

GRAVITATIONAL COLLAPSE AND SPACETIME SINGULARITIES

Physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star when it collapses under its own gravity at the end of its life cycle is one of the most important questions in gravitation theory and relativistic astrophysics, and is the foundation of blackhole physics.

General relativity predicts that continual gravitational collapse gives rise to a spacetime singularity, which may be hidden inside an event horizon or visible to external observers. This book investigates these issues, and shows how such visible ultra-dense regions arise naturally and generically as an outcome of dynamical gravitational collapse. Quantum gravity may take over in these regimes to resolve the classical spacetime singularity. The quantum effects from a visible extreme gravity region could then propagate to external observers, providing a useful laboratory for quantum gravity, and implying interesting consequences for ultra-high energy astrophysical phenomena in the universe.

This volume will be of interest to graduate students and academic researchers in gravitation physics and fundamental physics, as well as in astrophysics and cosmology. It includes a review of recent research into gravitational collapse, and several examples of collapse models are worked out in detail.

Pankaj S. Joshi conducts research at the Tata Institute of Fundamental Research, Mumbai. His research interests include gravitation physics, spacetime structure and quantum gravity, and cosmology and relativistic astrophysics. He has published many research papers and books in these areas, and has held visiting faculty positions in several countries, lecturing and doing research on these topics.

Professor Joshi has an excellent international reputation for his work in the field of gravitation theory. His extensive analysis of general relativistic gravitational collapse has been widely recognized as providing significant insights into the final end states of a continual collapse, formation of visible singularities, and nature of cosmic censorship and blackholes.

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PANKAJ S. JOSHI

 $Tata\ Institute\ of\ Fundamental\ Research,$ $Mumbai,\ India$





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> To my parents, Arunadevi Shantilal Joshi and Shantilal Ramshankar Joshi



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Preface

The physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star, when it collapses under its own gravity at the end of its life cycle, is one of the most important questions in gravitation theory and relativistic astrophysics today. The applications and basic theory of blackholes vigorously developed over the past decades crucially depend on this outcome.

A sufficiently massive star many times the size of the Sun would undergo a continual gravitational collapse on exhausting its nuclear fuel, without achieving an equilibrium state such as a neutron star or white dwarf. The singularity theorems in general relativity then predict that the collapse gives rise to a spacetime singularity, either hidden within an event horizon of gravity or visible to the external universe. The densities and spacetime curvatures get arbitrarily high and diverge at these ultra-strong gravity regions. Their visibility to outside observers is determined by the causal structure within the dynamically developing collapsing cloud, as governed by the Einstein field equations. When the internal dynamics of the collapse delays the horizon formation, these become visible, and may communicate physical effects to the external universe. These issues are investigated here, and the treatment is aimed at showing how such visible ultra-dense regions arise naturally and generically as the outcome of a dynamical gravitational collapse in Einstein gravity. While it predicts the existence of visible singularities; classical general relativity may no longer hold in these very late stages of the collapse, and quantum gravity may take over to resolve the classical spacetime singularity. The quantum effects from a visible, the extreme gravity region could then propagate to outside observers to provide a useful laboratory for quantum gravity. Blackholes need not form in such a scenario and there may be interesting consequences for ultra-high energy astrophysical phenomena in the universe.

The general theory of relativity, which has strong experimental support, is used here, and its basics and useful features of spacetimes are reviewed. The necessary tools are developed as needed, but a prior familiarity with general relativity would help. It is a pleasure to thank many friends and colleagues



x Preface

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