LETTERS TO THE EDITORS

MATHEMATICS

Gravitational Waves in General Relativity

THE emission of gravitational waves from a finite isolated axially symmetrical material system in otherwise empty space has been investigated by consideration of the metric :

$$ds^2 = \left(\frac{V}{r}e^{2\beta} - r^2 e^{2\gamma} U^2\right) du^2 + 2e^{2\beta} du dr +$$
$$2r^2 U e^{2\gamma} du d\theta - r^2 (e^{2\gamma} d\theta^2 + \sin^2\theta\, e^{-2\gamma} d\varphi^2)$$

Einstein vacuum field equations have been solved by an expansion in negative powers of r which represents radial distance in a well-defined sense. In this expansion it has been assumed that only outgoing waves are present. The expression :

$$\tfrac{1}{2} \lim_{r \to \infty} \int_0^\pi (r - V) \sin\theta d\theta$$

represents mass in the static case, and forms a suitable generalization of this static concept to the dynamical case.

The following results have been found :

(1) If a system is static before some instant, then undergoes motion, and eventually again returns to a static state, then its mass in the final state is less than its mass in the initial state.

(2) There exists a class of possible motions of the source in which the curvature tensor (measured by its physical components) falls off like r^{-3}. In this case no radiation is sent out and no loss of mass occurs. The existence of such a class of motions is related to Infeld's[1] result that a set of freely moving particles does not radiate. This case differs from the stationary one by, among other characteristics, lim $(r - V)$ being a function of θ though not of u.

(3) There exists a class of motions of the source in which the curvature tensor falls off like r^{-2}. In this case the power received by a set of small receivers covering a large sphere surrounding the source tends to zero as the radius of the sphere tends to infinity, since small receivers absorb energy at a rate proportional to the square of the curvature tensor. Nevertheless the source loses mass, because at any distance from the source a suitably constructed large receiver could absorb a finite amount of energy.

(4) For general motions the curvature tensor falls off like r^{-1}, but, because of (2), there is no close relation between the mass loss of the source and the energy that could be absorbed by distant small receivers. This point underlines the non-localizable nature of gravitational energy.

I am much indebted to Dr. F. A. E. Pirani for innumerable discussions that have done much to improve my understanding of the problem.

Full details of the work (carried on partly in collaboration with Dr. M. G. J. van der Berg, Department of Applied Mathematics, University of Liverpool) will be published elsewhere.

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[1] Infeld, L., *Ann. Phys.* (New York), 6, 341 (1959).

PHYSICS

Time-Dependent Tensile Strength of Solids

DURING the course of studying the strain-rate effects of a number of irradiated and non-irradiated polymeric solids as well as some metallic solids (refs. 1 and 2 and unpublished results) a common relationship between stress and strain-rate has been observed. For any given value of strain the stress obtained at different rates of straining is linearly related with the logarithm of the strain-rate. Mathematically, if ε is the total given strain at any given temperature T, then the stress σ is related with the strain-rate $\dot{\varepsilon}$ in the following manner :

$$\dot{\varepsilon} = \dot{\varepsilon}_0 e^{\frac{\sigma - \sigma_0}{m}} \Big|_{\varepsilon} \qquad (1)$$

where $\dot{\varepsilon}_0$ and σ_0 are constants independent of the strain ε, and m, which is also independent of the strain, is the slope of the straight line when the log of strain-rate is plotted against the stress. Or in another form, when the rate of straining is constant :

$$t = \frac{\varepsilon}{\dot{\varepsilon}_0} e^{\frac{\sigma_0 - \sigma}{m}} \Big|_{\dot{\varepsilon}} \qquad (2)$$

Depending upon the value of ε, which is a measure of molecular configuration of the solid in terms of deformation[3], the time-dependent strength can be evaluated.

As one special case, if we let :

$$\dot{\varepsilon} = \dot{\varepsilon}_b = \frac{\varepsilon_b}{t_b}$$

then from (1) :

$$t_b = \frac{\varepsilon_b}{\dot{\varepsilon}_0} e^{\frac{\sigma_0 - \sigma_b}{m}} \qquad (3)$$

where t_b is the time to fracture, ε_b is the fracture strain and σ_b the stress maintained until fracture occurs. This agrees well with the current theoretical and experimental developments of the subject[4-8].

On the basis of considering the oscillation and breaking of atomic bonds, the time-dependent strength properties of solids has been reported and may be expressed as follows[4] :

$$t_b = \frac{1}{\omega} e^{\frac{E - 2{\cdot}8\frac{\delta}{n}\sigma_b}{kT}} \qquad (4)$$

where t_b is the time taken for the sample to break under the tensile stress σ_b, ω corresponds to some bond oscillation frequency, n is the number of bonds per unit area the normal of which is in parallel to these bonds, E is the bond energy, 2δ is approximately the distance the bond will stretch before