putting $H = 30$ gauss and $\rho = 3 \times 10^{-8}$ g. cm.⁻³ (as average) we obtain

$$V = 5 \times 10^4 \text{ cm. sec.}^{-1}, \quad (1.4)$$

the extreme values $H_p = 40$; $\rho = 10^{-8}$ and $H_e = 20$, $\rho = 10^{-7}$ giving $V = 11 \times 10^4$ and 2×10^4 cm. sec.⁻¹.

2. *Generation of magneto-hydrodynamic waves through the granulation.*—Any motion in the photosphere gives rise to waves transmitted with the velocity (1.1). According to some observations the granulae are displaced with a velocity of about 3 km. sec.⁻¹. It is possible that this is a real velocity but maybe the motion is only apparent. If real it corresponds to an average kinetic energy of

$$\frac{1}{2}\rho v^2 = \frac{1}{2}(3 \times 10^{-8})(3 \times 10^5)^2 = 1.4 \times 10^3 \text{ erg cm.}^{-3}.$$

The granulation is observed as a difference in brilliance of different small parts of the photosphere. The maximum light difference is about $\Delta = 15$ per cent corresponding to a temperature difference $\frac{1}{4}\Delta$. If this is caused by adiabatic compression of an ideal gas the change in pressure must be $(5/2)(1/4)\Delta = 9$ per cent. As the pressure in the photosphere is 10^4 to 10^5 dynes cm.⁻², the pressure fluctuation corresponds to a difference in potential energy of 10^3 to 10^4 erg cm.⁻³. We should expect the turbulence changes in the potential energy to be of the same order of magnitude as those in the kinetic energy. This indicates that the velocities of about 3 km. sec.⁻¹ (which, as we have seen, corresponds to about 10^3 erg cm.⁻³) may be real.

In a magneto-hydrodynamic wave the magnetic energy equals the kinetic energy and also the pressure difference Δp associated with the wave:

$$(\mu/8\pi)H'^2 = \frac{1}{2}\rho v^2 = \Delta p. \quad (2.1)$$

Here H' means the magnetic field of the wave, which is superimposed upon the general field H_0 ; v means the material velocity in the wave. As the granulation reveals the existence of fluctuations in the kinetic and potential energy of the order of 10^3 to 10^4 erg cm.⁻³ we must expect a varying magnetic field between $H' = \sqrt{(8\pi \times 10^3)} = 160$ and $H' = \sqrt{(8\pi \times 10^4)} = 500$ gauss.

* *Observatory*, 66, 230, 1946.

† H. Alfvén, *loc. cit.*

Because of the granulation the general solar magnetic field must be superimposed—in the photosphere—by an irregularly varying magnetic field with an amplitude of a few hundred gauss.

Such fields would give a Zeeman effect broadening of spectral lines of about 0.01 Å. It would probably be rather difficult though not impossible to find this effect by observations.

3. *Transmission of the magneto-hydrodynamic waves.*—The turbulence of the photosphere is usually thought to be due to an instability which is located in the photosphere. This is not absolutely certain. The instability may also be situated at some depth below the photosphere, because a turbulence produced at some depth would be transmitted as magneto-hydrodynamic waves to the photosphere in the same way as the turbulence in the solar core causes sunspots at the solar surface after a wave transmission. Against the assumption that the cause of the granulation is situated below the photosphere speaks the fact that the granulation is independent of the latitude. Near the equator, where the magnetic field is horizontal a turbulence at some depth could not so easily be transmitted to the photosphere, so with this assumption we should expect a decrease in granulation close to the equator.

The magneto-hydrodynamic waves may be generated in the photosphere itself—as perhaps is most probable—or in a deeper layer; in any case they must be transmitted upwards from the photosphere along the magnetic lines of force. When they travel upwards in the solar atmosphere, their wave velocity changes especially because of the change in mass density ρ . This is equivalent to a change in refractive index, and if the change is very rapid a partial reflection of the waves may take place, which would mean that only a fraction of the energy reaches the chromosphere or corona. Decisive for the reflection is the ratio between the scale-height z_0 (defined through $z/z_0 = \epsilon n \rho / \rho_0$) and the wave-length λ . As

$$\lambda = \frac{2\pi V}{\omega} = \frac{\sqrt{\pi H_0}}{\omega \sqrt{\rho}}, \quad (3.1)$$

we have (if the magnetic lines of force are vertical as near the poles)

$$\frac{z_0}{\lambda} = \frac{\omega}{\sqrt{\pi H_0}} z_0 \sqrt{\rho}. \quad (3.2)$$

According to current views concerning the solar atmosphere we have:

Number of particles/cm. ³ n	Density g./cm. ³ ρ	Scale-height z_0 cm.	Magnetic field at axis H_p gauss	$z_0 \sqrt{\rho} / H_p$
10^{16}	1.6×10^{-8}	2×10^7	40	63
10^{14}	1.6×10^{-10}	5×10^7	40	16
10^{12}	1.6×10^{-12}	1.5×10^8	40	4.7
10^{10}	1.6×10^{-14}	10^9	40	3.2
10^8	1.6×10^{-16}	10^{10}	20	6
10^6	1.6×10^{-18}	1.5×10^{11}	0.5	380

The minimum value of $(z_0 \sqrt{\rho}) / H_p$ is about 3 sec.

Rydbeck* has treated the reflection of waves (magneto-hydrodynamic and

* O. Rydbeck, *Trans. Chalmers' Univ. Tech.*, Gothenburg, 1947 (in press).

electromagnetic) in a medium with variable refractive index. His formulae make it possible to calculate the reflection for cases when the refractive index can be approximated to certain functions. As we are mainly interested in the order of magnitude, it may be sufficient to use the result he has obtained for a medium where the density ρ falls exponentially with the scale-height z_0 . In this case no appreciable reflection occurs until

$$\lambda \simeq 4\pi z_0.$$

If the magnetic lines of force are not vertical we have to substitute $\lambda \cos \alpha$ (where α is the angle between the field and the vertical) for λ . Hence no considerable reflection takes place if the wave is smaller than

$$\lambda = 4\pi z_0 / \cos \alpha. \quad (3.3)$$

If ϕ is the latitude and H_p the field at the axis, we have $\cos \alpha = 2 \sin \phi (1 + 3 \sin^2 \phi)^{-\frac{1}{2}}$ and $H_0 = \frac{1}{2} H_p (1 + 3 \sin^2 \phi)^{\frac{1}{2}}$. Hence we have

$$\omega = \sqrt{\pi} \frac{z_0}{\lambda} \frac{H_0}{z_0 \sqrt{\rho}} = \frac{\sqrt{\pi}}{4\pi} \frac{H_p}{z_0 \sqrt{\rho}} \sin \phi. \quad (3.4)$$

For the minimum value of $z_0 \sqrt{\rho} / H_p$ we obtain

$$\omega = 0.047 \sin \phi. \quad (3.5)$$

Frequencies above this are not considerably reflected.

If the damping and reflection is neglected the amplitude of v varies as $\rho^{-\frac{1}{2}}$ and that of H' as $\rho^{\frac{1}{2}}$ (whereas the wave velocity varies as $\rho^{-\frac{1}{2}}$ and, moreover, is proportional to H_0)*. When the waves move upwards into the chromosphere and the corona, H' decreases, and as it is unobservable already in the photosphere it is likely to be of little importance higher up. The (material) velocity v , on the contrary, increases when ρ decreases and if the damping of the waves were negligible it would reach very high values up in the corona.

In fact, already in the inner corona, where the density is 10^{-8} of that in the photosphere, v would have increased 100 times, i. e. to 300 km. sec.⁻¹. This cannot be correct because the Doppler effect broadening of the spectral lines in the inner corona gives only about 20 km. sec.⁻¹. Consequently a damping must take place and this is also what could be expected theoretically, as will be shown in the next section.

As in a wave, the kinetic energy w_k equals the magnetic energy w_H , the energy transmitted upwards from the photosphere is

$$U = 2w_H \times V.$$

As we have found, V is of the order of 5×10^4 cm. sec.⁻¹ and $w_H = 10^3$ to 10^4 erg cm.⁻³ sec.⁻¹. Thus we find that U is of the order 10^8 to 10^9 erg cm.⁻² sec.⁻¹. This is of the order of one per cent of the total energy radiated by the Sun (5×10^{10} erg cm.⁻² sec.⁻¹).

4. *The absorption of the waves.*—A magneto-hydrodynamic wave is always associated with an electric current. A sine wave in the z -direction

$$H' = A \sin \omega(t - z/V), \quad (4.1)$$

contains a current with density

$$i = A \frac{c\omega}{H_0} \sqrt{\left(\frac{\rho}{4\pi\mu}\right)} \cos \omega \left(t - \frac{z}{V}\right). \quad (4.2)$$

* C. Walén, *loc. cit.*

The current flows in the x -direction if the induced magnetic field H' goes in the y -direction and the initial magnetic field H_0 is parallel to the z -axis.

If the conductivity σ is finite, the current produces a Joule heating w_j of the liquid. Its mean for one whole period is

$$w_j = \frac{1}{\sigma} (\text{mean of } i^2) = \frac{1}{2} \frac{A^2 \omega^2 \rho}{H^2 4\pi\mu}. \quad (4.3)$$

As energy is dissipated in this way the wave becomes damped with the damping exponent given by (1.3). In this formula we shall introduce the conductivity σ .

Chapman and Cowling* have calculated the conductivity of an ionized gas. According to Cowling† the presence of a magnetic field reduces the conductivity perpendicular to the field, especially at low pressures. Writing his formula in Gaussian units and introducing $p_e = nkT$ ($k = 1.37 \times 10^{-16}$) we obtain for the conductivity σ perpendicular to the magnetic field H

$$\frac{c^2}{\sigma} = 6.8 \times 10^{13} T^{-3/2} \bar{Z} + 0.58 \times 10^{26} \frac{H^2}{Zn^2} T^{3/2}, \quad (4.4)$$

where T is the temperature, \bar{Z} the mean ionization, n the number of electrons per cubic centimetre.† As our current i is perpendicular to the given field H_0 as well as to induced field H' we ought to use the conductivity perpendicular to the field. As long as the wave amplitude A is small ($A \ll H_0$), the magnetic field to be introduced into Cowling's formula equals H_0 . At large amplitudes, the matter becomes more complicated. According to (4.1) and (4.2), H' is zero when i is maximum, which reduces the influence of H' . As we are mainly concerned with the order of magnitude of the conductivity we use H_0 . This may give too high values of the conductivity, if the amplitude is large.

Let us calculate the damping exponent α for completely ionized hydrogen. We introduce $\rho = n \times m_H$ (m_H = mass of the hydrogen atom) and $\mu = 1$ into (1.3). With the help of (4.4) we obtain

$$\alpha = \frac{\omega^2}{H_0^2} [b_1 H_0^2 n^{-1} T^{3/2} + b_2 n^{3/2} T^{-3/2}] \quad (4.5)$$

with $b_1 = 2.2 \times 10^{-10} \text{ cm.}^{-3} g^{\dagger} \text{ sec. deg.}^{-3/2}$; $b_2 = 2.6 \times 10^{-22} \text{ cm.}^2 \text{ sec.}^{-1} g^{3/2} \text{ deg.}^{3/2}$.

In this formula we introduce $H_0 = 30$ gauss. The frequency of the waves probably covers a wide range down to the limit given by (3.5). Taking an average value for $\sin \phi$, this limit is about $\omega = 3 \times 10^{-2} \text{ sec.}^{-1}$, corresponding to a period of a few minutes. This is about the average life of the granulae, so we should expect this frequency, which is still not very much reflected, to be especially important. Fig. 1 shows how α varies with the density n for $T = 10,000$ deg. and 1,000,000 deg., corresponding roughly to the temperature of the photosphere and of the corona.

In the photosphere the value of α is 10^{-12} to $10^{-10} \text{ cm.}^{-1}$. As the height of the photosphere is a few hundred kilometres it is evident that the damping is negligible. Even in the chromosphere the damping is inconsiderable because the height of the chromosphere is about 10^9 cm. and α is smaller than 10^{-10} even if the temperature were as high as 10^6 degrees. In the inner corona ‡, however, where $n \sim 10^8 \text{ cm.}^{-3}$, α becomes 10^{-9} for $T = 10^6$ degrees, which means that the waves give off most of their energy in a layer of the thickness 10^9 cm. Thus the energy transmitted as

* T. G. Cowling, *Proc. Roy. Soc., A*, **183**, 453, 1945.

† The Hall current, for which Cowling gives an expression, may complicate the phenomena.

‡ The density of the corona at different heights is taken from S. Baumbach, *A.N.*, **263**, 212, 1937.

For the ratio between the total corona radiation and the radiation of the Sun if heated to the corona temperature (10^6 degrees), Waldmeier* gives the figure 2.5×10^{-12} . As the radiation is proportional to T^4 we find for the total radiation of the corona $2.5 \times 10^{-12} (10^6/6 \times 10^3)^4 = 0.2$ per cent of the solar radiation. This value is probably very uncertain, but it is of interest to observe that it is the same order of magnitude as found for the energy transmitted by the magneto-hydrodynamic waves. In any case we should not expect the corona to radiate more than a few per cent of the photosphere. Hence we may conclude that the mechanism outlined gives energy enough to heat the corona.

If we exclude the heating through meteors, it is difficult indeed to find an alternative to the heating through electric currents (heating through nuclear reactions being obviously insufficient). The electric currents can be supplied in different ways. The currents associated with "granulation waves" as considered above may supply the "normal" more or less constant heating. But no doubt currents could be produced also in other ways. In a series of papers the view has been propagated that the phenomena termed "solar activity" are essentially of electromagnetic or magneto-hydrodynamic nature.† The prominences can be understood as electric discharges (mainly along the magnetic lines of force) produced by electromotive forces generated through a process similar to unipolar induction.‡ The coronal arcs recently discovered by Lyot§, may be discharges of a similar kind. All these discharges give probably an essential additive heating of the chromosphere and especially of the corona, which is indicated by the correlation between the coronal shape and the occurrence of prominences. As was pointed out several years ago the corona may be an atmosphere, more or less in thermal equilibrium, heated electrically to an enormous temperature.||

According to what is said above, the material velocity of magneto-hydrodynamic waves is about 3 km. sec.^{-1} in the photosphere where the density is $n = 10^{15}$ to 10^{17} particles cm.^{-3} . In the chromosphere the density is about 10^{11} to 10^{14} cm.^{-3} , and as the material velocity is proportional to $\rho^{-\frac{1}{2}}$ the velocity ought to be about one power of ten as large as in the photosphere. A turbulent velocity of 15 to 20 km. sec.^{-1} is also really observed. The turbulence in the inner corona ought to be still larger, but there the waves are rapidly damped, which may explain that the observed values of the turbulence are only a little in excess of the chromospheric values.

6. *The density of the corona.*—Some years ago it was pointed out that the low density gradient of the corona may be explained simply as a consequence of the fact that the temperature is very high.¶ Thus the corona may be in gravitational equilibrium and it is not necessary to assume that it consists of "ejected gases" or is supported by the radiation pressure or some hypothetical force.

If we assume that no other force than gravitation acts upon the corona, we can calculate the temperature in each point of the corona from the density function.

* Waldmeier, *Mitt. Aargau. naturf. Ges.*, **22**, 199, 1945.

† H. Alfvén, *Ark. Mat. Astr. Fys.*, **29B**, No. 2, 1942, and **29A**, No. 12, 1943; C. Walén, *loc. cit.*; H. Alfvén, *M.N.*, **105**, 3, 1945; H. Alfvén, *Ark. Mat. Astr. Fys.*, **27A**, No. 20, 1940 and No. 25, 1941. (Sections 7-10 of No. 25 should be cancelled because the effect of the magnetic field is introduced in an erroneous way.)

‡ H. Alfvén, *loc. cit.*, No. 20, 1940.

§ B. Lyot, *Ann. Astrophys.*, **7**, 31, 1944.

|| H. Alfvén, *loc. cit.*, No. 25, 1941.

¶ H. Alfvén, *loc. cit.*

The latter has been derived by Baumbach* from all available photometric observations, under the assumption that the coronal light consists mainly of photospheric light scattered by free electrons in the corona. For the mean electronic density at the height $\eta = R/R_{\odot}$, Baumbach gives the empirical formula

$$N = 10^8(0.036\eta^{-1.5} + 1.55\eta^{-6} + 2.99\eta^{-16}) \text{ cm.}^{-3}. \quad (6.1)$$

Of course the charge of the electrons must be compensated by the same amount of positive charge (from positive ions). As hydrogen is likely to be predominant, we assume for an estimation of the order of magnitude that most of the ions in the corona are hydrogen ions. Thus the number of protons per cm.^3 amounts also to N .

As at least in the inner corona the density is high enough to ensure thermal equilibrium between the "molecules" (but of course not between molecules and quanta!), we can apply the common laws of kinetic gas theory. We assume that the mean energy of the molecules (in our case electrons and protons) amounts to $E = 3kT/2$. As there are $2N$ molecules the gas pressure is

$$p = (2/3)2NE. \quad (6.2)$$

If $m_H = 1.66 \times 10^{-24}$ g. is the mass of a hydrogen atom and $g_{\odot} = 2.74 \times 10^4$ cm. sec.⁻² is the acceleration at the Sun's surface, the gravitational force acting upon a cubic centimetre is $g_{\odot}Nm_H\eta^{-2}$. As we have assumed that this force is compensated by the pressure gradient, we have

$$\frac{dp}{R_{\odot}d\eta} = -\frac{g_{\odot}Nm_H}{\eta^2}. \quad (6.3)$$

Differentiating (6.2) we obtain from (6.2) and (6.3)

$$\frac{d}{d\eta}\left(\frac{E}{E_0}\right) + \frac{1}{N}\frac{dN}{d\eta}\frac{E}{E_0} = -\frac{1}{\eta^2}, \quad (6.4)$$

where

$$E_0 = \frac{3}{2}g_{\odot}R_{\odot}m_H = 2.38 \times 10^{-9} \text{ erg} = 1.49 \times 10^3 \text{ e. volts.}$$

From (6.4) we obtain

$$\frac{E}{E_0} = -\frac{1}{N}\int\frac{dN}{\eta^2}d\eta$$

or, according to (6.1),

$$\frac{E}{E_0} = \frac{0.036}{2.5}\eta^{-2.5} + \frac{1.55}{7}\eta^{-7} + \frac{2.99}{17}\eta^{-17} \\ = \frac{0.036}{0.036}\eta^{-1.5} + 1.55\eta^{-6} + 2.99\eta^{-16}.$$

The value of E/E_0 from this formula is shown in Fig. 2. The temperature in the inner corona is about constant, having the value $E/E_0 = 0.12$, which corresponds to about 2,000,000.

Taking account of the fact that the corona contains other gases than hydrogen the temperature becomes still higher.

As has been pointed out especially by Grotrian the coronal light is probably composed of the light from the real corona and a superposed radiation containing Fraunhofer lines. Recent polarization measurements by Öhman† confirm this

* S. Baumbach, *loc. cit.*, p. 121.

† Y. Öhman, *Observatory*, 66, 261, 1946.

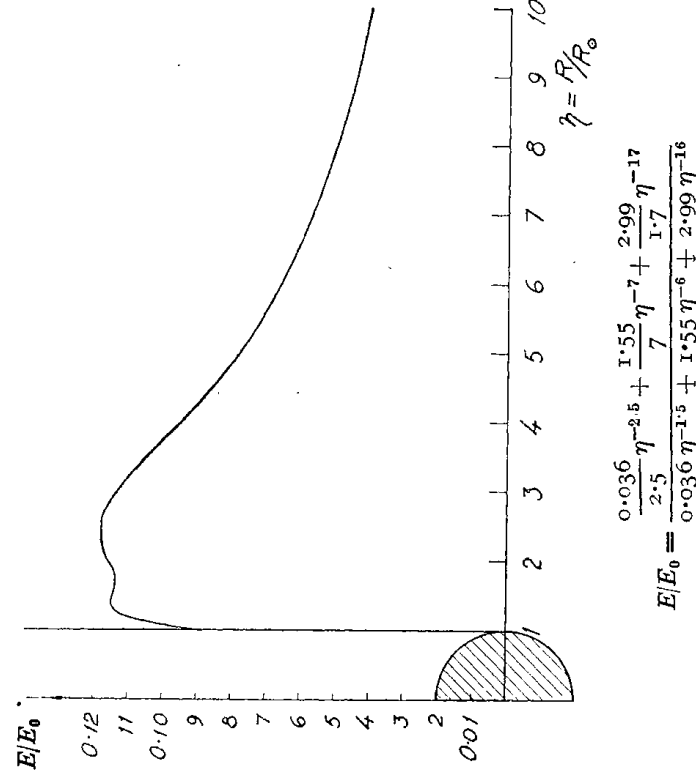


FIG. 2.

opinion. If a correction for this is applied the density gradient will certainly be considerably greater than calculated from Baumbach's formula. Hence the temperature is smaller than that found above, but probably still of the order 1,000,000 deg.

I wish to thank Professor Rydbeck of Gothenburg, for discussion of the reflection of waves in an inhomogeneous medium.

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1947 June 30.