

# GRASP with path-relinking for the quadratic assignment problem



Mauricio G.C. Resende, AT&T Research  
joint work with:  
Carlos A.S. Oliveira, U. of Florida  
Panos M. Pardalos, U. of Florida

Talk given at WEA 2004  
Angra dos Reis, Brazil  
May 2004

# Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

# Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

# Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

# Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

# Summary

- The quadratic assignment problem (QAP)
- GRASP for QAP
- Path-relinking for QAP
- Computational results
- Concluding remarks

# Quadratic assignment problem (QAP)

- Given  $N$  facilities  $f_1, f_2, \dots, f_N$  and  $N$  locations  $l_1, l_2, \dots, l_N$
- Let  $A^{N \times N} = (a_{i,j})$  be a positive real matrix where  $a_{i,j}$  is the flow between facilities  $f_i$  and  $f_j$
- Let  $B^{N \times N} = (b_{i,j})$  be a positive real matrix where  $b_{i,j}$  is the distance between locations  $l_i$  and  $l_j$

# Quadratic assignment problem (QAP)

- Given  $N$  facilities  $f_1, f_2, \dots, f_N$  and  $N$  locations  $l_1, l_2, \dots, l_N$
- Let  $A^{N \times N} = (a_{i,j})$  be a positive real matrix where  $a_{i,j}$  is the flow between facilities  $f_i$  and  $f_j$
- Let  $B^{N \times N} = (b_{i,j})$  be a positive real matrix where  $b_{i,j}$  is the distance between locations  $l_i$  and  $l_j$



# Quadratic assignment problem (QAP)

- Given  $N$  facilities  $f_1, f_2, \dots, f_N$  and  $N$  locations  $l_1, l_2, \dots, l_N$
- Let  $A^{N \times N} = (a_{i,j})$  be a positive real matrix where  $a_{i,j}$  is the flow between facilities  $f_i$  and  $f_j$
- Let  $B^{N \times N} = (b_{i,j})$  be a positive real matrix where  $b_{i,j}$  is the distance between locations  $l_i$  and  $l_j$

# Quadratic assignment problem (QAP)

- Let  $p: \{1,2,\dots,N\} \rightarrow \{1,2,\dots,N\}$  be an assignment of the  $N$  facilities to the  $N$  locations
- Define the cost of assignment  $p$  to be

$$c(p) = \sum_{i=1}^N \sum_{j=1}^N a_{i,j} b_{p(i),p(j)}$$

- QAP: Find a permutation vector  $p \in \prod_N$  that minimizes the assignment cost:

$$\min c(p): \text{subject to } p \in \prod_N$$

# Quadratic assignment problem (QAP)

- Let  $p: \{1,2,\dots,N\} \rightarrow \{1,2,\dots,N\}$  be an assignment of the  $N$  facilities to the  $N$  locations
- Define the cost of assignment  $p$  to be

$$c(p) = \sum_{i=1}^N \sum_{j=1}^N a_{i,j} b_{p(i),p(j)}$$

- QAP: Find a permutation vector  $p \in \prod_N$  that minimizes the assignment cost:

$$\min c(p): \text{subject to } p \in \prod_N$$

# Quadratic assignment problem (QAP)

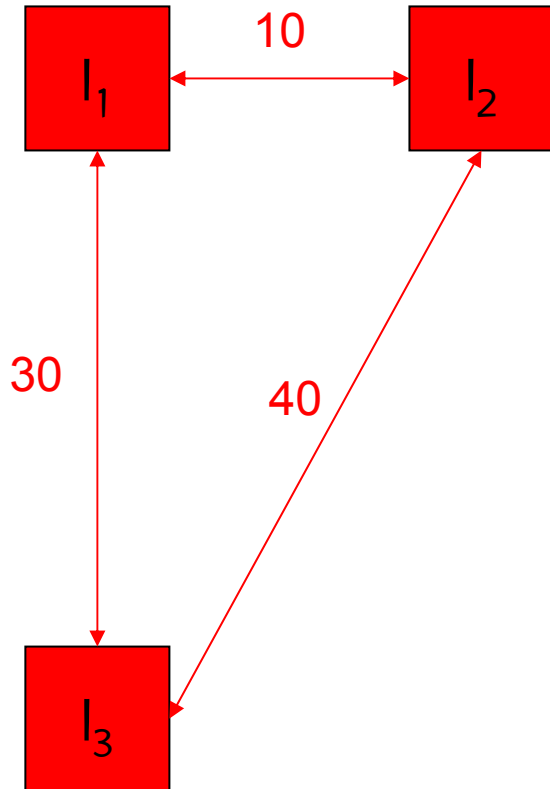
- Let  $p: \{1,2,\dots,N\} \rightarrow \{1,2,\dots,N\}$  be an assignment of the  $N$  facilities to the  $N$  locations
- Define the cost of assignment  $p$  to be

$$c(p) = \sum_{i=1}^N \sum_{j=1}^N a_{i,j} b_{p(i),p(j)}$$

- QAP: Find a permutation vector  $p \in \Pi_N$  that minimizes the assignment cost:

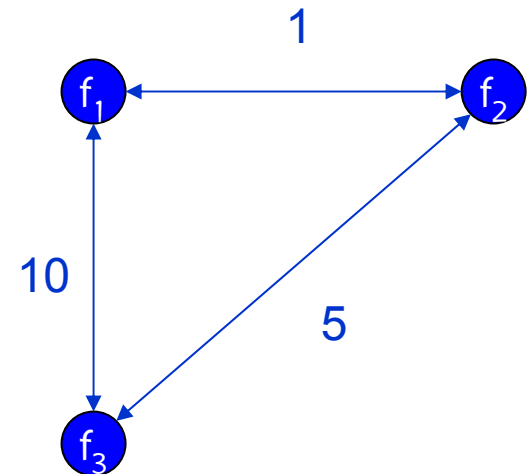
$$\min c(p): \text{subject to } p \in \Pi_N$$

# Quadratic assignment problem (QAP)



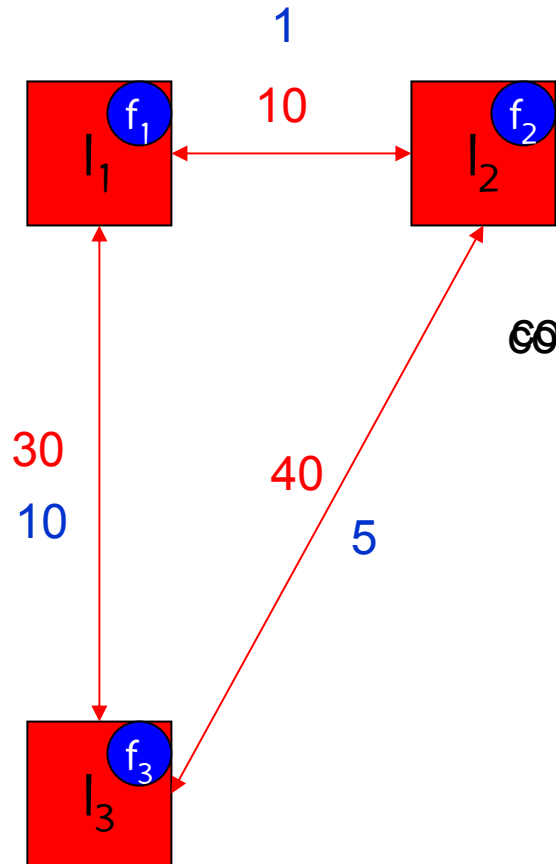
locations and distances

cost of assignment:  $10 \times 1 + 30 \times 10 + 40 \times 5 = 510$



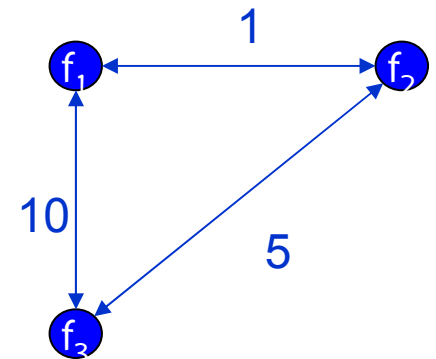
facilities and flows

# Quadratic assignment problem (QAP)



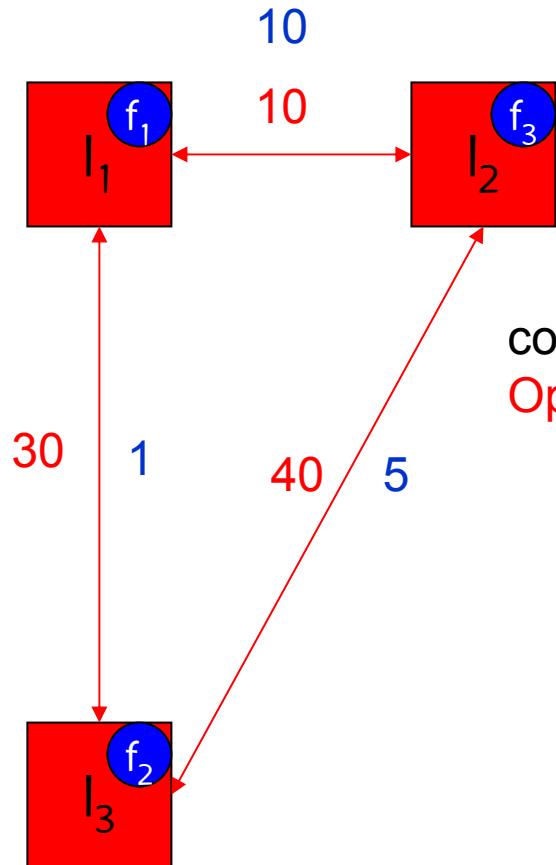
swap locations of facilities  $f_2$  and  $f_3$

cost of assignment:  $10 \times 10 + 30 \times 10 + 40 \times 5 = 630$



facilities and flows

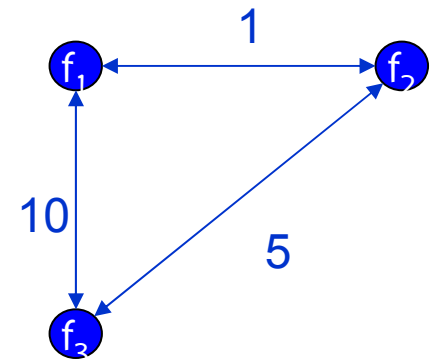
# Quadratic assignment problem (QAP)



swap locations of facilities  $f_1$  and  $f_3$

cost of assignment:  $10 \times 10 + 30 \times 5 + 40 \times 1 = 290$

Optimal!



facilities and flows

# GRASP for QAP

- GRASP \* multi-start metaheuristic: greedy randomized construction, followed by local search (Feo & Resende, 1989, 1995; Festa & Resende, 2002; Resende & Ribeiro, 2003)
- GRASP for QAP
  - Li, Pardalos, & Resende (1994): GRASP for QAP
  - Resende, Pardalos, & Li (1996): Fortran subroutines for dense QAPs
  - Pardalos, Pitsoulis, & Resende (1997): Fortran subroutines for sparse QAPs
  - Fleurent & Glover (1999): memory mechanism in construction



# GRASP for QAP

- GRASP \* multi-start metaheuristic: greedy randomized construction, followed by local search (Feo & Resende, 1989, 1995; Festa & Resende, 2002; Resende & Ribeiro, 2003)
- GRASP for QAP
  - Li, Pardalos, & Resende (1994): GRASP for QAP
  - Resende, Pardalos, & Li (1996): Fortran subroutines for dense QAPs
  - Pardalos, Pitsoulis, & Resende (1997): Fortran subroutines for sparse QAPs
  - Fleurent & Glover (1999): memory mechanism in construction

# GRASP for QAP

```
repeat {  
    x = GreedyRandomizedConstruction(▪);  
    x = LocalSearch(x);  
    save x as x* if best so far;  
}  
return x*;
```

# Construction

- Stage 1: make two assignments  $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$
- Stage 2: make remaining  $N-2$  assignments of facilities to locations, one facility/location pair at a time

# Construction

- Stage 1: make two assignments  $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$
- Stage 2: make remaining  $N-2$  assignments of facilities to locations, one facility/location pair at a time

# Stage 1 construction

- sort distances  $b_{i,j}$  in increasing order:

$$b_{i(1),j(1)} \leq b_{i(2),j(2)} \leq \dots \leq b_{i(N),j(N)} .$$

- sort flows  $a_{k,l}$  in decreasing order:

$$a_{k(1),l(1)} \geq a_{k(2),l(2)} \geq \dots \geq a_{k(N),l(N)} .$$

- sort products:

$$a_{k(1),l(1)} \cdot b_{i(1),j(1)}, a_{k(2),l(2)} \cdot b_{i(2),j(2)}, \dots, a_{k(N),l(N)} \cdot b_{i(N),j(N)}$$

- among smallest products, select  $a_{k(q),l(q)} \cdot b_{i(q),j(q)}$  at random:  
corresponding to assignments  $\{f_{k(q)} \rightarrow l_{i(q)} ; f_{l(q)} \rightarrow l_{j(q)}\}$

# Stage 1 construction

- sort distances  $b_{i,j}$  in increasing order:

$$b_{i(1),j(1)} \leq b_{i(2),j(2)} \leq \dots \leq b_{i(N),j(N)} .$$

- sort flows  $a_{k,l}$  in decreasing order:

$$a_{k(1),l(1)} \geq a_{k(2),l(2)} \geq \dots \geq a_{k(N),l(N)} .$$

- sort products:

$$a_{k(1),l(1)} \cdot b_{i(1),j(1)}, a_{k(2),l(2)} \cdot b_{i(2),j(2)}, \dots, a_{k(N),l(N)} \cdot b_{i(N),j(N)}$$

- among smallest products, select  $a_{k(q),l(q)} \cdot b_{i(q),j(q)}$  at random:  
corresponding to assignments  $\{f_{k(q)} \rightarrow l_{i(q)} ; f_{l(q)} \rightarrow l_{j(q)}\}$

# Stage 1 construction

- sort distances  $b_{i,j}$  in increasing order:

$$b_{i(1),j(1)} \leq b_{i(2),j(2)} \leq \dots \leq b_{i(N),j(N)} .$$

- sort flows  $a_{k,l}$  in decreasing order:

$$a_{k(1),l(1)} \geq a_{k(2),l(2)} \geq \dots \geq a_{k(N),l(N)} .$$

- sort products:

$$a_{k(1),l(1)} \cdot b_{i(1),j(1)}, a_{k(2),l(2)} \cdot b_{i(2),j(2)}, \dots, a_{k(N),l(N)} \cdot b_{i(N),j(N)}$$

- among smallest products, select  $a_{k(q),l(q)} \cdot b_{i(q),j(q)}$  at random:  
corresponding to assignments  $\{f_{k(q)} \rightarrow l_{i(q)} ; f_{l(q)} \rightarrow l_{j(q)}\}$

# Stage 1 construction

- sort distances  $b_{i,j}$  in increasing order:

$$b_{i(1),j(1)} \leq b_{i(2),j(2)} \leq \dots \leq b_{i(N),j(N)} .$$

- sort flows  $a_{k,l}$  in decreasing order:

$$a_{k(1),l(1)} \geq a_{k(2),l(2)} \geq \dots \geq a_{k(N),l(N)} .$$

- sort products:

$$a_{k(1),l(1)} \cdot b_{i(1),j(1)}, a_{k(2),l(2)} \cdot b_{i(2),j(2)}, \dots, a_{k(N),l(N)} \cdot b_{i(N),j(N)}$$

- among smallest products, select  $a_{k(q),l(q)} \cdot b_{i(q),j(q)}$  at random:  
corresponding to assignments  $\{f_{k(q)} \rightarrow l_{i(q)} ; f_{l(q)} \rightarrow l_{j(q)}\}$



# Stage 2 construction

- If  $\Omega = \{(i_1, k_1), (i_2, k_2), \dots, (i_q, k_q)\}$  are the  $q$  assignments made so far, then

- Cost of assigning  $f_j \rightarrow l_i$  is

$$c_{j,l} = \sum_{i,k \in \Gamma} a_{i,j} b_{k,l}$$

- Of all possible assignments, one is selected at random from the assignments having smallest costs and is added to  $\Omega$

# Stage 2 construction

- If  $\Omega = \{(i_1, k_1), (i_2, k_2), \dots, (i_q, k_q)\}$  are the  $q$  assignments made so far, then

- Cost of assigning  $f_j \rightarrow l_i$  is

$$c_{j,l} = \sum_{i,k \in \Gamma} a_{i,j} b_{k,l}$$

- Of all possible assignments, one is selected at random from the assignments having smallest costs and is added to  $\Omega$

# Stage 2 construction

- If  $\Omega = \{(i_1, k_1), (i_2, k_2), \dots, (i_q, k_q)\}$  are the  $q$  assignments made so far, then

- Cost of assigning  $f_j \rightarrow l_i$  is

$$c_{j,l} = \sum_{i,k \in \Gamma} a_{i,j} b_{k,l}$$

- Of all possible assignments, one is selected at random from the assignments having smallest costs and is added to  $\Omega$

Sped up in Pardalos, Pitsoulis, & Resende (1997) for QAPs with sparse A or B matrices.

# Swap based local search

- a) For all pairs of assignments  $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$ , test if swapped assignment  $\{f_i \rightarrow l_l ; f_j \rightarrow l_k\}$  improves solution.
- b) If so, make swap and return to step (a)

# Swap based local search

- a) For all pairs of assignments  $\{f_i \rightarrow l_k ; f_j \rightarrow l_l\}$ , test if swapped assignment  $\{f_i \rightarrow l_l ; f_j \rightarrow l_k\}$  improves solution.
- b) If so, make swap and return to step (a)

repeat (a)-(b) until no swap improves current solution

# Path-relinking

- Path-relinking:
  - Intensification strategy exploring trajectories connecting elite solutions: Glover (1996)
  - Originally proposed in the context of tabu search and scatter search.
  - Paths in the solution space leading to other elite solutions are explored in the search for better solutions:
    - selection of moves that introduce attributes of the guiding solution into the current solution

# Path-relinking

- Path-relinking:
  - Intensification strategy exploring trajectories connecting elite solutions: Glover (1996)
  - Originally proposed in the context of tabu search and scatter search.
  - Paths in the solution space leading to other elite solutions are explored in the search for better solutions:
    - selection of moves that introduce attributes of the guiding solution into the current solution

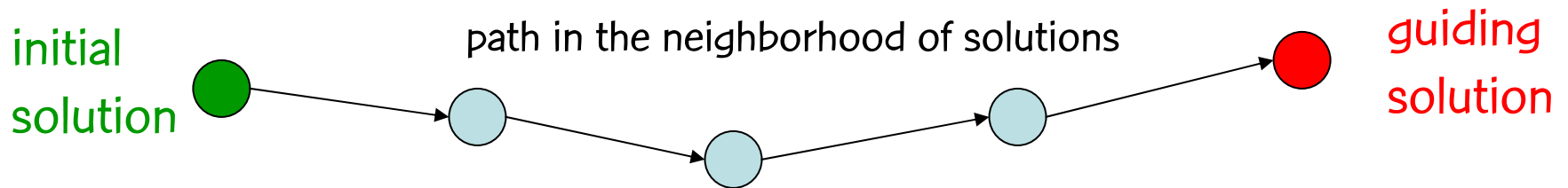
# Path-relinking

- Path-relinking:
  - Intensification strategy exploring trajectories connecting elite solutions: Glover (1996)
  - Originally proposed in the context of tabu search and scatter search.
  - Paths in the solution space leading to other elite solutions are explored in the search for better solutions:
    - selection of moves that introduce attributes of the guiding solution into the current solution



# Path-relinking

- Exploration of trajectories that connect high quality (elite) solutions:



# Path-relinking

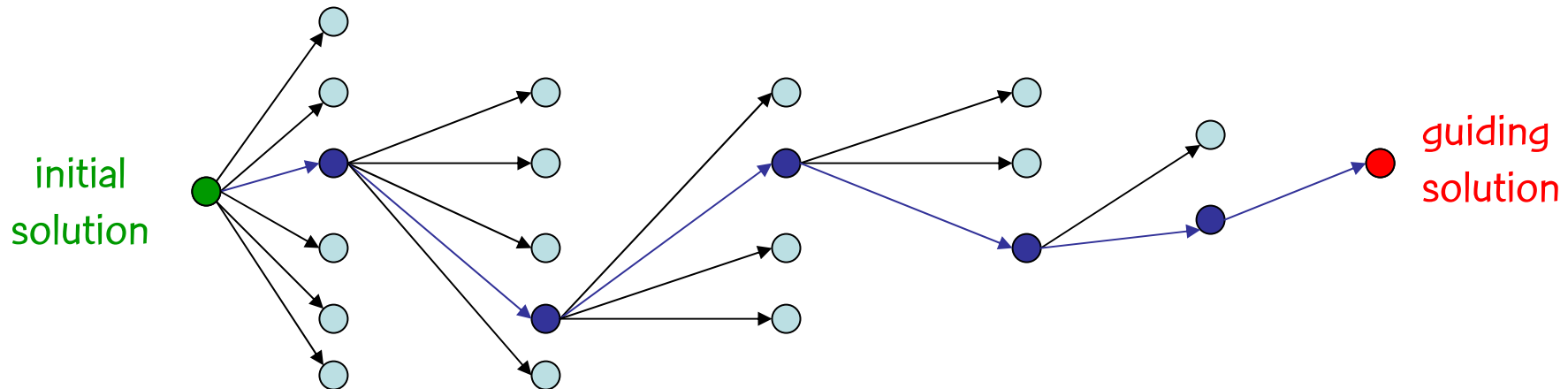
- Path is generated by selecting moves that introduce in the **initial solution** attributes of the **guiding solution**.
- At each step, all moves that incorporate attributes of the guiding solution are evaluated and the best move is selected:

initial  
solution ●

● guiding  
solution

# Path-relinking

- Path is generated by selecting moves that introduce in the initial solution attributes of the guiding solution.
- At each step, all moves that incorporate attributes of the guiding solution are evaluated and the best move is selected:



# Path-relinking

Combine solutions  $x$  and  $y$

$\Delta(x,y)$ : symmetric difference between  $x$  and  $y$

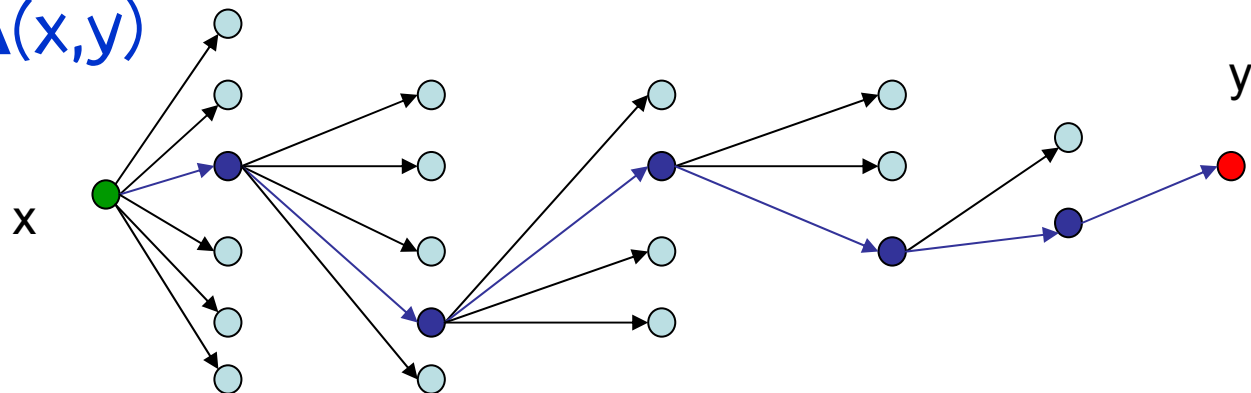
while (  $|\Delta(x,y)| > 0$  ) {

    evaluate moves corresponding in  $\Delta(x,y)$

    make best move

    update  $\Delta(x,y)$

}



# GRASP with path-relinking

- Originally used by Laguna and Martí (1999).
- Maintains a set of elite solutions found during GRASP iterations.
- After each GRASP iteration (construction and local search):
  - Use GRASP solution as initial solution.
  - Select an elite solution uniformly at random: guiding solution.
  - Perform path-relinking between these two solutions.

# GRASP with path-relinking

- Originally used by Laguna and Martí (1999).
- Maintains a set of elite solutions found during GRASP iterations.
- After each GRASP iteration (construction and local search):
  - Use GRASP solution as initial solution.
  - Select an elite solution uniformly at random: guiding solution.
  - Perform path-relinking between these two solutions.

# GRASP with path-relinking

- Originally used by Laguna and Martí (1999).
- Maintains a set of elite solutions found during GRASP iterations.
- After each GRASP iteration (construction and local search):
  - Use GRASP solution as **initial solution**.
  - Select an elite solution uniformly at random: **guiding solution**.
  - Perform path-relinking between these two solutions.

# GRASP with path-relinking

Repeat for Max\_Iterations:

Construct a greedy randomized solution.

Use local search to improve the constructed solution.

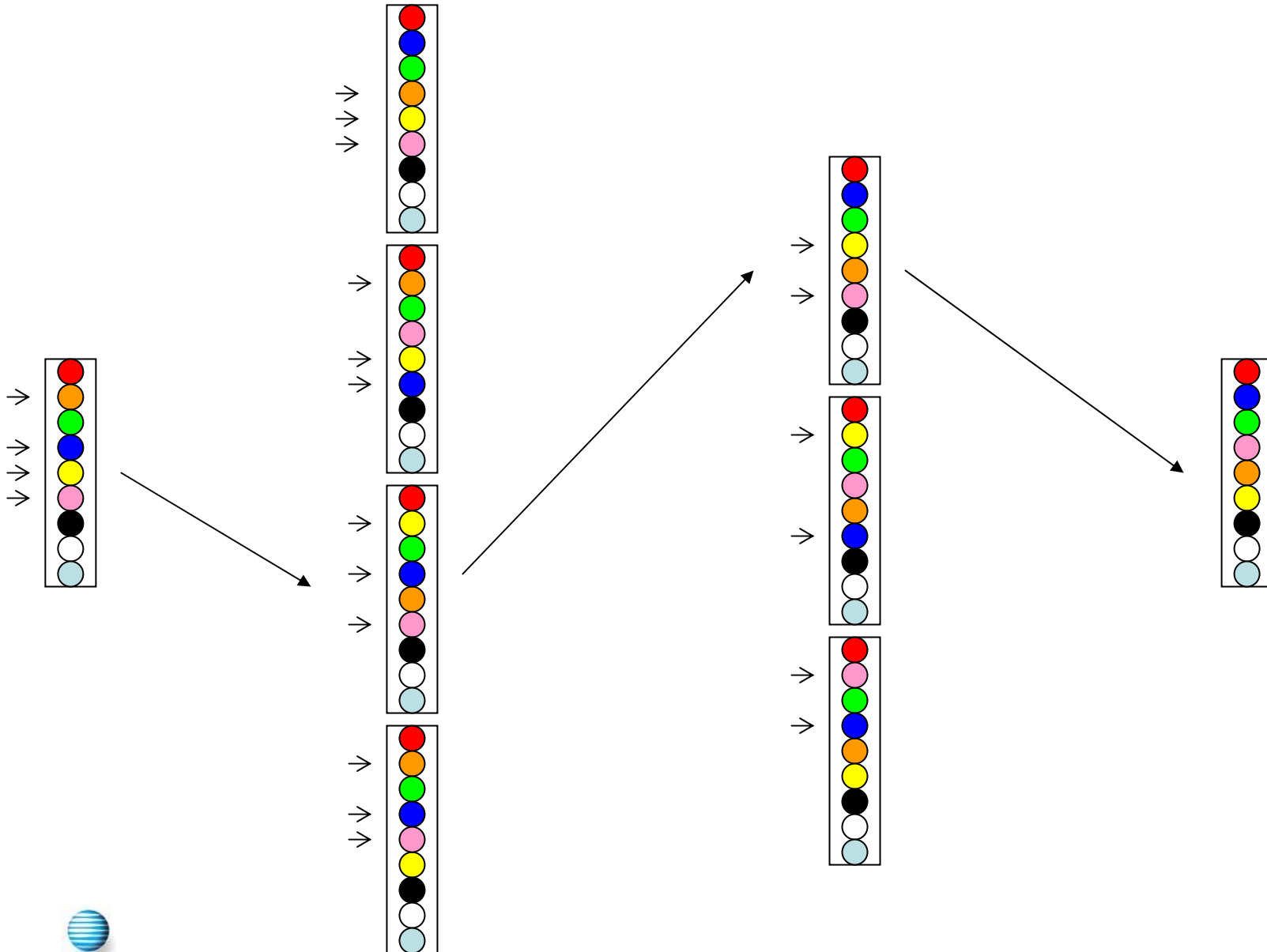
Apply path-relinking to further improve the solution.

Update the pool of elite solutions.

Update the best solution found.

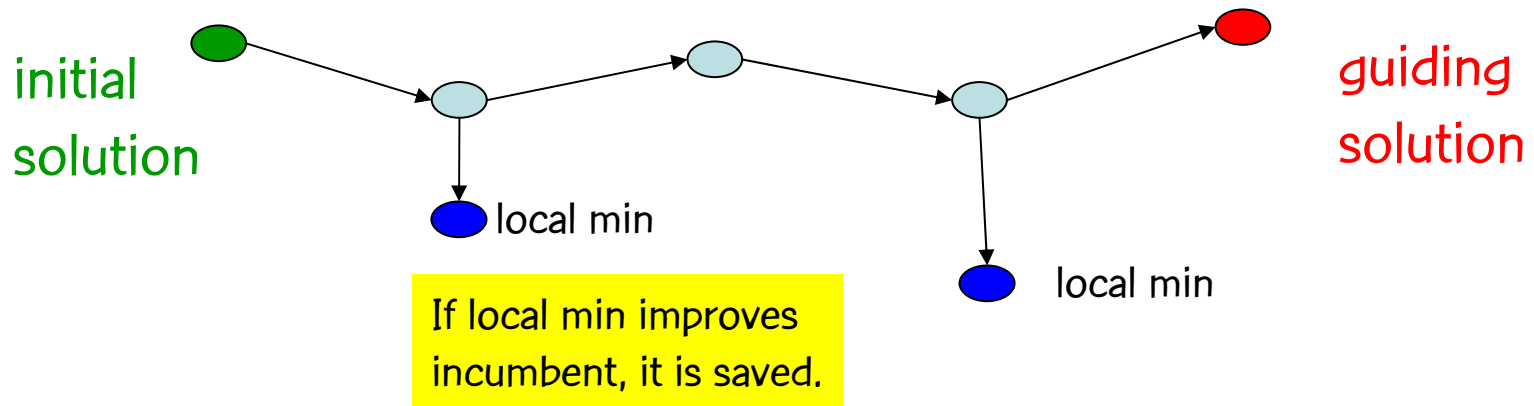


# PR for QAP (permutation vectors)



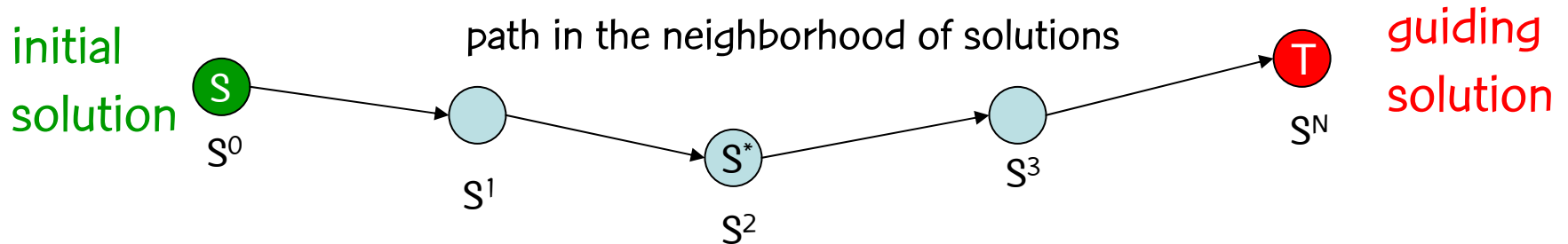
# Path-relinking for QAP

If swap improves solution: local search is applied



# Path-relinking for QAP

Results of path relinking:  $S^*$



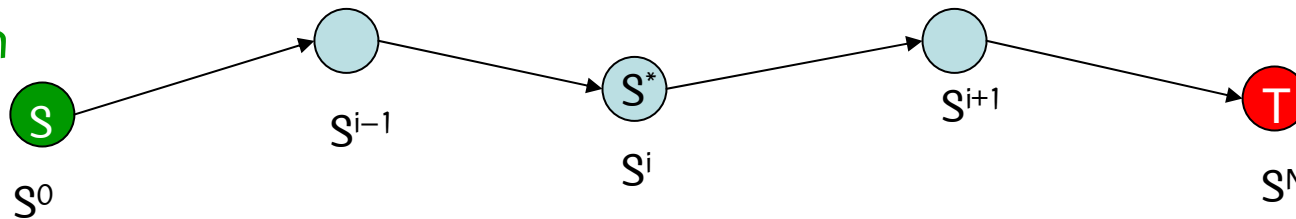
If  $c(S^*) < \min \{c(S), c(T)\}$ , and  $c(S^*) \leq c(S^i)$ , for  $i=1, \dots, N$ , i.e.  $S^*$  is best solution in path, then  $S^*$  is returned.

# Path-relinking for QAP

$S^i$  is a local minimum w.r.t. PR:

$c(S^i) < c(S^{i-1})$  and  $c(S^i) < c(S^{i+1})$ , for all  $i=1, \dots, N$ .

initial  
solution



guiding  
solution

If path-relinking does not improve  $(S, T)$ , then if  $S^i$  is a best local min w.r.t. PR: return  $S^* = S^i$

If no local min exists, return  $S^* = \operatorname{argmin}\{S, T\}$

# PR pool management

- $S^*$  is candidate for inclusion in pool of elite solutions ( $P$ )
- If  $c(S^*) < c(S^e)$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- Else, if  $c(S^*) < \max\{c(S^e), S^e \in P\}$  and  $|\Delta(S^*, S^e)| \geq 3$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- If pool is full, remove  $\operatorname{argmin} \{ |\Delta(S^*, S^e)|, \forall S^e \in P \text{ s.t. } c(S^e) \geq c(S^*) \}$

# PR pool management

- $S^*$  is candidate for inclusion in pool of elite solutions ( $P$ )
- If  $c(S^*) < c(S^e)$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- Else, if  $c(S^*) < \max\{c(S^e), S^e \in P\}$  and  $|\Delta(S^*, S^e)| \geq 3$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- If pool is full, remove  $\operatorname{argmin} \{ |\Delta(S^*, S^e)|, \forall S^e \in P \text{ s.t. } c(S^e) \geq c(S^*) \}$

# PR pool management

- $S^*$  is candidate for inclusion in pool of elite solutions ( $P$ )
- If  $c(S^*) < c(S^e)$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- Else, if  $c(S^*) < \max\{c(S^e), S^e \in P\}$  and  $|\Delta(S^*, S^e)| \geq 3$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- If pool is full, remove  $\operatorname{argmin} \{ |\Delta(S^*, S^e)|, \forall S^e \in P \text{ s.t. } c(S^e) \geq c(S^*) \}$

# PR pool management

- $S^*$  is candidate for inclusion in pool of elite solutions ( $P$ )
- If  $c(S^*) < c(S^e)$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- Else, if  $c(S^*) < \max\{c(S^e), S^e \in P\}$  and  $|\Delta(S^*, S^e)| \geq 3$ , for all  $S^e \in P$ , then  $S^*$  is put in  $P$
- If pool is full, remove  $\operatorname{argmin} \{ |\Delta(S^*, S^e)|, \forall S^e \in P \text{ s.t. } c(S^e) \geq c(S^*) \}$



# PR pool management

S is initial solution for path-relinking: favor choice of target solution T with large symmetric difference with S.

This leads to longer paths in path-relinking.

Probability of choosing  $S^e \in P$ :

$$p(S^e) = \frac{|\Delta(S, S^e)|}{\sum_{R \in P} |\Delta(S, R)|}$$

# Experimental results

- Compare GRASP with and without path-relinking.
- New GRASP code in C outperforms old Fortran codes: we use same code to compare algorithms
- All QAPLIB (Burkhard, Karisch, & Rendl, 1991) instances of size  $N \leq 40$
- 100 independent runs of each algorithm, recording CPU time to find the best known solution for instance

# Experimental results

- Compare GRASP with and without path-relinking.
- New GRASP code in C outperforms old Fortran codes: we use same code to compare algorithms
- All QAPLIB (Burkhard, Karisch, & Rendl, 1991) instances of size  $N \leq 40$
- 100 independent runs of each algorithm, recording CPU time to find the best known solution for instance

# Experimental results

- Compare GRASP with and without path-relinking.
- New GRASP code in C outperforms old Fortran codes: we use same code to compare algorithms
- All QAPLIB (Burkhard, Karisch, & Rendl, 1991) instances of size  $N \leq 40$
- 100 independent runs of each algorithm, recording CPU time to find the best known solution for instance

# Experimental results

- Compare GRASP with and without path-relinking.
- New GRASP code in C outperforms old Fortran codes: we use same code to compare algorithms
- All QAPLIB (Burkhard, Karisch, & Rendl, 1991) instances of size  $N \leq 40$
- 100 independent runs of each algorithm, recording CPU time to find the best known solution for instance

# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync



# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- **Size of elite set: 30**
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

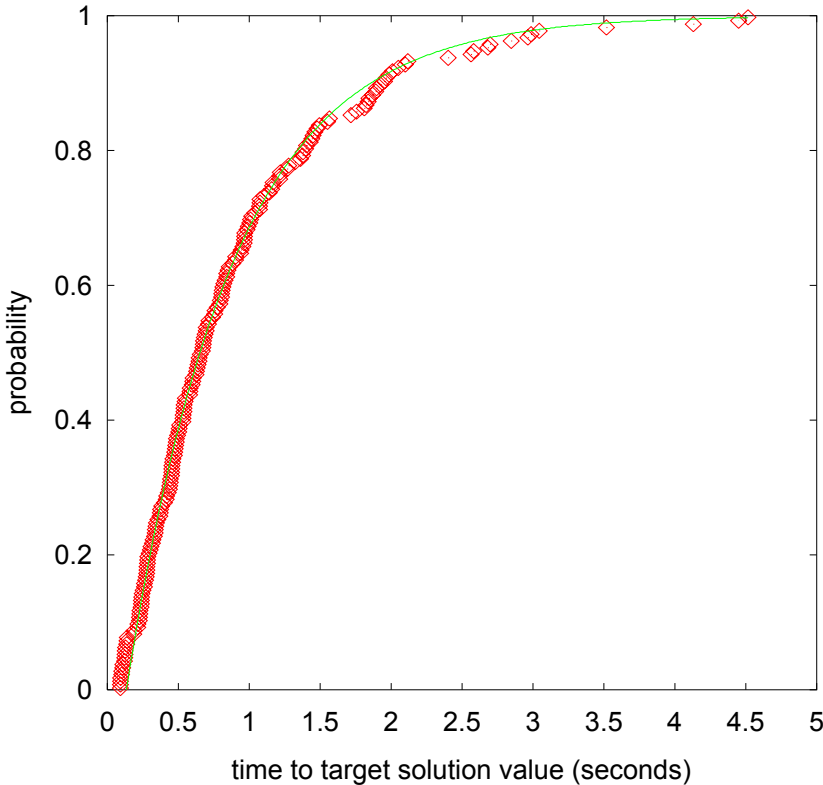
# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

# Experimental results

- SGI Challenge computer (196 MHz R10000 processors (28) and 7 Gb memory)
- Single processor used for each run
- GRASP RCL parameter  $\alpha$  chosen at random in interval  $[0,1]$  at each GRASP iteration.
- Size of elite set: 30
- Path-relinking done in both directions (S to T to S)
- Care taken to ensure that GRASP and GRASP with path-relinking iterations are in sync

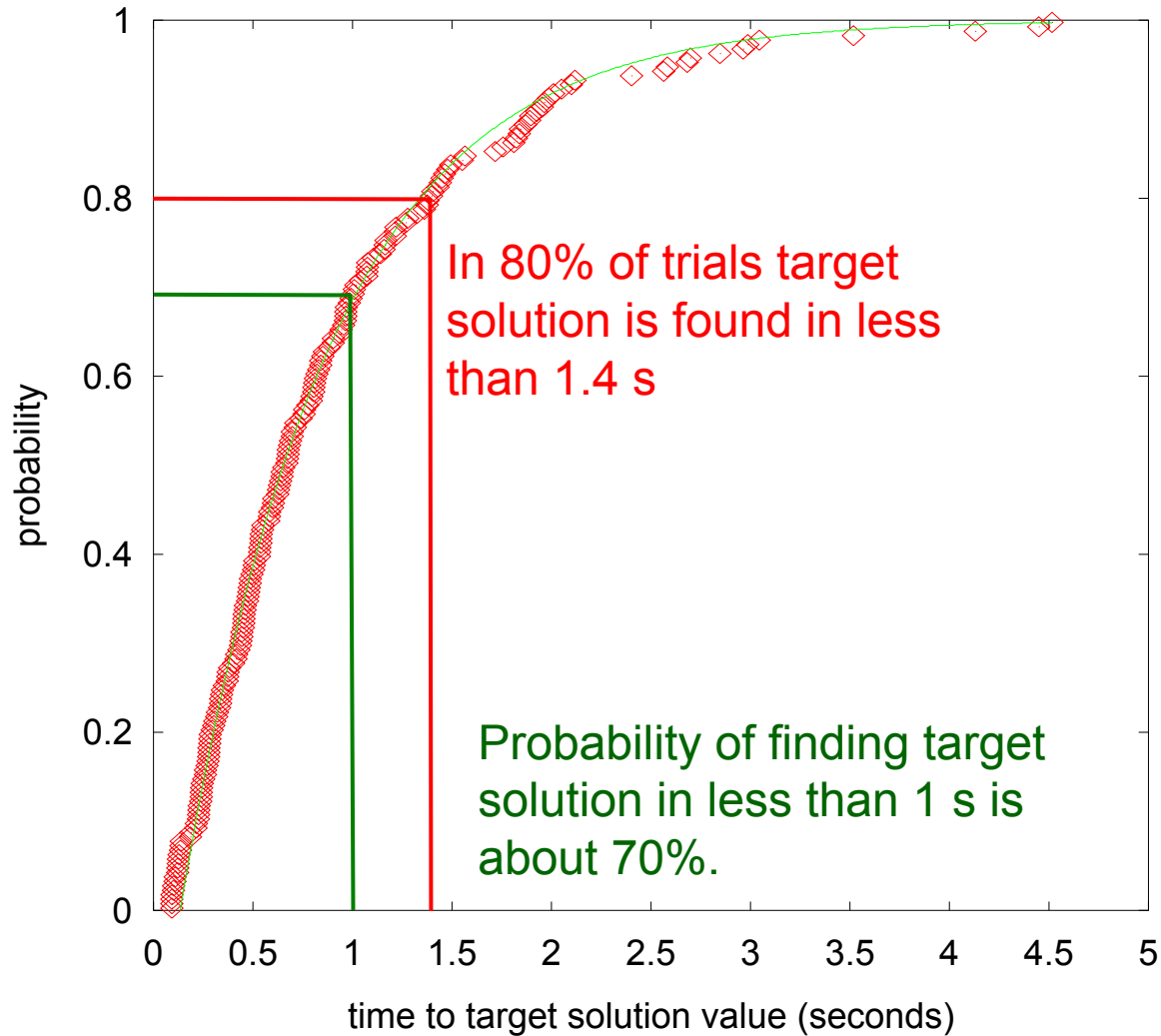
# Time-to-target-value plots



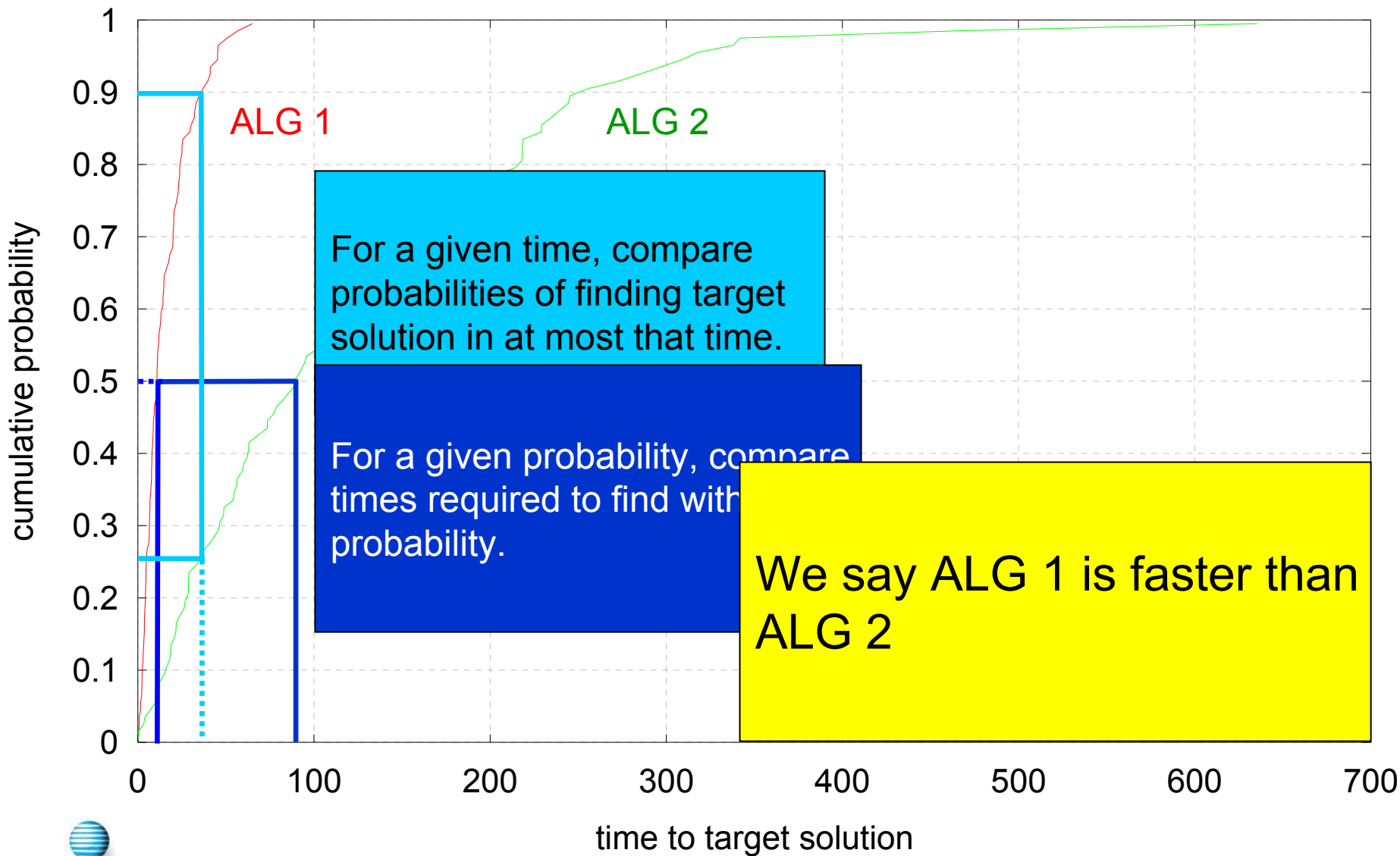
Sort times such that  
 $t_1 \leq t_2 \leq \dots \leq t_{100}$  and plot  
 $\{t_i, p_i\}$ , for  $i=1, \dots, N$ , where  
 $p_i = (i - .5) / 100$

Random variable **time-to-target-solution value** fits a two-parameter exponential distribution (Aiex, Resende, & Ribeiro, 2002).

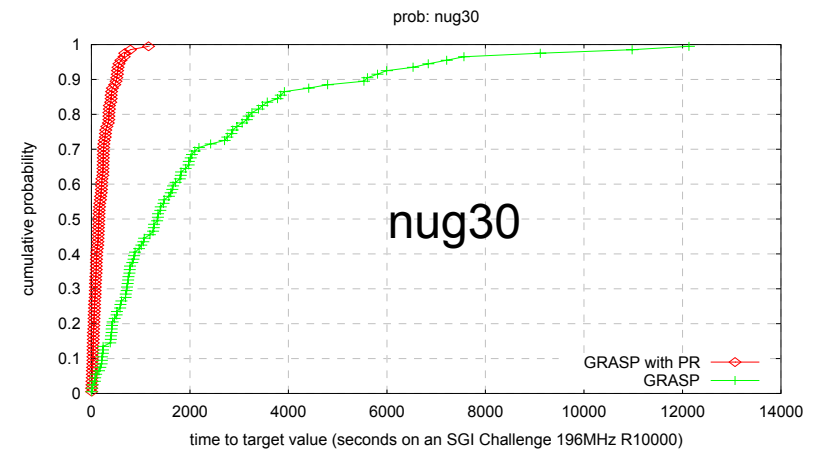
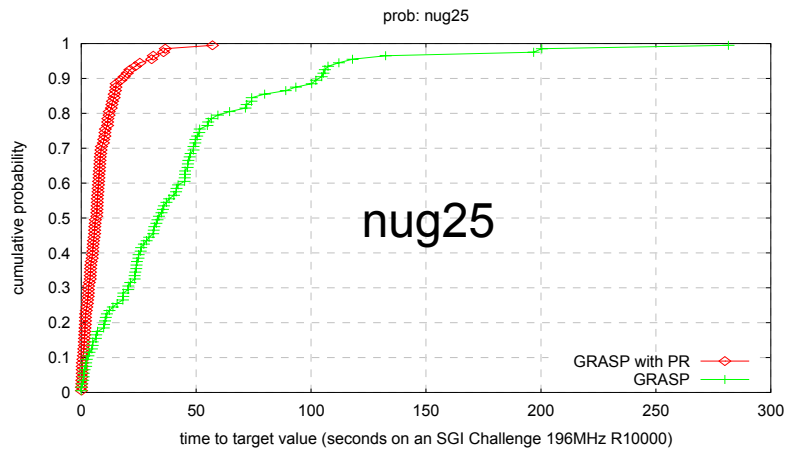
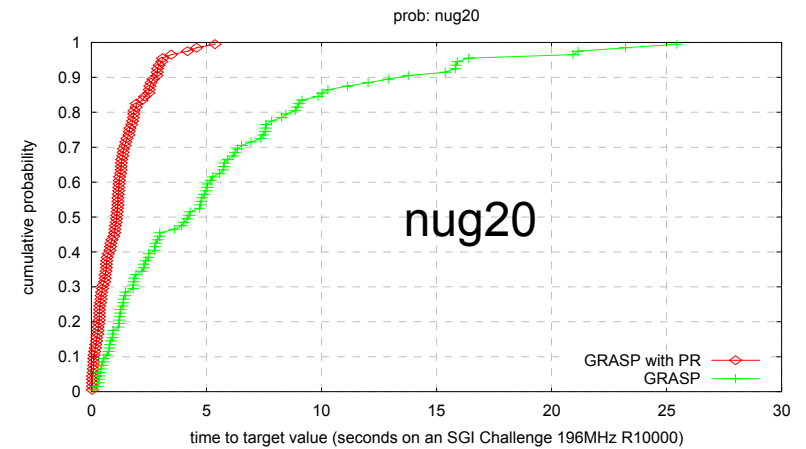
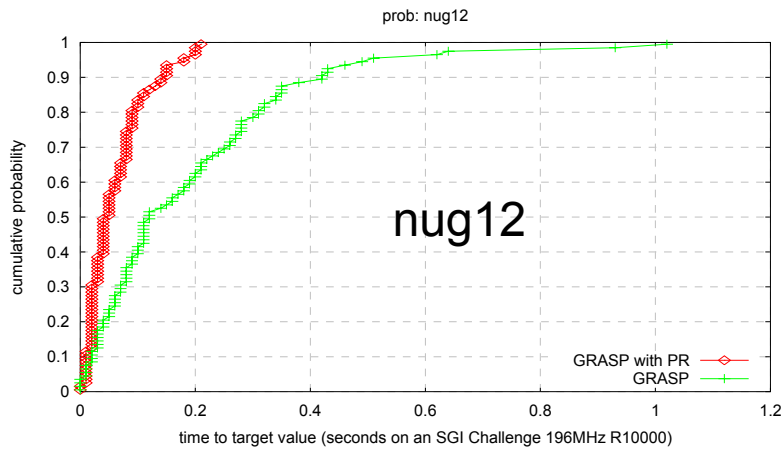
# Time-to-target-value plots



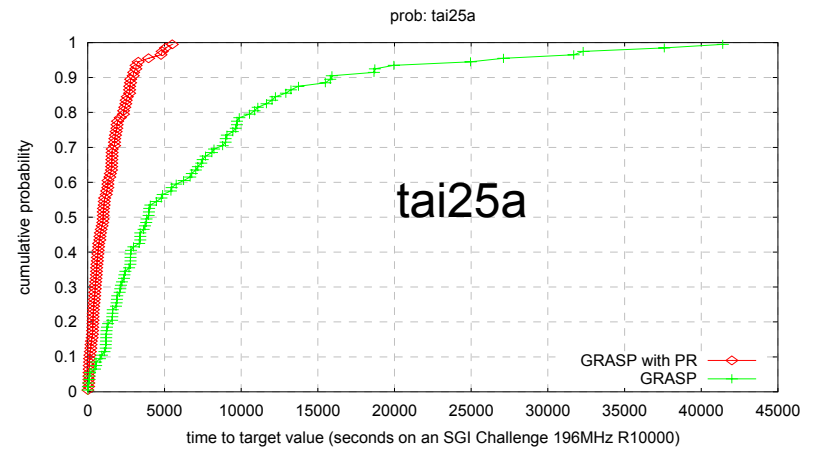
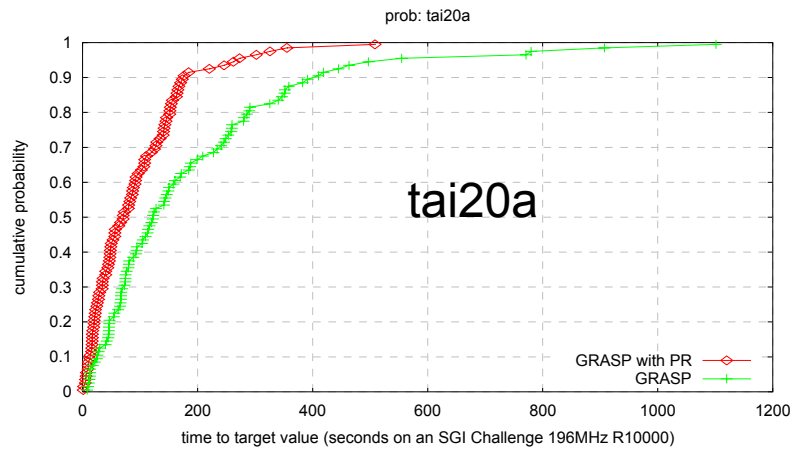
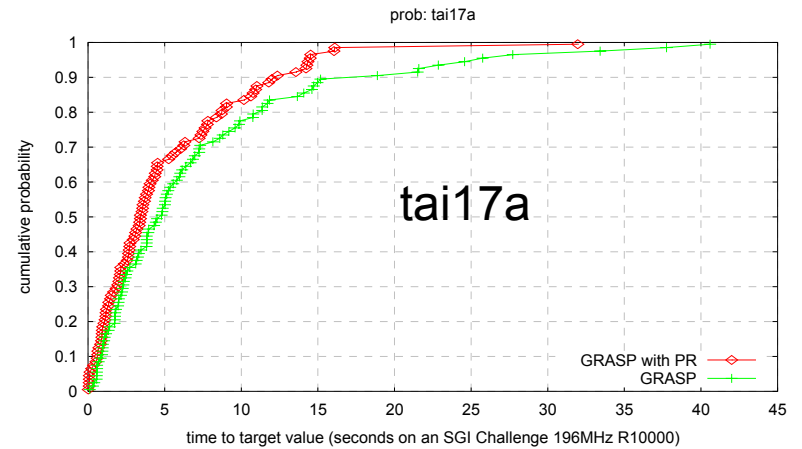
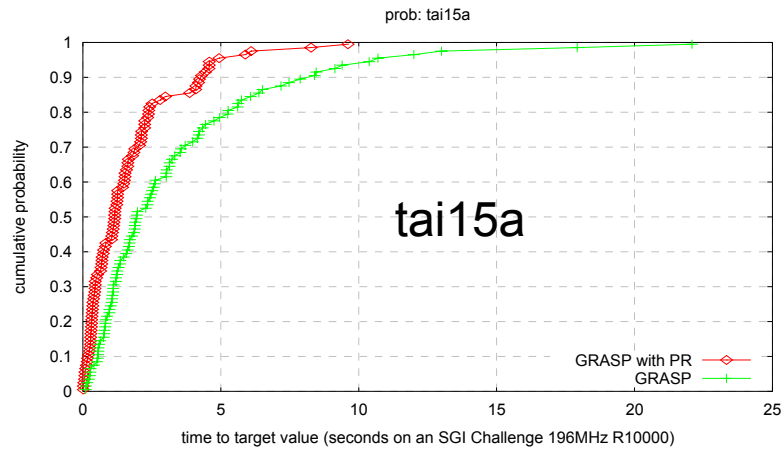
# Time-to-target-value plots



# C.E. Nugent, T.E. Vollmann and J. Ruml [1968]

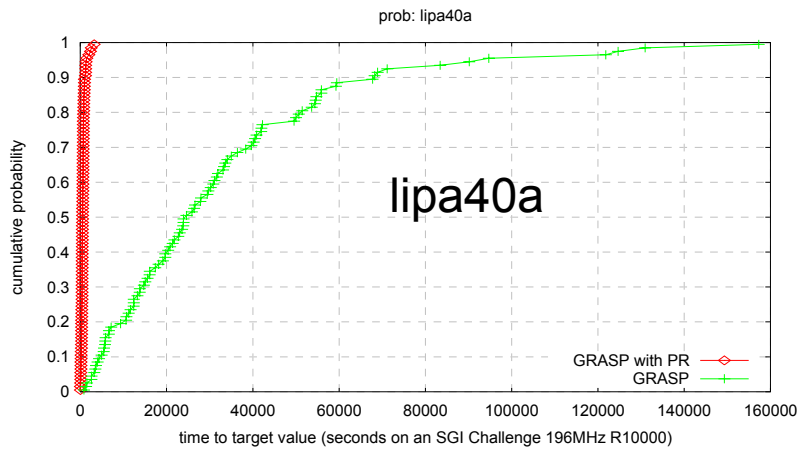
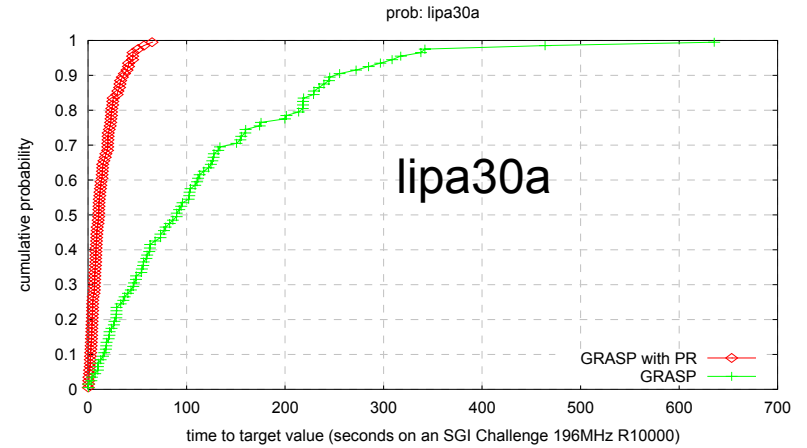
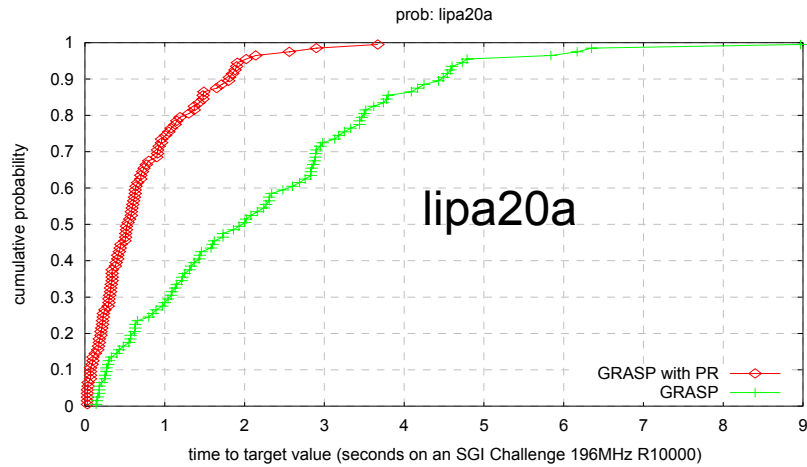


# E.D. Taillard [1991, 1994]

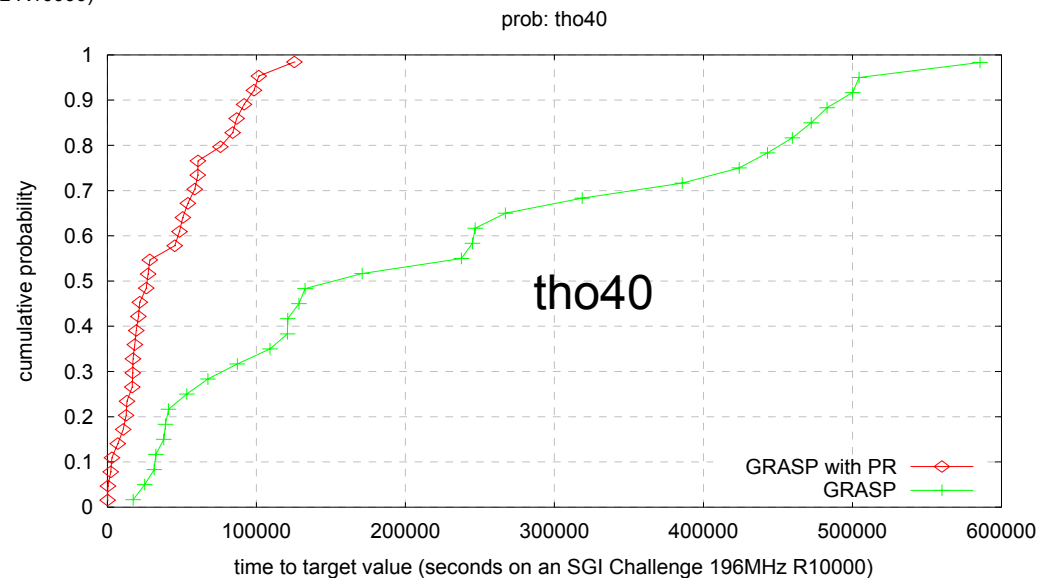
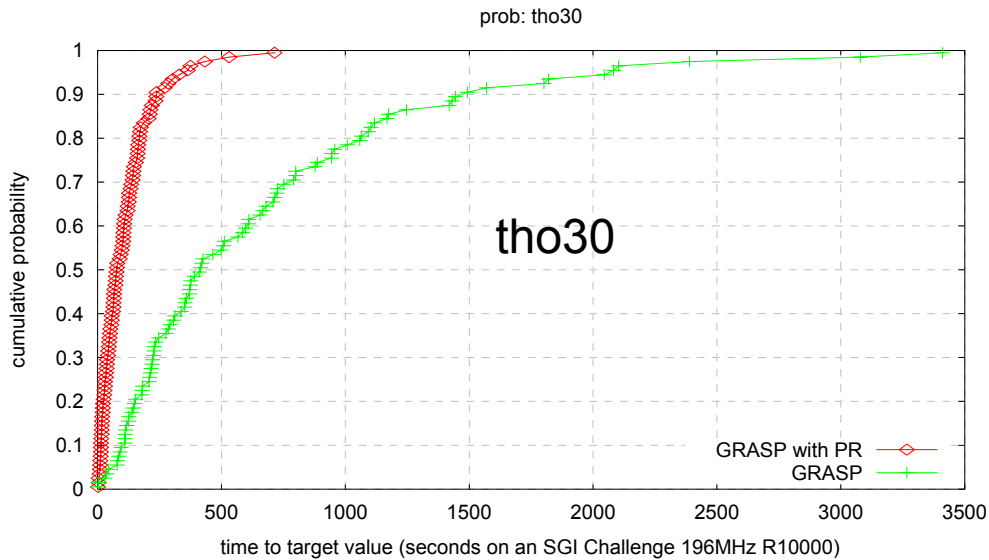




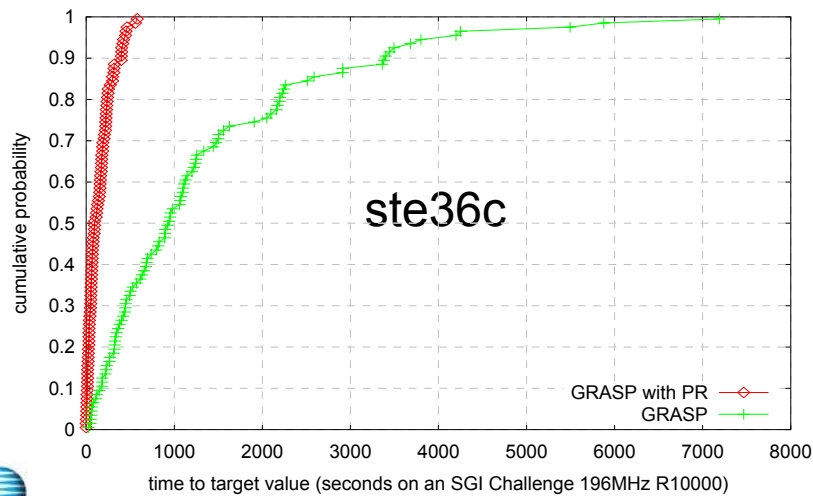
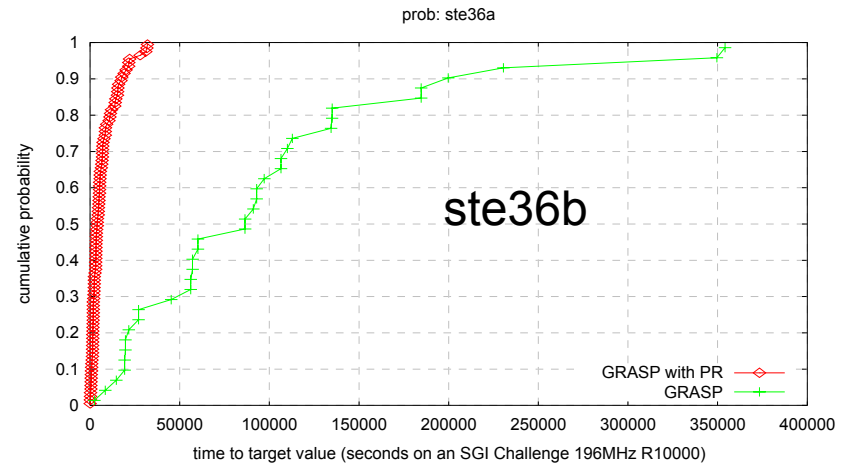
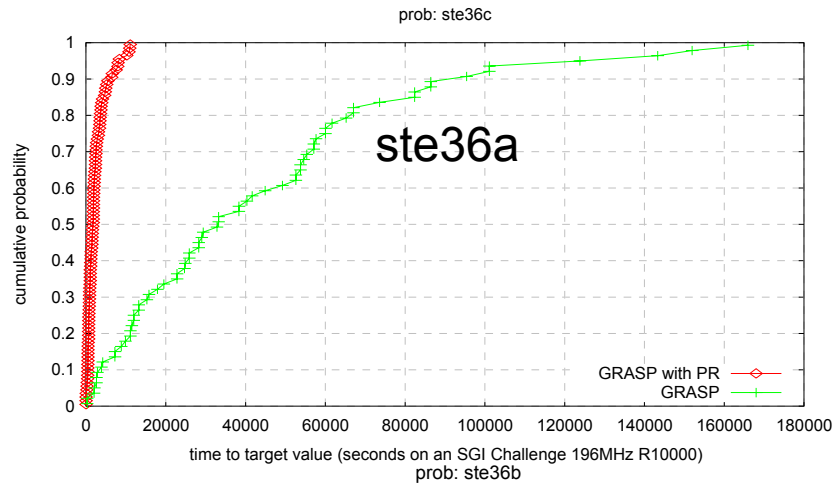
# Y. Li and P.M. Pardalos [1992]



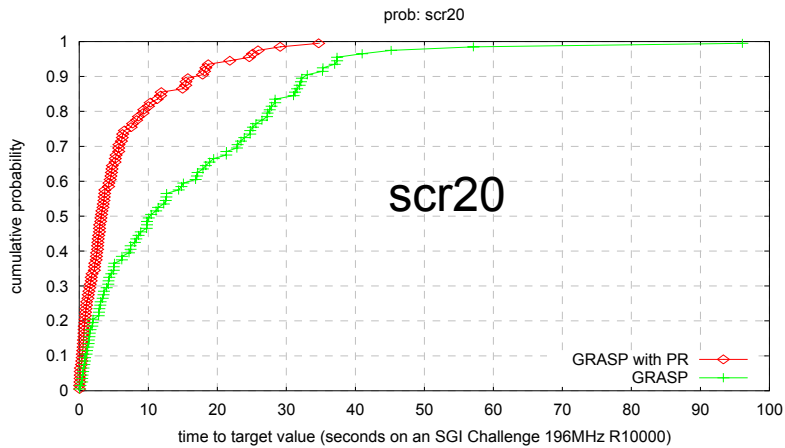
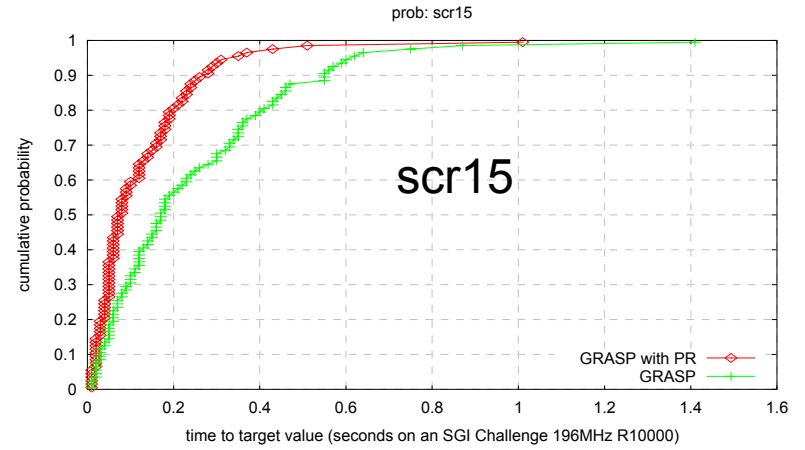
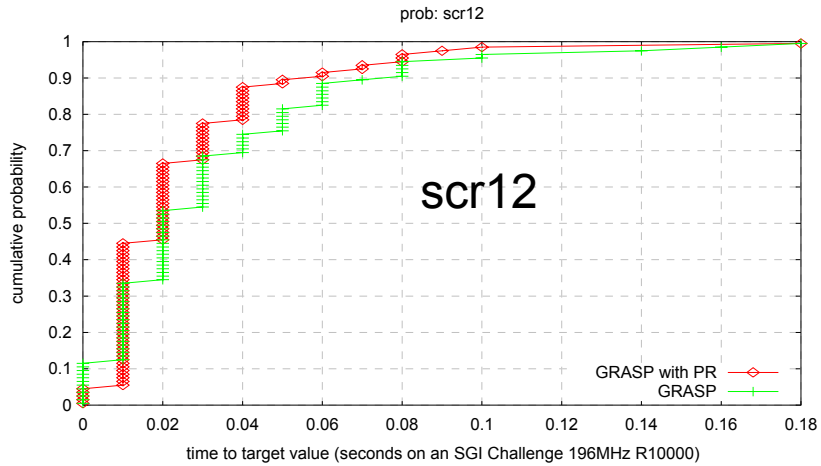
# U.W. Thonemann and A. Bölte [1994]



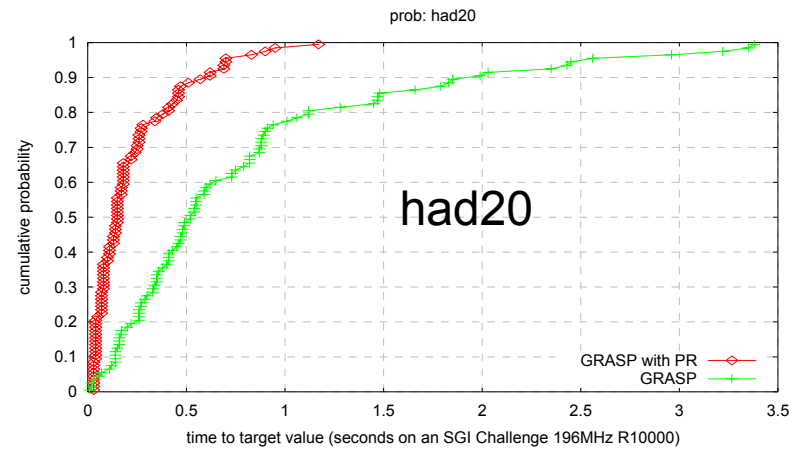
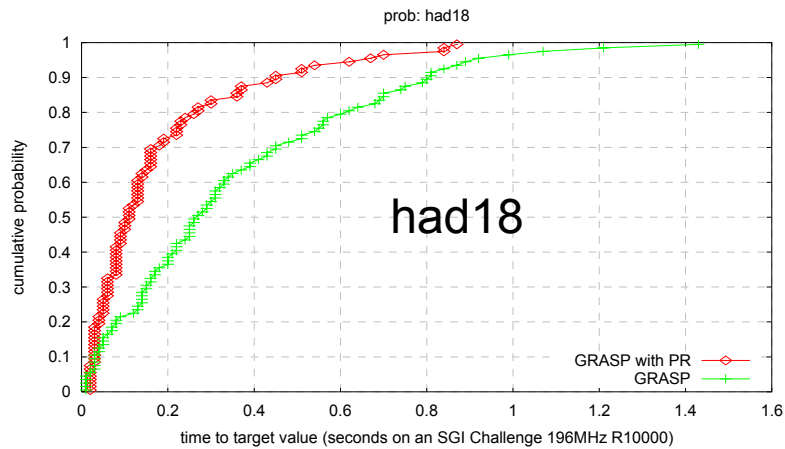
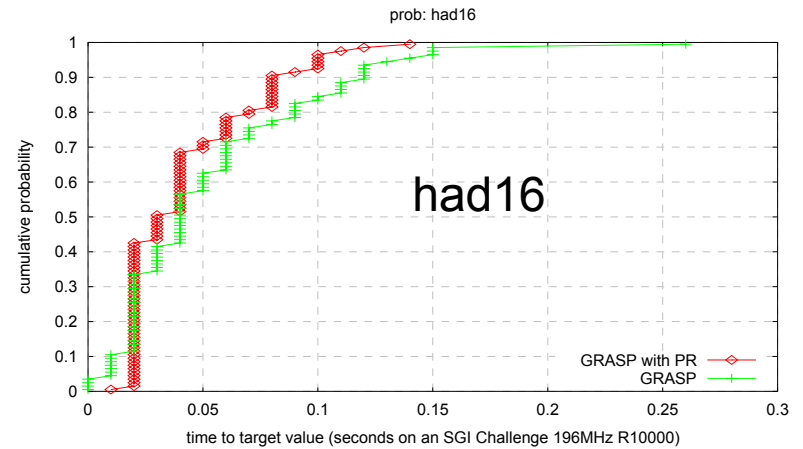
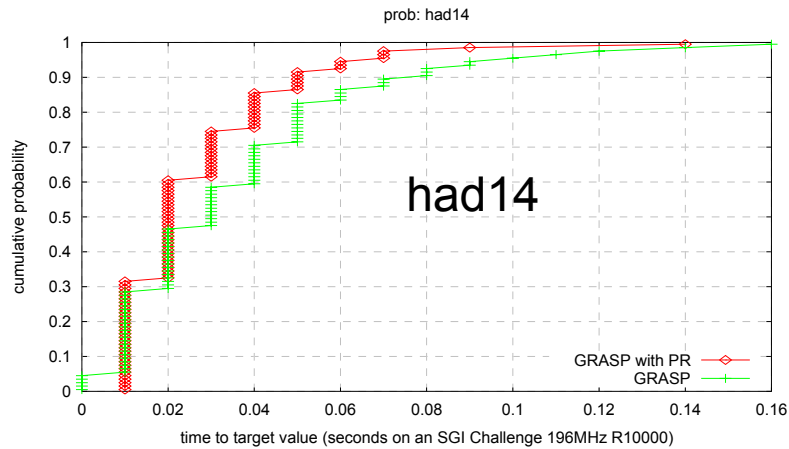
# L. Steinberg [1961]



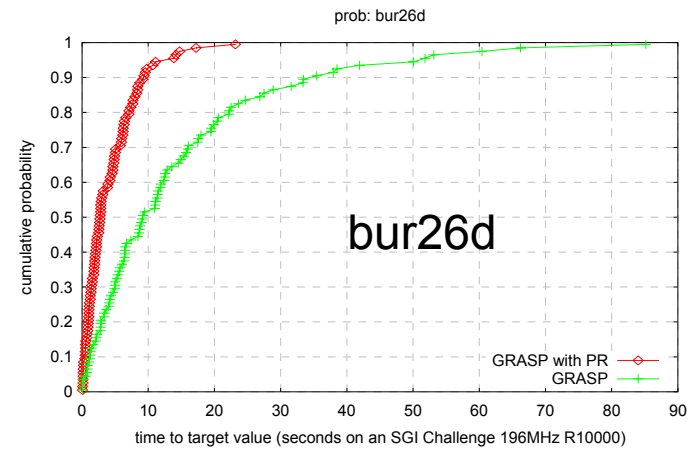
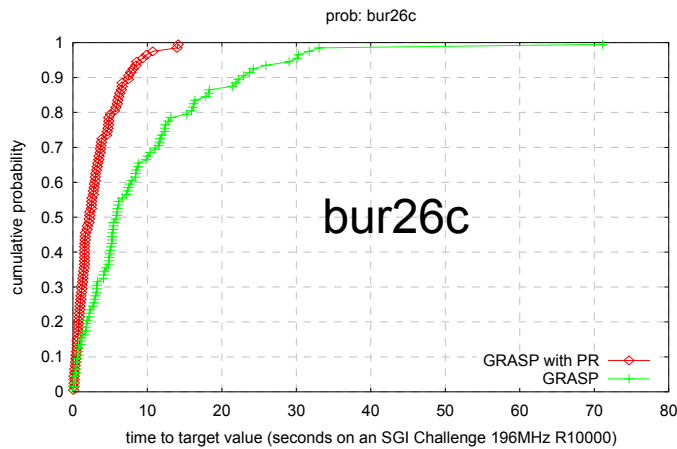
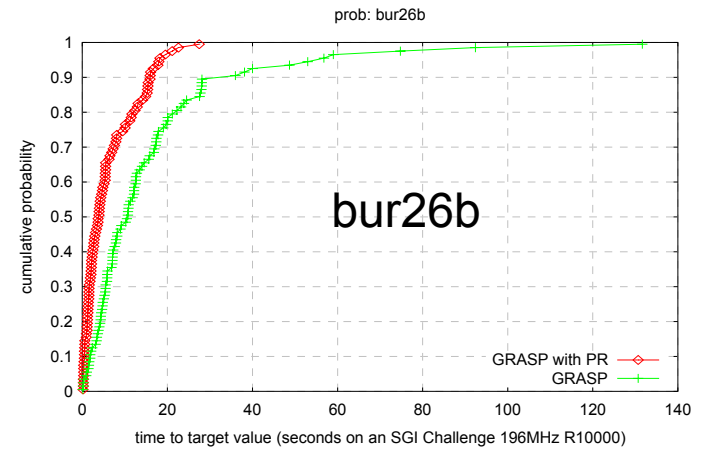
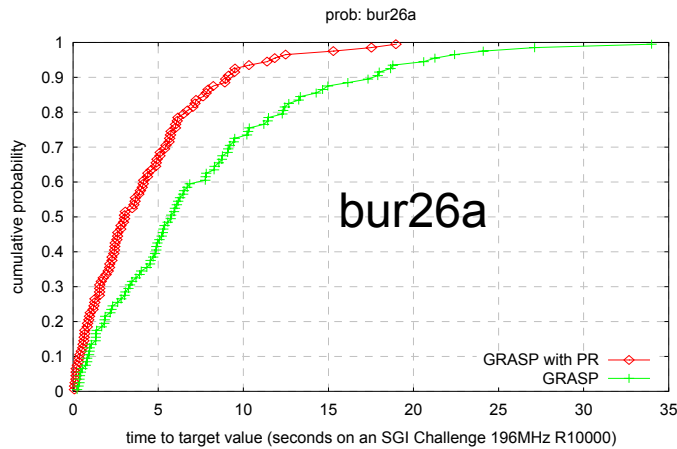
# M. Scriabin and R.C. Vergin [1975]



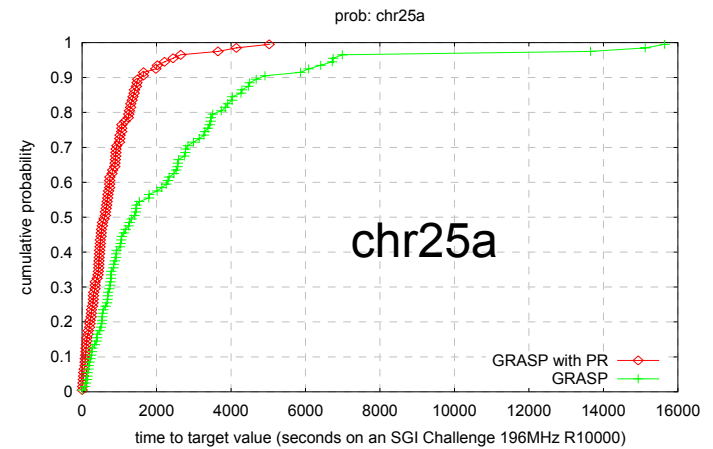
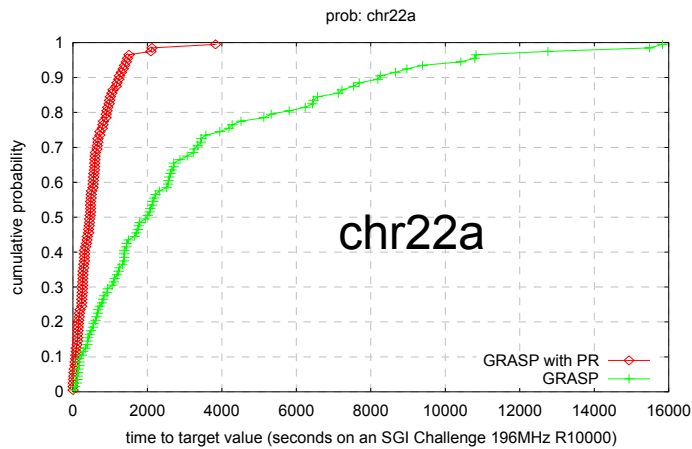
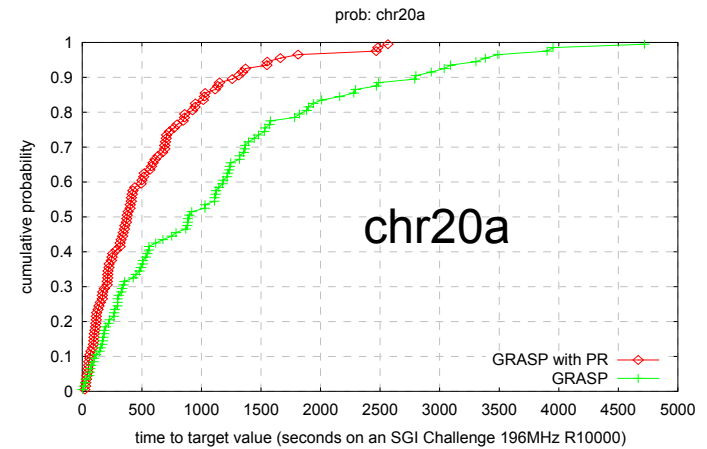
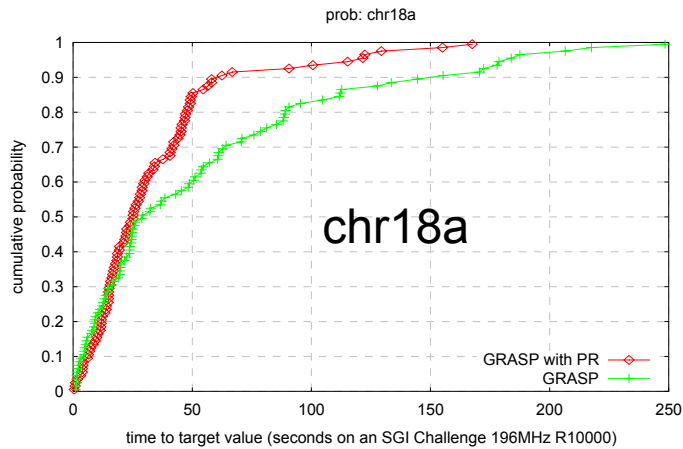
# S.W. Hadley, F. Rendl and H. Wolkowicz [1992]



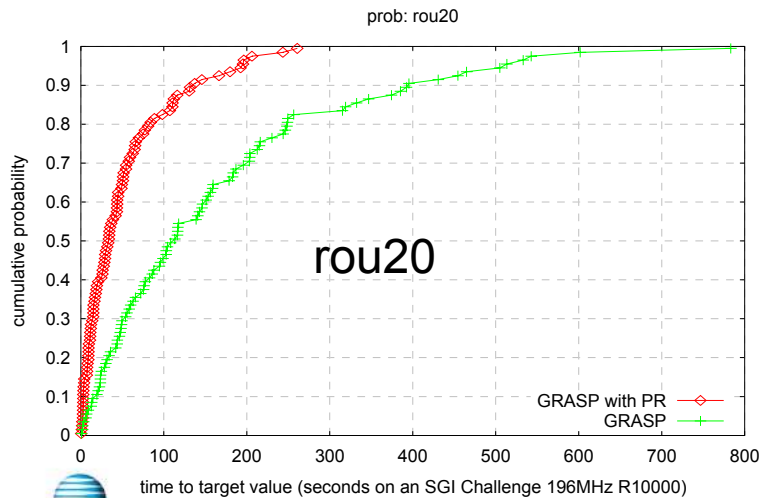
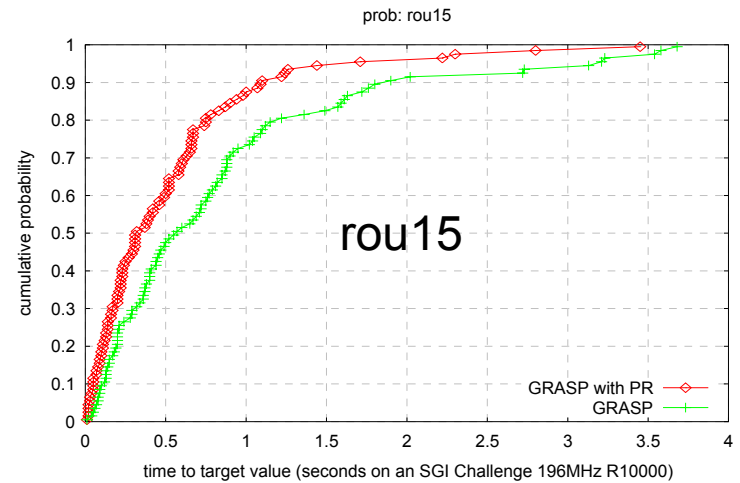
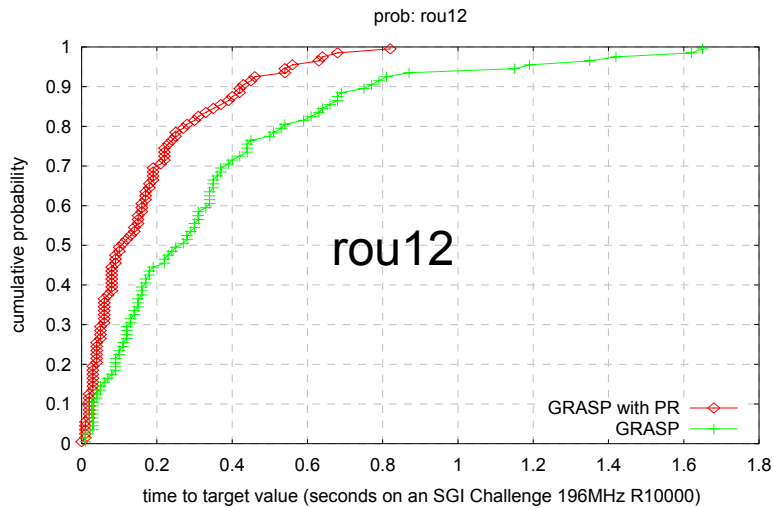
# R.E. Burkard and J. Offermann [1977]



# N. Christofides and E. Benavent [1989]

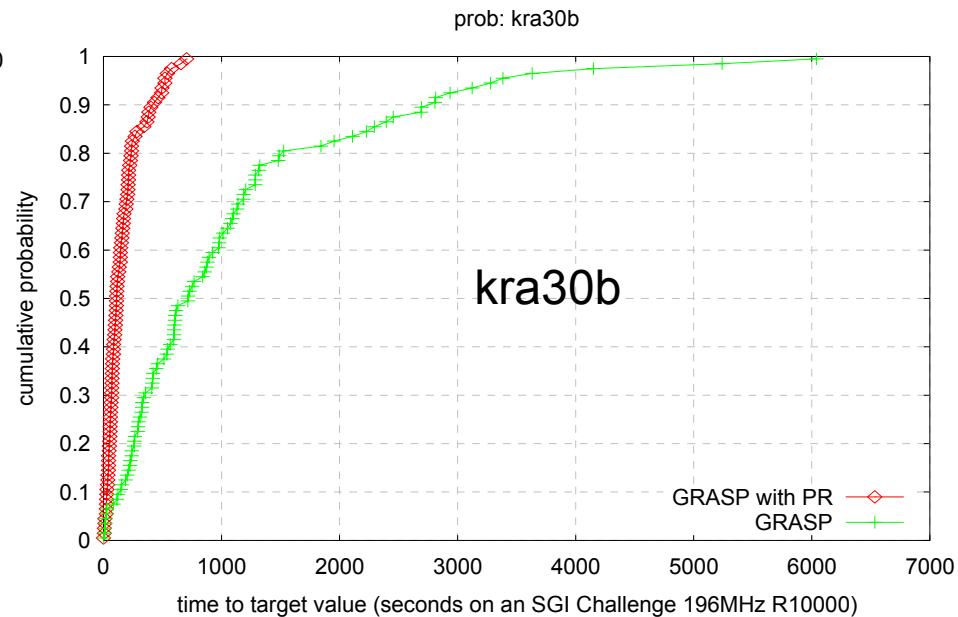
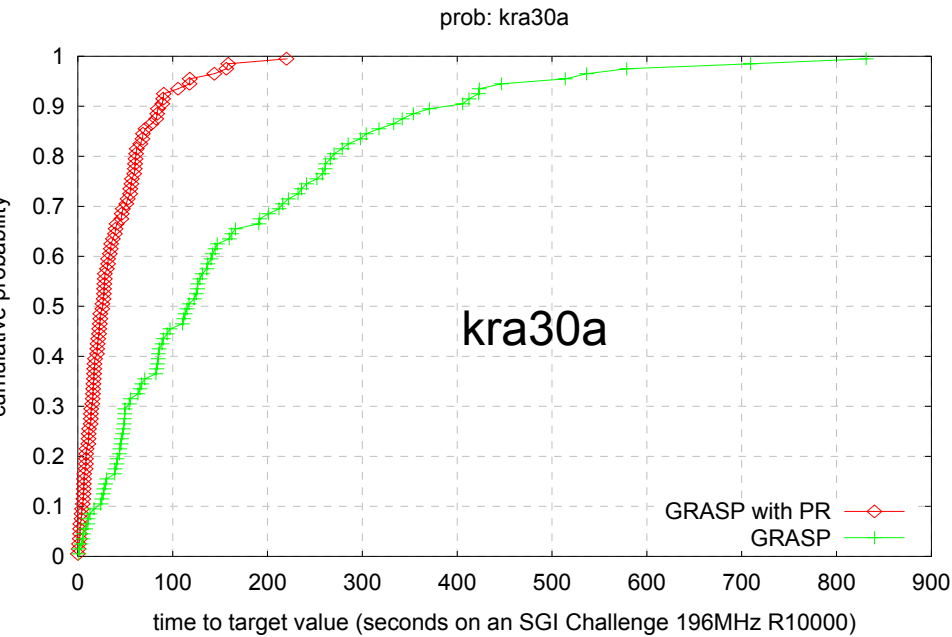


# C. Roucairol [1987]

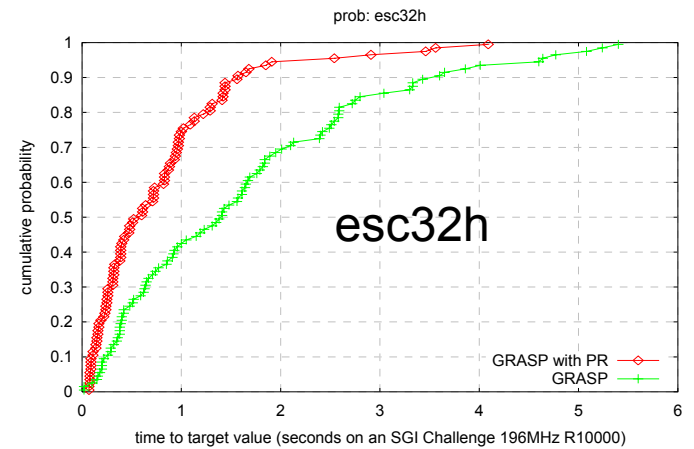
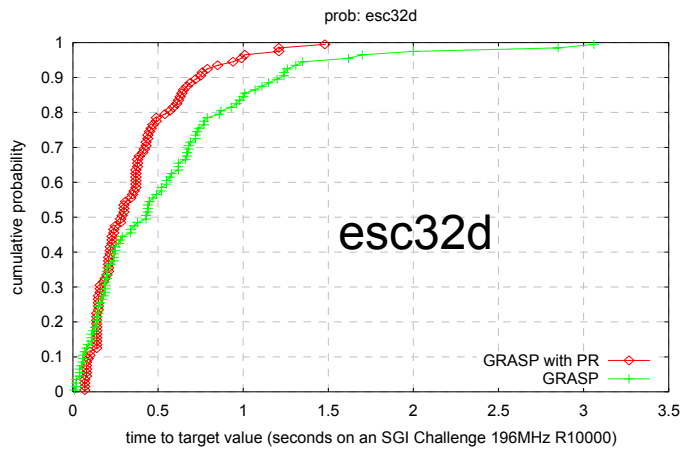
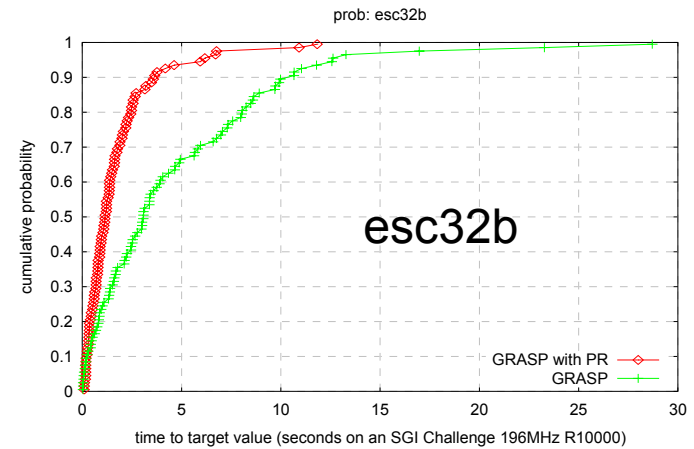
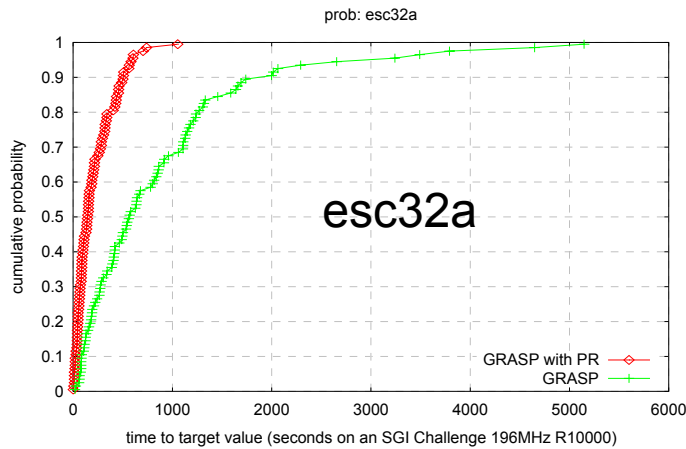




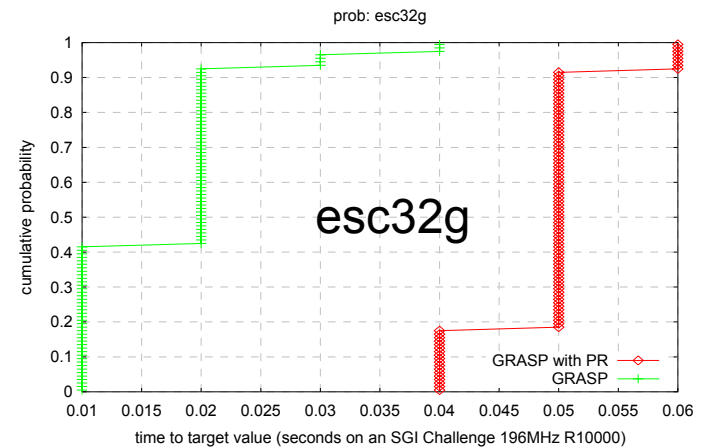
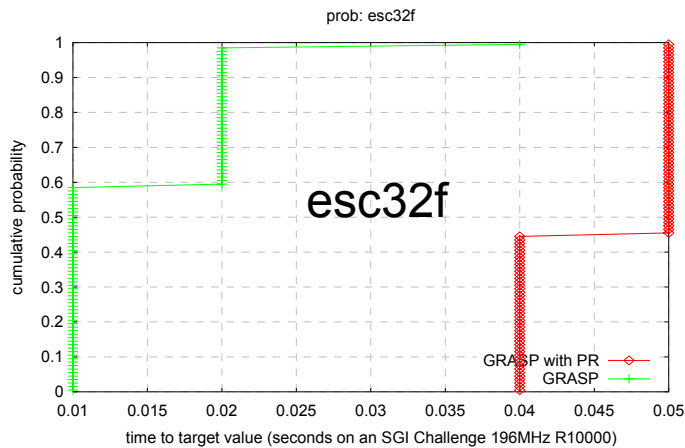
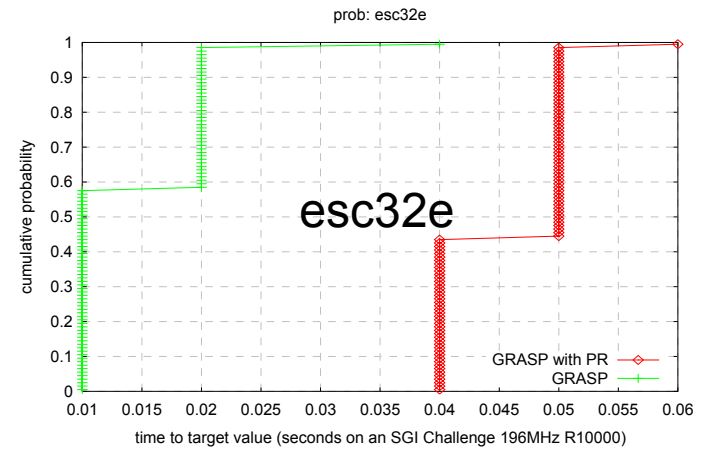
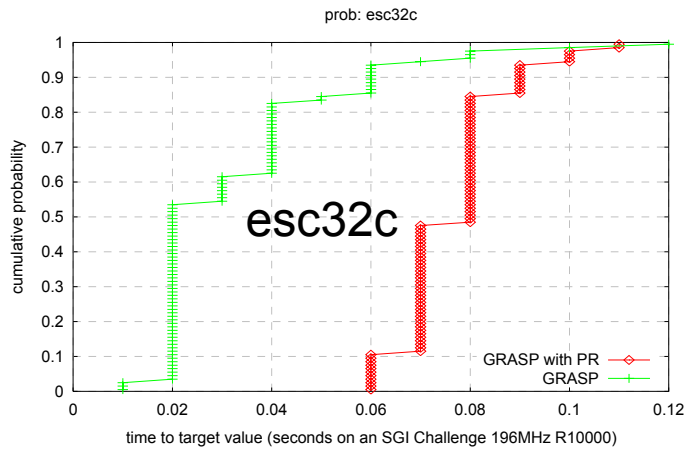
# J. Krarup and P.M. Pruzan [1978]



# B. Eschermann and H.J. Wunderlich [1990]



# B. Eschermann and H.J. Wunderlich [1990]



# Concluding remarks

- New heuristic for the QAP is described.
- Path-relinking shown to improve performance of GRASP on almost all instances.
- Journal paper will also compare GRASP+PR with other heuristics for QAP on larger instances from QAPLIB.
- Experimental results and code are available at <http://www.research.att.com/~mgcr/exp/gqapspr>

# Concluding remarks

- New heuristic for the QAP is described.
- Path-relinking shown to improve performance of GRASP on almost all instances.
- Journal paper will also compare GRASP+PR with other heuristics for QAP on larger instances from QAPLIB.
- Experimental results and code are available at <http://www.research.att.com/~mgcr/exp/gqapspr>

# Concluding remarks

- New heuristic for the QAP is described.
- Path-relinking shown to improve performance of GRASP on almost all instances.
- Journal paper will also compare GRASP+PR with other heuristics for QAP on larger instances from QAPLIB.
- Experimental results and code are available at <http://www.research.att.com/~mgcr/exp/gqapspr>

# Concluding remarks

- New heuristic for the QAP is described.
- Path-relinking shown to improve performance of GRASP on almost all instances.
- Journal paper will also compare GRASP+PR with other heuristics for QAP on larger instances from QAPLIB.
- Experimental results and code are available at <http://www.research.att.com/~mgcr/exp/gqapspr>

# My coauthors



Carlos A. S. Oliveira



Panos M. Pardalos