Graph Coloring Based Physical-Cell-ID Assignment for LTE Networks

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ABSTRACT

Autoconfiguration of the radio parameters is a key feature for next generation mobile networks. Especially for LTE the NGMN Forum has brought it up as a major requirement. It is indispensable that algorithms used for autoconfiguration terminate quickly and do not cause infinite iterative reconfigurations within the network.

Reference signal sequences are among the most important radio parameters for LTE, which are comparable to scrambling codes in 3G networks. In LTE they additionally serve as Cell Identifiers on the Physical Layer. Each cell is assigned one of the 504 available Physical Cell Identifiers. For proper operation the assignment has to be as well collision as also confusion free. Due to the high number and the layered structure of the cells within the network such an assignment is a complex task.

In addition to this complexity each change of the Physical Cell ID of an operational cell causes a service interruption in the cell, which has to be avoided. The approach presented maps the ID assignment problem to the well known and well understood problem of graph coloring. It is shown that an efficient initial assignment even for complex networks is possible. Cells added during the subsequent network growth, can already be confused when inserted into the network. In this case the IDs of the operational cells causing the confusion must be changed.

As a next logical step the incremental approach shows how the properties of the colored graph can be used for extending the network with new cells, with only minimal interruption while still retaining the properties of a colored graph.

Categories and Subject Descriptors

H.m [**Information Systems Applications**]: Miscellaneous

General Terms

Algorithms, Management, Reliability

Keywords

LTE, Management, Physical Cell ID, Graph Coloring

1. INTRODUCTION

Autoconfiguration of radio parameters is a requested feature for the upcoming LTE Networks. The NGMN Forum [1], an association of mobile network operators, is the main driver behind requirements for LTE.

The importance and the high interest of the topic in industry and academia is also reflected in the large number of international research projects dealing with autoconfiguration and autonomic network management. For example the EU funded SOCRATES Project [2], which has a large stake in autoconfiguration of LTE radio parameters.

Autoconfiguration requires algorithmic approaches which provide guaranteed results with little overhead and little disruptions on the operational system. LTE reference signal sequences, which are comparable to 3G scrambling codes are of special interest for autoconfiguration. They serve as a regional unique identifier on the physical level because reading and decoding a reference signal takes less than 5 ms. Without an assigned Physical Cell ID (PHY ID) no radio communication is possible within a cell, as the User Equipment (UE) cannot detect the cell.

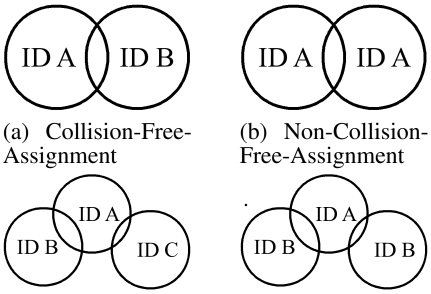
The reference signal sequences are constructed from a two dimensional pseudo random sequence and a two dimensional orthogonal sequence. There are 168 pseudo random sequences which are seen as cell identity groups. Each of them has three orthogonal sequences. This leads to 504 available Phy IDs [3], which should be enough for a proper assignment. However this set of IDs is partitioned into smaller sets for several reasons [4], which raises the requirement for an efficient distribution.

These facts lead to two important requirements for PHY ID assignments of neighboring cells. The assignment of IDs has to be collision and confusion free [5] (c.f. Figure 1) to allow a proper operation.

Two cells are considered to be neighbors if they have a common coverage area. Cells that form a continuous coverage area are called a cluster.

- **Collision Free:** Two neighboring cells may not have the same PHY ID. The following examples point out complications which may arise from colliding PHY IDs.

  User Equipment (UE) monitors constantly the received signals. In an area that is covered by multiple cells this process leads to the detection of potential handover candidates and then based on the signal strength to handover decisions. In case of identical PHY IDs the UE will discard the weaker of the received signals as it is regarded as a reflection or a multipath propagation. Which means the potential handover candidate is not recognized. As soon as the signal of the potential cell is stronger than the cell the UE is currently attached



(a) Collision-Free-Assignment (b) Non-Collision-Free-Assignment (c) Confusion-Free- Assignment (d) Non-Confusion-Free-Assignment

Figure 1: Collision and Confusion

to, it will try to use the stronger signal and the connection will fail.

If the UE has lost the connection to the serving cell it may not be able to attach to either one of the cells. The same may happen if a Mobile Terminal is turned on in such an area.

- **Confusion Free:** A cell may not have two neighbors with identical PHY IDs. An eNodeB has a table, with all information needed for handovers. The most important information are the PHY ID, the Global Cell ID (GCID) and the X2 information of potential handover candidates. If the UE reports a potential handover candidate, the eNodeB checks if this information is available for the given PHY ID. In case a cell has two neighbor cells with identical PHY IDs but only one of those cells is part of the neighbor relation table, handovers to the cell not in the table will fail. For the network monitoring it appears as if a fraction of the handovers to the cell identified by a particular PHY ID fails, since it is not possible to distinguish these cells.

Such an assignment is a complex task due to the high number and the layered structure of the cells in an LTE Network. In addition to this complexity each change of the Physical Cell ID causes a service interruption, which has to be avoided. The approach presented maps the ID assignment problem to the well known and well understood problem of graph coloring. It is shown that an efficient initial assignment even for complex networks is possible and the chance to solve issues on a local level is retained very long.

The PHY ID assignment is not only an issue in an initial rollout phase when all cells simultaneously have to be assigned a new PHY ID but also during incremental network growth. Mobile networks are constantly adapted to meet operator requirements. Cell sizes are changed, new cells are added, not only at the border of the coverage area but also in areas where higher capacity is needed. Such new cells have to be assigned a collision and collision free PHY ID. Newly introduced cells can already be confused by neighboring cells, c.f. Section 2.3. Such a confusion can only be removed if other, already operational, cells are reconfigured. A second approach shows how the properties of the colored graph can be used for extending the network with new cells, which causes only minimal interruption and still retains the properties of a colored graph like a minimal usage of colors.

Different methods have been proposed to achieve a proper assignment of the PHY IDs [5]. These approaches either assign the same or random PHY IDs to the cells [6]. Especially during network growth, cells are sometimes assigned a temporary PHY ID from a set of IDs which are only used until the final IDs are computed. Most of these approaches use radio measurements to detect collisions and confusions. Such approaches require changes at the UEs and the way radio measurements are done. Additionally, the time needed to find a consistent configuration is hard to predict. It is not even sure if a valid configuration can be found or if the changes will cause oscillating reconfigurations or reconfiguration loops which propagate through large parts of the network. Such problems could occur if a step by step optimization is performed. The system detects a first problem which is treated. Accidentally the solution for the first problem causes another problem which is detected and treated in a later step...

The approaches presented here map the problem of Cell ID Assignment to a classical mathematical problem. After a mapping to the graph coloring problem all known properties of the graph coloring are also valid for the problem of a collision and confusion free PHY ID assignment. This also allows the usage of existing algorithms with known properties. Depending on the used algorithms there will be guarantees concerning complexity and the possibility of a successful execution. These approaches can be executed centrally in the OAM System[1] for higher performance and reliability. Graph coloring has a history within Mobile Network Management. For example Frequency distribution for 2G systems has been computed using T-Coloring[7]. These approaches are too complex for the problem of PHY ID assignment.

Section 2 contains a detailed description of the approach in several steps, the third section contains an evaluation and the last section leads to a conclusion.

2. APPROACH

The presentation of the approach is divided into three separate subsections, which describe PHY ID assignment for the initial network wide configuration, the assignment for new cells during incremental network growth and the confusion repair phase.

2.1 Network Wide ID Assignment

Graph coloring is the problem to color the nodes of a graph in a way that two nodes that are connected with an edge are not assigned the same color, with a minimal number of colors. To find a minimal number of colors is known to be a NP-Complete problem [8]. This minimal number is called the Chromatic Number. For some special types of graphs the Chromatic Number is known in advance. For example bipartite graphs can be colored with only two colors, planar graphs with four colors. For non-trivial graphs the question whether a graph is planar or even bipartite is complex to solve. But it is known from[9] that the Chromatic Number is less or equal to the degree of the graph +1, which can serve as a worst case assumption for the required number of colors.

From the definition it is obvious that finding a collision free PHY ID assignment or a valid coloring for a graph are equivalent problems.

To be able to perform the graph coloring, the structure of the cell layout is transformed into a graph with the following steps:

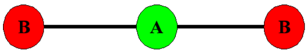
- Cells are depicted as vertices
- Vertices that represent neighboring cells in the network are connected by an edge

The resulting graph is then colored. The colors are translated to PHY IDs which are subsequently used for Radio Parameter Configuration.

The coloring algorithm used in this approach is the very simple extended greedy algorithm by Welsh and Powell [9], which will

[1] Operation Administration and Maintenance System

(a) Neighboring Cells connected by edges.



(b) Colored graph

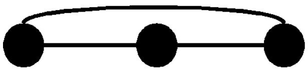
Figure 2: Graph for Collision-Free Assignment

find a solution if there is one. But this algorithm gives no guarantees on the optimality of this solution. The evaluation shows that the results are good enough for the problem of PHY ID assignment in LTE Networks.

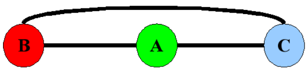
Up to this point the issue of confused cells has not been solved. For example a valid coloring of the graph uses only two colors (A,B), cf. Figure 2b. Although the coloring is valid and collision free, the center cell is confused.

The graph is extended with edges to the neighbors of the neighbors which solves the problem (c.f. Figure 3).

If the colored version of this graph is translated into PHY IDs for the cell deployment it is assured that all neighbors of a cell are assigned different PHY IDs it is as well collision as confusion free.



(a) Edges to neighbors of neighbors.



(b) Colored Graph

Figure 3: Graph for Collision and Confusion-Free Assignment

Additional properties that come with the usage of graph coloring theory can be taken into account. It can be checked wether an assignment is possible and a worst case estimation of required colores based on the chromatic number can be given[9]. This estimation can be compared to the number of available IDs to check whether a assignment attempt can be successful. The evaluation will lead to a better, more accurate estimation.

It has been shown that graph coloring is a good way to compute a valid PHY ID assignment for a complete or only a cluster of a network. For network growth the described approach would introduce too much overhead, as for each new cell all cells in the cluster could be affected. The next section provides a solution how the results and the properties of graph coloring theory can be used for incremental network growth.

2.2 Incremental Network Growth

The standard algorithm is not reasonable for incremental network growth as it opposes a lot of overhead and possible service interruptions through a possibly high number of PHY ID reconfigurations in the network.

To restrict the changes to small parts of the network an iterative process is defined that works on small partitions of the graph, while still retaining the properties of the colored graph, for example the minimal usage of colors.

Phy IDs are grouped into ID Sets. The following ID Sets are defined to facilitate the description of the algorithm:

- $\mathbf{NSet}$: The PHY IDs of the direct neighbors of the new cell
- $\mathbf{NNSet}$: The PHY IDs of the neighbors of the NSet
- $\mathbf{NNNSet}$: The PHY IDs of the neighbors of the NNSet
- $\mathbf{IDSet_u}$: All used PHY IDs in the network
- $\mathbf{IDSet_a}$: All PHY IDs that are available for the network

The subgraph that is considered contains the new cells as center and it's $NSet$, $NNSet$ and $NNNSet$. It has to be assured that the Phy IDs of the nodes in the graph are not changed by another process until all steps have been finished.

To retain a minimal ID usage, the new PHY ID should ideally be chosen from the $NNNSet$. If this is not possible and $\mathbf{IDSet_u}$ is not equal $\mathbf{IDSet_a}$ then the lowest PHY ID from $\mathbf{IDSet_a}$ that has not been used in $NSet$ or $NNSet$ is assigned.

The following steps describe the algorithm:

1. Check if there are PHY IDs in the $NNNSet$ that have not been used in the $NSet$ or $NNSet$ of the new cell. $NNNSet \setminus \{NSet \cup NNSet\} \neq \emptyset$
   1.1. If the result set is nonempty choose the PHY IDs from the result
   1.2. If the result set is empty perform next step.
2. Check if the $IDSet_u$ contains IDs that have not been used in the $NSet$ or $NNSet$ of the new cell $IDSet_u \setminus \{NSet \cup NNSet\} \neq \emptyset$
   2.1. If the result set is nonempty select the PHY ID from the result set
   2.2. If the result set is empty, continue
3. If $IDSet_a$ and $IDset_u$ are identical try a replanning of the complete network
   3.1. Otherwise introduce a new PHY ID by choosing the lowest PHY ID from $IDSet_a \setminus IDSet_u$

PHY ID distribution can be influenced with additional policies. Operators can provide specific groups of IDs[4] which meet specific requirements for example to ensure uniqueness in border regions. Additional policies can change the way an ID is chosen instead of choosing only the lowest PHY ID available, for example:

- Choose the ID that has been used least, most
- Choose an ID with minimal numerical distance or has not been used in the same location area
- Choose an ID with certain physical properties

The PHY ID chosen above is guaranteed to be collision free and to not confuse any of the neighboring cells, but it can already be confused. This situation is treated separately in Section 2.3.

In contrast to reality, this approach assumes cells to be perfect circles. A typical phenomenon in mobile networks are overshooting cells. Cells that cover areas which were not anticipated, and therefore have not anticipated neighbors. This problem obviously has a direct impact on the collision and confusion free assignment. Once the problem is detected it can easily be solved:

- Insert an edge in the graph between the overshooting cells and the newly detected neighbors
- Reassign the cell a new PHY ID

Other operator requirements like guard spaces for the reusage of PHY IDs or similar can easily be realized by introducing additional edges to define nodes as neighbors in the graph.

2.3 Confusion Removal

Newly introduced cells can easily be confused. Such a confusion is the result if the new cell connects two cells that have the same PHY ID, as shown in Figure 4. Which is possible if the cells have previously not been neighbors or neighbors of neighbors. The algorithm from Section 2.2 guarantees that the selected PHY ID of the new cell has neither collisions nor causes confusions of the surrounding cells. The new cell is confused if a PHY ID occurs more than once in the $NSet$.

Detection and removal are performed by the following steps:

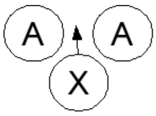


Figure 4: Newly introduced cell gets confused by existing cells with identical PHY ID

1. For each PHY ID used in the NSet of the new node create a list with the cells that use this particular PHY ID
2. Sort each of the lists in the decreasing order of the degree of the nodes
3. Remove the last element from the lists. If the list is nonempty this is a confusion that has to be removed. In case a confusion has to be treated all but one cells have to change their PHY ID, which is the removed one.
4. Change the PHY IDs of all nodes remaining in the lists. Starting with the node with the highest degree. The strategy to start with the cell with the highest degree is derived from an graph coloring algorithm developed at the Masseh College [10].

The cell that causes the confusion is assigned a new PHY ID based on the algorithm for network growth. Due to the properties of the colored graph the algorithm the altered cell cannot be confused, so no additional confusion check required.

3. EVALUATION

In order to validate the theoretical approach the algorithms were implemented and tested against information from extended real world networks.

Two data sets were used for the evaluation. The first set contains the geo positions of Vodafone Germanys' 3G sites for parts of eastern Germany especially in the greater area of Berlin. This information is available as a kmz-file[2] from [11]. This information was extended with an estimated but realistic cell radius according to their surrounding area. For example a radius between 1000 and 3500 meters for urban areas like the greater area of Berlin. The cells were approximated by circles.

The cell radius is important as it is used together with the geographical position of the sites to compute coverage areas and from that neighbors in the network. Cell coverage areas and neighbors are usually estimated by network planning tools. They use simulations which include antenna and radio parameters, geographical conditions etc. and give more precise estimations of the cell layout.

The second data set is an artificial network with 750 cells. The relative positions of the cells to each other are realistic for a dense urban mobile network. Cell sizes were varied to create different scenarios, which ranged from a network with many but small cells to a worst case scenario with very large cells. In the case with large cell sizes but distances that are typical for a dense urban network, each cell has a very large number of neighbors.

The evaluation targets the following questions:

- How many PHY IDs are used for a confusion and collision free assignment?
- How many neighbors does a cell have?
- How many cells form a cluster?
- Is there a relation between number of neighbors of the new cell, degree of the cluster and PHY IDs needed for a valid assignment?
- What are the differences if a complete network is assigned PHY IDs compared to an incremental growth of the same network?

[2] Zipped Keyhole Markup Language Format

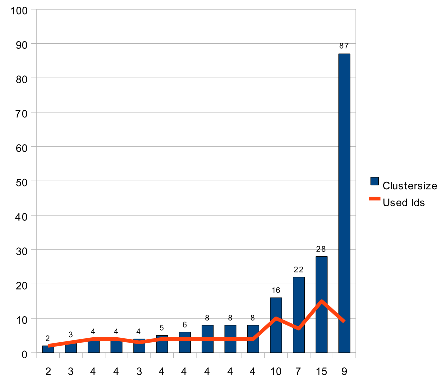


Figure 5: Comparison: Cluster Size, Number of used PHY IDs

Figure 5 compares clustersizes with the number of PHY IDs required. The x-axis shows the number of IDs used, the y-axis the cluster size. For small clusters the number of colors is close to the number of cells. This is obvious because small in small clusters most of the cells are within NSet and NNSet. For larger clusters the number of PHY IDs used is comparably small.

Although in this setup maximal 15 IDs are used, there are other examples where a larger number is required. Another setup needs 358 PHY IDs to color a cluster with 688 cells. The worst case cluster with 750 cells needs as much as 510 PHY IDs. This shows that even large and complex clusters can successfully be assigned IDs collision and confusion free using the graph coloring based approach.

For a selection of clusters Table 1 shows the maximal number of direct neighbors for a cell within a cluster, the degree of the used graph, the number of PHY IDs used for the collision and confusion free assignment with the distinction between initial coloring and a coloring during network growth. The table has been divided into two tables for layout reasons.

Table 1: Comparison: Maximal number of neighbors per cell, Used PHY IDs and degree of the used Graph

# Neighbors	1	2	2	3	3	3	3
# PHY IDs	2	3	3	4	4	4	4
# PHY IDs(Growth)	2	3	3	4	4	4	4
Degree of Graph	1	2	3	5	3	4	7

# Neighbors	6	8	9	14	18	344	503
# PHY IDs	7	9	10	15	20	358	510
# PHY IDs(Growth)	8	10	11	15	20	360	506
Degree of Graph	11	15	15	23	31	498	653

The graph coloring properties[9] give an upper bound for the number of IDs required. For small clusters this upper bound is a good estimation for the overall number required, which is quite obvious from the design of the graph. If this estimation would be true even for large clusters an assignment would not be possible. But the numbers for the larger cluster show, that a better estimation is: **The number of IDs required is at least Number of direct Neighbors + 1, or very close to this number.** This estimation leads to the conclusion, that there could be a valid assignment for the cluster shown in the last column.

These results allow operators to give a more precise estimation whether an assignment is possible or not, based on number of neighbors and degree of the graph. If the number of neighbors is larger than the number of available colors an assignment is impossible. In case the number of neighbors and degree are smaller than the number of available colors an assignment is possible. In case the degree is larger but the number of neighbors lower a rough estimation can be given.

The algorithm for the network growth tries to retain the properties of the colored graph without affecting more than the neighbors of the new cell. It is expected that this leads to a negative impact on the number of IDs used. To compare the quality of the results the clusters have been built cell by cell, as described in Section 2.2. The results show that the impact is rather small, only one or two additional IDs are used. Large, complex clusters with almost all Cells contained in the NSet are even assigned less IDs.

The reason is the large number of neighbors, which easily can lead to confusions of the new cells. Therefore the assignment is recomputed very often, which leads to a more optimized assignment that uses less PHY IDs. To assign the PHY IDs to the cluster no complete reassignment was necessary.

4. CONCLUSION

Over several steps it has been shown how PHY ID assignment for LTE Networks can benefit from graph coloring theory. How the problem of a collision and confusion free assignment can be solved by mapping it to the classical graph coloring problem. How the real world cell deployment can be assigned a valid, confusion and collision free assignment, by mapping it to a simple graph which then is colored using standard algorithms.

In a first step the transformation of the cell layout into a graph that guarantees a collision and confusion free assignment of PHY IDs was shown. The main disadvantage with this approach is, that for every change in the network the algorithm has to be rerun for all of the cells, which could cause reconfiguration of a large number of nodes. In an operational network reconfigurations of the active cell's PHY IDs should be avoided as they cause service interruptions.

In order to avoid service interruptions in large parts of the network a second approach for network growth was presented. It retains the properties of the colored graph but also tries to restrict its' operation to a very small area around the new cell. Only direct neighbors of a cell are changed if needed. This restriction allows also parallel introduction of cells at different positions in the network if the processes do not interfere with each other. The approach is divided into two simple steps. The first step allows a collision and confusion free selection of the new cells' PHY ID, but the new cell could then already be confused by neighboring cells. Such a confusion may arise if the new cell connects cells that have not been neighbors yet. The second step depicts a solution that solves such confusions.

The results showed that the approach for evolutionary network growth returns equally good results as the one that executes a basic graph coloring algorithm on the complete network.

The evaluation of the approaches showed that even for complex network scenarios the results are very good. The number of used PHY IDs is minimized, which means that the 504 IDs available in LTE networks mostly will be sufficient. And also partitions of the ID space are sufficient. More than that the results showed that for a first estimation of the required number of PHY IDs the coloring algorithm of Powell and Welsh gives a worst case assumption. But in all cases the number of required PHY IDs was very close to the maximal number of neighbors of a single cell within the cluster. This means even if the degree of the graph+1 is larger than the number of available PHY IDs a collision and confusion free assignment is possible as long as the maximal number of neighbors is less than the number of available PHY IDs.

Compared to other solutions these centrally executed approaches show several advantages:

- A success estimation is possible in advance
- The assignments are guaranteed collision and confusion free
- It is guaranteed that the assignment will come to a result if the graph can be assigned a valid coloring
- There is no chance of oscillating reconfigurations

Another major advantage is, that the approach does not rely on radio measurements. To obtain such measurements User Equipment has to be used within the cells. This is either accomplished with dedicated and therefore expensive drive tests or by using the measurements of the customer UEs that are used within the cells. Since the approach does not rely on radio measurements it is guaranteed that the algorithm will come to an end with either a valid coloring or the conclusion that a collision and confusion free assignment of IDs is not possible with the given network layout.

In the future additional properties beside minimal reuse of the PHY IDs can be introduced. This will be done by changing the specification of the ID Sets, the algorithms use to select new IDs or the graph by adding or removing edges. Also operator policies can be used to influence the distribution of the PHY IDs in the network. For example to assign certain IDs to certain cell types while still retaining collision and confusion free assignment.

Transferring the approach from a purely OAM centralized to a partly or fully decentralized approach could be a direction to go. But decentralized graph coloring is much more complex therefore a decentralized approach has to prove that its' benefits outweigh the larger complexity.

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