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Gravitational instability and star formation in disc galaxies

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ABSTRACT
We present a general star formation law in which the star formation rate depends upon the efficiency α and the time-scale τ of star formation, and the gas component σ_g of the surface mass density. The current nominal Schmidt exponent n_S for our model is $2 < n_S < 3$. Based on a gravitational instability parameter Q_A and another dimensionless parameter $f_P = (P/G\sigma_c^2)^{1/2}$, where $P =$ pressure and $\sigma_c =$ column density of molecular clouds, we suggest a general equation for the star formation rate which depends upon the relative contributions of the two parameters for various physical circumstances. We find that Q_A turns out to be a better parameter for the star formation scenario than does the Toomre Q-parameter. The star formation rate in the solar neighbourhood is found to be in good agreement with values inferred from previous studies. In the closed box approximation model, we obtain a relationship between the metallicity of the gas and the efficiency of star formation. Our model calculations of metallicity in the solar neighbourhood agree with earlier estimates. We conclude that the metallicity dispersion for stars of the same age may result from a change in the efficiency with which different stars in the sample were processed. For no significant change of metallicity with age, we suggest that all stars in the sample were born with similar efficiencies.

Key words: instabilities – stars: formation – galaxies: evolution – galaxies: general – galaxies: ISM.

1 INTRODUCTION

It was realized by Kennicutt (1989) that there is a non-linear increase in the star formation rate near the threshold surface density corresponding to the Q-parameter. The star formation rate (R) is high in spiral arms, mostly because there is a lot of gas present. The efficiency varies by a lesser amount, such as a factor of 2. For example, in M51 and NGC 6946 (Rydbeck, Hjalmarson & Rydbeck 1985; Lord 1987; Tacconi-Garman 1988), the gas densities in the spiral arms are larger than elsewhere by a factor of 2, indicating deviations in the usual power-law exponent ($n \gg 2$) of Schmidt (1959, 1963). In fact, star formation in many spiral galaxies under extreme conditions of low gas density and low disc self-gravity presents a challenge to all current theories for disc star formation (Ferguson et al. 1996). New star formation laws have therefore been proposed (see e.g. Talbot & Arnett 1975; Dopita 1985; Wyse 1986; Silk 1987, Köppen & Fröhlich 1997).

However, the concept of a global star formation law has been put in doubt (Hunter & Gallagher 1986). For a general star formation scenario, one may refer to Zinnecker & Tscharnuter (1984) and Zinnecker (1989, and reference therein). Many interesting variants on the simple star formation laws include, for example, self-propagating star formation (stochastic) (Gerola & Seiden 1978; Seiden & Gerola 1982; Dopita 1985; Coziol 1996), self-propagating star formattion (Arimoto 1989; Hensler & Burkert 1990a,b), and star formation bursts (stochastic) (Matteucci & Tosi 1985). Krügel & Tutukov (1993) and Tutukov & Krügel (1995) have used a one-zone dynamical code without radial dependence of the variables to study the conditions for bursts of star formation. In the latter paper, using a one-zone code, they studied types of bursts of star formation in a galactic nucleus that were different from periodic bursts. Furthermore, the surface gas density threshold for star formation has been discussed by Kennicutt (1989).

Wyse & Silk (1989) have discussed an extended Schmidt model with R-dependence on the surface gas density σ_g and the local angular frequency $\Omega(r)$ for atomic and molecular gases respectively, with $n = 1$ and 2. Wang & Silk (1994) have recently presented a self-consistent model (considering the total gas surface density) for global star formation based on the gravitational instability parameter $Q < 1$ due to Toomre (1964). In the solar neighbourhood, the model agrees with (i) the observed star formation rate, (ii) the metallicity distribution among G dwarfs, and (iii) the age–metallicity relation for F dwarfs. The model results may be compared to the star formation rate in galactic discs with a Schmidt law with an exponent of about 2. The star formation rate also depends on the epicyclic frequency. A natural cut-off for $Q = 1$ in the star formation rate results. However, Wang & Silk's analysis is heavily based on the $Q < 1$ criterion, which has been questioned in relation to non-radial instabilities in galactic discs which may play a more fundamental role when a magnetic field supported by azimuthal gas motions (thus the resulting thermal instability is not related to Q at all) is taken into consideration (Elmegreen 1993). We note [see e.g. figs 4 and 6 of Wang & Silk (1994)] that the star formation does proceed in the regions where $Q \geq 1$. A natural question to ask is: how does star formation occur when $Q \geq 1$ and consequently the system has attained a state of gravitational equilibrium? We attempt here to answer this question precisely, and to provide a scenario to circumvent this natural cut-off in the star formation process (see Section 3 for details). The regulation of Q near its threshold value has been discussed by Dopita (1985) and Silk (1992). Silk (1995) has argued that local self-regulation of star formation may help to explain the initial mass function of stars, and that global self-regulation can account for the rate of star formation. The effects of the environment on the gas content and rotation curves of disc galaxies may play a crucial role in determining star formation rates and histories.

A review of recent observations of the history of star formation and its relevance to galaxy formation and evolution has been given by Kennicutt (1996). For the evolution of the global star formation history measured from the *Hubble Deep Field*, one may refer to Connolly et al. (1997). The gravitational instability of galactic discs has also been studied by Elmegreen (1995a), Fall & Efstathiou (1980), Cowie (1981), Ikeuchi, Habe & Tanaka (1984) and Bizyaev (1997), while gravitational instabilities in the presence of turbulence have been discussed by Bonnazzola et al. (1987) and Leorat, Passot & Pouquet (1990). It is found that supersonic turbulence may be strong enough (in some cases) to counteract the Jeans criterion for gravitational instability. As a result, it may stop gravitational collapse. In this scenario, star formation takes place in molecular cloud complexes at places where the turbulence evolves into the subsonic phase.

Elmegreen (1995b) has discussed critical column densities for gravitational instabilities and for cooling to diffuse cloud temperatures. It has been shown that the fundamental scales for star formation in the outer regions of galaxies (in the spiral arms) and in the resonance rings are related to the local unstable length-scale. Since the critical gas density for gravitational instability scales as the local density, the inner regions of galaxies have higher star formation rates beyond the threshold density.

The consideration of magnetic field changes the velocity dispersion by a factor of $\sqrt{2}$ for $Q > 1$ (i.e. the stable region). Incorporating this with the fact that there is shear instability of magnetized gas in the azimuthal direction, one is led to think that $Q < 1$ may not be the only criterion for cloud formation that results in star formation. An alternative suggestion for cloud formation, as a result of energy dissipation accompanied by shear instability which leads to star formation (even if $Q > 1$), has been given (Elmegreen 1991a, 1993; see below for details). Macroscopic thermal instabilities and various cloud formation mechanisms have been reviewed by Elmegreen (1991b). We assume that the instability parameter suggested by Elmegreen (1993), i.e. $Q_A < 1$ (instead of $Q < 1$), is the criterion that determines the occurrence of significant cloud formation instabilities. A natural consequence of our analysis is that star formation proceeds in the regions where one has $Q \geq 1$. It may be noted that in these regions (the system being gravitationally stable) an altogether different cloud formation mechanism (leading to star formation), as suggested by Elmegreen (1993), is required. Later we shall present evidence in support of our assumption. The outline of this paper is as follows. We give a general law for the star formation rate in Section 2. In Section 3, we suggest a general equation for the star formation rate which depends upon two fundamental parameters, Q_A and f_P (defined in the text). We also give a comparison of the star formation rate in the solar neighbourhood and the time-scale of gas depletion. Variations in the star formation rate and metallicity distribution in the solar neighbourhood are discussed in Section 4. Section 5 presents a discussion and summary of our results.

2 STAR FORMATION RATE

We write the star formation rate in the form

$$R = \alpha(\sigma_g/\tau), \tag{1}$$

where α is the efficiency of star formation, τ is the time-scale of star formation, and σ_g is the surface density of the gas, composed of atomic and molecular components. Clearly, τ^{-1} is related to the growth rate of the instability (Goldreich & Lynden-Bell 1965). In the present analysis, we do not aim to discuss the instability criteria and their relevance to star formation (although these are certainly interesting topics of research at present); instead we aim to obtain a general star formation law with a small number of adjustable parameters. We assume neither infall nor radial flow in the disc. We consider gravitational instability owing to axisymmetric perturbations [for non-axisymmetric case, one may refer to Goldreich & Lynden-Bell (1965)], with a magnetic field in the azimuthal direction which gives rise to shear instability in a magnetized gas. The growth rate of the instability is now expressed as

$$w^2 = k^2 v_{\rm eff}^2 - 2\pi G\sigma_g k + \kappa^2, \tag{2}$$

where k is the wavenumber and κ is the epicyclic frequency. $v_{\rm eff}$ is the effective velocity dispersion for the ambient Alfvén speed, such that

$$v_{\rm eff} = (v^2\gamma_{\rm eff} + v_{\rm Alf}^2)^{1/2}; \tag{3}$$

v is the velocity dispersion without a magnetic field, and

$$\gamma_{\rm eff} = \frac{\gamma w - w_c(1 + s - 2r)}{w + w_c(3 - s)}. \tag{4}$$

Here γ is the ratio of two specific heats, and w_c is the cooling rate [see e.g. Elmegreen (1993) for details]. In equation (4), r and s are the powers of the density and velocity dispersion in the heating rate function. Thermal instability follows if $r < (1 + s)/2$. For small r, the equation of state is soft and the effective value of the ratio of specific heats is small. For $r = 1$, $\gamma_{\rm eff} \sim 0.40 \pm 0.01$; for $r = 2$, $\gamma_{\rm eff} \sim 1.05 \pm 0.01$ and the gas is harder to deform. When $r < 0.5$ (for $s = 0$), the gas is always thermally unstable. The case $r = 0$ is thermally unstable and has a large growth rate. For $r > 0.5$ (and $s = 0$), the gas is thermally table (Elmegreen 1991a, 1994). The parameter Q is written as $Q = kv_{\rm eff}/\pi G\sigma_g$. Gravitational instability requires both that $Q < 1$ and that k be smaller than a critical value:

$$k_{\rm cr} = \frac{\pi G\sigma_g}{v_{\rm eff}^2}[1 + (1 - Q^2)^{1/2}]. \tag{5}$$

Owing to thermal instability, if $\gamma_{\rm eff}$ reaches large negative values (such that $\gamma_{\rm eff} < 0$), it implies no critical (or minimum) wavelength for gravitational perturbation in the radial direction. This makes $Q^2 < 0$. However, we do have a maximum wavelength of the perturbation. Thus equation (2) shows the absence of the Q-threshold for azimuthal instability, which means that all Q-values provide unstable growth. The Q-threshold may appear only if $\gamma_{\rm eff}(w)$ becomes a constant. Therefore, for the present treatment, we demand that

$Q_A \equiv 2\sqrt{2}Av_{\rm eff}/\pi G\sigma_{\rm g} < 1$ for growth of gravitational instability, but we are well aware that thermal and shear instabilities (along the azimuthal direction) are capable of determining cloud formation leading to star formation, even if $Q > 1$.

The maximum of w^2 occurs at

$$k_{\rm max} = 2\sqrt{2}A/v_{\rm eff}Q_A \quad (6)$$

(A is the Oort shear constant), which provides the maximum growth rate as (Wang & Silk 1994)

$$|w_{\rm max}| = \frac{2\sqrt{2}A(1-Q_A^2)^{1/2}}{Q_A}. \quad (7)$$

Since $\tau \simeq |w_{\rm max}|^{-1}$, one obtains from equations (1) and (7) that

$$R = \frac{\alpha(2\sqrt{2}A)\sigma_{\rm g}(1-Q_A^2)^{1/2}}{Q_A}. \quad (8)$$

Following Wang & Silk (1994), we define a function $f_{\rm c} = \sigma_{\rm g}/\sigma_{\rm c}$, where $\sigma_{\rm c}$ is the column density of individual molecular clouds. However, the relationship between individual cloud formation and star formation is complicated. Even the cloud formation process is not well known. The assumption that star formation results from gravitational instability naturally demands a relationship with the cloud formation process. Elmegreen (1993) has shown that gravitational instabilities generally form giant molecular clouds faster than they would form via random collisions. Cloud formation followed by star formation in the interstellar medium is certainly not the purpose of our investigation. Under the assumption that only gravitational instability is predominant, small cloud collisions may lead to large molecular clouds, wherein star formation ensues. It is then natural to think that, within an order of magnitude, the cloud formation time-scale (or, equivalent, the cloud collision time-scale) and the growth time-scale of the local instability are similar. Within this scenario, Wang & Silk (1994) derived the expression for the collision time between two clouds. We thus make use of their result, and write the collision time between two clouds as

$$t_{\rm coll}^{-1} = \frac{\sigma_{\rm g}(2\sqrt{2}A)}{\sigma_{\rm c}Q_A}. \quad (9)$$

In view of the above, $t_{\rm coll}^{-1} \sim w_{\rm max}$, and we obtain

$$Q_A \sim (1-f_{\rm c}^2)^{1/2}. \quad (10)$$

It should be noted that this may not reflect the general properties of the interstellar medium: e.g. other types of instability (namely thermal and Parker instabilities) might also contribute and affect the time-scale of star formation (and subsequently other physical quantities). Substituting equation (10) into equation (8), the star formation rate is now expressed as

$$R = \frac{\alpha(2\sqrt{2}A)\sigma_{\rm g}f_{\rm c}}{(1-f_{\rm c}^2)^{1/2}}. \quad (11)$$

Finally, in this form equation (11) now assumes the conversion from column density to density using the galactic scaleheight. Let us write equation (11) in the form

$$\frac{\partial \ln R}{\partial \ln \sigma_{\rm g}} = 1 + \frac{\partial \ln A}{\partial \ln \sigma_{\rm g}} + \frac{\partial}{\partial \ln \sigma_{\rm g}}\left\{\ln\left[\frac{f_{\rm c}}{(1-f_{\rm c}^2)^{1/2}}\right]\right\}$$

or

$$n_{\rm S} \equiv \frac{\partial \ln R}{\partial \ln \sigma_{\rm g}} = 2 + \frac{\partial \ln A}{\partial \ln \sigma_{\rm g}} + \frac{f_{\rm c}^2}{1-f_{\rm c}^2}, \quad (12)$$

where $n_{\rm S}$ stands for the nominal Schmidt exponent. The second term in equation (12) appears because, for spiral waves, the epicyclic frequency is expressed through

Table 1. Variation of shear constant A with surface density.

Distance (kpc)	A (km s^{-1} kpc^{-1})	$\ln A$	$\sigma_{\rm g}$ ($M_\odot$ pc^{-2})	$\ln \sigma_{\rm g}$
1	105	2.0212	100	2.0000
2	30	1.4771	3	0.4771
3	20.9	1.3202	5	0.6990
4	19.7	1.2945	10	1.0000
5	19.1	1.2820	10.5	1.0212
6	18.2	1.2601	10.2	1.0086
7	17.2	1.2355	10	1.0000
10	13.8	1.1399	7	0.8451
12	11.5	1.0607	5	0.6990
14	9.6	0.9823	4	0.6021
16	7.9	0.8976	3	0.4771
18	6.5	0.8129	2	0.3010
20	5.44	0.7356	1	0.0000

$$\kappa = \kappa_0(\sigma_{\rm g}/\sigma_0)^{1/2}, \quad (13)$$

and the shear constant A is

$$A = A_0(2 - \sigma_{\rm g}/\sigma_0). \quad (14)$$

The non-axisymmetric gravitational perturbation of a magnetic gaseous disc has been discussed by Elmegreen (1987), who obtained equations (13) and (14). Here A_0 and σ_0 represent equilibrium values of the shear rate and the surface mass density (see also Waller & Hodge 1991). It is easy to see that for, vanishing shear constant, equation (12) reduces to equation (19) of Wang & Silk (1994). It may be regarded as generalized version of Wang & Silk's equation in the sense that there is an additional term on the right-hand side which is certainly non-zero. We calculate the second term on the right-hand side of equation (12), i.e. $\partial \ln A/\partial \ln \sigma_{\rm g} \sim 0.54$, using the least-squares method. The data reported in Table 1 have been taken from Einasto (1979) and Wang & Silk (1994). Since mostly $f_{\rm c}$ is very small compared with unity (see e.g. Table 5, later) for the present Galactic disc, we conclude that the nominal Schmidt exponent $n_{\rm S}$ for our model corresponds to $2 < n_{\rm S} < 3$ for the Galaxy. For the usual Schmidt law, $n_{\rm S}$ lies between 1 and 2. Other normal spiral galaxies of Milky Way type are supposed to follow the same law.

3 THE GENERAL EQUATION FOR STAR FORMATION

We suggest that two fundamental parameters (Elmegreen 1993) which determine star formation may be put in the form

$$R = \alpha a + \beta a f_P, \quad (15)$$

where $a = 2\sqrt{2}A\sigma_{\rm g}^2/Q_A\sigma_{\rm c}$ and $f_P = (P/G\sigma_{\rm c}^2)^{1/2}$; here β is another parameter resulting from energy dissipation, and P is the pressure. The dimensionless pressure f_P (defined originally by Elmegreen 1993) is the square root of the ratio of the cloud collision rate to the gravitational instability rate, and so is a measure of the relative importance of cloud collisions. In our analysis presented here, we make use of some interesting results from Elmegreen (1993). Both Q_A and f_P now determine the star formation rate. When both are large (i.e. $Q_A > 1$ and $f_P \gg 1$), either thermal instability (macroscopic) triggers star formation, or cooling (which is very effective) reduces Q_A until gravitational instabilities take over. When both are small ($Q_A < 1$ and $f_P \ll 1$), gravitational instabilities form clouds quickly but star formation is hampered owing to a lack of energy dissipation. However, when $Q_A \geq 1$ and $f_P \ll 1$, star formation proceeds via random cloud collisions triggered by thermal instability, and the rate R is determined by the second term in equation (15). This is believed to occur at galactic radii $r < 4$ kpc and > 8 kpc, where one observes $Q \geq 1$ [for the observed Q-distribution in the Galaxy, see e.g. Wang & Silk (1994)]. When $Q_A \leq 1$ and $f_P \gg 1$, gravitational instability is primarily responsible for both cloud and star formation at all radii. In this case, the first term in equation (15) determines R. It is found that, at all radii, star formation is governed by the relative effectiveness of these terms. It also becomes evident that $Q_A < 1$ (or $Q < 1$) is not an absolute criterion for star formation; instead, star formation proceeds continuously until the required ingredients are provided and the physical conditions are met. In fact, one observes significant star formation even when $Q_a > 1$ in the Galaxy. Thus the process of star formation can be visualized through equation (15). It may be noted that our model does not take into account the galactic bulge component (Oort 1977), which might contribute to galactic gas dynamics in the inner region inside 0.1 kpc.

We write the star formation rate as

$$R = \frac{1}{(1-\delta)}\frac{\mathrm{d}\sigma_\mathrm{g}}{\mathrm{d}t}, \tag{16}$$

where δ is the fraction of mass returned to the interstellar medium from the stars. From equations (15) and (16) we obtain

$$\frac{\alpha(2\sqrt{2}A)\sigma_\mathrm{g}^2}{Q_A\sigma_\mathrm{c}} + \frac{\beta(2\sqrt{2}A)\sigma_\mathrm{g}^2}{Q_A\sigma_\mathrm{c}} f_P = \frac{1}{(1-\delta)}\frac{\mathrm{d}\sigma_\mathrm{g}}{\mathrm{d}t}. \tag{17}$$

Assume that the parameters f_P and σ_c are independent of time. Let us write equation (17) in the form

$$[\alpha(1-\delta)(2\sqrt{2}A) + \beta(1-\delta)(2\sqrt{2}A)f_P]\,\mathrm{d}t = \frac{(1-f_\mathrm{c}^2)^{1/2}}{f_\mathrm{c}^2}\,\mathrm{d}f_\mathrm{c}. \tag{18}$$

Integrate equation (18) to obtain

$$\frac{t}{\tau_\alpha} + \frac{t}{\tau_\beta} f_P = \frac{(1-f_\mathrm{c}^2)^{1/2}}{f_\mathrm{c}} - \sin^{-1}(f_\mathrm{c}) + \text{constant}, \tag{19}$$

where we have used

$$\tau_\alpha^{-1} = \alpha(1-\delta)(2\sqrt{2}A), \quad \tau_\beta^{-1} = \beta(1-\delta)(2\sqrt{2}A). \tag{20}$$

If we express $f_\mathrm{g} = \sigma_\mathrm{g}/\sigma_\mathrm{i}$, $f_\mathrm{ci} = \sigma_\mathrm{i}/\sigma_\mathrm{c}$, where the subscript i denotes initial values of quantities, we can write equation (19) as

$$\frac{t}{\tau_\alpha} + \frac{t}{\tau_\beta} f_P = -\frac{(1-f_\mathrm{g}^2 f_\mathrm{ci}^2)^{1/2}}{f_\mathrm{g} f_\mathrm{ci}} - \sin^{-1}(f_\mathrm{g} f_\mathrm{ci}) + \text{constant}. \tag{21}$$

At $t = 0$, $\sigma_\mathrm{g} = \sigma_\mathrm{i}$ ($f_\mathrm{g} = 1$), we obtain the value of the costant in equation (21) as

$$\text{constant} = \frac{(1-f_\mathrm{ci}^2)^{1/2}}{f_\mathrm{ci}} + \sin^{-1}(f_\mathrm{ci}). \tag{22}$$

Now, equation (21) becomes

$$(1+f_P)\frac{t}{\tau} = -\frac{(1-f_\mathrm{g}^2 f_\mathrm{ci}^2)^{1/2}}{f_\mathrm{g} f_\mathrm{ci}} + \frac{(1-f_\mathrm{ci}^2)^{1/2}}{f_\mathrm{ci}} - \sin^{-1}(f_\mathrm{g} f_\mathrm{ci}) + \sin^{-1}(f_\mathrm{ci}), \tag{23}$$

where we have used $\tau_\alpha = \tau_\beta = \tau$. The contribution of the second parameter may be observed on the right-hand side of equation (23). For $f_P = 0$, equation (23) reduces (except for a minus sign) to the equation derived by Wang & Silk (1994) [cf. equation (23) in Wang & Silk]. When $f_\mathrm{g} \ll 1$ near the centre of the disc, we find

$$\sigma_\mathrm{g} \sim \frac{\sigma_\mathrm{c}}{1+f_P}\left(\frac{\tau}{t}\right). \tag{24}$$

Towards the centre, A increases, which shows that the gas surface density decreases (τ varies inversely with A.) Large values of f_P for diffuse clouds again guarantee the depletion of gas at the centre. For the outer parts of the disc, $f_\mathrm{g} \sim 1$, and, after expanding various terms in equation (23) and neglecting higher order terms, we obtain the following:

$$\sigma_\mathrm{g} = \sigma_\mathrm{i}\left[(1+f_P)f_\mathrm{ci}\left(\frac{t}{\tau}\right) + 1\right]. \tag{25}$$

In view of the large values of τ and $f_\mathrm{ci} \ll 1$, the gas density scales as the initial density.

We now proceed to obtain critical column densities based on κ and on the new parameter Q_A. We write

$$\sigma_{\mathrm{cr},\kappa} = \frac{\kappa v_\mathrm{eff}}{\pi G}, \qquad \sigma_{\mathrm{cr},A} = \frac{2\sqrt{2}A v_\mathrm{eff}}{\pi G}. \tag{26}$$

Assume a rotation curve of the form $v \propto r^\mu$ ($\mu = 0$ for a flat curve). We obtain

$$\frac{\sigma_{\mathrm{cr},A}}{\sigma_{\mathrm{cr},\kappa}} = \frac{1-\mu}{(1+\mu)}. \tag{27}$$

It is found that, for $\mu = 0$, both densities agree. For large μ (i.e. departures from flatness), however, $\sigma_{\mathrm{cr},A}$ becomes smaller than $\sigma_{\mathrm{cr},\kappa}$. For example, for M33, $\mu = 0.3$ (Newton 1980), which yields

$$\sigma_{\mathrm{cr},A} = 0.61\sigma_{\mathrm{cr},\kappa}. \tag{28}$$

Observations of σ_g (Wilson, Scoville & Rice 1991) for this galaxy are better explained if one takes $\sigma_{\mathrm{cr},A}$ as the threshold density rather than $\sigma_{\mathrm{cr},\kappa}$ (see also Elmegreen 1993). Thus Q_A emerges as a better parameter, as far as disc instabilities are concerned, than the Toomre Q-parameter for star formation. This is also supported by the ratio of the two threshold densities (see e.g. Table 2).

Table 2 shows that, for the highly non-linear region of rotation velocity (i.e. $\mu \ll 1$), the threshold density based on Q_A is lowered (relatively), favouring the instability for star formation. On the other hand, the threshold density based on the Q-parameter is higher (by about one order of magnitude) in this region. This shows that the Q-parameter is relatively less efficient for star formation. We therefore conclude that, in the non-linear regime of the rotation velocity curve, the Q-parameter is less effective than the Q_A-parameter for triggering the process.

We have computed the ratio Q_A/Q (see e.g. Table 3, data taken

Table 2. Variation of the ratio of the threshold densities with index μ.

μ	$\sigma_{\mathrm{cr},A}/\sigma_{\mathrm{cr},\kappa}$
0.005	0.99
0.05	0.93
0.10	0.86
0.15	0.80
0.20	0.73
0.30	0.61
0.40	0.51
0.50	0.41
0.60	0.32
0.70	0.23
0.80	0.12
0.90	0.07

Table 3. The radial variation of Q_A/Q for the Galaxy (data from Einasto 1979).

r (kpc)	A ($\times 10^{-16.5}\,\mathrm{s}^{-1}$)	$-B$ ($\times 10^{-16.5}\,\mathrm{s}^{-1}$)	κ ($\times 10^{-16.5}\,\mathrm{s}^{-1}$)	Q_A/Q
1	105	62	203.5	1.5
2	30	55	136.8	0.6
3	20.9	44.5	107.9	0.5
4	19.7	34.1	85.7	0.6
5	19.1	26.1	68.7	0.8
6	18.2	20.1	55.5	0.9
7	17.2	15.6	45.2	1.1
10	13.8	7.9	26.2	1.5
12	11.5	5.5	19.3	1.7
14	9.6	4.3	15.5	1.7
16	7.9	3.6	12.9	1.7
18	6.5	3.3	11.4	1.6
20	5.44	3.11	10.3	1.5
30	2.91	2.40	7.1	1.2
50	1.59	1.53	4.4	1.0
75	1.06	1.00	2.9	1.0

from Einasto 1979) at various radial distances in the Galaxy. B denotes the second Oort constant. A plot of Q_A/Q with radial distance from the centre is shown in Fig. 1. It is found that Q_A displays almost the same behaviour as the observed Q-distribution (cf. fig. 6 of Wang & Silk 1994), with remarkably good agreement for the range 1–15 kpc, and the ratio stays at $Q_A/Q \gtrsim 1$ beyond 30 kpc. Thus Q_A and Q agree beyond 30 kpc, i.e. in the flat rotation curve region. We obtain the same result from the data shown in Table 2. In fact, for a disc radius below 30 kpc, the deviations in the two parameters become significant which shows the relative merit of the Q_A-parameter over the Q-parameter in keeping track of physical processes such as star formation and other nuclear activity.

3.1 Comparison of observations for the Galaxy

We assume a constant initial mass function in the solar neighbourhood (Miller & Scalo 1979; Scalo 1986), and take the following input parameters: initial surface density $\sigma_{i,\odot} \simeq \sigma_{g,\odot} + \sigma_{s,\odot} \simeq 50\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}$(where $\sigma_{g,\odot}$ and $\sigma_{s,\odot}$ are gas and star densities normalized to solar values) (Kuijken & Gilmore 1989; Bahcall, Flymn & Gould 1992), $\sigma_g \simeq 10\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}$ (McKee 1990), $f_g \sim 0.2, f \sim 0.05$ (Elmegreen 1993), $t =$ age of the Galaxy $= 15$ Gyr, $\alpha = 0.1$ (Myers et al. 1986), Oort shear constant $A = 15\,\mathrm{km\,s}^{-1}\,\mathrm{kpc}^{-1}$ (Kerr & Lynden-Bell 1986), and $\delta = 0.3$ (Miller & Scalo 1979; Scalo 1986). We obtain the time-scale of star formation as $\tau = 0.38$ Gyr. We calculate $f_{ci,\odot}$ using equation (23) as $f_{ci,\odot} \sim 0.10$. After substituting these values into equation (11), we obtain the star formation rate as $R = 3.8\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}\,\mathrm{Gyr}^{-1}$. This is in agreement with Scalo (1986) who infers $R \simeq 1$–$4\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}\,\mathrm{Gyr}^{-1}$ within an uncertainty of about a factor of 3.

We find that our model provides a star formation rate that is in good agreement with the inferred rate in the solar neighbourhood. It should be noted that our model is sensitive enough to the efficiency α introduced in equation (1), which is, however, determined by the star formation time-scale τ. Parametric freedom for α and f_P, even when $Q_A \gtrsim 1$ (i.e. when non-gravitational instabilities are dominant), provides a general star formation scenario. Our model thus presents a generalization of Wang & Silk's model with a dependence of the star formation rate on the Oort shear constant A. In contrast to Wang & Silk, we find a continuous (in the sense of Q-values) star formation rate obeying a similar but different criterion (i.e. $Q_A < 1$) of gravitational instability for gaseous discs. In fact, the competitive nature of the two terms in equation (15) helps one to visualize the essence of continuity in the star formation process. We discuss this scenario in more detail in Section 3.3.

3.2 Time-scale of gas depletion

From equations (1) and (16) we obtain

$$\frac{d\sigma_g}{\sigma_g} = -\alpha(1-\delta)\tau^{-1}\,dt, \tag{29}$$

$$\ln \sigma_g = -\alpha(1-\delta)\tau^{-1}t + \text{constant}. \tag{30}$$

At $t = 0$, $\sigma_g(r,t) = \sigma_g(r,0)$, which yields

$$\sigma_g(r,t) = \sigma_g(r,0)\exp[-\alpha(1-\delta)\tau^{-1}t] \tag{31}$$

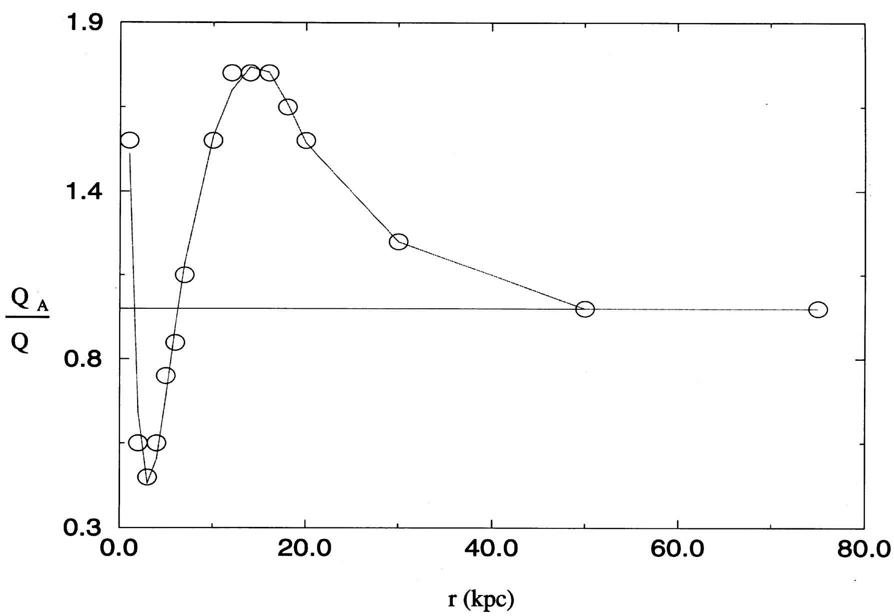

Figure 1. The radial variation of Q_A/Q for the Galaxy. We have taken data from Einasto (1979), and have expressed the Q_A-parameter as in Elmegreen (1993).

(see also Lynden-Bell 1975; Güsten & Mezger 1983). Now we can write the e-folding time as

$$t_d = \frac{1}{\alpha(1-\delta)\tau^{-1}}. \tag{32}$$

For our input parameters, the depletion time t_d for the model is $t_d \simeq 5.4$ Gyr. For an age of 15 Gyr of the Galaxy, the present gas fraction is ~10 per cent of its initial value, assuming that there has been little variation over the last 5 Gyr (Dopita 1985, 1987).

3.3 The f_P-parameter and star formation

The f_P-parameter introduced in equation (15) requires further analysis, as regards the process of star formation. It is dimensionless and measures the fraction of diffuse clouds to self-gravitating clouds. Low values of f_P ($f_P \sim 0.01$) mean that clouds are dense and self-gravitating. In this case, the physics of star formation is largely determined by the first term in equation (15). If, however, $f_P \sim 100$ as for example in the inner Galaxy, where the pressure becomes high (Elmegreen & Elmegreen 1987; Polk et al. 1988; see also Vogel, Kulkarni & Scoville 1988 for M51), diffuse molecular clouds collide and cool, leading to high-mass cloud formation. Nevertheless, this does not mean that such regions evolve into large star-forming clouds. In fact, gravitational instabilities are more efficient (as compared with diffuse cloud collisions) at producing high-mass star-forming clouds. In this case, however, local energy dissipation occurs through diffuse cloud collisions (Elmegreen 1989). A major difficulty for star formation triggered by gravitational instability appears when Q_A and f_P are both large. In this case, only thermal instability is responsible for switching on the star formation process. Murray & Lin (1989) have stressed the dominant role of thermal instability over gravitational instability for a protoglobular cluster where fragmentation (into protostars) is initiated by the former. Low f_P-values may also result when the pressure becomes low (i.e. in the outer spiral arms of galaxies where the gravity is not sufficient to form large molecular clouds) and star formation proceeds via shear instability. This instability does depend upon Q_A. Still, Q_A has to be relatively small to guarantee unstable radial motion, which in turn facilitates dense cloud formation.

Table 4. The radial variation of surface density for the Galaxy (data from Einasto 1979, Lacey & Fall 1985 and Wang & Silk 1994).

d (kpc)	$\ln \sigma_i (M_\odot\,pc^{-2})$	$\ln \sigma_g (M_\odot\,pc^{-2})$
0.001	5.5	
0.01		
0.1	4.4	
1.0	3.0	2.0
2.0	3.0	0.5
3	2.8	0.7
4	2.7	1.0
5	2.5	1.0
6	2.4	1.0
7	2.2	1.0
10	1.7	0.9
12	1.4	0.7
14	1.0	0.6
16	0.7	0.5
18	0.4	0.3
20	0.1	0.0

4 VARIATION IN THE STAR FORMATION RATE

We assume the cloud mass density in the solar neighbourhood, $\sigma_c \sim 100\,M_\odot\,pc^{-2}$, to be constant (Larson 1981). We further assume that $f_c \sim 0.01$ at $d = 1$ kpc (since σ_g at 1 kpc is $\sim 100\,M_\odot\,pc^{-2}$, which makes $f_c = 1$, yielding an infinite R) to keep the star formation rate reasonably large in the model. The components of surface density [data taken from Einasto (1979), Lacey & Fall (1985) and Wang & Silk (1994)] are given in Table 4 and plotted in Fig. 2 at various distances from the Galactic Centre.

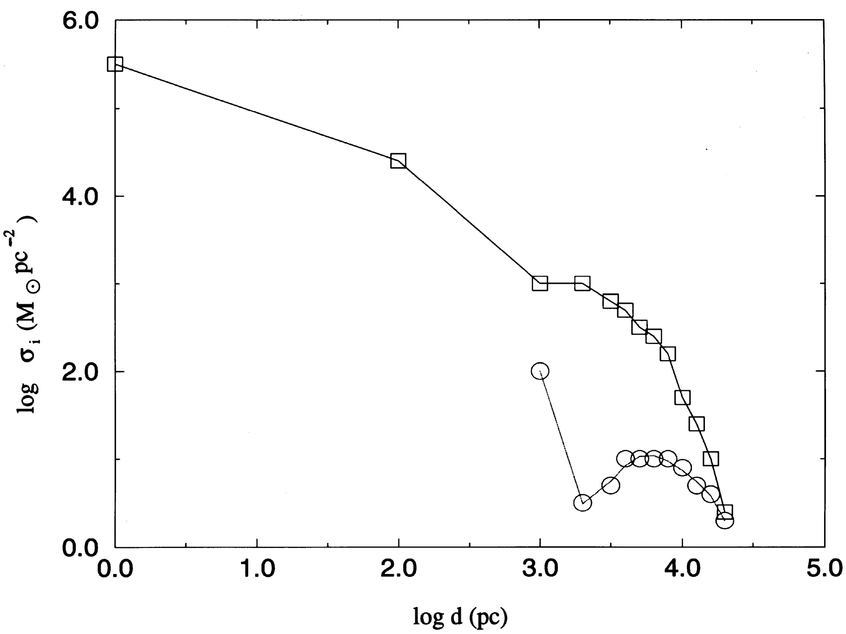

Figure 2. The radial variation of the total surface density $\sigma_i (M_\odot\,pc^{-2})$ and the gas surface density $\sigma_g (M_\odot\,pc^{-2})$ for the Galaxy. Circles denote data taken from Lacey & Fall (1985) and Wang & Silk (1994); squares are data from Einasto (1979).

Table 5. The radial variation of star formation rate for the Galaxy.

d (kpc)	A ($\times 10^{-16.5}\,s^{-1}$)	σ_i ($M_\odot\,pc^{-2}$)	f_c	$R/R_\odot$
1	105	1016.3	0.01	78.6
2	30	851.1	0.03	56.5
3	20.9	633.9	0.05	48.8
4	19.7	452.9	0.10	66.0
5	19.1	318.4	0.11	49.5
6	18.2	222.3	0.10	29.9
7	17.2	154.9	0.10	19.7
10	13.8	50.9	0.07	3.6
12	11.5	23.6	0.05	1.0
14	9.6	10.7	0.04	0.3
16	7.9	4.9	0.03	0.1
18	6.5	2.3	0.02	0.02
20	5.44	1.2	0.01	0.005

We infer from Fig. 2 that the Einasto model shows a $\sigma_i \sim r^{-0.8}$ dependence for $r \leq 6$ kpc and deviates for $r > 6$ kpc [see also e.g. Kundt (1990) for a variant of the Galactic mass distribution]. Now, we aim to discuss the variation of the star formation rate normalized to that in the solar neighbourhood, and we therefore calculate $R/R_\odot$ (see e.g. Table 5) using data from Einasto (1979) Lacey & Fall (1985) and Wang & Silk (1994) and plot it in Fig. 3 at various Galactocentric distances. From Figs 2 and 3 we infer that the star formation rate varies like the gas component of the surface density.

A minimum in σ_g occurs at ~3 kpc where we also observe a minimum in the star formation rates. Thereafter, σ_g increases again and reaches a maximum at ~4 kpc where we observe a corresponding increase and maximum in $R/R_\odot$. Our model agrees with Wang & Silk's model, but we obtain a larger Schmidt exponent (see e.g. equation 12).

The star formation rates inferred from (i) pulsar data (Lyne, Manchester & Taylor 1985), (ii) observations of supernova remnants (Guibert, Lequeux & Viallefond 1978), and (iii) Lyman continuum photon observations from H II regions (Güsten & Mezger 1983) are consistent with our model at all radial distances. For example, the higher rate of star formation traced by the Lyman continuum near 4 kpc agrees with our model calcualtions. This is demonstrated by the maximum in Fig. 3 at 4 kpc from the Galactic Centre. In view of comments (Wyse & Silk 1989) regarding the higher star formation rate of Güsten & Mezger compared with those given by Scalo (1988) (i.e. the estimate may be higher by an order of magnitude), and also the fact that it does not match with the star formation profile obtained by other techniques (Rana & Wilkinson 1986), our values are apparently more accurate.

We assume that $\tau = 0.45$ Gyr. Using parameters as described in Section 3.1, we calculate the efficiency α of star formation as a function of distance from the Galactic Centre. This is achieved by obtaining σ_g and R using Figs 2 and 3 at a particular distance, thereby resulting in a value of α for that distance. It is interesting to observe that α changes in the solar neighbourhood. Small values of α at 1 kpc may be understood to arise because of shear instability which removes the growth of perturbations. Star formation can proceed if the self-gravitational collapse time becomes shorter than the shear time (~0.01 Gyr). However, a relatively large value of α out to 10 kpc does not lead to a large star formation rate $R/R_\odot$ (see Fig. 3), owing to the paucity of gas. In fact, the density σ_i drops below the observed value (Wilson et al. 1991) of the critical density at 14 kpc, where we expect a turn-off of star formation owing to gravitational instability. This is also supported by a significant depletion of gas at this distance (see e.g. Fig. 2). The striking feature of our result is that α changes in the solar neighbourhood (an efficiency gradient ~$0.0057\,kpc^{-1}$) by an amount ~0.02. It thus seems natural to think that the efficiency gradient is responsible for the radial abundance gradients that are reported in many disc galaxies (Edmunds & Pagel 1984; Diaz & Tosi 1984; Tosi & Diaz 1985). The fact that metallicity gradients may arise from changes in the efficiency of star formation was suggested previously by Lacey & Fall (1985). We aim to confirm this suggestion from our calculations also. There is hardly a need to invoke radial flows (see the discussion in Scalo 1988) in this scenario.

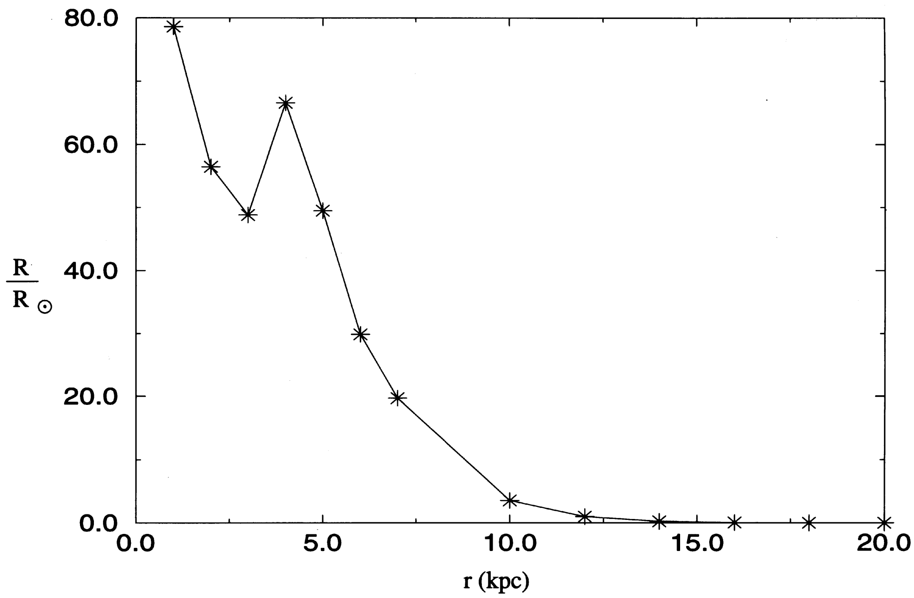

Figure 3. The star formation rate normalized to its value in its solar neighbourhood. The data are from Einasto (1979), Lacey & Fall (1985) and Wang & Silk (1994).

4.1 Metallicity gradient versus efficiency gradient

Following the Pagel & Patchett (1975) (see also e.g. Pagel & Edmunds 1981) model of chemical evolution of the Galaxy in the solar neighbourhood, we define ξ, a mass ratio in the form of long-lived stars, and p as the yield of heavy elements which represents mass ejected per unit mass of long-lived stars (cf. Searle & Sargent 1972; Talbot & Arnett 1973a). For our model, $\sigma_s = \xi\sigma_i$, $\sigma_g = (1-\xi)\sigma_i$, $\sigma_s = \xi/(1-\xi)\sigma_g$ and

$$\frac{d\sigma_s}{dt} = -\alpha\left(\frac{\sigma_g}{\tau}\right) = \left(\frac{\xi}{1-\xi}\frac{d\sigma_g}{dt}\right). \tag{33}$$

Equation (33) gives

$$\frac{d}{dt}(\ln\sigma_g) = -\frac{\alpha(1-\xi)}{\tau\xi}. \tag{34}$$

Integration of equation (34) yields

$$\ln\sigma_g = -\frac{\alpha(1-\xi)}{\xi}\frac{t}{\tau} + \text{constant}. \tag{35}$$

At $t = 0$, $\sigma_g = \sigma_i$, and hence the constant $= \ln\sigma_i$. Thus equation (35) takes the form

$$\frac{\sigma_g}{\sigma_i} = (1-\xi) = \exp\left[-\frac{\alpha(1-\xi)}{\xi}\frac{t}{\tau}\right]. \tag{36}$$

Therefore

$$\tau(t) = -\frac{\alpha(1-\xi)}{\xi}[\ln(1-\xi)]^{-1}t. \tag{37}$$

The metallicity Z is expressed as (Pagel & Patchett 1975)

$$Z = p\ \ln\left(\frac{1}{1-\xi}\right) = \frac{p\alpha(1-\xi)}{\xi}\frac{t}{\tau}. \tag{38}$$

We see that τ is now a function of time and is given by equation (37). The time evolution of Z may be written as

$$\frac{dZ}{dt} = \frac{p}{\tau} = \frac{p\xi}{\alpha(1-\xi)[\ln(1-\xi)]^{-1}t}. \tag{39}$$

From equation (38) we infer that Z varies linearly with both time and the efficiency of star formation. We assume that $\xi \simeq 0.8$ (Talbot & Arnett 1973a) and $p \simeq 0.7\,Z_\odot$ (Wang & Silk 1993) to calculate Z in the solar neighbourhood. For an efficiency of $\alpha \sim 0.07$, we find $Z \simeq 1.13\,Z_\odot$ for the solar age. This is in agreement with the plot of metallicity in the solar neighbourhood presented by Wyse & Silk (1989, fig. 2d). The present model thus providess the time evolution of the metallicity which, however, depends upon the efficiency of star formation.

For an efficiency run of $\alpha \sim 0.07$, 0.08, 0.09 and 0.10, we find $Z/Z_\odot \simeq 1.23$, 1.17, 1.13 and 1.09 respectively, which are independent of the Galactic age provided of course that the parameters p and ξ do not change with time. In other words, disc ageing alters $\tau(t)$ such that $t/\tau(t)$ remains constant (for a fixed α), and hence there is no change in $Z/Z_\odot$. Our calculations show that the metallicity decreases with increasing α at a given age. In the solar neighbourhood, this may be understood as being due to the paucity of gas, favouring a relatively low star formation rate at large distances (see Fig. 3) and therefore low metal production (see also Friel & Janes 1993). We note that the observed run of metallicity of G–K dwarfs in our Galaxy is very sensitive to the chemical composition of stars of the same age (Tinsley 1975). Janes & McClure (1972) have suggested an enhancement in the dispersion owing to chemical inhomogeneities in the Galaxy (Talbot & Arnett 1973b). The structure of the Galactic disc and the presence of population gradients are considered by Ferrini et al. (1994). For a radial distribution of abundances in galaxies, refer to Molla, Ferrini & Diaz (1996), who have also discussed the chemical evolution of the solar neighbourhood (see e.g. Pardi, Ferrini & Matteucci 1995).

However, the fact, noted by Tinsley (1975), that the observed dispersion (see also Hearnshaw 1972 in metallicity for stars of the same age may either be partly due to chemical inhomogeneities (of the interstellar medium) or result from altogether different causes essentially favours our analysis. We find that the metallicity dispersion for stars of the same age may be due to a variation of the efficiency α with which different sample stars were processed. This confirms the assumption of Rana & Wilkinson (1986) that the metallicity dispersion is due to stellar processing only. It is found that α depends upon the star formation rate and also the gas component of the surface density σ_g. We conclude that α predominantly determines the observed dispersion, and plays a key role in the metal enrichment or otherwise of the interstellar medium.

At various disc ages (at a given radial distance), there occurs a change in α which causes metallicity dispersion. We note that α also changes at various distances from the Galactic Centre, which results in spatial metallicity gradients. We find that the apparent metallicity dispersions with either age or distance depend upon α. An [O/H] versus age plot (Wyse & Silk 1989, fig. 2d; see also Carlberg et al. 1985) shows a barely significant metallicity gradient at all disc ages (cf. Friel & Janes 1993). We suggest that all of the sample stars might have evolved with almost the same efficiency. Thus the important result of this analysis is the confirmation of the suggestion by Lacey & Fall (1985) and Richtler (1995) regarding metallicity gradients. For a comprehensive treatment of radial abundance gradients in spiral discs and the age–metallicity relation in different stellar populations, refer to Edvardsson et al. (1993) and Pagel (1994). An interesting modern analysis of the kinematics and abundance distribution for our Galaxy has been given by Gilmore, Wyse & Kuijken (1989). Matteucci (1996) has exhaustively reviewed the evolution of the abundances of heavy elements in gas and stars (indicating observational and theoretical constraints) in galaxies of different morphological types. After similar work by Tinsley (1980), his paper provides a good review of progress in the understanding of physical processes regulating the chemical evolution of galaxies. The formation and evolution of our Galaxy are also discussed. For a review on abundance ratios and Galactic chemical evolution, see McWilliam (1997). The chemical evolution of the solar neighbourhood according to the standard infall model, using data on Type II supernovae, is summarized by Thomas, Greggio & Bender (1998).

5 CONCLUSIONS

We have discovered that the theory of Elmegreen (1993) regarding star formation appears more robust than the Q-criterion. This is because, unless $Q \le 1$, gravitational instability does not permit star formation. However, when $Q > 1$, the system becomes gravitationally stable and consequently star formation via large cloud formation is not feasible. A natural question to ask is: how does star formation proceed when Q enters the stable regime? This in fact has led to an alternative criterion for cloud formation (discussed in the text), leading to star formation as originally suggested by Elmegreen (1993). Accordingly, when the magnetic field is taken into consideration, the velocity dispersion changes and thus Q is pushed into the stable regime. At this stage, non-gravitational instabilities (e.g. thermal instability, shear instability) dominate over gravitational instability. We infer from Fig. 1 that the dependence of Q_A/Q on distance from the Galactic Centre describes the relative merit of the Q_A-parameter over the Q-parameter beyond 6 kpc. Observations of σ_g for M33 are in better agreement with theory when one regards Q_A as the gravitational instability parameter (see e.g. Wilson et al. 1991). It is found that both Q_A and Q agree beyond 30 kpc.

We have obtained a generalized version of Wang & Silk's (1994) equation [see e.g. Section 2, equation (12); and Section 3, equation (23)] in the sense that (i) there is an additional non-zero term in equation (12), and (ii) in view of equation (15), one arrives at a natural escape from the cut-off criterion for star formation. We have also shown that the nominal Schmidt exponent n_S is given by $2 < n_S < 3$ in our model. We suggest a general equation (e.g. equation 15) for the star formation rate, consisting of two terms: the first term dominates when $Q_A < 1$ and $f_P \ll 1$; the second term dominates when $Q_A \geq 1$ and $f_P \ll 1$. Apparently, the relative sizes of these terms determine the star formation scenario (as discussed in Section 3) at all radial distances. $Q < 1$ (or $Q_A < 1$) is not an absolute criterion for star formation. For our model, we obtain star formation rates that are in good agreement with values inferred by Scalo (1986). We find that our models are sensitive enough to the efficiency α and time-scale τ of star formation.

We suggest that essentially the efficiency gradient is the cause of the radial abundance gradients that are reported in many disc galaxies. Under the approximation of the closed box model, we have derived the time evolution of τ and also the metallicity $Z(t)$. Both $\tau(t)$ and $Z(t)$ are functions of α, p (the yield of heavy elements) and the mass ratio ξ. We notice hardly any metallicity change as the disc ages which, however, reflects the fact that stellar processing occurs at a fixed α. The metallicity dispersion for stars of the same age many be caused by variations in α. We conclude that α is predominantly responsible for the metallicity dispersion and also for the metal enrichment of the interstellar medium. A simple model, as discussed above, provides some important characteristics of our Galactic disc, although, as suggested by Tinsley (1980), the star formation is a complicated function of numerous physical parameters: e.g. gas density, gas sound speed, shock frequency, shock strength, gas rotation, shear constant A, magnetic field, gas metal abundance and background star density. It is, however, difficult to predict the actual dependence of R on these parameters. One therefore studies some form of R and its consequent effect on the chemical and photometric evolution. Finally, the model predictions have been compared with observations.

We note that the star formation rate was probably higher in the central part of the disc of our Galaxy at an early epoch of evolution. It should be noted that hydrodynamical simulations of the formation and evolution of a galaxy may be performed by incorporating our model formulation of the star formation rate and metallicity. Model predictions, when compared with observations of other galaxies, would provide evidence of its robustness and accuracy.

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