# GRAPHS AND THEIR USES 

by

## Oystein Ore

Yale University
Revised and updated edition prepared by
Robin J. Wilson
The Open University, England


34

THE MATHEMATICALASSOCIATION
OF A M ERIC A

## Contents

Introduction ..... 3
Chapter 1 What Is a Graph? ..... 5
1.1 Team Competitions ..... 5
1.2 Null Graphs and Complete Graphs ..... 7
1.3 Isomorphic Graphs ..... 9
1.4 Planar Graphs ..... 12
1.5 Planar Problems ..... 14
1.6 The Number of Edges in a Graph ..... 18
1.7 Interval Graphs ..... 21
Chapter 2 Connected Graphs ..... 24
2.1 Connected Components ..... 24
2.2 The Problem of the Bridges of Königsberg ..... 26
2.3 Eulerian Graphs ..... 27
2.4 Finding Your Way ..... 30
2.5 Hamiltonian Cycles ..... 31
2.6 Puzzles and Graphs ..... 33
Chapter 3 Trees ..... 37
3.1 Trees and Forests ..... 37
3.2 Cycles and Trees ..... 39
3.3 The Connector Problem ..... 41
3.4 The Travelling Salesman Problem Revisited ..... 44
3.5 Bracing Frameworks ..... 46
3.6 Streets and Squares ..... 49
Chapter 4 Matchings ..... 53
4.1 The Jobs and the Applicants ..... 53
4.2 Other Formulations ..... 56
4.3 Round-Robin Matchings ..... 59
Chapter 5 Directed Graphs ..... 63
5.1 Team Competitions Re-examined ..... 63
5.2 The Problems of One-Way Traffic ..... 63
5.3 Degrees ..... 70
5.4 Genetic Graphs ..... 72
5.5 Finding the Shortest Route ..... 78
Chapter 6 Questions Concerning Games and Puzzles ..... 83
6.1 Puzzles and Directed Graphs ..... 83
6.2 The Theory of Games ..... 86
6.3 The Sportswriter's Paradox ..... 90
Chapter 7 Relations ..... 95
7.1 Relations and Graphs ..... 95
7.2 Special Conditions ..... 97
7.3 Equivalence Relations ..... 101
7.4 Partial Orders ..... 105
Chapter 8 Planar Graphs ..... 109
8.1 Conditions for Planar Graphs ..... 109
8.2 Euler's Formula ..... 113
8.3 Graph Relations and Dual Graphs ..... 115
8.4 The Platonic Solids ..... 117
8.5 Mosaics ..... 121
Chapter 9 Map Coloring ..... 125
9.1 The Four Color Problem ..... 125
9.2 The Five Color Theorem ..... 129
9.3 Coloring Maps on Other Surfaces ..... 133
Solutions ..... 136
Bibliography ..... 146
Glossary ..... 148
Index ..... 151

