Graphs and Homomorphisms

Pavol Hell

Simon Fraser University, Burnaby, B.C., Canada

and

Jaroslav Nesetfil

Charles University, Prague, The Czech Republic



CONTENTS

1	Intr	oduction	1
	1.1	Graphs, digraphs, and homomorphisms	1
	1.2	Homomorphisms preserve adjacency	3
	1.3	Homomorphisms generalize colourings	6
	1.4	The existence of homomorphisms	10
	1.5	Homomorphisms generalize isomorphisms	16
	1.6	Homomorphic equivalence	18
	1.7	The composition of homomorphisms	20
	1.8	Homomorphisms model assignments and schedules	27
	1.9	Remarks	33
	1.10	Exercises	34
2	Products and retracts		
	2.1	The product	37
	2.2	Dimension	40
	2.3	The Lovasz vector and the Reconstruction Conjecture	43
	2.4	Exponential digraphs	46
	2.5	Shift graphs	47
	2.6	The Product Conjecture and graph multiplicativity	50
	2.7	Projective digraphs and polymorphisms	57
	2.8	The retract	58
	2.9	Isometric trees and cycles	60
	2.10	Reflexive absolute retracts	64
	2.11	Reflexive dismantlable graphs	68
		Median graphs	72
		Remarks	76
	2.14	Exercises	78
3	The	partial order of graphs and homomorphisms	81
	3.1	The partial orders C and Cs	81
	3.2	Representing ordered sets	82
	3.3	Incomparable graphs and maximal antichains in Cs	85
	3.4	Sparse graphs with specified homomorphisms	89
	3.5	Incomparable graphs with additional properties	93
	3.6	Incomparable graphs on <i>n</i> vertices	94
	3.7	Density	96
	3.8	Duality and gaps	98
	3.9	Maximal antichains in C	101
		Bounds	101
	0.10		

CONTENTS

Index			239	
Re	222			
	6.6	Exercises	219	
	6.5	Remarks	218	
	6.4	Oriented and acyclic colourings	212	
	6.3	T-colourings	210	
	6.2	Fractional colourings	200	
-	6.1	Circular colourings	192	
6	Colouring—variations on a theme			
	5.10	Exercises	187	
	5.9	Remarks	186	
	5.8	Generalized split graphs	183	
	5.7	Trigraph homomorphisms	178	
	5.6	List homomorphisms and retractions	170	
	5.5	Pair consistency and majority functions	166	
	5.4	Duality and consistency	161	
	5.3	Digraph homomorphisms and CSPs	151	
	5.2	Dichotomy for graphs	142	
5	5.1	e	142	
5	Test	ting for the existence of homomorphisms	142	
	4.10	Exercises	139	
	4.9	Remarks	138	
	4.8	Some categories are not rich enough	135	
	4.7	A combinatorial obstacle to representation	132	
	4.6	Representation	128	
	4.5	Categories	122	
	4.4	The replacement operation	113	
	4.2 4.3	An excursion to infinity	109	
	4.1	Rigid digraphs	109	
	1 ne 4.1	structure of composition Introduction	109 109	
		Exercises	100	
	3 1 1	Remarks	106	