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Graphic Organizers Applied to Secondary Algebra Instruction for Students with Learning Disorders

Bob Ives

University of Georgia

Students who have particular difficulty in mathematics are a growing concern for educators. Graphic organizers have been shown to improve reading comprehension and may be applied to upper level secondary mathematics content. In two systematic replications, one randomly assigned group was taught to solve systems of linear equations through direct instruction and strategy instruction. The other group was taught with the same methods with the addition of a graphic organizer. Students who received instruction with the graphic organizers outperformed those who received instruction without the organizers. They also better understood the related concepts as measured by immediate posttests in both replications. The difference in understanding concepts was maintained on a 2–3 week posttest.

INTRODUCTION

> Many important mathematicians and scientists have had great difficulty with lower mathematics but excel at higher mathematics—which is less mechanical, less memory based—but often more visual, more logical based, more conceptual, more philosophical. Einstein had such difficulties—as did Stephen Wolfram, the founder of Wolfram Research and the inventor of the high level, general purpose professional mathematics software program called "Mathematica." (West, 2000, p. 25)

History provides examples of several distinguished mathematicians for whom basic mathematics skills, such as memorization of mathematics facts and rote application of algorithms, were both tedious and difficult. In contrast, these scholars were much more successful with the flexible, sometimes more graphic, thinking required for more advanced mathematics problem solving. In 6 years of experience as a secondary mathematics teacher of students with learning disabilities, I saw the same pattern in many of my students. Quite a few of those students performed within the average range across curricula such as algebra, precalculus, and even calculus. Yet these same students often had significant deficits in language and reading skills, and struggled to memorize basic mathematics facts.

Mainstream classroom instruction in mathematics assumes adequate learner language and reading competence (Bley & Thornton, 1995; Moses & Cobb, 2001; Rivera, 1998). This reliance presents a challenge for teaching students who demonstrate language learning disorders. Geary, 2000, 2003 identified three different subtypes related to calculation disabilities, or dyscalculia. One of Geary's subtypes was characterized by difficulty following procedures. A second involved spatial deficits. The third of Geary's subtypes attributes students' difficulties with calculation to deficits in semantic memory. Geary has been able to demonstrate a high level of comorbidity between this third type of dyscalculia and diagnosed reading disabilities. This comorbidity has also been demonstrated by other researchers (e.g., Jordan, Hanich, & Kaplan, 2003) and confirms that some relationship exists between some language skills and some mathematical skills. Given this relationship between some language disabilities and some mathematics disabilities, students with language difficulties may benefit from instruction that is less dependent on language skills.

Teachers and researchers have begun to explore instructional methods that are less dependent on reading and language comprehension than traditional mathematics instruction. For example, the concrete-semiconcrete-abstract (CSA) teaching sequence has been used to demonstrate basic mathematics concepts (Harris, Miller, & Mercer, 1995; Marzola, 1987; Miller & Mercer, 1993; Miller, Mercer, & Dillon, 1992; Peterson, Mercer, & O'Shea, 1988), as well as more advanced mathematics skills such as subtracting integers (Maccini & Ruhl, 2000), and the transformation of equations (Witzel, Mercer, & Miller, 2003).

Another instructional method that reduces the reading and language comprehension demands placed on students while solving mathematics problems is the use of schema diagrams. Several studies using this approach have reported improved mathematical problem solving with elementary and middle school students with mild disabilities (Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra et al., 1998). In the schema diagram approach, students are taught to distinguish between a few different types of mathematical problems. The students are also taught to match each problem type to a particular schema diagram that is provided to them, and then to use the diagram as a guide to solving the problem.

Requests for reprints should be sent to Bob Ives, Mail Stop 299/Department of Educational Specialties, University of Nevada, Reno, Nevada 89557–0251. Electronic inquiries may be sent to rives@unr.edu.

Although a valuable technique for mathematics instruction, the CSA sequence cannot be readily applied to all types of mathematics. In particular, higher level mathematics concepts and relationships often do not lend themselves to concrete models. Thus, the literature in mathematics intervention provides limited support for teachers who want to help students with these higher level mathematics concepts and relationships when those students demonstrate significant language and reading deficits.

A promising technique from the reading comprehension literature might provide assistance in the teaching of higher level mathematics to students with language and reading deficits. Students who are trying to understand concepts and relationships in their reading have been able to improve their reading comprehension by applying graphic organizers in a variety of ways (Alvermann & Swafford, 1989; Dunston, 1992; Moore & Readance, 1984; Rice, 1994; Robinson, 1998; Swafford & Alvermann, 1989). In these studies, graphic organizers are graphic arrangements of words, phrases, and sentences, and they may also include graphic elements such as arrows, and boxes. The graphic features are intended to indicate relationships between the verbal elements. The concept of graphic organizers can be expanded and modified to apply to mathematics content. In particular, the verbal elements can be replaced by mathematical symbols, expressions, and equations. In this way graphic organizers may be useful for helping students understand concepts and relationships that involve these mathematical symbols, expressions, and equations and that can be represented graphically.

The purpose of this investigation was to address the following three research questions.

- Q1 Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?
- Q2 Will the difference in performance cited in the first research question be maintained for 2–3 weeks after instruction and immediate posttesting are completed?
- Q3 Will the findings of the first question be replicated when graphic organizers are used to teach secondary students with learning disabilities or attention disorders to solve systems of three linear equations in three variables?

STUDY 1

A two-group comparison experimental design was used to investigate the effectiveness of using a graphic organizer to teaching secondary students with learning difficulties to solve systems of linear equations. Following the first investigation, a second study was conducted to provide a systematic replication related to the use of the same graphic organizer with different students learning a different but related skill.

Methods

Setting

The study took place in a private school in Georgia attended by 6th through 12th grade students with learning disabilities and attention disorders. Approximately 200 students attend the school. This site was selected because the school offers an environment in which all students in every class have been identified as having learning problems. With rare exceptions, class sizes are less than 10 students.

Participants

Of the 14 students in the graphic organizer (GO) group, 10 (71 percent) were male and 4 (29 percent) were female. Ten of these 14 students had been diagnosed with language-related disabilities (reading, writing, and/or general language). This distribution compares with that of the control; (CO) group of 16 in which 11 (69 percent) were male and 5 (31 percent) were female. In this group, 11 of the 16 students had been diagnosed with language-related disabilities (reading, writing, and/or general language). The ages of the GO group ranged from 13.6 to 19.3 years and averaged 15.9 years ($SD = 1.3$). For the CO group the age range was 14.7 to 17.9 years with a mean of 15.8 ($SD = 0.9$). There was one Asian-American student in the GO group. All other students were Caucasian American. English was the first language for all students. The intelligence (IQ) scores of the GO group, expressed as standard scores, ranged from 85 to 136 and averaged 100 ($SD = 15$). For the CO group the IQ range, in standard scores, was 80 to 143 with a mean of 102 ($SD = 18$). Table 1 reports socioeconomic status, grade level, and diagnoses for both groups. Socioeconomic status was estimated as the highest educational degree completed by either parent.

Instruments

Graphic Organizer. Figure 1 shows an example of a completed graphic organizer as a two by three (two rows and three columns) array of rectangular cells with Roman numeral column headings. This organizer was used in its entirety in Study 2. However, in Study 1 only columns II and I were used for these smaller systems of equations.

In a typical system of equations, the solving of the system involves working from cell to cell in a clockwise direction starting with the top left cell. The top row is used to combine equations in order to eliminate variables until an equation in one variable is produced. Once this equation is found, the bottom row serves to guide the finding of successive roots until the entire system is solved. Generally, both the relative positions of symbolic content elements to each other and their positions relative to the frame indicate relationships between the elements. For example, across the top row, each column only contains equations with the same number of variables as the heading of the column. As the solver works from left to right, moving across columns reinforces the concept of eliminating variables. An equation with only one variable

TABLE 1
Characteristics of Student Participants in Study 1

	Graphic Organizer Group	*Control Group*
Highest Parent Degree as N (%):		
HS	1 (7%)	0 (0%)
Assoc.	1 (7%)	2 (13%)
BA/BS	8 (57%)	9 (56%)
Master's	2 (14%)	5 (31%)
Doctoral	2 (14%)	0 (0%)
Grade Level as N (%):		
7	1 (7%)	0 (0%)
8	2 (14%)	4 (25%)
9	6 (43%)	7 (44%)
10	3 (21%)	4 (25%)
11	1 (7%)	1 (6%)
12	1 (7%)	0 (0%)
Diagnoses as N (%):		
ADHD	11 (79%)	12 (75%)
LD/Reading	5 (36%)	6 (38%)
LD/Language	3 (21%)	3 (19%)
LD/Mathematics	3 (21%)	2 (13%)
LD/Written	2 (14%)	2 (13%)
Tourette's	2 (14%)	1 (6%)
OCD	0 (0%)	2 (13%)

can be solved for a unique solution, whereas equations with more than one variable will typically have an infinite number of possible solutions. This concept is reinforced by the use of the far right column where equations only have one variable and can be moved down into the bottom row for solving. Thus the relative locations of the columns are related to important concepts in the solution of these systems. Similarly, while the top row is used for eliminating variables in equations, the bottom row is used to solve equations of one variable for specific solutions for each of the variables in the system. In the right column an equation of one variable is solved for a unique solution. In the second column this first solution is substituted into an equation with two variables in order to solve for the remaining variable in that equation. Finally, the first two solutions are substituted into an equation of three

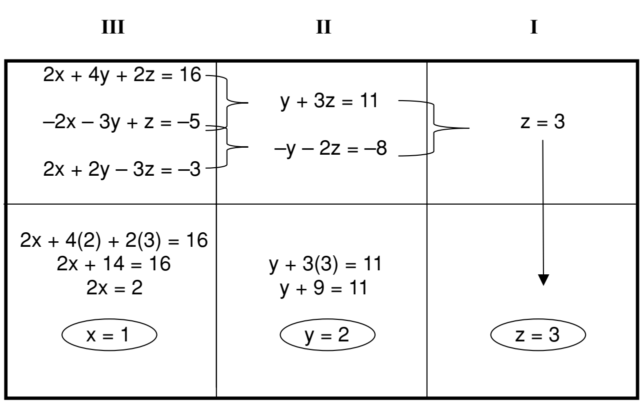

FIGURE 1 A completed graphic organizer for solving systems of linear equations in three variables.

variables from the third column in order to solve fore the last variable. While the columns reinforce the procedures, they also support the concept of using existing solutions to create new equations with only one variable that can be solved. These concepts, and others, are articulated and reinforced through the interaction of the instruction and the use of the graphic organizer, as described by Ives and Hoy (2003).

Test of Prerequisite Skills. A researcher-constructed test of prerequisite skills was administered to all students in this study. The results of this test were used to modify lessons to ensure that both groups were familiar with prerequisite skills relevant to solving systems of linear equations by using linear combinations. Parts of lessons that specifically addressed prerequisite skills were the same for both groups and did not include the use of graphic organizers. Prerequisite skills assessed by this measure included: (1) solving linear equations in one variable, (2) substituting a value in place of variable in linear equations of two variables so that they can be solved for the remaining variable, (3) combining linear equations with two variables, (4) multiplying linear equations in two variables by a constant, and (5) finding common multiples for two positive integers. All of these skills had also been covered in lower level mathematics classes or earlier in the Algebra I course.

Test of Content Skills. The researcher-constructed content skills test had two sections. The first section included a group of three short-answer questions designed to assess how well students could conceptually justify the procedures for solving systems of equations in two variables. These questions did not require any calculation. A sample question is:

How many solutions does each equation have?

$$5x = 35$$
$$3x - y = 16$$

The second section was comprised of four systems of equations to be solved. The first system required no multiplying of equations and began with two equations in two variables. An example of this kind of system is:

$$2x - y = 19$$
$$-2x + 4y = -4$$

The second system required multiplying equations but still began with two equations in two variables. For example:

$$3x - 2y = -11$$
$$2x + y = -5$$

The third system required multiplying equations and began with one equation in two variables and one equation in one variable. For example:

$$-2x - y = 14$$
$$3x = -27$$

The fourth system involved generalization. Three linear equations in three variables were given, but no multiplication of equations was required, and all three equations contained all three variables. Students had not been taught to solve these larger systems. For example:

$$2x - y + 3z = 7$$
$$-2x + 4y - 5z = -3$$
$$2x - 7y + 8z = 0$$

All of these systems were taken from popular textbooks and were selected to be typical of the kinds of systems solved by students in general education classes when they are being taught to solve systems of linear equations. All of the selected systems had integer coefficients and solutions. Two versions of this test were generated by creating twice as many of each type of item as was needed for a single test, and randomly assigning items by type to create two equivalent versions.

Teacher-Generated Assessment. Two weeks before instruction began, the teachers provided tests of the material that would be covered in class during Study 1 classes. These instruments reflected the teachers' performance expectations for the students and were used as an outcome variable to test for group differences in mean scores.

Procedures for Study 1

Advance Preparation

Lesson Planning. The lessons were constructed to teach all of the skills and difficulty levels represented on the teacher-generated tests. Lessons included elements of both strategy and direct instruction as defined and shown by Swanson, Hoskyn, and Lee (1999) to be effective for students with learning disabilities. Both strategy and direct instruction place great reliance on language skills. For example, based on their review of intervention literature, Swanson et al., 1999 noted that direct instruction typically included asking questions, providing repeated feedback, and administering probes. Strategy instruction typically included elaborate explanations, verbal modeling, reminders, dialogue, and asking questions. All of these components for both approaches are typically, if not necessarily, provided through language.

The first lesson was a review and assessment of prerequisite skills. The second lesson presented relatively simple examples of systems of equations, and the next two lessons introduced variations. The content of the lessons was modified on an ad hoc basis during the study in response to the progress that students were making with the material. This flexibility reflected a realistic teaching experience that recognized the importance of student needs. Adjustments to the lesson plan content were carried out in all six sections of the course and were not a systematic difference between sections. Detailed excerpts of similar lessons have been offered by Ives and Hoy (2003).

Classroom Acclimation. I attended all five sections of the course every day for at least 1 week prior to beginning the instructional phase of the experiment. During this time I occasionally provided tutoring and support characteristic of a teaching assistant to give the students time to become accustomed to my presence.

In-Class Procedures and Instruction

On the first day of instruction, the students completed the test of prerequisite skills. I read the instructions and problems aloud, and students were reassured that they were not expected to be able to do all of the problems but were encouraged to attempt as much of the test as they could. They were given ample time to complete whatever problems they were able to complete. Once the prerequisite tests were completed and collected, instruction began with a review of the prerequisite skills. Prepared lesson plans were carried out at the conclusion of the prerequisite skills review. Both groups received the same number of hours of instruction, the same number of practice problems, and the same homework assignments.

On the last day of instruction, the students completed one version of the content skills test. The choice of test version was counterbalanced across students within each group. The teachers administered their teacher-generated tests whenever they had been planned in the normal course of the classes. This occurred within a week of completion of the instructional phase of the study. Between 2 and 3 weeks after the instructional phase of the study was completed the students completed the second version of the content skills test.

Study 1 Results

Prerequisite Skills

Results of the test of prerequisite skills were analyzed to determine if groups differed in their level of preparedness for the new skills and concepts involved in solving systems of linear equations. The 14 items on the prerequisite skills tests were scored as right or wrong for a maximum of 14 points. Error analysis showed that most of the errors were arithmetic and not conceptual or procedural. The GO group mean (Mean = 11.36, $SD = 1.95$) was not statistically significantly different from the CO group mean (Mean 12.00, $SD = 1.49$) although the GO group mean was slightly lower ($F = .766$, $p = .391$). Thus, if the groups were not comparable in prerequisite skills, any difference would tend to favor the control group.

Language Control

Controlling language in instruction is particularly critical to ensure that the verbal instruction provided to students is comparable across conditions in order to better isolate the influence of the graphic organizer on the outcome variables. The goal in monitoring the relevant language used across the groups was to control for language differences as an intervening variable. For this reason, the teachers categorized oral instruction statements. The statements were grouped into 1 of 4 categories. The first category included any statement that indicated or asked the number of different variables in one or more equations. The second category of statements included any entry that addressed the question of whether items in two different equations matched, or were equal, in some way. The third category of statements included entries that questioned or stated whether an equation was solvable. The fourth category of entries included any statement or question that involved the number of equations being addressed. These four categories of statements relate to concepts involved in understanding the steps for solving systems of equations using linear combinations. In addition to controlling the overall levels of relevant language across groups, this coding compares relevant language use across groups related to each of these specific concepts separately.

Each classroom teacher was trained to carry out this categorization using definitions and examples for each of the four categories. The teacher rated each of the statements in a sample transcript. Each teacher's ratings were compared to prior ratings I had done on the same transcript. Discrepancies were discussed, and interrater agreement was calculated as the percent of exact matches when at least one of the raters scored an entry as belonging to 1 of the 4 categories. After initial training, the classroom teacher categorized each oral statement during instruction according to the same scheme while I carried out classroom instruction.

After initial training in the coding procedure, the high school teacher's interrater reliability with the sample precoded by the investigator was 93 percent. For the middle school teacher the interrater reliability was 96 percent. The coding done by these teachers during the instruction of new material (Days 2, 3, and 4 of the lesson plans) was averaged by class period for each group. Table 2 shows these averages for both the graphic organizer (GO) group and the control (CO) group. The last coding category does not apply to systems of two equations and two variables so there are no entries for that category in Study 1. The values are similar across the two groups, supporting the claim that verbal instruction was comparable for both groups.

TABLE 2
Characteristics of Student Participants in Study 2

	Graphic Organizer Group	*Control Group*
Highest Parent Degree as N (%):		
HS	1 (20%)	0 (0%)
Assoc.	0 (0%)	1 (20%)
BA/BS	3 (60%)	1 (20%)
Master's	1 (20%)	3 (60%)
Doctoral	0 (0%)	0 (0%)
Diagnoses as N (%):		
ADHD	3 (60%)	3 (60%)
LD/Reading	4 (80%)	1 (20%)
LD/Language	1 (20%)	1 (20%)
LD/Mathematics	1 (20%)	2 (40%)
LD/Written	2 (40%)	1 (20%)
Nonverbal	0 (0%)	1 (20%)

Data Analyses for Questions 1 and 2 in Study 1

Analysis of variance (ANOVA) was used for each comparison of means in both studies. All skewness and kurtosis values were within the range of ± 1.0. On the basis of these results I assumed there was an adequate approximation to normal distributions. The values for the Levene's Statistics (Huck & Cormier, 1996) ranged from .14 to 1.00. These values did not justify rejecting the null hypotheses that the variances were equal in each case. All ANOVAs were conducted using an alpha level of 0.10. Results from teacher-generated tests and investigator-generated tests were both used to address this question. There were at least three reasons for considering an alpha level for this study that is larger than the usual conventions. One was that this study investigated the effectiveness of an intervention for which little, if any research existed. As a result the cost of a Type II error was increased. If the study data failed to indicate that graphic organizers were effective, even though they actually were, then further investigation would be less likely and a useful intervention might be lost. A second reason was that this was a field based investigation rather than a laboratory study. Third, as is often the case in the field of special education, the sample size for this study was limited. For these reasons, an alpha level of .10 was used. Results are presented in terms of effect size and statistical significance (Ives, 2003).

To answer the first and second research questions (Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers? Will the difference in performance cited in the first research question be maintained for 2–3 weeks after instruction and immediate posttesting are completed?), for the teacher-generated tests, each teacher scored his or her own tests and assigned grades for individual items. Both teachers included the content for the study in a test that covered additional material. The two tests did not include identical items. However, both teachers included only systems of two linear equations in two variables in which all coefficients were single digit integers, and all solutions were made up of integers. For these reasons, the difficulty level was considered equivalent for the two tests and their results were combined for statistical analysis. The points each student received on the questions related to the study content were converted into percentages of the number of available points for those problems. The mean score for the GO group was statistically significantly higher than the mean score for the CO group on these teacher-generated tests ($F = 3.14$, $p = .087$, $\eta^2 = .101$). The effect size falls within the medium to large range suggested by Cohen (1988).

The investigator-generated content skills test resulted in two scores. The first three questions on the test were designed to test for understanding of the concepts behind the solution process. The last four questions on the content skills test required that the student solve systems of equations. Because the process of solving these systems of equations involved multiple steps, these systems were graded to allow partial credit. For each system, a point was earned for each new equation generated that contributed to the solution of the system. An additional point was earned for each correctly assigned value in the final solution. Scores from each of these sections were analyzed separately using the same approach as that used for the scores on the teacher-generated test.

Results for the concepts section of the investigator-generated test were compared across groups for the immediate posttest ($F = 7.86, p = .009, \eta^2 = .219$) and the follow-up or maintenance test ($F = 6.11, p = .020, \eta^2 = .179$). The mean scores for the GO group were statistically significantly higher than the mean scores for the CO group on concept sections of both the immediate posttests and the follow-up posttests. Both effect sizes are large.

Results for the system solving section of the investigator-generated test were compared across groups for the immediate posttest ($F = 0.19, p = .664, \eta^2 = .007$) and the follow-up or maintenance test ($F = 0.00$, $p = 1.000$, $\eta^2 = .000$). The mean scores for the two groups were not statistically significantly different.

TABLE 3
Class Period Averages for Graphic Organizer and Control Groups on Verbal Coding Categories in Study 1

Coding Category	*Graphic Organizer*	*Control*
Number of variables	7.7	6.7
Matching or equal items	3.3	3.7
Solvable?	3.7	4.0
Number of equations	0	0
Totals	14.7	14.3

STUDY 2

The purpose of the second study was to provide a systematic replication of the first study with a different population and related content. The same graphic organizer was used in both studies. This second study differed from the first in four important ways. First, the mathematics content was systems of three linear equations with three variables rather than two linear equations with two variables. Second, Study 2 included a much smaller number of student participants. Statistical analysis was not expected to produce statistically significant results because of the loss of power. Thompson's (1993, 1996) recommendation of following up statistical significance tests that did not reach significance with a "what if" analysis was planned. Third, no follow up test for maintenance was included. Finally, no teacher-generated test was included in the study. As a result, Question 2 was not tested for Study 2.

Methods

Participants

All 10 participants in both groups were male. The ages of the GO group ranged from 16.9 to 19.3 years and averaged 17.6 years ($SD = 0.4$). For the CO group the age range was 17.2 to 18.6 years with a mean of 17.8 ($SD = 0.3$). There was one Asian American student in the GO group. All other students were Caucasian American. English was the first language for all students. The intelligence (IQ) scores of the GO group, expressed as standard scores, ranged from 96 to 130 and averaged 107 ($SD = 14$). For the CO group the IQ range, in standard scores, was 91 to 124 with a mean of 100 ($SD = 16$). Each group included one senior and four juniors. Eight of the 10 participants had been diagnosed with reading and/or other language disabilities. Table 3 reports socioeconomic status, and diagnoses for both groups.

Instruments

As in Study 1, the graphic organizer itself was the critical instructional tool being tested in this study. An investigator-generated test of prerequisite skills was used to modify lessons to ensure that both groups were familiar with prerequisite skills relevant to solving systems of linear equations by using linear combinations. Sources of outcome data included the content skills test of concepts and system solving used to compare group performance. In addition, data were collected by the classroom teacher to analyze the consistency of language of the instruction across conditions.

Graphic Organizer. The graphic organizer for this study is shown in Figure 1. The difference between the graphic organizers for the two studies was simply that the graphic organizer for Study 2 included the third column on the left that the graphic organizer for Study 1 did not use.

Test of Prerequisite Skills. I constructed four items to test each of the following prerequisite skills: (1) solving linear equations in one variable, (2) substituting values in place of variables in linear equations until they can be solved for one remaining variable, (3) combining (adding) linear equations, (4) multiplying linear equations by a constant, and (5) finding common multiples. The prerequisite skills test for Study 2 covered the same set of skills as those covered on the prerequisite skills test for Study 1. However, some of the tasks included higher difficulty levels for Study 2. All of these skills are typically covered in lower level mathematics classes.

Test of Content Skills. The content test followed a format parallel to that used in Study 1. The test included two sections. The first section was a group of six short-answer questions designed to assess how well students understood the concepts that justify the procedures for solving these systems of equations. These concepts are related to the coding categories that were used to classify instructional statements and questions as described in the *Language Control* section of Study 1. Following the first six questions were four systems of equations to be solved. The first system required no multiplying of equations and began with three equations in three variables. The second system required multiplying equations but still began with three equations in three variables. The third system required multiplying equations and began with one equation in three variables and one equation in two variables. The fourth system involved a generalization to four linear equations in four variables. This problem required no multiplication of

TABLE 4
Averages per Class Period for Verbal Coding Categories in Control and Graphic Organizer Classes in Study 2

Coding Category	*Graphic Organizer*	*Control*
Number of variables	13.3	9.3
Matching or equal items	9.7	9.7
Solvable?	6.0	6.7
Number of equations	2.0	3.3
Totals	31.0	29.0

equations, and all four initial equations contained all four variables. All of the selected systems had integer coefficients and solutions.

Study 2 and Cross-Study Results

Language Control

Table 4 shows verbal coding results for the Algebra II teacher averaged across classes for both the GO group and the CO group. The values are reasonably similar across the two groups, supporting the claim that verbal instruction was comparable for both groups.

Data Analysis for Question 1 in Study 2

To answer the first research question (Will secondary students with learning disabilities or attention disorders who have been taught to solve systems of two linear equations in two variables with graphic organizers perform better on related skill and concept measures than students instructed on the same material without graphic organizers?), ANOVA was used to compare mean scores across the two groups on each section of the investigator-generated test. For the system-solving section of the test, the difference between mean scores was not statistically significant ($F = 1.09, p = .327, \eta^2 = .120$). This result is not surprising given that small number of participants. However, the difference in means is in the same direction as that from Study 1 and the effect sizes from both studies are quite similar in magnitude (.120 compared to .101). The effect size falls within the medium to large range suggested by Cohen (1988). Holding the effect size constant, the estimated number of participants necessary to make this result statistically significant is 26, or 13 in each group. This compares quite well with the results of Study 1.

Unexpectedly, these results were statistically significant based on an alpha level of .10 ($F = 11.26, p = .100, \eta^2 = .585$). In addition, the effect size is quite large. These results support the first question for system solving in Study 2, in contrast to the lack of support from the system solving results of the content skills test in Study 1.

Data Analyses for Question 3 Across the Two Studies

The third research question looked at whether results from the first study were replicated in the second study. Figure 2 compares the results for the content skills immediate posttests from both studies. Both studies produced differences favoring the GO groups on the concept sections of the tests. This is seen when comparing the first two columns from each group of four columns. In contrast, only Study 2 resulted in a difference

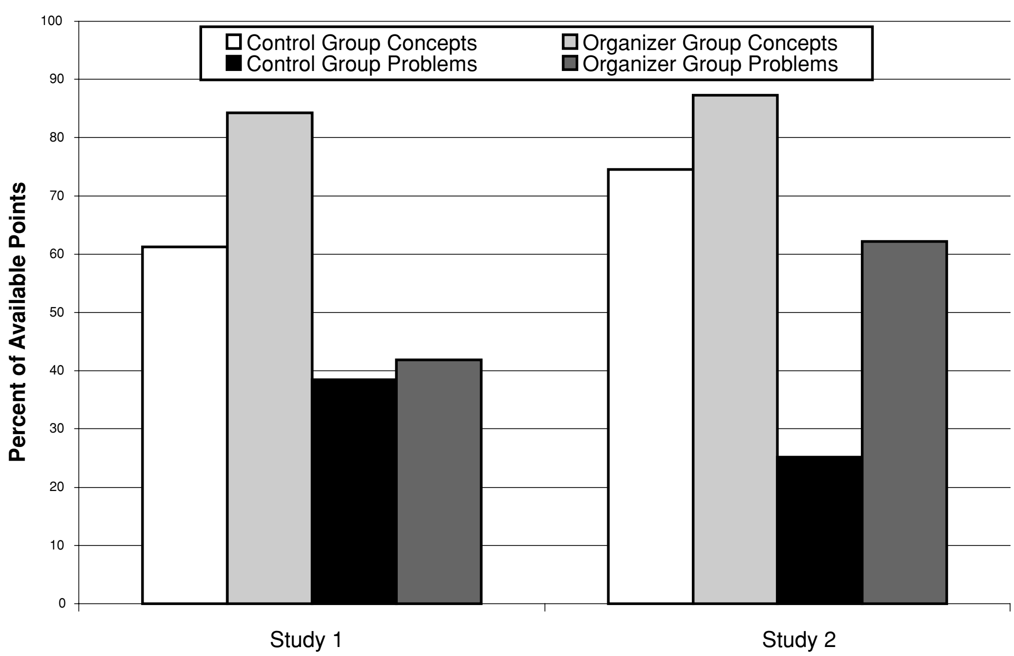

FIGURE 2 Comparison of content skills test results for control and graphic organizer groups across both studies.

between the groups on the system solving portions of the content skills tests. This difference does favor the GO group, thus supporting the first question. This comparison can be seen in the last two columns of each group of four. Of course, the first study did yield a statistically significant difference favoring the GO group on the teacher-generated test results. This indicates some partial support for the third question with respect to solving the systems.

DISCUSSION

The results of these two studies consistently showed that students who worked with the graphic organizers had a stronger grasp of the conceptual foundations for solving systems of linear equations than did the students who did not work with the graphic organizers based on their performance on the conceptual sections of the investigator-generated tests. This advantage in conceptual understanding was maintained over a few weeks in Study 1. With respect to student performance in actually solving these systems, results are less consistent. In no case did the students in the CO groups demonstrate better performance in solving systems of equation than the students in the GO groups did. However, students in the GO were significantly more successful at solving systems of equations in Study 2.

In the context of the inconsistent results for the system solving in Study 1, it may be helpful to consider the results from the system-solving section of the content skills test in Study 2. Unlike Study 1, this comparison of means yielded a statistically significant difference in spite of the fact that there were only five participants per group. This unexpected result may be attributable to an important difference between solving systems of equations with two variables as opposed to systems with three variables. A review of students' written responses on the system-solving sections of the content skills tests in both studies as well as the teacher-generated test in Study 1 support this possibility. In Study 1 only 2 students out of 14 in the GO group tried to use the graphic organizer consistently. In contrast, 4 of the 5 students in the GO group from Study 2 used the graphic organizer throughout the system solving section of the content skills test. This difference may be the result of system complexity. As systems get larger their complexity grows exponentially. Students in Study 1 may have seen the systems as manageable without using the graphic organizer, and students tackling the more complex systems in Study 2 may have seen more benefit to the approach because it was more difficult to complete these systems without the guidance provided by the graphic organizers.

While these results are encouraging, limitations related to these studies should be noted and addressed in future studies. Because instruction was provide by the investigator for the studies reported here, it is not clear what the results would be if classroom teachers used the graphic organizers themselves. Swanson, Hoskyn, and Lee (1999) have noted that effect sizes tend to be higher for intervention studies with students who have learning disabilities when the investigator implements the instruction rather than the regular classroom teachers. Regardless of the reason for this difference, the utility of interventions is lost if they are not adequately effective when classroom teachers implement them.

A second limitation related to these studies is that they both have rather small sample sizes of students with a mixture of disabilities, even though language disabilities predominate. Replications involving larger samples of students with more well-defined disabilities would help test whether students with specific reading, writing, and other language disabilities all benefit from instruction with graphic organizers.

In addition, these studies offer little data on the technical adequacy of the measures. Researcher-generated tests should have good content validity and social validity given that they represent how the students are being held accountable. Evidence for the content validity of the investigator-generated tests is offered as well. However, no broader construct validity data are available for any of these instruments beyond the data reported here.

Despite these limitations, the results of these studies suggest that using graphic organizers to teach higher level mathematics to students with language and attention problems leads to improved conceptual understanding of that mathematics content. The use of graphic organizers may also lead to improved system solving when the systems become complex enough to challenge the ability of students to keep the process organized without the organizers. These results are consistent with the assumption that students with language disorders may particularly benefit from instruction that provides content supported by nonverbal associations. Given the high comorbidity between students with calculation disabilities and those with reading disabilities, confirming this proposition would provide important evidence to guide mathematics instruction for these students.

These exploratory results are encouraging enough to warrant further investigation of the applicability of graphic organizers to other mathematics topics, and other classroom settings. Additionally, further investigations should focus on teaching higher level mathematics to students with high incidence cognitive disabilities, particularly when those disabilities involve language-related deficits.

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About the Authors

Bob Ives is an Assistant Professor at the University of Nevada, Reno and received his Ph.D. in Special Education from the University of Georgia. His research interests include learning disabilities in mathematics, special education assessment, and educational research methods. Dr. Ives is co-director of the Research in Romania program which supports educational reform while giving U.S. faculty and student opportunities to carry out educational research there.