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Gravitational-Wave Observations as a Tool for Testing Relativistic Gravity*

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ABSTRACT

Gravitational-wave observations can be powerful tools in the testing of relativistic theories of gravity. Future experiments should be designed to search for six different types of polarization, and for anomalies in the propagation speed of the waves: $|c_{\text{grav. waves}} - c_{\text{em waves}}| \gtrsim 10^{-7} c_{\text{em waves}}$. This letter outlines the nature and implications of such measurements.

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Several viable gravitation theories now exist that differ radically when describing strong gravitational fields, but that are identical to each other and to general relativity in the "post-Newtonian limit." During the next twenty years, one will probably not be able to distinguish these theories from general relativity or from each other by means of "solar-system experiments" (gravitational redshift, perihelion shift, light deflection, time delay, gyroscope precession, lunar-laser ranging, gravimetry, Earth rotation, ...). However, gravitational-wave experiments offer hope: These theories differ in their predictions of (i) propagation speed, and (ii) polarization properties of gravitational waves.

Propagation speed: Some of the competing theories[1-3] predict the same propagation speed for gravitational waves (c_g) as for light (c_{em}). But others[4,5] predict a difference that, in weak gravitational fields, is typically

$$(c_g - c_{em})/c \sim (1/c^2) \times |\text{Newtonian potential}|$$

$\sim 10^{-7}$, for waves travelling in our region of the Galaxy or in the field of the Virgo cluster. An experimental limit of $\lesssim 10^{-8}$ would disprove most such theories and would stringently constrain future theory-building. Perhaps the most promising way to obtain such a limit is by comparing arrival times for gravitational waves and for light that come from the onset of a supernova, or from some other discrete event. If current experimental efforts continue unabated, by 1980 one may detect gravitational-wave bursts from supernovae in the Virgo cluster (~ 3 supernovae per year). Then a limit of

$$|c_g - c_{em}|/c \lesssim 10^{-9} \times (\text{time lag precision})/(1\text{ week})$$

will be possible.

Polarization: All of the currently viable theories fall into a class called "metric theories of gravity."[6,7] Recently we have completed an analysis of the polarization properties of the most general weak, plane, null gravitational wave permitted by any metric theory. (Details will be published elsewhere.[8] Our considerations also apply to waves which are approximately, rather than exactly, null.) We find that the most general wave is composed of six modes of polarization (general relativity has only two), as follows.

Use coordinates txyz. Let the wave propagate in the +z direction. The wave is characterized by six amplitudes which depend only on "retarded time" u, where $u \equiv t - z/c$. Our analysis describes these amplitudes by two real functions $\Psi_2(u)$, $\Phi_{22}(u)$ and the real and imaginary parts of two complex functions $\Psi_3(u)$, $\Psi_4(u)$. These functions are related to those components of the Riemann tensor which determine the action of the wave on a detector[9] by

$$\Psi_2 = -\frac{1}{6} R_{zozo}$$

$$\Psi_3 = \frac{1}{2} (- R_{xozo} + i R_{yozo})$$

$$\Psi_4 = R_{yoyo} - R_{xoxo} + 2i R_{xoyo}$$

$$\Phi_{22} = - (R_{xoxo} + R_{yoyo}).$$

Figure 1 shows the action of each mode on a sphere of test bodies. Ψ_4 and Φ_{22} are purely transverse, Ψ_2 is purely longitudinal, and Ψ_3 is mixed.

These waves can be classified in a Lorentz-invariant manner according to the vanishing or nonvanishing of certain of the amplitudes. Imagine many observers in different Lorentz frames, some moving with respect to each other, but all measuring the same 4-momentum of the wave. The amplitudes transform between observers in a complicated way [cf. Eq. (1) below] but the waves fall into these invariant classes:

Class II_6. $\Psi_2 \not\equiv 0$. All observers in such Lorentz frames measure a nonzero amplitude in the Ψ_2 mode, and agree on the value of this amplitude. (But they will generally disagree about the presence or absence and amplitude of all other modes.)

Class III_5. $\Psi_2 \equiv 0 \not\equiv \Psi_3$. All observers agree on the absence of Ψ_2 and the presence of Ψ_3. (But they generally disagree about the presence or absence of Ψ_4 and Φ_{22}.)

Class N_3. $\Psi_2 \equiv 0 \equiv \Psi_3$; $\Psi_4 \not\equiv 0 \not\equiv \Phi_{22}$. All observers agree about the presence or absence of all modes.

Class N_2. $\Psi_2 \equiv 0 \equiv \Psi_3$; $\Psi_4 \not\equiv 0 \equiv \Phi_{22}$. All observers agree.

Class O_1. $\Psi_2 \equiv 0 \equiv \Psi_3$; $\Psi_4 \equiv 0 \not\equiv \Phi_{22}$. All observers agree.

Class II_6 is the most general; as one demands that successive amplitudes vanish identically, one descends to less and less general classes. The class of the most general permitted wave in some currently viable metric theories is: General relativity,[1] N_2; Dicke-Brans-Jordan,[2] N_3; Will-Nordtvedt,[3] III_5; Ni's new theory,[4] II_6; and Lightman-Lee,[5] II_6. All these but Dicke-Brans-Jordan have the same post-Newtonian limit as general relativity, for a reasonable choice of cosmological model.

We see that measuring the polarization of gravitational waves provides a sharp experimental test of theories of gravity. The class of the "correct" theory is at least as general as that of any observed wave. The observation of a wave more general than N_2 would contradict general relativity but would be consistent with other viable theories.[2-5] Weber[10] has initiated such experiments by searching for the Φ_{22} mode, with negative results.

To test theories, an experimenter must classify the waves that he detects. If he knows the direction of a wave a priori (e.g., from a particular supernova), he can directly extract the amplitude of each mode from his data and determine the class. If he does not know the direction, he <u>cannot</u> extract the amplitudes or determine the direction without applying some further assumption to his data (e.g., that the wave is no more general than N_3 and is therefore purely transverse). But he <u>can</u> usually say something definite about the class of the wave:

(i) If the driving forces in his detector are not in any one plane, the wave is II_6 or III_5.

(ii) If the driving forces are in a plane and are "pure monopole" [as in Fig. 1(c)], the wave is not N_2.

(iii) If the driving forces are in a plane and are "pure quadrupole" [as in Fig. 1(a)], the wave is not O_1.

(iv) Otherwise the wave is either II_6, III_5, or N_3.

We now sketch the arguments that lead to these results about polarization of gravitational waves in metric theories. Consider a weak, plane, null wave described by a linearized Riemann tensor, $R_{\alpha\beta\gamma\delta}(u)$, with $\nabla u \cdot \nabla u = 0$. Work in an approximately constant quasi-orthonormal null tetrad[11] $(\mathbf{k}, \boldsymbol{\ell}, \mathbf{m}, \overline{\mathbf{m}})$, where $\mathbf{k} = \nabla u$. The Bianchi identities imply that there are six functionally independent real components of the Riemann tensor; take them, in the notation of Newman and Penrose,[11] to be Ψ_2, Ψ_3, Ψ_4, Φ_{22}, as above. Consider the "little group[12]" of Lorentz transformations of the tetrad which fix $\mathbf{k}$: $\mathbf{k}' = \mathbf{k}$, $\mathbf{m}' = e^{i\varphi}(\mathbf{m} + \alpha\mathbf{k})$, $\boldsymbol{\ell}' = \boldsymbol{\ell} + \bar{\alpha}\mathbf{m} + \alpha\overline{\mathbf{m}} + \alpha\bar{\alpha}\mathbf{k}$, where α is complex and φ is a real phase. The action of E(2) on the Riemann tensor of a wave is

$$\begin{aligned} \Psi_2' &= \Psi_2 \\ \Psi_3' &= e^{-i\varphi}(\Psi_3 + 3\bar{\alpha}\Psi_2) \\ \Psi_4' &= e^{-2i\varphi}(\Psi_4 + 4\bar{\alpha}\Psi_3 + 6\bar{\alpha}^2\Psi_2) \\ \Phi_{22}' &= \Phi_{22} + 2\alpha\Psi_3 + 2\bar{\alpha}\bar{\Psi}_3 + 6\bar{\alpha}\alpha\Psi_2. \end{aligned} \tag{1}$$

The Lorentz-invariant classes of waves that are defined above correspond precisely to the different representations of E(2) that can arise through Eqs. (1).

The helicity (spin) decomposition of a wave is Lorentz-invariant only for classes N_3, N_2, and O_1. Theories in classes N_3, N_2, and O_1 provide a unitary representation of E(2) which is a direct sum of 1-dimensional massless particle representations,[12-14] containing at most spins 0, ± 2. Theories in classes II_6 and III_5 provide a reducible representation of E(2) which is not completely reducible and is therefore nonunitary[14]; it is likely that such theories cannot be quantized. No other representation of E(2) (such as one with "continuous spin[13]") can occur.

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FIGURE CAPTION

Fig. 1. The six polarization modes of a weak, plane, null gravitational wave permitted in the generic metric theory of gravity. Shown is the displacement that each mode induces on a sphere of test particles. The wave is propagating in the +z direction (arrow at upper right) and has time dependence cos ωt. The solid line is a snapshot at $\omega t = 0$, the broken line one at $\omega t = \pi$. There is no displacement perpendicular to the plane of the figure.

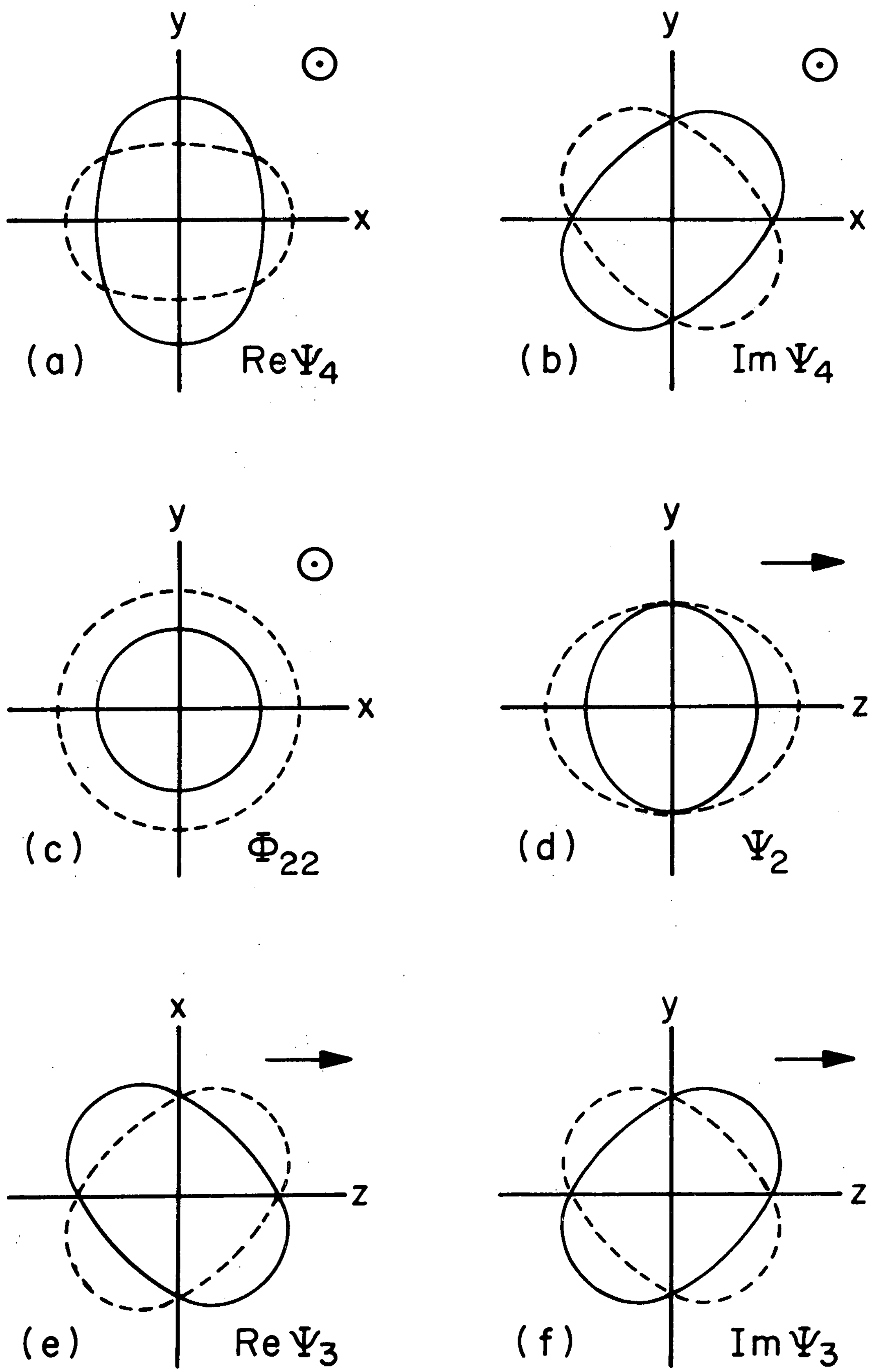
y
x
(a)
Re Ψ_4
y
x
(b)
Im Ψ_4
y
x
(c)
Φ_{22}
y
z
(d)
Ψ_2
x
z
(e)
Re Ψ_3
y
z
(f)
Im Ψ_3

Fig. 1