Gravitational waves from neutron stars: Promises and challenges

N. Andersson[1], V. Ferrari[2], D.I. Jones[1], K.D. Kokkotas[3],

B. Krishnan[4], J. Read[4], L. Rezzolla[4,5] & B. Zink[3]

[1]*School of Mathematics, University of Southampton, Southampton SO17 1BJ, UK*

[2]*Dipartimento di Fisica "G. Marconi",*

Sapienza Università di Roma and Sezione INFN ROMA1,

piazzale Aldo Moro 2, I-00185 Rome, Italy

[3]*Theoretical Astrophysics, Eberhard-Karls*

University of Tübingen, Tübingen 72076, Germany

[4] *Max-Planck-Institut für Gravitationsphysik,*

Albert-Einstein-Institut, Potsdam-Golm, Germany

[5] *Department of Physics and Astronomy,*

Louisiana State University, Baton Rouge, LA, USA

Abstract

We discuss different ways that neutron stars can generate gravitational waves, describe recent improvements in modelling the relevant scenarios in the context of improving detector sensitivity, and show how observations are beginning to test our understanding of fundamental physics. The main purpose of the discussion is to establish promising science goals for third-generation ground-based detectors, like the Einstein Telescope, and identify the various challenges that need to be met if we want to use gravitational-wave data to probe neutron star physics.

I. CONTEXT

Neutron stars are cosmic laboratories of exotic and exciting physics. With a mass of more than that of the Sun compressed inside a radius of about 10 kilometers, their density reaches beyond nuclear saturation. In essence, our understanding of these extreme circumstances requires physics that cannot be tested in terrestrial laboratories. Instead, we must try to use astrophysical observations to constrain our various theoretical models.

We already have a wealth of data from radio, X-ray and gamma-ray observations, providing evidence of an incredibly rich phenomenology. We have learned that neutron stars appear in many different disguises, ranging from radio pulsars and magnetars to accreting millisecond pulsars, radio transients and intermittent pulsars. Our models for these systems remain rather basic, despite four decades of effort going into modelling the violent dynamics of supernovae and gamma-ray bursts, trying to understand the radio pulsar emission mechanism, glitches, accreting systems etcetera. In the next few years we expect "gravitational-wave astronomy" to (finally) become reality. This is an exciting prospect, because gravitational-wave (GW) observations have the potential to probe several aspects of neutron star physics [1]. Moreover, the information gleaned will be complementary to electromagnetic observations. In particular, we would hope to be able to provide constraints on the state of matter at extreme densities.

In the last few years the first generation of large-scale interferometric GW detectors (LIGO, GEO600 and Virgo) have reached the original design sensitivity in a broad frequency window [2]. Achieving a sensitivity to detect a (dimensionless) strain amplitude of $h < 10^{-21}$ at a frequency around 100 Hz is obviously an enormously impressive feat of technology. Moreover, the detectors are very stable. A full years worth of (triple coincidence) high quality data was taken during the LIGO S5 run. This data is now being analysed and, even though there has not yet been a detection, the experiment has provided interesting information. The LIGO detectors are now running in an enhanced configuration. In the next five year period, they will be upgraded using advanced technology. Once this upgrade is complete, around 2015, the second generation of ground-based detectors will reach the level of sensitivity where the first detection can be expected (about one order of magnitude more sensitive than the first generation). Meanwhile, the discussion of third generation (3G) detectors has begun in earnest with the EU funded Einstein Telescope (ET) design study.

The aim of 3G detectors is to improve the broadband sensitivity by (roughly) another order of magnitude.

The main aim of this article is to give a brief overview of how future observational capabilities may impact on our understanding of neutron stars. We will try to identify the promising scenarios, and how we need to improve our theoretical models if we want to extract maximum neutron star science from an instrument like ET.

Neutron stars radiate gravitationally in a number of ways. The most promising scenarios involve:

Inspiralling binaries: The slow orbital evolution is well modelled within the post-Newtonian approximation, and a detection would allow the extraction of the individual masses, spins etcetera. The internal composition of the bodies will become important at some point before merger, but it is still not clear to what extent the compressibility, or tidal resonances, lead to observable features in the GW signal. For second generation detectors this issue may not be so important because the late stages of binary evolution will be difficult to detect anyway. The situation will be very different for 3G detectors, for which a key science target will be to extract as much physics information from these systems as possible.

Supernova core collapse: The violent dynamics associated with a supernova core collapse is expected to lead to GW emission through a number of channels [3, 4]. The large scale neutrino asymmetries associated with the standing accretion shock instability (SASI) should be relevant, the global dynamics of the collapsing core through the shock bounce, and the oscillations of the hot newly born compact object will also affect the signature. Current simulations suggest that core collapse events in our galaxy should be detectable with current technology. Simple scaling out to distances visible with ET suggests that, even though the event rate may still not be overwhelmingly impressive, the detection of GWs from supernovae provides a key target. In particular, it appears that different suggested supernova explosion mechanisms may lead to rather different GW signals.

Rotating deformed stars: Rotating neutron stars will radiate gravitationally due to asymmetries. The required deformation can be due to strain built up in the crust, or indeed the deep core if it has elastic properties, the magnetic field or arise as a result of accretion. Our current understanding of this problem is mainly based on attempts to establish how large a deformation the star can sustain, e.g. before the crust breaks. The best estimates suggest that the crust is rather strong [5], and that it could, in principle, sustain asymmetries as

large as one part in 10^5 [6]. However, it is important to understand that this does not in any way suggest that neutron stars will have deformations of this magnitude. The real problem is to provide a reasonable scenario that leads to the development of sizeable deformations. In this sense, accreting systems are promising because of the expected asymmetry of the accretion flow near the star's surface. Of course, accreting systems are quite messy so the required modelling is very hard.

Oscillations and instabilities: Neutron stars have rich oscillation spectra which, if detected, could allow us to probe the internal composition. The basic strategy for such "gravitational-wave asteroseismology" has been set out [7], but our models need to be made much more realistic if the method is to be used in practice. It is also important to establish why various oscillation modes would be excited in the first place, and obviously what level of excitation one would expect. In order to address this problem we need to be able to model potentially relevant scenarios like magnetar flares and pulsar glitches. It seems inevitable that such events will emit GWs at some level, but at the present time we do not have any realistic estimates (although see [8]). We need more work on these problems, even if the end result is that these systems are not promising sources. The most promising scenarios may be associated with unstable modes. There are a number of interesting instabilities, like the GW driven instability of the f- and r-modes, the dynamical bar-mode and low T/W instabilities, instabilities associated with a relative flow in a superfluid core etcetera. In recent years our understanding of these instabilities has improved considerably, but we are still quite far away from being able to make quantitative predictions.

As we will discuss, modelling these different scenarios is far from easy. Basically, our understanding of neutron stars relies on much poorly known physics. In order to make progress, we must combine supranuclear physics (the elusive equation of state) with magnetohydrodynamics, the crust elasticity, a description of superfluids/superconductors (which is relevant since these systems are "cold" on the nuclear physics temperature scale) and potentially exotic phases of matter like a deconfined quark-gluon plasma or hyperonic matter. Moreover, in order to be quantitatively accurate, all models have to account for relativistic gravity.

It is probably unrealistic to expect that we will be able to resolve all the involved issues in the next decade. Most likely, we will need better observational data to place constraints on the many theoretical possibilities. Thus, it is very important to consider what we can

hope to learn about the different emission mechanisms from future GW observations. This discussion is particularly relevant for ET (and other 3G detectors) where key design decisions still have to be made. In this context it is relevant to ask what we can hope to achieve with ET, but not (necessarily) with the second generation of detectors. How much better can you do with (roughly) a one order of magnitude improvement in broadband sensitivity? Are there situations where this improvement is needed to see the signals in the first place, or is it more a matter of doing better astrophysics by getting better statistics and an increased signal-to-noise to faciliate parameter extraction?

Note: various detector noise cuves are used in this article. The Advanced LIGO curves are taken from the LIGO technical document ligo-t0900288-v2 "Advanced LIGO anticipated sensitivity curves". The Virgo curve is taken from [9]. The ET curve was obtanied from the website of the Working Group 4 of the Einstein Telescope project (`https://workarea.et-gw.eu/et/WG4-Astrophysics/`). Note also that this review is not exhaustive in any sense. We have focussed on some of the (in our opinion) key aspects. We are often citing the most recent work rather than providing a complete history.

II.   BINARY INSPIRAL AND MERGER

The late stage of inspiral of a binary system provides an excellent GW source [10]. As the binary orbit shrinks due to the energy lost to radiation, the GW amplitude rises and the frequency increases as well. This inspiral "chirp" is advantageous for the observer in many ways. First of all, it is well modelled by post-Newtonian methods and does not depend (much) on the actual physics of the compact objects involved. In fact, much of the signal is adequately described by a point-mass approximation. From an observed binary signal one would expect to be able to infer the individual masses, the spin rates of the objects and the distance to the source.

A key fact that makes binary systems such attractive sources is that the amplitude of the signal is "calibrated" by the two masses. The only uncertainty concerns the event rate for inspirals in a given volume of space. Given this, it is natural to discuss the detectability of these systems in terms of the "horizon distance" d_h, the distance at which a given binary signal would be observable with a given detector. Let us assume that detection requires a signal-to-noise ratio (SNR) of 8, and focus on equal mass neutron star binaries (we take

each star to have mass $1.4 M_\odot$). For such systems, the current and predicted future horizon distance and the expected event rates are given in Table I below [11, 12].

detector	d_h	event rate
LIGO S5	30 Mpc	1 event per 25-400 yrs
Advanced LIGO/Virgo	300 Mpc	Several to hundreds of events per year
ET	3 Gpc	Tens to thousands of events per year

TABLE I: Horizon distances and estimated event rates for different generations of GW detectors.

From this data we learn a number of things. First of all, we see why it would be quite surprising if a binary neutron star signal were to be found in the LIGO S5 data. Given even the most optimistic estimated event rate from population synthesis models, these events would be rare in the observable volume of space. The situation changes considerably with Advanced LIGO/Virgo. Based on our current understanding, one would expect neutron star binaries to be seen once the detectors reach this level of sensitivity. However, it is also clear that if the most pessimistic rate estimates are correct, then we will not be able to gather a statistically significant sample of signals. Most likely, we will need detectors like ET to study populations.

A third generation of detectors is also likely to be required if we want to study the final stages of inspiral, including the merger. This is a very interesting phase of the evolution, given that the merger will lead to the formation of a hot compact remnant with violent dynamics. It may also trigger a gamma-ray burst [13, 14, 15, 16, 17]. Most of this dynamics radiates at kHz frequencies. The tidal disruption that leads to the merger occurs above 600 Hz or so and the oscillations of the remnant could lead to a signal at several kHz. However, the merger signal should be rich in information. In particular, it should tell us directly whether a massive neutron star or a black hole is formed, thus placing constraints on the (hot) neutron star equation of state (EOS). As we will discuss below, recent numerical relativity simulations [18, 19, 20, 21] illustrate the complexity of the merger signal. On the one hand, this is problematic because we are unlikely to reach a stage where we have truly reliable theoretical signal templates. On the other hand, one should be able to make progress by focussing on "robust" characteristics of the signal. However, it seems clear that we will need 3G detectors if we want to explore the science of the merger event. Combining

the expected binary inspiral rates with the results in Figure 1 we learn that, roughly, if the inspiral phase is observable with Advanced LIGO/Virgo then ET should be able to detect the merger. This is strong motivation for not compromising on the high-frequency performance of a 3G detector.

The final stages of binary evolution before merger may also encode the detailed properties of matter beyond nuclear saturation [22, 23]. As the binary tightens the equation of state begins to influence the GW signal, making it possible to extract additional information (other than the masses and spins). For mixed neutron-star/black-hole binaries, the characteristic frequency of tidal disruption, which terminates the inspiral in some cases, also depends on the equation of state [24, 25, 26, 27, 28]. The signal from a binary containing a strange quark star would also be quite distinct in this respect [29, 30, 31, 32]. For double neutron-star binaries, the tidal deformation due to the other body can modify the orbital evolution in late inspiral. The tidal phase corrections for most realistic equations of state, although invisible to Advanced LIGO/Virgo, make potentially distinguishable contributions to the signal in ET [33]. The deformability of a given mass neutron star is encoded in the so-called Love numbers, which depend on the equation of state and the resulting radius for a given mass [34, 35]. Roughly, larger neutron stars are deformed more easily. Further effects of matter dynamics, such as tidal resonances, may also contribute to the signal [36, 37, 38].

In order to illustrate the information that one may hope to extract from detecting the final merger of these sources we consider the GW emission produced by two equal-mass neutron stars. Detailed simulations of such systems have been presented in [19, 21]. Here we simply recall that the simulations were performed using high-resolution shock-capturing methods for the hydrodynamics equations and high-order finite-differencing techniques for the Einstein equations. The complete set of equations was solved using adaptive mesh-refinement techniques with "moving boxes" and the properties of the black holes produced in the merger were extracted using the isolated-horizon formalism. The initial data was obtained from a self-consistent solution of Einstein's equations in the conformally-flat approximation and represents a system of binary neutron stars in irrotational quasicircular orbits. The matter was evolved using two idealized EOS given either by the "cold" or polytropic EOS $p = K\rho^{\Gamma}$, or the "hot" (ideal fluid) EOS $p = (\Gamma - 1)\rho\,\epsilon$, where ρ is the rest-mass density, ϵ the specific internal energy, K the polytropic constant and Γ the adiabatic exponent. It should be noted that because the polytropic EOS is isentropic it is unrealistic for describing the post-merger

evolution. Nevertheless, it provides a reasonably realistic description of the inspiral phase, during which the neutron stars are expected to interact only gravitationally. Moreover, because these two EOSs are mathematically equivalent in the absence of shocks, one can employ the *same* initial data and thus compare directly the influence that the EOS has on the evolution. Also, because they represent "extremes" of the possible fluid behaviour, they offer insights into the expected range in dynamical behaviour.

In order to quantify the impact that different EOSs have on the GW signal we consider the power spectral density (PSD) of the effective amplitude $\tilde{h}(f)$

$$\tilde{h}(f) \equiv \sqrt{\frac{\tilde{h}_+^2(f) + \tilde{h}_\times^2(f)}{2}}, \tag{1}$$

where f is the gravitational-wave frequency and where

$$\tilde{h}_{+,\times}(f) \equiv \int_0^\infty e^{2\pi i f t} h_{+,\times}(t) dt \tag{2}$$

are the Fourier transforms of the gravitational-wave amplitudes $h_{+,\times}(t)$, built using only the (dominant) $l = m = 2$ multipole.

Fig. 1 shows the PSD of the $l = m = 2$ component of $\tilde{h}(f)f^{1/2}$ for a binary with total gravitational mass $M = 2.98\,M_\odot$ ("high-mass" binary, left panel) and for a binary with $M = 2.69\,M_\odot$ ("low-mass" binary, right panel) at a distance of $300\,\mathrm{Mpc}$. In both cases the binaries are evolved from an initial separation of 45 km. To emphasize the differences induced by the EOS, we show the PSDs when the binary is evolved with either the "cold" or the "hot" EOS. For comparison, we also provide the result for an equal-mass, nonspinning black-hole binary with total mass $M = 200\,M_\odot$ (also at a distance of $300\,\mathrm{Mpc}$). Finally, we show the phenomenological evolutionary tracks obtained by combining post-Newtonian and numerical relativity results [39, 40].

The PSD for the high-mass cold-EOS binary (blue curve in the left panel of Fig. 1) is quite simple to interpret. It shows, besides the large power at low frequencies corresponding to the inspiral, a peak at $f \approx 4\,\mathrm{kHz}$ corresponding to the rapid merger of the two neutron stars. For this cold EOS the merger is not influenced by increased pressure forces via shock heating, so the GW signal terminates abruptly with a prompt collapse to black hole and a cut-off corresponding to the fundamental quasi-normal mode (QNM) of the black hole at $f_{\mathrm{QNM}} \simeq 6.7\,\mathrm{kHz}$. In contrast, the PSD for the high-mass hot-EOS binary is more complex, with the inspiral peak being accompanied by a number of other peaks, the two most prominent, at
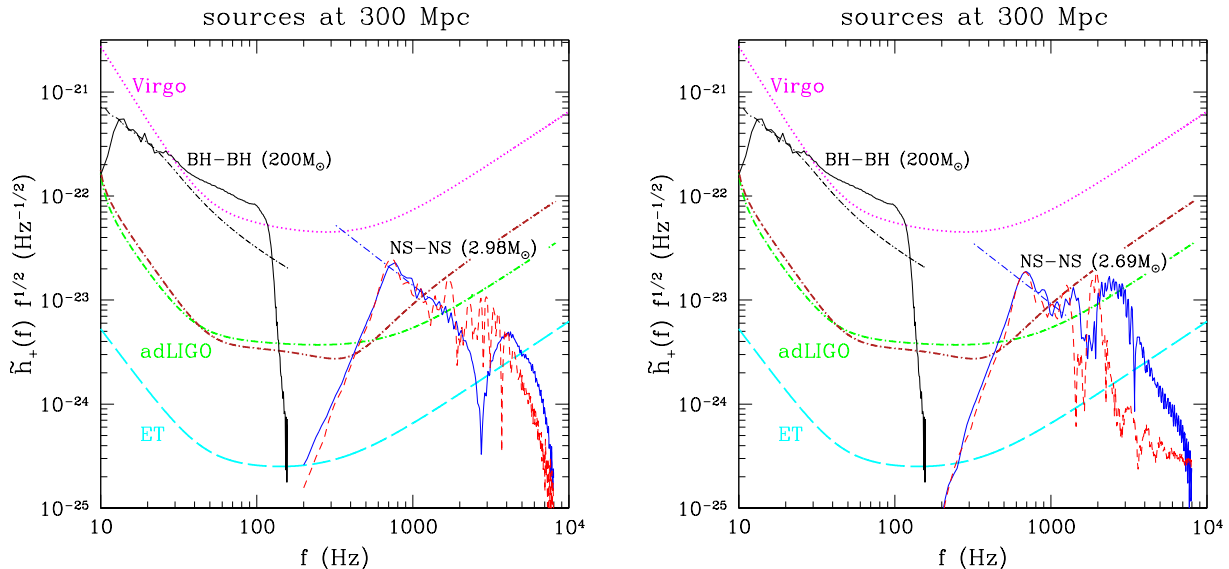
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FIG. 1: *Left panel*: PSD of the $l = m = 2$ component of $\tilde{h}(f)f^{1/2}$ for the high-mass binary when evolved with the "cold" (blue solid line) or with the "hot" (red dashed line) EOS. Shown for comparison is the corresponding PSD for an equal-mass, nonspinning black-hole binary with total mass $M = 200\,M_\odot$, which appears in the low-frequency part of the spectrum (black solid line). *Right panel*: The same as in the left panel but for the low-mass binary. In both panels we indicate (as dot-dashed lines) phenomenological evolutionary tracks obtained by combining PN and numerical relativity results [39, 40]. The results are compared to the sensitivity curves of present and future detectors; Virgo (dotted magenta line), advanced LIGO (zero-detuned configuration as a green dot-dashed line and a configuration optimized for binary neutron star inspirals as a brown dot-dashed line) and ET (as a long-dashed, cyan line).

$f \approx 1.75\,\mathrm{kHz}$ and $f \approx 3\,\mathrm{kHz}$, having almost comparable amplitude. These additional peaks are related to the post-merger phase and the dynamics of the hypermassive neutron star that is formed after the merger. The signal is sensitive to the dynamics of the cores of the two neutron stars, which merge and "bounce" several times before the hypermassive neutron star eventually collapses to a black hole, leaving a signature at $f \approx 4\,\mathrm{kHz}$. The fundamental QNM frequency at $f_{\mathrm{QNM}} \simeq 7.0\,\mathrm{kHz}$ marks the cutoff of the signal also in this case.

We can interpret the PSDs of the low-mass binaries in a similar way. The cold EOS results, in particular, show a very broad peak between $f \approx 2\,\mathrm{kHz}$ and $\approx 3.5\,\mathrm{kHz}$ related to the

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dynamics of the bar-shaped hypermassive neutron star that persists for several milliseconds after the merger. A small excess at $f \gtrsim 4\,\mathrm{kHz}$ is associated with the collapse to a black hole, whose fundamental QNM has a frequency of $f_{\mathrm{QNM}} \simeq 7.3\,\mathrm{kHz}$. Interestingly, the low-mass hot EOS PSD does not show the broad peak. Instead, there is a very narrow and high-amplitude peak around $f \approx 2\,\mathrm{kHz}$. This feature is related to a long-lived bar deformation of the hypermassive neutron star, which was evolved for ~ 16 revolutions without the hypermassive neutron star collapsing to a black hole. It should be noted that the simulations were performed assuming a rotational symmetry [19], which prevents the growth of the $m = 1$ modes which have been shown to limit the persistence of the bar-mode instability (see [41] for a detailed discussion). Although it is reasonable to expect that the bar deformation will persist for several milliseconds after merger, it is unclear whether this prominent peak will remain (and if so at what amplitude) when the simulations are repeated with more generic boundary conditions. The high-frequency part of the PSD for the low-mass hot-EOS binary obviously does not show the expected cut-off introduced by the collapse to black hole. An estimate based on the secular increase of the central density suggests that, in this case the collapse takes on a timescale of $\sim 110\,\mathrm{ms}$, much longer than the simulations.

Three main conclusions can be drawn from the results shown in Fig. 1. First of all, with the exception of very massive neutron stars (in which case the collapse of the hypermassive neutron star occurs essentially simultaneously with the merger), the GW signal from binary neutron stars is considerably richer (and more complex) than that from binary black holes. Secondly, while small differences between the two EOSs appear already during the inspiral, it is really the post-merger phase that is markedly different. Hence, an accurate description of the post-merger evolution is *essential* not only to detect this part of the signal, but also to extract information concerning the neutron star interior structure. Finally, the parts of the PSD that are most interesting and most likely to yield fundamental clues to the physics beyond nuclear density, are likely to be only marginally detectable by detectors like advanced LIGO/Virgo. As a result, 3G detectors like ET may provide the first realistic opportunity to use GWs as a Rosetta stone to decipher the physics of neutron star interiors.

III.   CORE COLLAPSE SUPERNOVAE AND HOT REMNANTS

Neutron stars are born when a massive star runs out of nuclear fuel and collapses under its own gravity. The dynamics of the stellar evolution after core bounce is tremendously complicated, and depends on the interplay of a number of physical mechanisms. This complex process can produce GWs through a number of different channels, some of which are connected to the dynamics of the proto-neutron star and its immediate environment (usually associated with high-frequency components of the signal), and others which depend on the convective zone behind the stalled shock front (which gives rise to low frequency signals). For slowly rotating iron cores, bounce and initial ringdown are expected to lead to signals with peak frequencies in the range of $700 - 900\,\mathrm{Hz}$ and dimensionless strain amplitudes of less than 5×10^{-22} at a distance of 10 kpc [42], i.e. within our Galaxy. Faster rotation amplifies the bounce signal: If the iron core has moderate rotation, the peak frequencies span the larger range of $400 - 800\,\mathrm{Hz}$ with amplitudes of 5×10^{-22} up to 10^{-20}. Very rapid rotation leads to bounce at subnuclear densities, and GW signals in a significantly lower frequency band of hundreds of Hz and strains around 5×10^{-21} at 10 kpc. Prompt convection occuring shortly after core bounce due to negative lepton gradients lead to Galactic signal amplitudes in the range of 10^{-23} to 10^{-21} at frequencies of $50 - 1000\,\mathrm{Hz}$ (all data is taken from from table 2 in [4], see also [43, 44]), whereas signals of convection in the proto-neutron star have strains of up to 5×10^{-23} in a somewhat larger range of frequencies. Neutrino-driven convection and a potential instability of the accretion-shock, the standing accretion shock instability (SASI), could be relevant sources as well, with strain amplitudes up to 10^{-22} at $100 - 800\,\mathrm{Hz}$. In addition, an acoustic mechanism has been proposed for supernova explosions [45] which is connected with low-order g-mode oscillations in the proto-neutron star. If this mechanism is active, very large strain amplitudes of up to 5×10^{-20} at 10 kpc could be reached in extreme cases [43].

A major uncertainty connected with supernova models is the initial state, in particular the angular momentum distribution in the iron core. Current expectations from stellar evolution calculations imply a very slowly rotating core as a canonical case [46]. This is further supported by the observation that neutron stars seem to be born with comparatively low rotation rates, and by recent evidence for massive loss of angular momentum of stars before the white-dwarf stage [47]. Strong GW signals can only be obtained by invoking processes
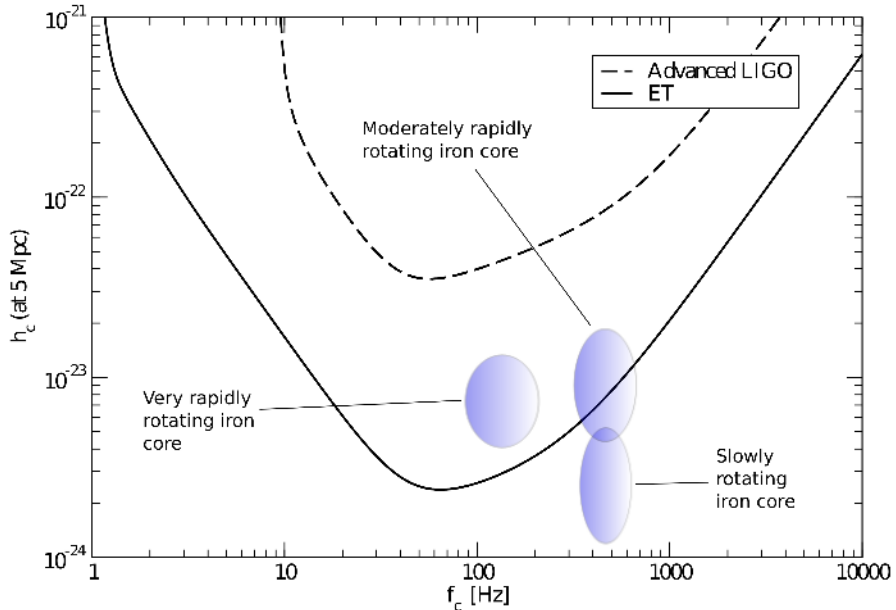
FIG. 2: Characteristic signal strengths, at a distance of 5 Mpc, associated with the bounce in core collapse, adapted from fig. 3 in [4]. Typical iron cores have low rotation rates and consequently weak core bounce signals. However, signals from systems with moderate or rapid rotation, which is expected to be present in a subset of progenitor stars, can potentially be detected with ET even at this distance.

which break the approximate spherical symmetry of the system. If the core rotates faster than expected, for example for collapsing stars that lead to gamma-ray bursts [48], then rotational instabilities may become relevant sources of GWs. Of course, these instabilities (or magnetic fields) must be effective enough to spin down young neutron stars after birth, in order to reconcile this scenario with the observed neutron star spin distribution. Also, magnetic wind-up may in some cases open channels to magnetically driven explosions, which could give rise to detectable signals.

The traditional bar-mode instability operates only at very high spin rates (measured in terms of the ratio of rotational kinetic to gravitational binding energy, $T/|W|$, the limit is about 0.25 in general relativity), which are not expected even in more extreme models [42]. Moreover, recent work [49] suggests that once it is active the instability does not

persist for long due to nonlinear mode-mode coupling. Long-lived bar configurations require extreme fine-tuning of the initial data, something that nature is rather unlikely to provide. However, if the proto-neutron star is sufficiently differentially rotating, so-called low $T/|W|$ instabilities may be operative [50]. This may lead to substantial deformations which could be detectable. The associated signal-to-noise ratio depends (essentially) on the number of cycles and the saturation level of the instability. A typical value for the maximal strain obtained in numerical simulations is 10^{-21} at $10\,\mathrm{kpc}$ with a frequency around $400 - 900\,\mathrm{Hz}$ [51]. Finally, secular instabilities (e.g. driven by the GW emission) could be active during or after the unbinding of the stellar envelope. We will discuss these instabilities later.

The different emission mechanisms all have quite characteristic signatures, so GW measurements would provide an unusually direct (probably the only besides neutrinos) way of probing the conditions inside core collapse supernovae [4]. However, the signal-to-noise ratio estimated from numerical simulations make a detection of an extragalactic core collapse supernova from a slowly rotating (canonical) iron core seem unlikely even with second generation detectors. Even if higher core rotation rates are assumed, detections will be possible maybe up to at most 1 or $2\,\mathrm{Mpc}$. Since the rates of (successful) supernovae are known from observations, we know that Galactic events happen every $30 - 100$ years. Even at a distance of 1 Mpc, that is for very optimistic GW estimates, the event rate is low enough that we may have to be lucky to even see a single event during the whole operation of Advanced LIGO. However, at a distance of $3 - 5$ Mpc, a range which could admit a detectable signal in a 3G detector, the event rate rate would be a few per year [52, 53]. Therefore, it is safe to say that investigating the core collapse supernova mechanism with GWs absolutely requires a 3G detector to obtain meaningful statistics.

The proto-neutron star that is born in a core collapse is a hot and rapidly evolving object. After the first tenths of seconds of the remnant's life, the lepton pressure in the interior decreases due to extensive neutrino losses, the mantle contracts and the radius reduces to about $20 - 30$ km [54]. This is known as a proto-neutron star. The subsequent evolution is "quasi-stationary", and can be described by a sequence of equilibrium configurations [55]. Initially, the diffusion of high-energy neutrinos from the core to the surface generates a large amount of heat within the star, while the core entropy approximately doubles. Within ~ 10 s the lepton content of the proto-neutron star is drastically reduced. While the star is very hot, neutrino pairs of all flavours are thermally produced and dominate the emission.

As the neutrinos continue to diffuse and cool the star, their average energy decreases and their mean free path increases. After a few tens of seconds, the mean free path becomes comparable to the stellar radius, the net lepton number in the interior has decreased to very low values, the temperature has dropped to around 10^{10} K, and the star becomes a neutrino-transparent neutron star. The strong entropy and temperature gradients which develop in the interior during this evolution strongly affect the star's oscillation frequencies.

As examples of this let us consider the fundamental mode and the first gravity g-mode, cf. the discussion in [56]. During the first 5 s of evolution the frequencies of these modes change dramatically. In the case of the f-mode, the frequency scales with the square root of the average density so it will more than double as the star shrinks to the final 10 km radius. Meanwhile, the g-modes depend on the strong temperature gradients that only prevail as long as the neutrinos are trapped. Detailed calculations [56, 57] confirm that, during the first few tens of seconds, the mode frequencies and damping times are considerably different from those of a cold neutron star. Moreover, dissipative processes "competing" with GW emission, (essentially due to neutrino viscosity, diffusivity, thermal conductivity, or thermodiffusion) have timescales of the order $t_{diss} \approx 10 - 20$ seconds (see [56] for details). For instance, for the f-mode the GW damping time t_f is such that $t_f < t_{diss}$ when $t \gtrsim 0.2$ seconds. Therefore, if some energy is initially stored into this mode, it will be emitted in GWs. In contrast, the g-mode is an efficient emitter of GWs only during the first second, since after this time its damping time becomes larger than t_{diss}.

As we will discuss later, it is straightforward to estimate the detectability of an oscillation mode once we know the associated frequency and damping time. Figure 3 shows results from [56] for the f- and the most promising g-mode of a proto-neutron star. The data corresponds to signals detectable with a signal-to-noise ratio of 8 in ET. For a galactic source, at a distance of 10 kpc, this would require an energy equivalent to $\Delta E_f \sim 2 \times 10^{-12}$ $M_\odot c^2$ and $\Delta E_g \sim 3 \times 10^{-11}$ $M_\odot c^2$ to be radiated through the f- and g-mode, respectively (note that the signal strength scales with the square root of the radiated energy). This level of energy releases is not unrealistic. Recent core collapse simulations suggest that a conservative estimate of the amount of energy emitted in GWs is of the order of $10^{-9} - 10^{-8}$ $M_\odot c^2$. (As a comparison and useful upper limit, the collapse to a black hole of an old and rapidly rotating neutron star would emit an energy in GWs of the order of 10^{-6} $M_\odot c^2$ [58].) If a small fraction of this energy goes into the excitation of the two modes shown in Figure 3, they
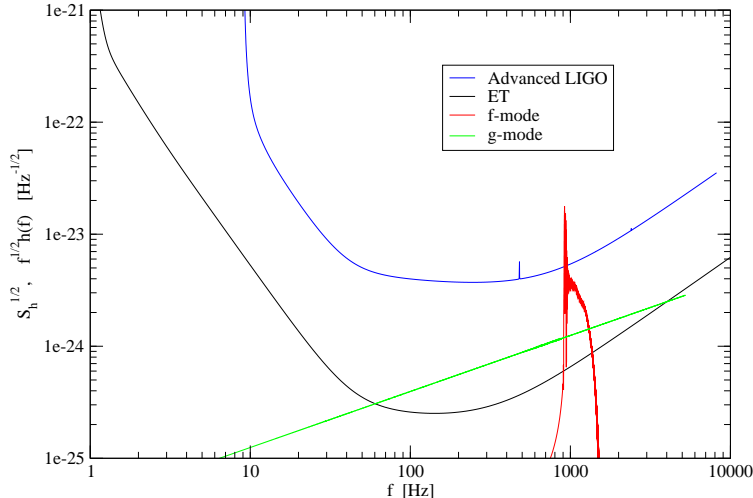
FIG. 3: This figure compares the strain amplitude of the signal emitted by a proto-neutron star oscillating in either the f-mode or the g-mode to the sensitivity of Advanced LIGO and ET. The signal is assumed to have an amplitude at the detector site which corresponds to an ET detection with a signal-to-noise ratio of 8 using matched filtering techniques. For a galactic source at distance of 10 kpc, this would require an energy equivalent to $\Delta E_f \sim 2 \times 10^{-12} \ M_\odot c^2$ and $\Delta E_g \sim 3 \times 10^{-11} \ M_\odot c^2$ to be radiated through the f- and g-mode, respectively. A key point of these results is that the oscillation spectrum evolves during the observation. It is important to establish to what extent future detections, e.g. with ET, may be able to extract the characteristics of these signals and allow us to probe the physics of newly born neutron stars.

may be observable. Moreover, given such a signal one would expect to be able to identify features in the waveform. For instance, one may infer the slope of the g-mode signal, or the detailed structure of the f-mode waveform. From these features one could hope to gain insight into physical processes occurring inside the star. In this way ET has the potential to be a powerful instrument for exploring the physics of newly born neutron stars.

IV. ROTATING DEFORMED NEUTRON STARS

Asymmetries, generated either by strains in the star's crust or by the magnetic field, are expected to slowly leak rotational energy away from spinning neutron stars. Such sources would be the GW analogue of radio pulsars. Indeed, the known radio pulsar population,
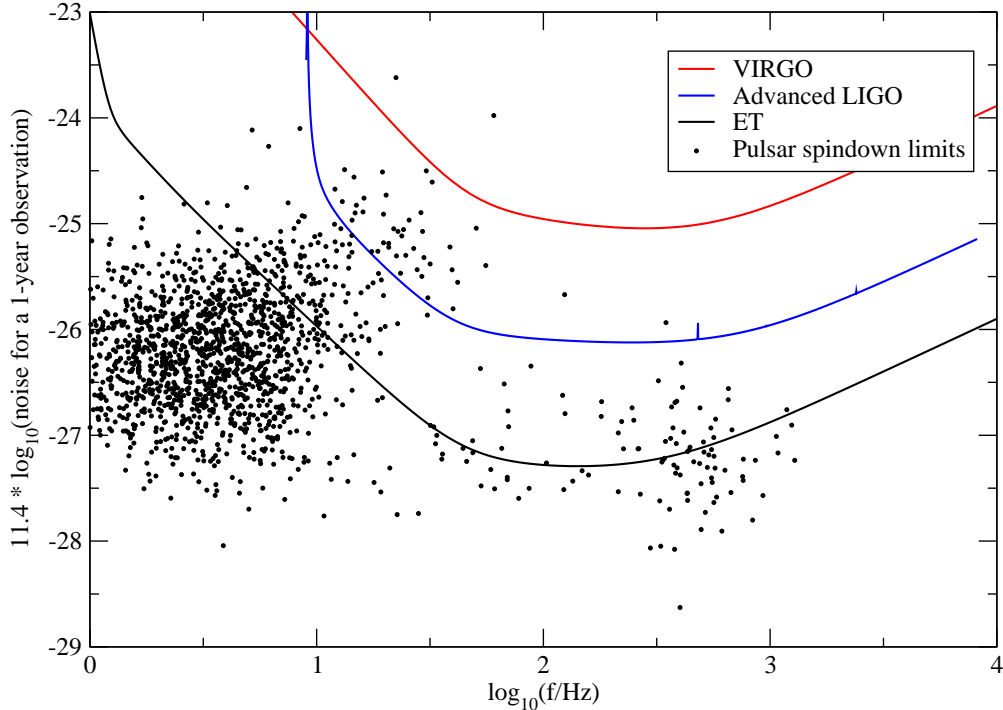
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FIG. 4: Upper bounds on GW amplitude from known pulsars assuming 100% conversion of spin-down energy into GWs. An integration time of one year is assumed.

together with the accreting low-mass X-ray binary systems, are prime candidates for GW detection via targeted searches, where the observed electromagnetic phase is used to guide the gravitational search. Equally interestingly, there may be a population of neutron stars currently invisible via electromagnetic observations, spinning down by GW emission. These require an all-sky blind search, the computational costs of which are very high, requiring the sort of computing power made available by the `Einstein@Home` project [59].

How likely is a detection of a spinning deformed star, and how does ET enter into the game? For targeted searches for stars of known distance, spin frequency and spin-down rate, one can place an upper bound on the GW emission by assuming that all of the kinetic energy being lost is being converted into GWs. Such a plot is shown in Figure 4 for the known pulsar population, with various detector noisecurves, including ET, shown for reference.

Superficially, the figure would suggest that several tens of pulsars might be detectable

by Advanced LIGO/Virgo, with many more potentially detectable by ET. However, this plot makes a dangerous assumption: by assuming 100% conversion of kinetic energy into GW energy, it implicitly requires neutron stars to be capable of supporting asymmetries of sufficient size to power the necessary GW emission. The crucial question then becomes: what level of asymmetry would one expect a neutron star to have? This is a complicated multi-faceted problem, where the answer depends not only on the properties of the star, but also on the star's evolutionary history. So far, theoretical modelling has mainly focused on establishing what the largest possible neutron star "mountain" would be [60]. Expressing this in terms of a (quadrupole) ellipticity, the most detailed modelling of crustal strains suggest that [6]

$$\epsilon < 2 \times 10^{-5} \left(\frac{u_{\mathrm{break}}}{0.1} \right), \tag{3}$$

where u_{break} is the crustal breaking strain. Recent molecular dynamics simulations [5] suggest that this may be as large as 0.1, much larger than had been anticipated. This would make the neutron crust super-strong! In comparison, terrestrial materials have $u_{\mathrm{break}} \approx 10^{-4} - 10^{-2}$. Basically, the available estimates suggest that the crust would break if the deformation were to exceed about 20 cm (on a 10 km star).

State-of-the-art calculations of the high density equation of state suggest that solid phases may also be present at higher densities, allowing the construction of stars with larger deformations. Based on a model of a solid strange quark star, Owen [61] estimates $\epsilon < 6 \times 10^{-4}(u_{\mathrm{break}}/10^{-2})$, while, based on a crystalline colour superconducting quark phase, Haskell et al estimate $\epsilon < 10^{-3}(u_{\mathrm{break}}/10^{-2})$ [62]. (Note that the molecular dynamics simulations of [5] do not apply to such exotic phases.) So, significantly larger mountains *might* be provided by nature, depending upon the high density equation of state.

The magnetic field will also tend to deform the star, but for typical pulsar field strengths the deformation is small [63, 64, 65]:

$$\epsilon \approx 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2 . \tag{4}$$

Note however that it is the *internal*, rather than the external field strength that counts. Also, the above estimate assumes a normal fluid core; a superconducting core complicates this picture, and could produce larger asymmetries. A simple estimate for a type II super-conducting core gives [66, 67]

$$\epsilon \approx 10^{-9} \left(\frac{B}{10^{12}\,\mathrm{G}} \right) \left(\frac{H_{\mathrm{crit}}}{10^{15}\,\mathrm{G}} \right), \tag{5}$$
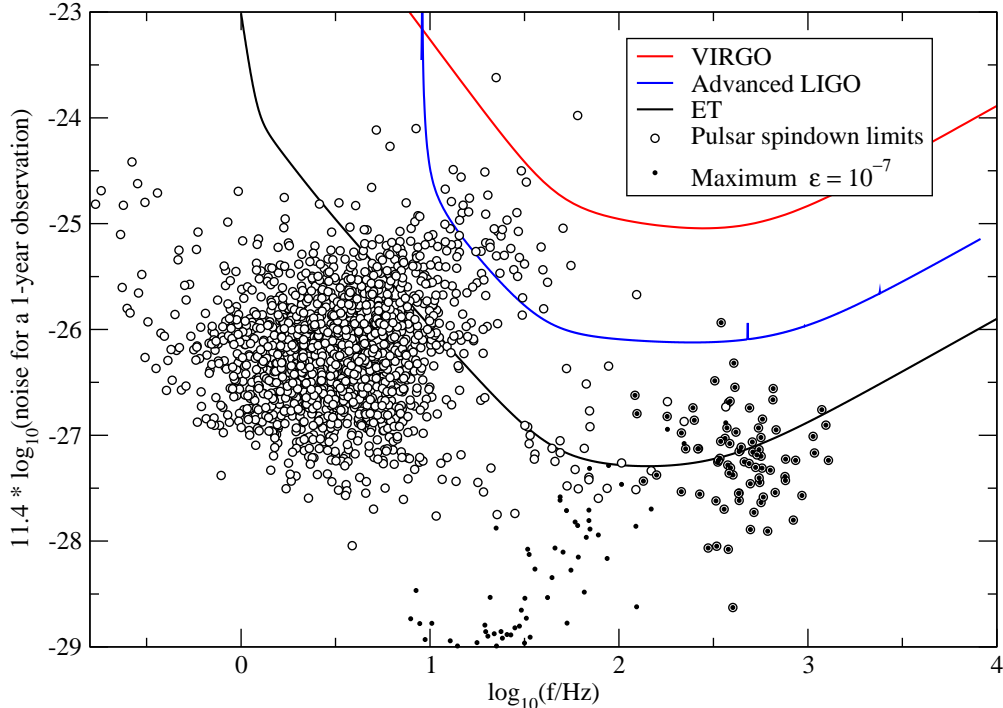
FIG. 5: Spindown upper limits on known radio pulsars. The open circles assume 100% coversion of spindown energy into GWs, regardless of how large an ellipticity is required to power such emission. In contrast, the open circles limit the allowed ellipticity to be no greater than 10^{-7}. An integration time of one year is assumed.

where H_{crit} is the so-called critical field strength [66].

Clearly, this is a rather complicated story, with different physical assumptions leading to very different possible maximum mountain sizes. To gain some understanding of how the maximum mountain size affects detection prospects, in Figure 5 we re-plot the spin-down limits of Figure 4, this time limiting the maximum mountain size to 10^{-7}, a possibly optimistic but certainly not unrealistic value. We plot the original spin down limits of Figure 4 as open circles, and the limits obtained by putting a 10^{-7} cut-off in ellipticity as solid circles.

Clearly, Figure 5 tells a rather different story from Figure 4. Most of the younger pulsars, on the left of the diagram, have dropped off completely: the ellipticites needed for them

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to radiate all of their kinetic energy via GWs are unphysically large. In contrast, the millisecond pulsars on the right hand side have stayed put: they are able to spin down via the GW channel with ellipticites smaller than 10^{-7}. Interestingly, many of the millisecond pulsars lie below the Advanced LIGO/Virgo noise curves but above the ET one. So, if nature supplies millisecond pulsars deformed at the level of one part in 10^7, ET may provide the key to detecting them.

This dependence on rather uncertain physics makes the prospect of a detection all the more exciting. In fact, observations of targeted radio pulsars are already providing interesting results. An observational milestone was reached recently, when LIGO data from the first 9 months of the S5 science run was used to beat the Crab pulsar spin-down limit [68]. Since then a larger data set, taken from the full S5 run, has been analysed; it was found that no more than 2% of the spin-down energy was being emitted in the GW channel, corresponding to an ellipticity bound of approximately $\epsilon < 10^{-4}$ [69]. This result shows that GW emission does not dominate the spin-down of the Crab. As argued in [70], there was no real possibility that 100% of the spin-down was GW powered, as pure gravitational spin-down would conflict with the Crab's measured braking index. However, the fact that the GW contribution to spin-down is less than 2% was not at all obvious. It tells us, for instance, that the Crab is not a maximally strained quark star.

How are these results likely to improve in the future? This is quite easy to estimate since the sensitivity of a search increases in inverse proportion to the detector noise level and as the square root of the observation time. This means that, in the case of the Crab, a search over two years of Advanced LIGO/Virgo data would be sensitive to mountains of about 10^{-5}, while ET may push the limit to 10^{-6}, a sufficiently small value that even a conventional neutron star crust may radiate at a detectable level. Similarly, ET may be able to detect deformations at the $\epsilon \sim 10^{-9}$ level in some of the millisecond pulsars. One would probably expect a signal to be detected before this level is reached, but unfortunately we do not know this for sure. The main challenge concerns the generation mechanism for deformations. Why is the neutron star deformed in the first place? This is an urgent problem that needs to be addressed by theorists. A key question concerns the size of the "smallest" allowed mountain. Based on current understanding, the interior magnetic field sets the lower limit, as given above. Clearly, for a typical pulsar this limit is $\epsilon \approx 10^{-12}$, too small to ever be detected. Of equal interest is the likely – as opposed to maximum – size of an elastic mountain, but this

is an even more difficult problem, depending not just upon the material properties of the crust but also on its "geological" history.

As far as evolutionary scenarios are concerned, accreting neutron stars in low-mass X-ray binaries have attracted the most attention. This is natural for a number of reasons. First of all, the currently observed spin distribution in these systems seems consistent with the presence of a mechanism that halts the spin-up due to accretion well before the neutron star reaches the break-up limit [71]. GW emission could provide a balancing torque if the accretion leads to non-axisymmetries building up in the crust [60, 71, 72, 73, 74]; the required deformation is certainly smaller than (3). Alternatively, accreting stars might rotate fast enough for some modes of oscillation to go unstable, again provide a gravitational spin-down torque to oppose the accretion spin-up [75, 76, 77]. The problem is that accreting systems are very messy. In particular, we do not understand the detailed accretion torque very well [78]. To make progress we need to improve our theoretical models, and we also need future high precision X-ray timing observations to help constrain the binary parameters.

Unlike say binary neutron star mergers or stellar collapse, the detection of long-lasting periodic GWs from a deformed neutron star would be a significant data analysis and computational problem even if we had perfect theoretical templates. This warrants an extended discussion of the data analysis problems that must be addressed before data from Advanced LIGO/Virgo and ET can be fully exploited for continuous wave signal detection and astrophysics. The difficulty arises because of a combination of the expected low signal-to-noise ratio and the very large parameter space of possible signal shapes. Here we briefly review existing search techniques and discuss the improvements necessary in the ET era, to detect continuous wave signals.

Ignore the various systematic signal model uncertainties for the moment, and consider searching for the signals using different techniques, ranging from optimal matched filtering to time-frequency methods. The GW phase model typically considered for these searches is, in the rest frame of the neutron star

$$\Phi(\tau) = \Phi_0 + 2\pi \left[f(\tau - \tau_0) + \frac{\dot{f}}{2}(\tau - \tau_0)^2 + \dots \right]. \tag{6}$$

Here τ is time in the rest frame of the star, Φ_0 the initial phase, τ_0 a fiducial reference time, and f, $\dot{f}$ are respectively the instantaneous frequency and its time derivative at τ_0. Going from τ to detector time t requires us to take into account the motion of the detector in the

solar system and, if the neutron star is in a binary system, the parameters of its orbit.

Using matched filtering, the signal-to-noise ratio builds up with the observation time T as $T^{1/2}$. More precisely, the minimum detectable amplitude h_0^{min} for a single template search at a frequency f for D detectors, each with a single sided noise power spectral density $S_n(f)$ is

$$h_0^{min} = 11.4\sqrt{\frac{S_n(f)}{DT}} \ . \tag{7}$$

Here we have chosen thresholds corresponding to a false alarm rate of 1% and a false dismissal rate of 10%, and averaged over all possible pulsar orientations and sky positions. For the isolated neutron stars discussed previously, if a fraction p of their spindown energy goes into GWs, the required observation time for a detection is proportional to $S_n(f)/(Dp)$. Thus, if we wish to probe the Crab pulsar at say the $p = 1\%$ level as opposed to the 6% limit set by the LIGO S5 search, we would need to observe six times longer. This longer observation time leads to much more stringent requirements on the accuracy of the signal model used in the search. Presently, one assumes that the GWs are locked to the EM phase. Do we really believe that this assumption holds over say, a 5 year observation period, corresponding to a frequency offset of $\sim 6 \times 10^{-9}\,\mathrm{Hz}$? Furthermore, there might be interesting physics causing the GW frequency to be slightly different from twice the radio signal frequency. The accuracy with which we can determine this deviation (and the parameters of any model describing these deviations) is proportional to the inverse of the signal-to-noise ratio (SNR). We could, even without a 3G detector, try to improve the SNR by increasing the observation time. However, a larger observation time typically implies a larger parameter space to search. This makes the search computationally harder and also adversely affects the statistical significance of any discoveries. This argues in favour of increasing the SNR by either increasing the number of detectors or making them more sensitive. The same holds, in fact to a much greater degree, for the wide parameter space searches as we now discuss.

If we do not know the signal parameters *a priori*, then we need to perform a search over a parameter space informed by astrophysical constraints. The most straightforward way of doing the search is to lay a grid of templates over this parameter space and to find the template leading to the largest value of the appropriate detection statistic. It turns out that in most cases, the number of templates increases very quickly with T. This search over a large number of templates has two main effects, both of which affect the sensitivity

adversely. The first is that the larger number of statistical trials leads to a correspondingly larger probability of a statistical false alarm or equivalently, to a larger threshold if we want to maintain a fixed false alarm rate. The second and more important effect is simply that the number of templates might become so large that the computational cost becomes a key factor limiting the largest value of T that we can consider. For isolated neutron stars, the relevant parameters in a blind search are the sky position and the frequency and higher time derivatives of the frequency. For a pulsar in a binary system, the orbital parameters need to be considered as well, leading to three additional parameters for circular orbits (five if we include eccentricity). A detailed analysis, including a catalogue of accreting neutron stars of potential interest for GW searches can be found in [79].

This data analysis challenge needs to be addressed with better techniques and larger computational resources. Examples of more efficient data analysis techniques are the so-called semi-coherent methods which break up the data set of duration T into N smaller segments of length T_{coh} and combine the results of a coherent matched filter search from each segment. While such a method would be suboptimal when we are not computationally limited, it would lead to a closer to optimal method in the presence of computational constraints. Examples of such methods are described in [80, 81, 82], and examples of GW searches employing them are [59, 83, 84, 85]. The sensitivity of these searches is

$$h_0^{min} = \frac{k}{N^{1/4}}\sqrt{\frac{S_n(f)}{DT_{coh}}} \qquad (8)$$

where k is typically ~ 30. The goal here is to make T_{coh} as large as computational resources allow. Further developments of these techniques (a notable example is [86]), and the ability to follow up candidate events enabling multi-stage hierarchical searches (see e.g. [87]) will be necessary for both isolated and binary systems. A different kind of technique is based on cross-correlating data from multiple detectors. These are again less sensitive than coherent matched filtering, but they are computationally easy and highly robust against signal uncertainties. A description of an algorithm tailored towards continuous wave signals can be found in [88]. It is shown that this method is, in a suitably generalized sense, also an example of a semi-coherent technique. This approach is expected to be especially valuable for binary systems where there are potential uncertainties in the signal model. It has, in fact, already been applied to LIGO S4 data [89].

The continued development of larger computational platforms, and increasing the avail-

able computational resources faster than the baseline improvement predicted by Moore's law will be critical. A good example of a large platform tailored towards such searches is the `Einstein@Home` project, and the increased use of GPUs is another. On the timescale relevant for ET we expect to see significant improvement of our computational resources.

One crucial point is that there is more to the detection process than simply observing the amplitude of the signal – higher precision will allow you to extract more physics. Even in the simple case of a steadily rotating star, getting an accurate fix on the phase allows detailed comparison between the electromagnetic and gravitational signals, something which can provide information (in a model dependent way) on the coupling between the stellar components and possibly on the nature of pulsar timing noise [90]. Alternatively, if the rotating star undergoes free precession or contains a pinned superfluid component misaligned with the principal axes, there will be radiation at multiple harmonics, some of which might be significantly weaker than others [91, 92]. Observing these harmonics, and comparing their sizes, will yield yet more information on the stellar interior [93]. Higher quality observations can also be used to measure the distances to (sufficiently nearby) pulsars [94], thus allowing an estimate of the ellipticity to be made using GW data alone and helping us to build a three dimensional picture of the source distribution. The key point for instruments like ET is this: it is not enough to simply detect a signal – the higher the signal-to-noise, the more detailed a picture of the waveform can be built up. This provides strong motivation for a 3G search for GWs, even if second generation detectors prove successful in identifying signals.

V. OSCILLATIONS AND INSTABILITIES

In principle, the most promising strategy for constraining the physics of neutron stars involves observing their various modes of oscillation. We have already discussed the particular case of newly born neutron stars. The problem for mature neutron stars is similar, in the sense that different families of modes can be (more or less) directly associated with different core physics. For example, the fundamental f-mode (which should be the most efficient GW emitter) scales with average density, while the pressure p-modes overtones probe the sound speed throughout the star, the gravity g-modes are sensitive to thermal/composition gradients and the w-modes represent oscillations of spacetime iself. A mature neutron star also has elastic shear modes in the crust [95] and superfluid modes associated with regions where

various constituents form large scale condensates [96]. Magnetic stars may have complex dynamics due to the internal magnetic field. Finally, in a rotating star, there is a large class of inertial modes which are restored by the Coriolis force. One of these rotationally restored modes is the r-mode. The r-mode is particularly interesting because it may be driven unstable by the emission of gravitational radiation.

While the asteroseismology strategy seems promising [7, 97, 98, 99, 100, 101] (provided we can model neutron stars at a sufficient level of realism), it is clear that it relies on how well future GW detectors will be able to detect the various pulsation modes. It is quite easy to make this question quantitative since a typical GW signal from a neutron star pulsation mode will take the form of a damped sinusoid, with oscillation frequency f and damping time t_d. That is, we would have

$$h(t) = \mathcal{A}e^{-(t-T)/t_d} \sin[2\pi f(t - t_0)] \quad \text{for } t > t_0 \tag{9}$$

where t_0 is the arrival time of the signal at the detector (and $h(t) = 0$ for $t < t_0$). Using standard results for the GW flux the amplitude $\mathcal{A}$ of the signal can be expressed in terms of the total energy radiated in the oscillation,

$$\mathcal{A} \approx 7.6 \times 10^{-24} \sqrt{\frac{\Delta E_\odot}{10^{-12}} \frac{1 \text{ s}}{t_d}} \left(\frac{1 \text{ kpc}}{d}\right) \left(\frac{1 \text{ kHz}}{f}\right) . \tag{10}$$

where $\Delta E_\odot = \Delta E/M_\odot c^2$. Finally, the signal-to-noise ratio for this signal can be estimated from

$$\left(\frac{S}{N}\right)^2 = \frac{4Q^2}{1 + 4Q^2} \frac{\mathcal{A}^2 t_d}{2S_n} \tag{11}$$

where the "quality factor" is $Q = \pi f t_d$ and S_n is the spectral noise density of the detector. It is worth noting that, for oscillations lasting longer than the observation time T (i.e. when $Q \gg 1$) we regain the continuous-wave result that the signal-to-noise ratio improves as the square root of T.

So far, the key parameters are the oscillation frequency and the damping time of the mode. We now need an astrophysical scenario that excites the oscillations. Moreover, this scenario has to be such that it can be modeled using linear perturbation theory (with a "slowly" evolving background configuration as reference). This means that one would not expect to deal with the violent dynamics immediately following the formation of a hot neutron star, either after binary merger or core collapse. As we have already discussed, these problems are the realm of nonlinear simulations. Once the (proto-)neutron star settles down to a

relatively slow evolution, the mode-problem becomes relevant. Of course, in this regime it is less obvious that the oscillations will be excited to large amplitude. Hence, the interest in the various instabilities that may affect the star as it evolves is natural. We can also ask general questions concerning the amount of energy that would need to be channeled through the various modes, and whether these energy levels are astrophysically "reasonable". An optimistic scenario would perhaps consider that as much as $10^{-6} M_\odot c^2$ may be radiated (this would be the energy level expected from collapse to a black hole [58]). However, this number is likely only relevant (if at all) for the oscillations of a hot remnant. For mature neutron stars, significantly lower energy levels should be expected. We can take as a bench-mark the energy involved in a typical pulsar glitch, in which case one might (still optimistically?) radiate an energy equivalent to $10^{-13} M_\odot c^2$ [102].

As far as instabilities are concerned, the GW driven instability of the r-mode remains (after more than a decade of scrutiny) the most promising [103, 104, 105, 106, 107, 108]. The r-mode instability window depends on a balance between GW driving and various dissipation mechanisms. In principle, this provides a sensitive probe of the core physics. To illustrate this let us consider a simple model of a neutron star composed of neutrons, protons and electrons, ignoring issues to do with the crust physics, superfluidity, magnetic fields etcetera. If we take the overall density profile to be that of an $n = 1$ polytrope (a simple yet useful approximation) then the characteristic growth timescale for the $l = m = 2$ r-mode is

$$t_{\mathrm{gw}} \approx 50 \left(\frac{1.4 M_\odot}{M} \right) \left(\frac{10 \ \mathrm{km}}{R} \right)^4 \left(\frac{P}{1 \ \mathrm{ms}} \right)^6 \ \mathrm{s} \tag{12}$$

where P is the spin-period of the star. In the simplest model the unstable mode is damped by shear and bulk viscosity. At relatively low temperatures (below a few times 10^9 K) the main viscous dissipation mechanism arises from momentum transport due to particle scattering, modelled as a macroscopic shear viscosity. In a normal fluid star neutron-neutron scattering provides the most important contribution. This leads to a typical damping time

$$t_{\mathrm{sv}} \approx 7 \times 10^7 \left(\frac{1.4 M_\odot}{M} \right)^{5/4} \left(\frac{R}{10 \ \mathrm{km}} \right)^{23/4} \left(\frac{T}{10^9 \ \mathrm{K}} \right)^2 \ \mathrm{s} \tag{13}$$

In other words, at a core temperature of 10^9 K the damping timescale is longer than a year. At higher temperatures bulk viscosity is the dominant dissipation mechanism. Bulk viscosity arises as the mode oscillation drives the fluid away from beta equilibrium. It depends on the extent to which energy is dissipated from the fluid motion as weak interactions

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try to re-establish equilibrium. This is essentially a resonant mechanism that is efficient when the oscillation timescale is similar to the reaction timescale. At higher and lower frequencies (or, equivalently, temperatures) the bulk viscosity mechanism shuts off. This resonance is apparent in the schematic instability window shown in Figure 6. In the Cowling approximation one can show that the bulk viscosity damping timescale (relevant just above 10^9 K, i.e. in region 3 in Figure 6) is given by

$$t_{\mathrm{bv}} \approx 3 \times 10^{11} \left(\frac{M}{1.4 M_\odot} \right) \left(\frac{10 \text{ km}}{R} \right) \left(\frac{P}{1 \text{ ms}} \right)^2 \left(\frac{10^9 \text{ K}}{T} \right)^6 \text{ s} \qquad (14)$$

It is easy to see that, while this damping is inefficient at 10^9 K it is very efficient at 10^{10} K.

From these estimates we learn that the r-mode instability will only be active in a certain temperature range. To have an instability we need t_{gw} to be smaller in magnitude than both t_{sv} and t_{bv}. We find that shear viscosity will completely suppress the r-mode instability at core temperatures below 10^5 K. This corresponds to region 4 in Figure 6. Similarly, bulk viscosity will prevent the mode from growing in a star that is hotter than a few times 10^{10} K. This is the case in region 2 of Figure 6. However, if the core becomes very hot (as in region 1 in Figure 6), then the nuclear reactions that lead to the bulk viscosity are strongly suppressed. For our chosen model, this region is likely not very relevant since neutron stars will not remain at such extreme temperatures long enough that an unstable mode can grow to a large amplitude. Having said that, one should be aware that the "resonance temperature" where the bulk viscosity is strong is highly model dependent and there may well be situations where region 1 is relevant. Finally, in the intermediate region 3 in figure 6, there is a temperature window where the growth time due to gravitational radiation is short enough to overcome the viscous damping and the mode is unstable.

This schematic picture holds for all modes that are driven unstable by GWs, but the instability windows are obviously different in each case. Detailed calculations show that viscosity stabilizes all f-modes below $\Omega_c \approx 0.95 \Omega_K$ in a Newtonian star, while the r-modes are stable below $\Omega_c \approx 0.04 \Omega_K$. (Note that the instability windows are likely to be significantly smaller in more realistic models.) For the $n = 1$ polytrope that we have considered the break-up limit Ω_K would correspond to a spin period of around 1.2 ms (for uniform rotation). Because of the potentially very large instability window, the r-modes have been studied in a variety of contexts, at many different levels of detail. (It is perhaps worth noting that, with the exception of [109, 110, 111, 112], most of the relevant studies have not accounted for
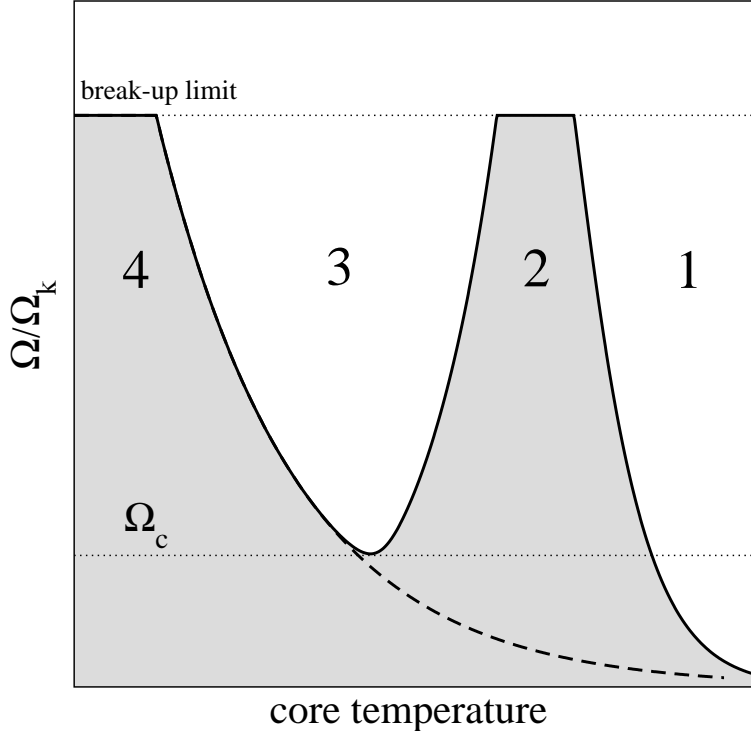
FIG. 6: This figure provides a schematic illustration of the r-mode instability window. At low temperatures (region 4) dissipation due to shear viscosity (dashed curve) counteracts the instability. At temperatures of the order of 10^{10} K bulk viscosity suppresses the instability (region 2). The associated critical temperature is due to a "resonance" between the oscillation and nuclear reaction timescales. At very high temperatures (region 1) the nuclear reactions that lead to the bulk viscosity are suppressed and an unstable mode can, in principle, grow. However, this region may only be relevant for the first few tens of seconds following the birth of a neutron star. The main instability window is expected at temperatures near 10^9 K (region 3), above a critical rotation Ω_c. Provided that gravitational radiation drives the unstable mode strongly enough the instability should govern the spin-evolution of a neutron star. The instability window may change considerably if we add more detailed pieces of physics, like superfluidity and the presence of hyperons, to the model.

relativistic gravity.) We now (think we) know that key issues concern the interaction with magnetic fields in the star [113, 114, 115], the damping due to the vortex mediated mutual friction in a superfluid [116, 117, 118, 119], the role of turbulence [120], the boundary layer at the crust-core interface [121, 122, 123, 124] and exotic bulk viscosity due to the presence of hyperons [125, 126] or deconfined quarks [127, 128] in the deep neutron star core. These

problems are all very challenging. In addition, we need to model the GW signal from an unstable r-mode. This is also difficult because, even though the r-mode growth phase is adequately described by linear theory, nonlinear effects soon become important [129]. Detailed studies show that the instability saturates at a low amplitude due to coupling to other inertial modes [130, 131, 132, 133, 134, 135]. The subsequent evolution is very complex, as is the associated GW signal [136, 137, 138].

While we improve our understanding of this mechanism, we should not forget about the f-mode instability. Basically, we know that the strongest f-mode instability in a Newtonian model is associated with the $l = m = 4$ modes [139]. The situation is different in a relativistic model where the $l = m = 2$ modes may also be unstable [140]. The quadrupole modes are more efficient GW emitters and so could lead to an interesting instability. Moreover, the supposed "killer blow" of superfluid mutual friction suppression [141] only acts below the critical temperature for superfluidity. This means that, even in the worst case scenario, the f-mode could experience an instability in very young neutrons stars (provided that they are born spinning fast enough) [142]. This problem clearly needs further attention. In particular, it should be noted that the various dissipative mechanisms have only been discussed in Newtonian gravity. This means that we have no realistic estimates of the damping of the unstable quadrupole mode. Moreover, not much effort has gone into understanding the associated saturation amplitude and potential GW signal.

To model a truly realistic oscillating neutron star may be difficult, but the potential reward is considerable. This is clear from recent results for the quasiperiodic oscillations that have been observed in the tails of magnetar flares. These oscillations have been interpreted as torsional oscillations of the crust. These crust modes should be easier to excite than poloidal oscillations since they do not involve density variations. Three giant flare events, with peak luminosities in the range 10^{44} - 10^{46} erg s^{-1}, have been observed so far; SGR 0526-66 in 1979, SGR 1900+14 in 1998, and SGR 1806-20 in 2004. Timing analysis of the last two events revealed several quasiperiodic oscillations in the decaying tail, with frequencies in the expected range for toroidal crust modes [143, 144, 145, 146, 147]. If this interpretation is correct, then we are already doing neutron star asteroseismology! Subsequent calculations have shown how observations may constrain both the mass, radius and crust thickness of these stars [148, 149]. Eventually, it may also be possible to probe the magnetic field configuration [150]. There are, however, important conceptual issues that we need to resolve

in order to make progress on this problem. A key question concerns the existence of a magnetic continuum [151, 152, 153, 154, 155, 156, 157, 158, 159] and its potential effect on the dynamics of the system.

Our current models may not be particularly reliable, but they should motivate us to improve our understanding of the key physics (like the interior magnetic field [160], superfluid phases [161, 162] and the dynamical coupling between the crust and the core [163]). It is also possible that the magnetar events, such as the 27/12/2004 burst in SGR 1806-20, generate GWs. Indeed, LIGO data has already been searched for signals from soft-gamma ray repeaters [164, 165, 166], including searches at frequencies at which f-modes might be expected to radiate. No detections have yet been made. Is this surprising? Given our current uncertainty as to just which parts of the star are set in motion following a burst, probably not. Basically, pure crust oscillations would not generate strong GWs due to the low density involved. The situation may change if the dense core fluid is involved in the oscillation, e.g. through magnetic coupling [151, 167], but in that case we do not yet know what the exact signature of the event would be so it is difficult to search for. It would most likely not be appropriate to search for a signal at the observed frequencies in the X-ray data. We need to make progress on a number of issues if we want to understand these events. In absence of better information perhaps the best we can do is "look under the light".

VI. CHALLENGES

GW astronomy promises to provide insights into the "dark side" of the Universe. Because of their density, neutron stars are ideal GW sources and we hope to be able to probe the extreme physics of their interior with future detectors. The potential for this is clear, in particular with third generation detectors like the Einstein Telescope. However, in order to detect the signals and extract as much information as possible, we need to improve our theoretical models considerably. It seems natural to conclude this survey of scenarios by discussing some of the main challenges that we need to meet in the next few years.

For binary inspirals, we need to work out when finite size effects begin to affect the evolution. We should consider tidal resonances and compressibility in detail and ask at what level they affect the late stages of inspiral. It is also important to quantify to what extent we will be able to "read off" neutron star physics from a detected signal.

For hot young remnants, resulting from binary mergers or core collapse events, we need to refine our large scale numerical simulations. It is important to understand that, despite great progress in recent years, these simulations are not yet in the truly "convergent" regime. Key technical issues (like how one should model, and keep track of, the fluid to vacuum transition at the surface) still need to be resolved. Future simulations must use "realistic" equations of state, and consider evolving composition, heat/neutrino cooling and magnetic fields with as few "cheats" as possible. To some extent this effort may be helped by the fact that many effects that are important in a mature neutron are less so on the dynamical timescales that lend themselves to large-scale simulations.

For rotating deformed neutron stars, we need to go beyond producing upper limits on possible signal strengths, and address the much more subtle issue of their *likely* deformations. It will also be vital for the source modelling community to engage further with the high energy physics/QCD theorists, whose models of matter at high density could prove crucial in interpreting future detections. Finally, the data analysis challenge involved in looking for the long-lived signals expected from rotating stars is formidable — further work on reducing the computational burden of these searches is crucial.

In parallel, we need to build on our current understanding of neutron star oscillations and instabilities. This effort should aim at accounting for as much of the interior physics as possible. If we want to model proto-neutron stars then we need to refine our understanding of heat conduction (in general relativity) and the role of the trapped neutrinos. For mature neutron stars we need to model the crust, which will contain a "decoupled" neutron superfluid. The complex composition of the core, with potentially several exotic phases of matter, poses a number of challenges. To meet these we need to improve our models for multi-fluid systems. It is also key to understand the relevance of proton superconductivity. After all, the magnetic field will always play some role. For magnetars it may, in fact, be the dominant factor. Finally, it is crucial that we find a way to model systems that evolve on a secular timescale. The archetypal problem may be the nonlinear saturation and subsequent evolution of an unstable r-mode. At the present time, the available models are based on the explicit calculation of, and transfer of energy between, coupled modes. It will not be easy to extend this calculation to more realistic neutron star models, and it seems natural to ask if there is some alternative way of modelling such problems.

Finally, we need a clearer phenomenological understanding of pulsar glitches, accreting

neutron stars, magnetar flares etcetera. It is natural to develop this as part of an effort in multi-messenger astronomy [168, 169], a theme we have only touched upon briefly. Electro-magnetic and GW data have in fact already been usefully combined. For instance, LIGO data was used to show that the gamma ray burst GRB070201, which originated in the di-rection of the spiral arms of the Andromeda galaxy, was *not* powered by a neutron star inspiral at the distance of Andromeda [170]. The improved sensitivity of 3G detectors will allow for more frequent and more powerful statements to be made, covering a whole host of electromagnetic observations. In particular, it seems appropriate to ask what we can learn from more detailed radio observations with SKA and Lofar (for example). Future X-ray data should also continue to provide key information — just think how much we have learned from RXTE. In fact, accurate neutron star timing data is key for GW science. Of course, we should not simply wait for our colleagues in other fields to give us answers. We should ask if our modelling effort can help shed light on phenomena in "mainstream" astronomy. Given the accurate neutron star models that we are now developing, it would seem obvious that this is the case.

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