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Gravitino Production in the Inflationary Universe and the Effects on Big-Bang Nucleosynthesis

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Gravitino production and decay in the inflationary universe are reexamined. Assuming that the gravitino mainly decays into a photon and a photino, we calculate the upperbound on the reheating temperature. Compared to previous works, we have essentially improved the following two points: (i) the helicity $\pm(3/2)$ gravitino production cross sections are calculated by using the full relevant terms in the supergravity lagrangian, and (ii) the high energy photon spectrum is obtained by solving the Boltzmann equations numerically. Photo-dissociation of the light elements (D, T, ^{3}He, ^{4}He) leads to the most stringent upperbound on the reheating temperature, which is given by (10^6-10^9) GeV for the gravitino mass 100 GeV $-$ 1 TeV. On the other hand, requiring that the present mass density of photino should be smaller than the critical density, we find that the reheating temperature have to be smaller than $(10^{11}-10^{12})$ GeV for the photino mass $(10-100)$ GeV, irrespectively of the gravitino mass. The effect of other decay channels is also considered.

§1. Introduction

When one thinks of new physics beyond the standard model, supersymmetry (SUSY) is one of the most attractive candidates. Cancellation of quadratic divergences in SUSY models naturally explains the stability of the electroweak scale against radiative corrections.[1),2)] Furthermore, if we assume the particle contents of the minimal SUSY standard model (MSSM), the three gauge coupling constants in the standard model meet at $\sim 2\times 10^{16}$ GeV,[3),4)] which strongly supports the grand unified theory (GUT).

In spite of these strong motivations, no direct evidence for SUSY (especially superpartners) has been discovered yet. This means that the SUSY is broken in nature, if it exists. Although many efforts have been made to understand the origin of the SUSY breaking, we have not understood it yet. Nowadays, many people expect the existence of *local* SUSY (i.e., supergravity) and try to find a mechanism to break it spontaneously in this framework. In the broken phase of the supergravity, super-Higgs effect occurs and the gravitino, which is the superpartner of graviton, acquires mass by absorbing the Nambu-Goldstone fermion associated with the SUSY breaking sector. In a softly broken SUSY models induced by minimal supergravity, we expect that the mass of the gravitino $m_{3/2}$ lies in the same order of those of squarks and sleptons since the following (tree level) super-trace formula among the mass matrices $\mathcal{M}_J{}^2$ holds,[5)]

$$\mathrm{Str}\mathcal{M}^2 \equiv \sum_{\text{spin } J}(-1)^{2J}(2J+1)\mathrm{tr}\mathcal{M}_J{}^2 \simeq 2(n-1)m_{3/2}^2\,, \tag{1}$$

where n is the number of chiral multiplet. For example in models with the minimal

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kinetic term, this is the case and all the SUSY breaking masses of squarks and sleptons are equal to gravitino mass at the Planck scale. But contrary to our theoretical interests, we have no hope to see the gravitinos directly in the collider experiments since the interaction of gravitino is extremely weak.

On the other hand, if we assume the standard big-bang cosmology, the mass of gravitino is severely constrained. If the gravitino is unstable, it may decay after the big-bang nucleosynthesis (BBN) and produces an unacceptable amount of entropy, which conflicts with the predictions of BBN. In order to keep success of BBN, the gravitino mass should be larger than $\sim 10\,\mathrm{TeV}$ as Weinberg first pointed out.[6] Meanwhile, in the case of stable gravitino, its mass should be smaller than $\sim 1\,\mathrm{keV}$ not to overclose the universe.[7] Therefore, the gravitino mass between $\sim 1\,\mathrm{keV}$ and ~ 10 TeV conflicts with the standard big-bang cosmology.

However, if the universe went through inflation, we may avoid the above constraints[8] since the initial abundance of gravitino is diluted by the exponential expansion of the universe. But even if the initial gravitinos are diluted, the above problems still potentially exist since gravitinos are reproduced by scattering processes off the thermal radiation after the universe has been reheated.[9)~17] The number density of secondary gravitino is proportional to the reheating temperature and hence, upperbound on the reheating temperature should be imposed not to overproduce gravitinos. Therefore, even assuming inflation, a detailed analysis must be done to obtain the upperbound on the reheating temperature. The case of stable gravitino has been analyzed in Refs. 13), 15), 17) and we will not deal with it.

In this paper, we assume that gravitino is unstable and derive an upperbound on the reheating temperature. The analysis of the cosmological regeneration and decay of unstable gravitino has been done in many articles. These previous works show that the most stringent upperbound on the reheating temperature comes from the photo-dissociation of the light nuclei (D, T, ^{3}He, ^{4}He). Once gravitinos are produced in the early universe, most of them decay after BBN since the lifetime of gravitino with mass $O(100\,\mathrm{GeV}-10\,\mathrm{TeV})$ is $O((10^8-10^2)\mathrm{sec})$. If gravitinos decay radiatively, emitted high energy photons induce cascade processes and affect the result of BBN. Not to change the abundances of light nuclei, we must constrain the number density of the gravitino, and this constraint is translated into the upperbound on the reheating temperature.

In order to analyze the photo-dissociation process, we must calculate the following two quantities precisely; the number density of the gravitino produced after the reheating of the universe, and the high energy photon spectrum induced by radiative decay of gravitino. But the previous estimations of these values are incomplete. As for the number density of gravitino, most of the previous works follow the result of Ref. 11), where the number density is underestimated by a factor ~ 4. Furthermore, in many articles, the spectrum of high energy photon, which determines the photo-dissociation rates of light elements, are calculated by using a simple fitting formula. In this paper, we treat these effects precisely and found the more stringent upperbound on the reheating temperature than the previous calculations.

The plan of this paper is as follows. In § 2, we discuss the interactions of the gravitino. In § 3, we calculate the gravitino production rate in the early universe. In

§ 4, the high energy photon spectrum induced by the radiative decay of gravitino is obtained by solving the Boltzmann equations numerically. The results are shown in § 6. Other cosmological constraints are considered in § 7 and § 8 is devoted to discussion.

§ 2. Interaction of gravitino

Before investigating the effects of the gravitino on the inflationary universe, let us discuss the interaction of the gravitino briefly. From the supergravity lagrangian,[5)] we can obtain the relevant interaction terms of gravitino ψ_μ with gauge multiplets (A_μ, λ) and chiral multiplets (ϕ, χ),

$$\mathcal{L}=\frac{i}{8M}\bar{\lambda}\gamma_\mu[\gamma_\nu, \gamma_\rho]\psi_\mu F_{\nu\rho}+\left\{\frac{1}{\sqrt{2}M}\bar{\psi}_{\mu L}\gamma_\nu\gamma_\mu\chi_L D_\nu\phi^\dagger+\text{h.c.}\right\}, \tag{2}$$

where $M=M_{\text{pl}}/\sqrt{8\pi}\simeq 2.4\times 10^{18}\,\text{GeV}$ (with M_{pl} being the Planck mass).*) Note that other interaction terms including gravitino field are not important for our analysis since their contributions are suppressed by a factor of M^{-1}.

Combining Eq. (2) with the renormalizable part of the SUSY lagrangian, we have calculated the helicity $\pm 3/2$ gravitino production cross sections and the results are shown in Table I. Note that the cross sections for the processes (B), (F), (G) and (H) are singular because of the t-channel exchange of gauge bosons. These singularities should be cut off when the effective gauge boson mass m_{eff} due to the plasma effect is taken into account. Following Ref. 11), we take $\delta\equiv(1\mp\cos\theta)_{\text{min}}=(m_{\text{eff}}^2/2T^2)$ where θ is a scattering angle in the center-of-mass frame, and in our numerical calculations, we choose $\log(m_{\text{eff}}^2/T^2)=0$.

The effective total cross section in thermal bath Σ_{tot} is defined by

Table I. Total cross sections for the helicity $\pm(3/2)$ gravitino production process. Spins of the initial states are averaged and those of the final states are summed. f^{abc} and T_{ij}^a represent the structure constants and the generator of the gauge group, respectively. Note that for the processes (B), (F), (G) and (H), we cut off the singularities due to the t-, u-channel exchange of gauge bosons, taking $(1\pm\cos\theta)_{\text{min}}=\delta$ where θ is the scattering angle in the center-of-mass frame.

Process		$\sigma=(g^2/64\pi M^2)\times$
(A)	$A^a+A^b\to\psi+\lambda^c$	$(8/3)\lvert f^{abc}\rvert^2$
(B)	$A^a+\lambda^b\to\psi+A^c$	$4\lvert f^{abc}\rvert^2\{-(3/2)+2\log(2/\delta)+\delta-(1/8)\delta^2\}$
(C)	$A^a+\phi_i\to\psi+\chi_j$	$4\lvert T_{ji}^a\rvert^2$
(D)	$A^a+\chi_i\to\psi+\phi_j$	$2\lvert T_{ji}^a\rvert^2$
(E)	$\chi_i+\phi_j^*\to\psi+A^a$	$4\lvert T_{ji}^a\rvert^2$
(F)	$\lambda^a+\lambda^b\to\psi+\lambda^c$	$\lvert f^{abc}\rvert^2\{-(62/3)+16\log[(2-\delta)/\delta]+22\delta-2\delta^2+(2/3)\delta^3\}$
(G)	$\lambda^a+\chi_i\to\psi+\chi_j$	$4\lvert T_{ji}^a\rvert^2\{-2+2\log(2/\delta)+\delta\}$
(H)	$\lambda^a+\phi_i\to\psi+\phi_j$	$\lvert T_{ji}^a\rvert^2\{-6+8\log(2/\delta)+4\delta-(1/2)\delta^2\}$
(I)	$\chi_i+\bar{\chi}_j\to\psi+\lambda^a$	$(8/3)\lvert T_{ji}^a\rvert^2$
(J)	$\phi_i+\phi_j^*\to\psi+\lambda^a$	$(16/3)\lvert T_{ij}^a\rvert^2$

*) In Ref. 11), interactions between the gravitino and the chiral multiplets (the second term in Eq. (2)) are ignored in calculating the cross sections for the gravitino production processes.

$$\Sigma_{\text{tot}}=\frac{1}{2}\sum_{x,y,z}\eta_x\eta_y\sigma_{(x+y\to\psi_\mu+z)}\,, \tag{3}$$

where $\sigma_{(x+y\to\psi_\mu+z)}$ is the cross section for the process $x+y\to\psi_\mu+z$, $\eta_x=1$ for incoming bosons, $\eta_x=3/4$ for fermions. For the MSSM particle content, Σ_{tot} is given by

$$\Sigma_{\text{tot}}=\frac{1}{M^2}\{2.50g_1^2(T)+4.99g_2^2(T)+11.78g_3^2(T)\}\,, \tag{4}$$

where g_1, g_2 and g_3 are the gauge coupling constants of the gauge group $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. Note that in high energy scattering processes, effect of the renormalization group flow of the gauge coupling constants should be considered. Using the one loop β-function of MSSM, solution to the renormalization group equation of gauge coupling constants is given by

$$g_i(T)\simeq\left\{g_i^{-2}(M_Z)-\frac{b_i}{8\pi^2}\log\left(\frac{T}{M_Z}\right)\right\}^{-1/2} \tag{5}$$

with $b_1=11$, $b_2=1$, $b_3=-3$. In this paper, we use the above formula.

From Eq. (2), we can also get the decay rate of the gravitino. In this paper, we only consider $\psi_\mu\to\gamma+\tilde{\gamma}$ decay mode, for which the decay rate is given by

$$\Gamma=\frac{m_{3/2}^3}{32\pi M^2}\left\{1-\left(\frac{m_{\tilde{\gamma}}}{m_{3/2}}\right)^2\right\}^3\left\{1+\frac{1}{3}\left(\frac{m_{\tilde{\gamma}}}{m_{3/2}}\right)^2\right\}\,, \tag{6}$$

where $m_{\tilde{\gamma}}$ is the photino mass. In the case $m_{\tilde{\gamma}}\ll m_{3/2}$, this decay rate corresponds to the lifetime

$$\tau_{3/2}(\psi_\mu\to\gamma+\tilde{\gamma})=3.9\times10^8\left(\frac{m_{3/2}}{100\,\text{GeV}}\right)^{-3}\ \text{sec}\,. \tag{7}$$

In the cosmological applications, the gravitino lifetime $\tau_{3/2}$ determines the decay time of the gravitino, i.e., the gravitino decays when the Hubble time becomes of the same order of $\tau_{3/2}$. Thus the temperature of the background photon at the gravitino decay time becomes lower as the lifetime gets longer.

§ 3. Gravitino production in the early universe

After the universe has reheated, gravitinos are reproduced by the scattering processes of the thermal radiations and decay with the decay rate of order of $m_{3/2}^3/M_{\text{pl}}^2$. Since the interaction of gravitino is very weak, gravitino cannot be thermalized if the reheating temperature T_R is less than $O(M_{\text{pl}})$. In this case, Boltzmann equation for the gravitino number density $n_{3/2}$ can be written as

$$\frac{dn_{3/2}}{dt}+3Hn_{3/2}=\langle\Sigma_{\text{tot}}v_{\text{rel}}\rangle n_{\text{rad}}^2-\frac{m_{3/2}}{\langle E_{3/2}\rangle}\frac{n_{3/2}}{\tau_{3/2}}\,, \tag{8}$$

where H is the Hubble parameter, $\langle\cdots\rangle$ means thermal average, $n_{\text{rad}}\equiv\zeta(3)T^3/\pi^2$ represents the number density of the scalar boson in thermal bath, v_{rel} is the relative velocity of the scattering radiations ($\langle v_{\text{rel}}\rangle=1$ in our case), and $m_{3/2}/\langle E_{3/2}\rangle$ is the averaged Lorentz factor. Note that the first term of the right-hand side (r.h.s) of

Eq. (8) represents contribution from the gravitino production process, and the second one comes from the decay of gravitino. In Eq. (8), we have omitted the terms which represent the inverse processes since their contributions are unimportant at low temperature that we are interested in. In the radiation dominated universe, H is given by

$$H \equiv \frac{\dot{R}}{R} = \sqrt{\frac{N_* \pi^2}{90 M^2}} T^2 , \tag{9}$$

where R is the scale factor and N_* is the total number of effectively massless degrees of freedom, respectively. For the particle content of MSSM, $N_*(T_R) \sim 228.75$ if T_R is much larger than the masses of the superpartners, and N_* $(T \ll 1\,\text{MeV}) \sim 3.36$.

At the time right after the end of the reheating of the universe, the first term dominates the r.h.s. of Eq. (8) since gravitinos have been diluted by the de Sitter expansion of the universe. Using yield variable $Y_{3/2} \equiv n_{3/2}/n_{\text{rad}}$ and ignoring the decay contributions, Eq. (8) becomes

$$\frac{dY_{3/2}}{dT} = -\frac{\langle \Sigma_{\text{tot}} v_{\text{rel}} \rangle n_{\text{rad}}}{HT} , \tag{10}$$

where we have assumed the relation

$$RT = \text{const} . \tag{11}$$

Ignoring the small T-dependence of Σ_{tot}, we can solve Eq. (10) analytically. Integrating Eq. (10) from the reheating temperature T_R to T $(T_R \gg T)$ and multiplying the dilution factor $N_S(T)/N_S(T_R)$, the yield of gravitino is found to be

$$Y_{3/2}(T) = \frac{N_S(T)}{N_S(T_R)} \times \frac{n_{\text{rad}}(T_R) \langle \Sigma_{\text{tot}} v_{\text{rel}} \rangle}{H(T_R)} . \tag{12}$$

For the MSSM particle content, $N_S(T_R) \sim 228.75$ and $N_S(T \ll 1\,\text{MeV}) \sim 3.91$. Equation (12) shows that $Y_{3/2}$ is proportional to T_R. From Eq. (12), we can derive the simple fitting formula for $Y_{3/2}$,

$$Y_{3/2}(T \ll 1\,\text{MeV}) \simeq 2.14 \times 10^{-11} \left(\frac{T_R}{10^{10}\,\text{GeV}} \right) \left\{ 1 - 0.0232 \log \left(\frac{T_R}{10^{10}\,\text{GeV}} \right) \right\} , \tag{13}$$

where the logarithmic correction term comes from the renormalization group flow of the gauge coupling constants. The difference between the exact formula (12) and the above approximated one is within $\sim 5\%$ $(\sim 25\%)$ for $10^6\,\text{GeV} \lesssim T_R \lesssim 10^{14}\,\text{GeV}$ $(10^2\,\text{GeV} \lesssim T_R \lesssim 10^{19}\,\text{GeV})$. Note that the numerical value of $Y_{3/2}$ in our case is about $4-5$ times larger than the result in Ref. 11) due to the difference of $\langle \Sigma_{\text{tot}} v_{\text{rel}} \rangle$. Some comments on this difference are given in § 8.

As the temperature of the universe drops and H^{-1} approaches $\tau_{3/2}$, the decay term becomes the dominant part of the r.h.s. of Eq. (8). Ignoring the scattering term, Eq. (8) can be rewritten as

$$\frac{dY_{3/2}}{dt} = -\frac{Y_{3/2}}{\tau_{3/2}} , \tag{14}$$

where we have taken $m_{3/2}/\langle E_{3/2}\rangle=1$ since gravitinos are almost at rest. Using Eq. (12) as a boundary condition, we can solve Eq. (14) and the solution is

$$Y_{3/2}(t)=\frac{n_{3/2}(t)}{n_{\rm rad}(t)}=\frac{N_S(T)}{N_S(T_R)}\times\frac{n_{\rm rad}(T_R)\langle\Sigma_{\rm tot}v_{\rm rel}\rangle}{H(T_R)}\exp\left(-\frac{t}{\tau_{3/2}}\right), \tag{15}$$

where the relation between t and T can be obtained by solving Eq. (9) with Eq. (11),

$$t=\frac{1}{2}\sqrt{\frac{90M^2}{N_*\pi^2}}T^{-2}. \tag{16}$$

§ 4. Radiative decay of gravitino

Radiative decay of gravitino may affect BBN. We analyze this effect assuming that the gravitino ψ_μ mainly decays to a photon γ and a photino $\tilde{\gamma}$.

In order to investigate the photo-dissociation processes, we must know the spectra of the high energy photon and electron induced by the gravitino decay. In this section, we will derive these spectra by solving the Boltzmann equations numerically.

Once high energy photons are emitted in the gravitino decay, they induce cascade processes. In order to analyze these processes, we take the following radiative processes into account. (I) The high energy photon with energy ϵ_γ can become e^+e^- pair by scattering off the background photon if the energy of the background photon is larger than m_e^2/ϵ_γ (with m_e being the electron mass). We call this process double photon pair creation. For sufficiently high energy photons, this is the dominant process since the cross section or the number density of the target is much larger than other processes. Numerical calculation shows that this process determines the shape of the spectrum of the high energy photon for $\epsilon_\gamma\gtrsim m_e^2/22T$. (II) Below the effective threshold of the double photon pair creation, high energy photons lose their energy by the photon-photon scattering. But in the limit $\epsilon_\gamma\to 0$, the total cross section for the photon-photon scattering is proportional to ϵ_γ^3 and this process loses its significance. Hence finally, photons are thermalized by (III) pair creation in the nuclei, or (IV) Compton scattering off the thermal electron. And (V) emitted high energy electrons and positrons lose their energy by the inverse Compton scattering off the background photon. Furthermore, (VI) the source of these cascade processes is the high energy photons emitted in the decay of gravitinos. Note that we only consider the decay channel $\psi_\mu\to\gamma+\tilde{\gamma}$ and hence the energy of the incoming photon $\epsilon_{\gamma0}$ is monochromatic.

The Boltzmann equations for the photon and electron distribution function f_γ and f_e are given by

$$\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}=\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm DP}+\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm PP}+\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm PC}+\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm CS}+\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm IC}+\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm DE}, \tag{17}$$

$$\frac{\partial f_e(E_e)}{\partial t}=\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\rm DP}+\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\rm PC}+\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\rm CS}+\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\rm IC}, \tag{18}$$

where DP, PP, PC, CS, IC and DE represent double photon pair creation, photon-photon scattering, pair creation in nuclei, Compton scattering, inverse Compton scattering, and the contribution from the gravitino decay, respectively. Full details are shown in Appendix A.

In order to see the photon spectrum, we have to solve Eqs. (17) and (18). Since the decay rate of gravitino is much smaller than the scattering rates of other processes, gravitinos can be regarded as a stationary source of high energy photon at each moment. Therefore, we only need a stationary solution to Eqs. (17) and (18) with non-zero $(\partial f_\gamma/\partial t)|_{\mathrm{DE}}$ at each temperature.*) Note that Eqs. (17) and (18) are linear equations of f_γ and f_e, and hence, once Eqs. (17) and (18) have been solved with some reference value of $(\partial \tilde{f}_\gamma/\partial t)|_{\mathrm{DE}}$ we can reconstruct the photon spectrum for arbitrary value of $(\partial f_\gamma/\partial t)|_{\mathrm{DE}}$ with T and $\epsilon_{\gamma 0}$ fixed,

$$f_\gamma(\epsilon_\gamma)=\tilde{f}_\gamma(\epsilon_\gamma)\times\frac{(\partial f_\gamma/\partial t)|_{\mathrm{DE}}}{(\partial \tilde{f}_\gamma/\partial t)|_{\mathrm{DE}}}. \tag{19}$$

For each T and $\epsilon_{\gamma 0}$, we have calculated the reference spectra $\tilde{f}_\gamma(\epsilon_\gamma)$ and $\tilde{f}_e(E_e)$ by solving Eqs. (17) and (18) numerically with the condition,

$$\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}=\frac{\partial f_e(E_e)}{\partial t}=0. \tag{20}$$

Typical spectra are shown in Fig. 1 in which we show the case $\epsilon_{\gamma 0}=100$ GeV and 10 TeV, $T=100$ keV, 1 keV, 10 eV and the incoming flux of the high energy photon is normalized to be

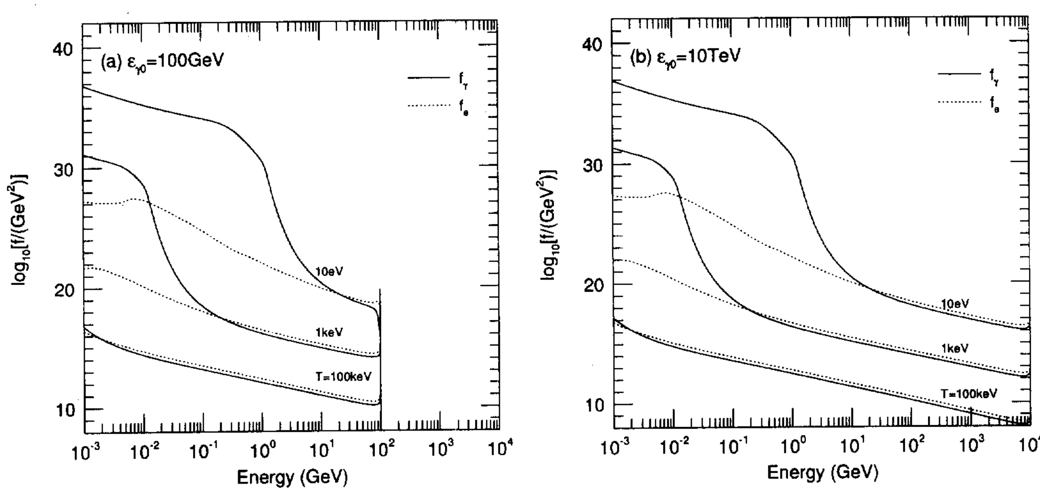


Fig. 1. Typical spectra of photon (the solid lines) and electron (the dotted lines). We take the temperature of the background photon to be $T=100$ keV, 1 keV, 10 eV, and the energy of the incoming high energy photon $\epsilon_{\gamma 0}$ is (a) 100 GeV and (b) 10 TeV. Normalization of the initial photon is given by $\epsilon_{\gamma 0}\times(\partial \tilde{f}_\gamma(\epsilon_\gamma)/\partial t)|_{\mathrm{DE}}=\delta(\epsilon_\gamma-\epsilon_{\gamma 0})\ \mathrm{GeV}^5$.

*) This approximation is justified if the scattering rates of high energy photons and electrons are sufficiently larger than the expansion rate of the universe. This condition is satisfied in the present situation.

$$\epsilon_{\gamma 0} \times \frac{\partial \tilde{f}_\gamma(\epsilon_\gamma)}{\partial t}\bigg|_{\mathrm{DE}} = \delta(\epsilon_\gamma - \epsilon_{\gamma 0})\, \mathrm{GeV}^5 . \tag{21}$$

The behaviors of the photon spectra can be understood in the following way. For a given temperature T, in the region $\epsilon_\gamma \gtrsim m_e^2/22T$, the photon number density is extremely suppressed since the rate of double photon pair creation process is very large. Just below this threshold value, the shape of the photon spectrum is determined by the photon-photon scattering process, and if the photon energy is sufficiently small, the Compton scattering with the thermal electron is the dominant process for photons. Note that the photon and electron spectra are determined almost only by the total amount of energy injection. That is, the initial energy dependence of the low energy spectra is negligible. This is consistent with the previous work.[16] In Fig. 2, we compare our photon spectrum with the results of the simple fitting formula used in Ellis et al.[16),*)] The fitting formula in Ref. 16) is derived from the numerical results given in Refs. 18) and 19) in which, however, the effect of the Compton scattering is not taken into account. Our results indicate that the number of Compton scattering events is comparable to that of the inverse Compton events for such low energy region ($\epsilon_\gamma \leq m_e^2/80T$), since the number density of the high energy electron is extremely smaller than that of the high energy photon. Therefore, the deformation of the photon spectrum by Compton scattering is expected below the threshold of the photon-photon scattering.

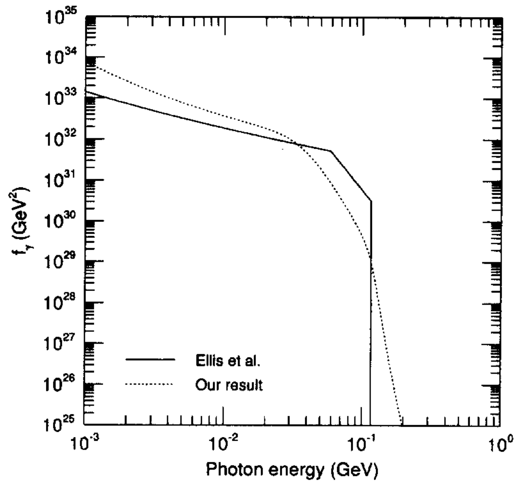


Fig. 2. Photon spectrum derived from the fitting formula used in Ref. 16) is compared with our result. We take the temperature of the background photon to be 100 eV and the normalization of the incoming flux is the same as in Fig. 1. The solid line is the result of fitting formula, and the dotted line is our result with $\epsilon_\gamma = 100$ GeV.

§ 5. BBN and photo-dissociation of light elements

BBN is one of the great successes of the standard big bang cosmology. It is believed that light elements of mass number less than 7 are produced at an early stage of the universe when the cosmic temperature is between 1 MeV and 10 keV. Theoretical predictions for abundances of light elements are excellently in good agreement with those expected from observations if the baryon-to-photon ratio η_B is about

*) Although the photon spectrum is not explicitly given in Ref. 16), we obtain it from their photon production spectrum divided by the Compton scattering rate $\langle n_e \sigma_{\mathrm{CS}} v_{\mathrm{rel}} \rangle$ (where n_e is the number density of the electron and σ_{CS} the cross section for the Compton scattering process) as Ellis et al. did.[16] Since the cross section has energy dependence, the resultant spectrum ($\propto \epsilon_\gamma^{-0.9}$) becomes softer than that for photon production ($\propto \epsilon_\gamma^{-1.5}$).

Table II. Photo-dissociation reactions.

Reaction	Threshold (MeV)	References
$D+\gamma\to n+p$	2.225	[21]
$T+\gamma\to n+D$	6.257	[22], [23]
$T+\gamma\to p+n+n$	8.482	[23]
$^3He+\gamma\to p+D$	5.494	[24]
$^3He+\gamma\to p+D$	7.718	[24]
$^4He+\gamma\to p+T$	19.815	[25]
$^4He+\gamma\to n+{}^3He$	20.578	[26]
$^4He+\gamma\to p+n+D$	26.072	[27]

3×10^{-10}.

However the presence of gravitino might destroy this success of BBN. Gravitino may have three effects on BBN. First the energy density of gravitino at $T\simeq1$ MeV speeds up the cosmic expansion and leads to increase the n/p ratio and hence ^{4}He abundance. Second, the radiative decay of gravitino reduces the baryon-to-photon ratio and results in too baryon-poor universe. Third, the high energy photons emitted in the decay of gravitino destroy the light elements. Among three effects, photo-dissociation by the high energy photons is the most important for gravitino with mass less than ~1 TeV. In the following we consider the photo-dissociation of light elements and discuss other effects in § 7.

The high energy photons emitted in the decay of gravitinos lose their energy during multiple electro-magnetic processes described in the previous section. Surviving soft photons can destroy the light elements (D, T, ^{3}He, ^{4}He) if their energy is greater than the threshold of the photo-dissociation reactions. We consider the photo-dissociation reactions listed in Table II. For the process $D(\gamma, n)p$, we use the cross section in analytic form which is given in Ref. 21), and the cross sections for other reactions are taken from the experimental data (for references, see Table II). We neglect $^4He(\gamma, D)D$ and $^4He(\gamma, 2p\,2n)$ since their cross sections are small compared with the other reactions. Furthermore, we do not include the photo-dissociation processes for ^{7}Li and ^{7}Be because the cross section data for ^{7}Be is not available and hence we cannot predict the abundance of ^{7}Li a part of which comes from ^{7}Be.

The time evolution of the light elements is described by

$$\frac{dn_D}{dt}=-n_D\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{D\to a}(\epsilon_\gamma)f_\gamma(\epsilon_\gamma)+\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{a\to D}(\epsilon_\gamma)n_af_\gamma(\epsilon_\gamma)\,, \tag{22}$$

$$\frac{dn_T}{dt}=-n_T\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{T\to a}(\epsilon_\gamma)f_\gamma(\epsilon_\gamma)+\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{a\to T}(\epsilon_\gamma)n_af_\gamma(\epsilon_\gamma)\,, \tag{23}$$

$$\frac{dn_{^3He}}{dt}=-n_{^3He}\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{^3He\to a}(\epsilon_\gamma)f_\gamma(\epsilon_\gamma)+\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{a\to{}^3He}(\epsilon_\gamma)n_af_\gamma(\epsilon_\gamma)\,, \tag{24}$$

$$\frac{dn_{^4He}}{dt}=-n_{^4He}\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{^4He\to a}(\epsilon_\gamma)f_\gamma(\epsilon_\gamma)+\sum_i\int_{E_i}d\epsilon_\gamma\sigma^i_{a\to{}^4He}(\epsilon_\gamma)n_af_\gamma(\epsilon_\gamma)\,, \tag{25}$$

where $\sigma^i_{a\to b}$ is the cross section of the photo-dissociation process i: $a+\gamma\to b+\cdots$ and E_i is the threshold energy of reaction i. When the energy of the high energy photon is relatively low, i.e. $2\,\mathrm{MeV} \lesssim \epsilon_\gamma \lesssim 20\,\mathrm{MeV}$ the D, T and ^{3}He are destroyed and their aboundances decrease. On the other hand, if the photons have high energy enough to destroy ^{4}He, it seems that such high energy photons only decrease the abundance of all light elements. However, since D, T and ^{3}He are produced by the photo-dissociation of ^{4}He whose abundance is much higher than the other elements, their abundances can increase or decrease depending on the number density of high energy photon. When the number density of high energy photons with energy greater than $\sim 20\,\mathrm{MeV}$ is extremely high, all light elements are destroyed. But as the photon density becomes lower, there is some range of the high energy photon density at which the overproduction of D, T and ^{3}He becomes significant. And if the density is sufficiently low, the high energy photon does not affect the BBN at all.

From various observations, the primordial abundances of light elements are estimated[20] as

$$0.22 < Y_p \equiv \left(\frac{\rho_{^4\mathrm{He}}}{\rho_B}\right)_p < 0.24\,, \tag{26}$$

$$\left(\frac{n_\mathrm{D}}{n_\mathrm{H}}\right)_p > 1.8\times 10^{-5}\,, \tag{27}$$

$$\left(\frac{n_\mathrm{D}+n_{^3\mathrm{He}}}{n_\mathrm{H}}\right)_p < 1.0\times 10^{-4}\,, \tag{28}$$

where $\rho_{^4\mathrm{He}}$ and ρ_B are the mass densities of ^{4}He and baryon. The abundances of light elements modified by gravitino decay must satisfy the observational constraints above. In order to make precise predictions for the abundances of light elements, the evolutional equations (22)$\sim$(25) should be incorporated with the nuclear network calculation of BBN. Therefore, we have modified Kawano's computer code[28] to include the photo-dissociation processes.

From the above arguments it is clear that there are at least three free parameters, i.e. mass of gravitino $m_{3/2}$, reheating temperature T_R and the baryon-to-photon ratio η_B. Furthermore we also study the case in which gravitino has other decay channels. In the present paper we do not specify other decay channel. Instead, we introduce another free parameter B_γ which is the branching ratio for the channel $\psi_\mu \to \gamma + \tilde{\gamma}$. Therefore we must study the effect of gravitino decay on BBN in four dimensional parameter space. However in the next section it will be shown that the baryon-to-photon ratio η_B is not an important parameter in the present calculation because the allowed value for η_B is almost the same as that in the standard case (i.e. without gravitino).

§ 6. Results

A. $B_\gamma = 1$ case

First we investigate the photo-dissociation effect when all gravitinos decay into photons and photinos ($B_\gamma = 1$). We take the range of three free parameters as $10\,\mathrm{GeV} \leq m_{3/2} \leq 10\,\mathrm{TeV}$, $10^5\,\mathrm{GeV} \leq T_R \leq 10^{13}\,\mathrm{GeV}$ and $10^{-10} \leq \eta_B \leq 10^{-9}$. In this calculation, we assume that the photino is massless. The contours for the critical abundances of the

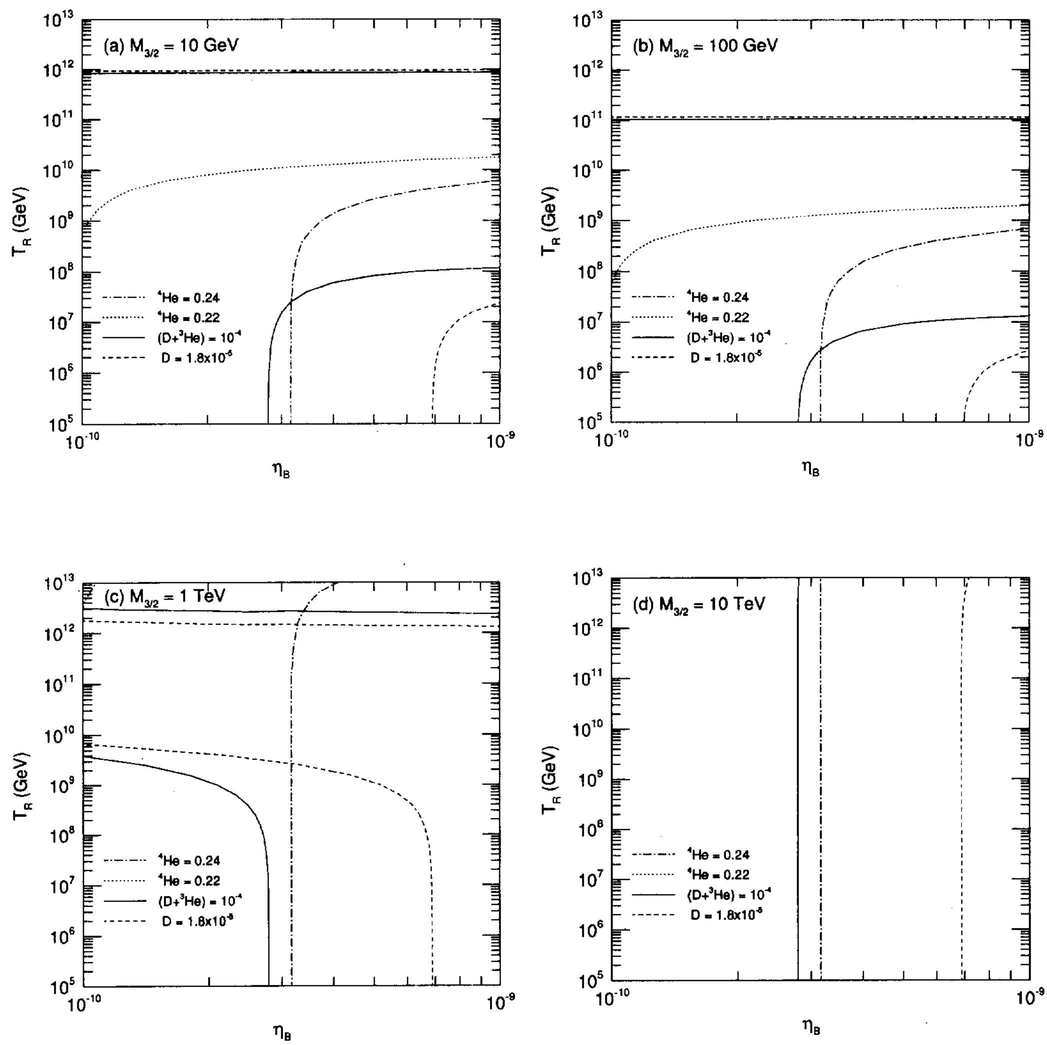
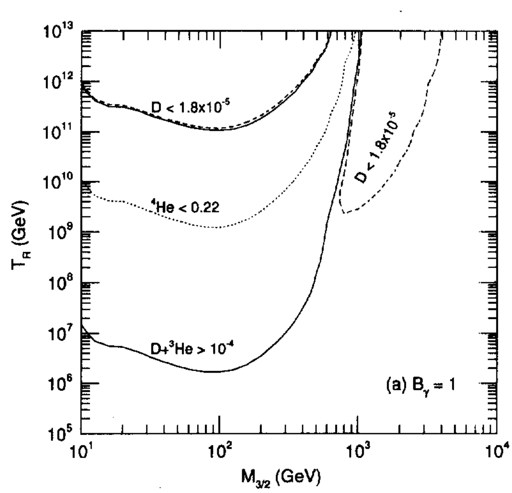
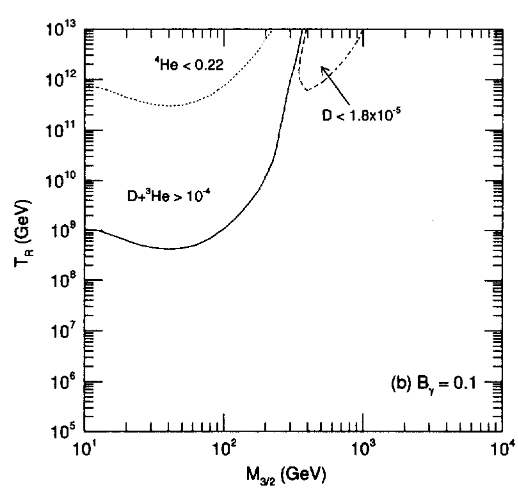


Fig. 3. Contours for critical abundance of light elements in the η_B-T_R plane for (a) $m_{3/2}$=10 GeV, (b) $m_{3/2}$=100 GeV, (c) $m_{3/2}$=1 TeV and (d) $m_{3/2}$=10 TeV.

light elements D, (D+^{3}He) and ^{4}He in the η_B-T_R plane are shown in Fig. 3 for (a) $m_{3/2}$ =10 GeV, (b) 100 GeV, (c) 1 TeV and (d) 10 TeV, respectively. For low reheating temperature ($T_R \lesssim 10^6$ GeV), the number density of the gravitino is very low and hence the number density of the induced high energy photons is too low to affect the BBN. Therefore the resultant abundances of light elements are the same as those in the standard BBN. The effect of the photo-dissociation due to gravitino decay becomes significant as the reheating temperature increases.

As seen in Figs. 3, the allowed range of the baryon-to-photon ratio is almost the same as that without gravitino for $m_{3/2} \lesssim 1$ TeV, i.e., very narrow range around $\eta_B \sim 3\times10^{-10}$ is allowed. However for $m_{3/2} \sim 1$ TeV and $T_R \sim 10^9$ GeV or $m_{3/2} \sim 1$ TeV and $T_R \sim 10^{12}$ GeV, lower values of η_B are allowed (Fig. 3(c)). In this case, the critical photon energy ($\sim m_e^2/22T$) for double photon pair creation process is lower than the threshold of photo-dissociation reaction of ^{4}He at the decay time of the gravitino. Therefore, $T_R \lesssim 10^{12}$ GeV, the aboundance of ^{4}He is not affected by the gravitino decay. Then the emitted photons only destroy ^{3}He and D whose abundances would be larger than the observational constraints for low baryon density if gravitino did not exist. Therefore one sees the narrow allowed band at $T_R = 10^9$ GeV where only a small number of ^{3}He and D are destroyed to satisfy the constraints (27) and (28). For $T_R \gtrsim 10^{12}$ GeV, since a large number of high energy photons are produced even above the threshold of double photon pair creation, a part of ^{4}He are destroyed to produce ^{3}He and D, which leads to the very narrow allowed region at $T_R \sim 10^{12}$ GeV. However even in this special case, the upper limit of reheating temperature changes





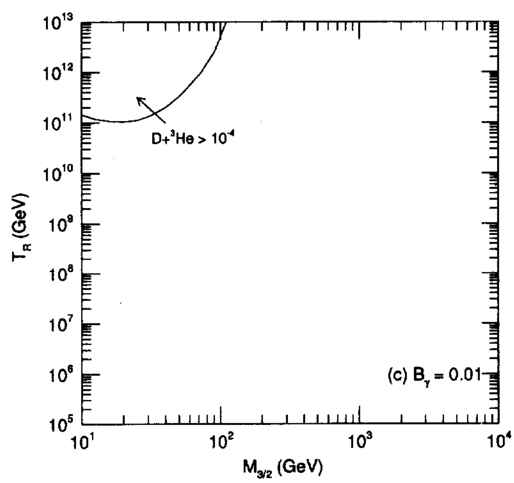


Fig. 4. Allowed regions in the $m_{3/2}$-T_R plane for (a) $B_\gamma=1$, (b) $B_\gamma=0.1$ and (c) $B_\gamma=0.01$. In the region above the solid curve ^{3}He and D are overproduced, the abundance of ^{4}He is less than 0.22 above the dotted curve and the abundance of D is less than 1.8×10^{-5} above the dashed curve.

very little between $\eta_B=10^{-10}$ and $\eta_B\sim 3\times 10^{-10}$. This allows us to fix $\eta_B=3.0\times 10^{-10}$ in deriving the upperbound on the reheating temperature.

The allowed regions that satisfy the observational constraints (26)~(28) are also shown in Figs. 4 in the $m_{3/2}$-T_R plane for $\eta_B=3\times 10^{-10}$. In Figs. 3 and 4(a) one can see four typical cases depending on T_R and $m_{3/2}$.

- $m_{3/2}\lesssim 1$ TeV, $T_R\lesssim 10^{11}$ GeV:
  In this case the lifetime of the gravitino is so long that the critical energy for the double photon process ($\sim m_e^2/22T$) at the decay time of gravitino is higher than the threshold of the photo-dissociation reactions for ^{4}He. Thus ^{4}He is destroyed to produce T, ^{3}He and D. (Since T becomes ^{3}He by β-decay, hereafter we mean T and ^{3}He by the word "^{3}He".) Since the reheating temperature is not so high, the number density of gravitino is not high enough to destroy all the light elements completely. As a result, ^{3}He and D are produced too much and the abundance of ^{4}He decreases. To avoid the overproduction of ^{3}He and D, the reheating temperature should be less than $\sim(10^6-10^9)$ GeV.
- $m_{3/2}\lesssim 1$ TeV, $T_R\gtrsim 10^{11}$ GeV:
  The lifetime is long enough to destroy ^{4}He and the gravitino abundance is very large since the reheating temperature is extremely high. As the result, all the light elements are destroyed. This parameter region is strongly excluded by the observation.
- 1 TeV $\lesssim m_{3/2}\lesssim 3$ TeV:
  The lifetime becomes shorter as the mass of gravitino increases, and the decay occurs when the double photon pair creation process works well. If the cosmic temperature at $t=\tau_{3/2}$ is greater than $\sim m_e^2/22E_{^4\mathrm{He}}$ (where $E_{^4\mathrm{He}}\sim 20$ MeV represents the typical threshold energy of ^{4}He destruction processes), ^{4}He abundance is almost unaffected by the high energy photons as can be seen in Fig. 3(c). In this parameter region, the overproduction of (D+^{3}He) cannot occur since ^{4}He is not destroyed. In this case, the destruction of D is the most important to set the limit of the reheating temperature. This gives the constraint of $T_R\lesssim(10^9-10^{12})$ GeV.
- $m_{3/2}\gtrsim 3$ TeV:
  In this case the decay occurs so early that all high energy photons are quickly thermalized by the double photon process before they destroy the light elements. Therefore the effect on BBN is negligible. Figure 3(d) is an example of this case. The resultant contours for abundances of light elements are almost identical to those without the decay of gravitino.

B. *$B_\gamma<1$ case*

So far we have assumed that all gravitinos decay into photons and photinos. But if other superpartners are lighter than the gravitino, the decay channels of gravitino increases and the branching ratio for the channel $\psi_\mu\to\gamma+\tilde{\gamma}$ becomes less than 1. In this case, various decay products affect the evolution of the universe and BBN. In this paper, instead of studying all decay channels, we consider only the $\gamma+\tilde{\gamma}$ channel with taking the branching ratio B_γ as another free parameter. With this simplification, the effect of all possible decay products other than photon is not taken into account. Therefore the resultant constraints on the reheating temperature and the mass of gravitino should be taken as the conservative constraints since other decay products may destroy more light elements and make the constraints more stringent.

Although we have four free parameters in the present case, the result for $B_\gamma=1$ implies that the allowed range of T_R and $m_{3/2}$ is obtained if we take the baryon-to-photon ratio to be 3×10^{-10}. Since our main concern is to set the constraints on T_R and $m_{3/2}$, we can safely fix η_B $(=3\times10^{-10})$.

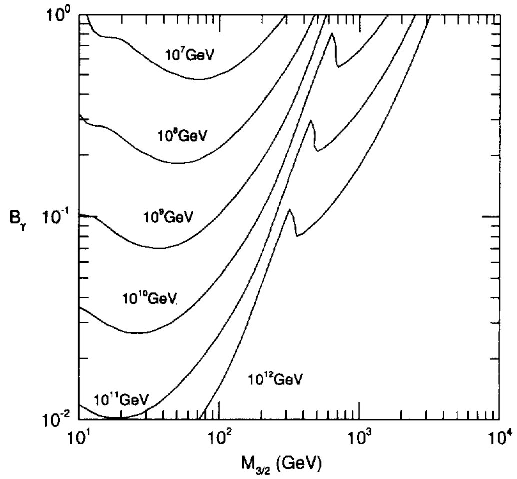


Fig. 5. Contours for the upper limits of the reheating temperature in the $m_{3/2}$-B_γ plane. The numbers in the figure denote the limit of the reheating temperature.

The constraints for $B_\gamma=0.1$ and $B_\gamma=0.01$ is shown in Figs. 4(b) and (c) which should be compared with Fig. 4(a) ($B_\gamma=1$ case). Since the number density of the high energy photons is proportional to B_γ, the constraint on the reheating temperature becomes less stringent as B_γ decreases. In addition, the total lifetime of gravitino is given by

$$\tau_{3/2}=\tau_{3/2}(\psi_\mu\rightarrow\gamma+\tilde{\gamma})\times B_\gamma\,. \tag{29}$$

Thus the gravitinos decay earlier than that for $B_\gamma=1$ case and the constraints from (^{3}He +D) overproduction become less stringent. This effect can be seen in Fig. 4(b), where the constraint due to the overproduction of (^{3}He+D) has a cut at $m_{3/2}\simeq400$ GeV compared with ~1 TeV for $B_\gamma=1$.

In Fig. 5, the contours for the upperbound on reheating temperature are shown in the $m_{3/2}$-B_γ plane. One can see that the stringent constraint on T_R is imposed for $m_{3/2}\lesssim100$ GeV even if the branching ratio is small. As mentioned before this constraint should be regarded as the conservative one and the actual constraint may become more stringent by the effect of other dacay products, which will be investigated elsewhere.

§ 7. Other constraints

In the previous section, we have considered the constraints from the photo-dissociation of light elements. But as we have seen, if the mass of gravitino is larger than a few TeV, gravitino decay does not induce light element photo-dissociation and no constraints have been obtained. In the case of such a large gravitino mass, we must consider other effects of gravitino.

If we consider the present mass density of photinos produced by the gravitino decay, we can get the upperbound on the reheating temperature. In SUSY models with R-invariance (which is the usual assumption), the lightest superparticle (in our case, photino) is stable. Thus the photinos produced by the decay of gravitinos survive until today, and they contribute to the energy density of the present universe. Since one gravitino produces one photino, we can get the present number density of the photino:

$$n_{\tilde{\gamma}}=Y_{3/2}(T\ll 1\,\mathrm{MeV})\times\frac{\zeta(3)}{\pi^2}T_0^3\,, \tag{30}$$

where T_0 is the present temperature of the universe. The density parameter of the photino $\Omega_{\tilde{\gamma}}\equiv m_{\tilde{\gamma}}n_{\tilde{\gamma}}/\rho_c$ can be easily calculated, where $m_{\tilde{\gamma}}$ is the photino mass, $\rho_c\simeq 8.1\times 10^{-47}h^2\,\mathrm{GeV}^4$ is the critical density of the universe and h is the Hubble parameter in units of 100 km /sec/Mpc. If we constrain that $\Omega_{\tilde{\gamma}}\le 1$ in order not to overclose the universe, the upperbound on the reheating temperature is given by

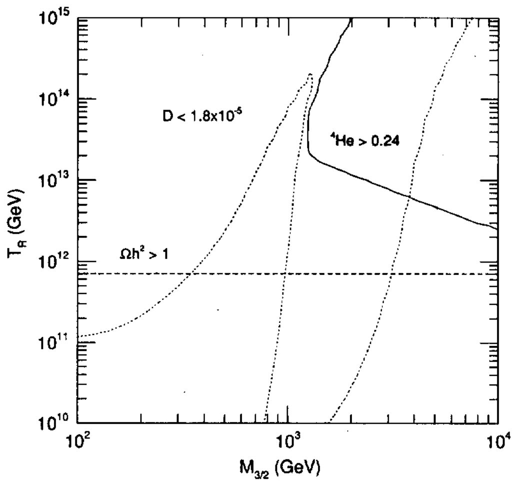


Fig. 6. Upperbound on the reheating temperature. Dashed line represents the constraint from the present mass density of photino. Solid curve represents the upperbound requiring ^{4}He<0.24. Constraints from D photo-dissociation are also shown by dotted line.

$$T_R\le 2.7\times 10^{11}\left(\frac{m_{\tilde{\gamma}}}{100\,\mathrm{GeV}}\right)^{-1}h^2\,\mathrm{GeV}\,, \tag{31}$$

where we have ignored the logarithmic correction term of Σ_{tot}. To set the upperbound on the reheating temperature, we need to know the mass of photino. If one assumes the gaugino-mass unification condition, the lower limit on the mass of photino is 18.4 GeV.[29] Then we can get the following upperbound on the reheating temperature:

$$T_R\le 1.5\times 10^{12}h^2\,\mathrm{GeV}\,. \tag{32}$$

Note that this bound is independent of the gravitino mass and branching ratio.

Another important constraint comes from the effect on the cosmic expansion at BBN. As mentioned before, if the density of gravitino at nucleosynthesis epochs becomes high, the expansion of the universe increases, which leads to more abundance of ^{4}He. We study this effect by using modified Kawano's code and show the result in Fig. 6. In the calculation, we take $\eta_B=2.8\times 10^{-10}$ and $\tau_n=887$ sec (where $\tau_n=(889\pm 2.1)$ sec is the neutron lifetime[34]) so that the predicted ^{4}He abundance is minimized without conflicting the observational constraints for other light elements. The resultant upperbound on the reheating temperature is given by

$$T_R\lesssim 2\times 10^{13}\,\mathrm{GeV}\left(\frac{m_{3/2}}{1\,\mathrm{TeV}}\right)^{-1}, \tag{33}$$

for $m_{3/2}>1$ TeV.

§ 8. Discussion

We have investigated the production and the decay of gravitino, in particular, the effect on BBN by high energy photons produced in the decay. We have found that the stringent constraints on reheating temperature and mass of gravitino.

Let us compare our result with those in other literatures. Our number density of gravitino produced in the reheating epochs of the inflationary universe is about four times larger than that in Ellis et al.[11] Since Ellis et al.[11] note that they have neglected the interaction terms between gravitino and chiral multiplets (which is the second term in Eq. (2)), they might underestimate the total cross section for the production of gravitino. All previous works concerning gravitino problem were based on the gravitino number density given by Ellis et al.[11] Therefore our constraints are more stringent than others. In addition, we include all standard nuclear reactions as well as photo-dissociation processes in our calculation. Therefore the production of (D+^{3}He) contains both contributions from standard BBN and photo-dissociation of ^{4}He. Since only the photo-production is taken into account in Ref. 11), our constraint from (D+^{3}He) overproduction is more stringent.

Furthermore, our photon spectrum is different from that in Ref. 16) as shown in Fig. 2.*) The spectrum adopted by Ref. 16) has more power to destroy light elements above the threshold for the photon-photon scattering and less power below the threshold. In Refs. 18) and 19), Compton scattering process is not taken into account in calculating the photon spectrum which Ellis et al.[16] used to derive a fitting formula for the high energy photon spectrum. Therefore, it is expected that the difference comes mainly from the neglect of Compton scattering off thermal electron, which is the most dominant process for the relatively low energy photons ($\epsilon_\gamma \lesssim m_e^2/80T^2$). Our spectrum is also different from that in Ref. 14), i.e. our spectrum has larger amplitude especially for heavy gravitino case, and hence our constraint on T_R seems to be more stringent.**) The method taken in Ref. 14) is full numerical integration (over both time and momentum) of the complicated Boltzmann equations and need many time steps to get the final spectrum since the typical interaction time is much smaller than the cosmic time. Therefore, it may easily contain cumulative numerical errors. Since we obtain the steady state solution at each epoch, there are no cumulative errors in the present spectrum. Therefore we believe that the spectrum that we have obtained is more precise than those used in other works.

In summary, we have investigated the photo-dissociation processes of light elements due to the high energy photons emitted in the decay of gravitino and set the upperbound on the reheating temperature by using precise production rate of gravitino and the spectrum of high energy photon. Together with other constraints (the present mass density of photino and the enhancement of cosmic expansion due to

*) However, the upperbound on T_R does not change significantly even if we use the spectrum given in Ref. 16).

**) It should be noted that the constraints given in Ref. 14) are obtained by using a simple approximation and data different from (26)~(28).

gravitino) we have obtained the following constraint,

$$T_R \lesssim 10^{6-7}\,\mathrm{GeV}\,,\quad m_{3/2} \lesssim 100\,\mathrm{GeV}\,, \tag{34}$$

$$T_R \lesssim 10^{7-9}\,\mathrm{GeV}\,,\quad 100\,\mathrm{GeV} \lesssim m_{3/2} \lesssim 1\,\mathrm{TeV}\,, \tag{35}$$

$$T_R \lesssim 10^{9-12}\,\mathrm{GeV}\,,\quad 1\,\mathrm{TeV} \lesssim m_{3/2} \lesssim 3\,\mathrm{TeV}\,, \tag{36}$$

$$T_R \lesssim 10^{12}\,\mathrm{GeV}\,,\quad 3\,\mathrm{TeV} \lesssim m_{3/2} \lesssim 10\,\mathrm{TeV}\,. \tag{37}$$

This provides a severe constraint in building the inflation models based on supergravity. In this paper we have also studied the gravitino which decays into other channels by taking the branching ratio as a free parameter. Although this gives conservative upperbound on the reheating temperature, the precise constraints cannot be obtained unless various processes induced by other decay products are fully taken into account. This will be done in the future work.[37]

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Appendix A
—— Boltzmann Equation ——

In order to calculate the high energy photon spectrum, we must estimate the cascade processes induced by the radiative decay of the gravitinos. In our calculation, we have taken the following processes into account; (I) double photon pair creation, (II) photon-photon scattering, (III) pair creation in nuclei, (IV) Compton scattering off thermal electron, (V) inverse Compton scattering off background photon, and (VI) radiative decay of the gravitinos. The Boltzmann equations for this cascade processes are given by

$$\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t} = \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{DP}} + \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{PP}} + \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{PC}} + \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{CS}} + \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{IC}} + \left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{DE}}, \tag{A1}$$

$$\frac{\partial f_e(E_e)}{\partial t} = \left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\mathrm{DP}} + \left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\mathrm{PC}} + \left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\mathrm{CS}} + \left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\mathrm{IC}}. \tag{A2}$$

Below, we see contributions from each process in detail.

(I) Double photon pair creation $[\gamma+\gamma\to e^+ + e^-]$

For the high energy photon whose energy is larger than $\sim m_e^2/22T$, double photon pair creation is the most dominant process.

The total cross section for the double photon pair creation process σ_{DP} is given by

$$\sigma_{\mathrm{DP}}(\beta)=\frac{1}{2}\pi r_e^2(1-\beta^2)\left\{(3-\beta^4)\log\frac{1+\beta}{1-\beta}-2\beta(2-\beta^2)\right\}, \tag{A3}$$

where $r_e=\alpha/m_e$ is the classical radius of electron and β is the electron (or positron) velocity in the center-of-mass frame. Using this formula, one can write down $(\partial f_\gamma/\partial t)|_{\mathrm{DP}}$ as

$$\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\mathrm{DP}}=-\frac{1}{8}\frac{1}{\epsilon_\gamma^2}f_\gamma(\epsilon_\gamma)\int_{m_e/\epsilon_\gamma}^{\infty}d\bar{\epsilon}_\gamma\frac{1}{\bar{\epsilon}_\gamma^2}\bar{f}_\gamma(\bar{\epsilon}_\gamma)\int_{4m_e^2}^{4\epsilon_\gamma\bar{\epsilon}_\gamma}ds\, s\sigma(\beta)\Big|_{\beta=\sqrt{1-(4m_e^2/s)}}. \tag{A4}$$

The spectrum of the final state electron and positron is obtained in Ref. 30), and $(\partial f_e/\partial t)|_{\mathrm{DP}}$ is given by

$$\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\mathrm{DP}}=\frac{1}{4}\pi r_e^2 m_e^4\int_{E_e}^{\infty}d\epsilon_\gamma\frac{f_\gamma(\epsilon_\gamma)}{\epsilon_\gamma^3}\int_0^{\infty}d\bar{\epsilon}_\gamma\frac{\bar{f}_\gamma(\bar{\epsilon}_\gamma)}{\bar{\epsilon}_\gamma^2}G(E_e,\epsilon_\gamma,\bar{\epsilon}_\gamma), \tag{A5}$$

where $\bar{f}_\gamma$ represents the distribution function of the background photon at temperature T,

$$\bar{f}_\gamma(\bar{\epsilon}_\gamma)=\frac{\bar{\epsilon}_\gamma^2}{\pi^2}\times\frac{1}{\exp(\bar{\epsilon}_\gamma/T)-1}, \tag{A6}$$

and function $G(E_e,\epsilon_\gamma,\bar{\epsilon}_\gamma)$ is given by

$$\begin{aligned}G(E_e,\epsilon_\gamma,\bar{\epsilon}_\gamma)=&\frac{4(E_e+E_e')^2}{E_eE_e'}\log\frac{4\bar{\epsilon}_\gamma E_eE_e'}{m_e^2(E_e+E_e')}-8\frac{\bar{\epsilon}_\gamma\epsilon_\gamma}{m_e^2}\\&+\frac{2\{2\bar{\epsilon}_\gamma(E_e+E_e')-m_e^2\}(E_e+E_e')^2}{m_e^2E_eE_e'}\\&-\left\{1-\frac{m_e^2}{\bar{\epsilon}_\gamma(E_e+E_e')}\right\}\frac{(E_e+E_e')^4}{E_e^2E_e'^2},\end{aligned} \tag{A7}$$

with

$$E_e'=\epsilon_\gamma+\bar{\epsilon}_\gamma-E_e.$$

(II) Photon-photon scattering $[\gamma+\gamma\to\gamma+\gamma]$

If the photon energy is below the effective threshold of the double photon pair creation, photon-photon scattering process becomes significant. This process is analyzed in Ref. 19) and for $\epsilon_\gamma'\lesssim O(m_e^2/T)$, $(\partial f_\gamma/\partial t)|_{\mathrm{PP}}$ is given by

$$\begin{aligned}\left.\frac{\partial f_\gamma(\epsilon_\gamma')}{\partial t}\right|_{\mathrm{PP}}=&\frac{35584}{10125\pi}\alpha^2r_e^2m_e^{-6}\int_{\epsilon_\gamma'}^{\infty}d\epsilon_\gamma f_\gamma(\epsilon_\gamma)\epsilon_\gamma^2\left\{1-\frac{\epsilon_\gamma'}{\epsilon_\gamma}+\left(\frac{\epsilon_\gamma'}{\epsilon_\gamma}\right)^2\right\}^2\int_0^{\infty}d\bar{\epsilon}_\gamma\bar{\epsilon}_\gamma^3\bar{f}_\gamma(\bar{\epsilon}_\gamma)\\&-\frac{1946}{50625\pi}f_\gamma(\epsilon_\gamma')\alpha^2r_e^2m_e^{-6}\epsilon_\gamma'^3\int_0^{\infty}d\bar{\epsilon}_\gamma\bar{\epsilon}_\gamma^3\bar{f}_\gamma(\bar{\epsilon}_\gamma).\end{aligned} \tag{A8}$$

For a larger value of ϵ_γ', we cannot use this formula. But in this energy region, photon-photon scattering is not significant because double photon pair creation determines the shape of the photon spectrum. Therefore, instead of using the exact formula, we take m_e^2/T as a cutoff scale of $(\partial f_\gamma/\partial t)|_{\mathrm{PP}}$, i.e., for $\epsilon_\gamma'\le m_e^2/T$ we use Eq. (A8) and for $\epsilon_\gamma'>m_e^2/T$ we take

$$\left.\frac{\partial f_\gamma(\epsilon'_\gamma > m_e^2/T)}{\partial t}\right|_{\rm PP} = 0\,. \tag{A9}$$

Note that we have checked the cutoff dependence of spectra is negligible.

(III) Pair creation in nuclei $[\gamma + N \to e^+ + e^- + N]$

Scattering off the electric field around nucleon, the high energy photon can produce electron positron pair if the photon energy is larger than $2m_e$. Denoting total cross section of this process $\sigma_{\rm PC}$, $(\partial f_\gamma/\partial t)|_{\rm NP}$ is given by

$$\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm NP} = -n_N \sigma_{\rm PC} f_\gamma(\epsilon_\gamma)\,, \tag{A10}$$

where n_N is the nucleon number density. For $\sigma_{\rm PC}$, we use the approximate formula derived by Maximon.[31]

Differential cross section for this process $d\sigma_{\rm PC}/dE_e$ is given in Ref. 32), and $(\partial f_e/\partial t)|_{\rm NP}$ is given by

$$\left.\frac{\partial f_e(E_e)}{\partial t}\right|_{\rm NP} = n_N \int_{E_e+m_e}^{\infty} d\epsilon_\gamma \frac{d\sigma_{\rm PC}}{dE_e} f_\gamma(\epsilon_\gamma)\,. \tag{A11}$$

(IV) Compton scattering $[\gamma + e^- \to \gamma + e^-]$

Compton scattering is one of the processes by which high energy photons lose their energy. Since the photo-dissociation of light elements occurs when the temperature drops below ~ 0.1 MeV, we can consider the thermal electrons to be almost at rest. Using the total and differential cross section at the electron rest frame $\sigma_{\rm CS}$ and $d\sigma_{\rm CS}/dE_e$, one can derive

$$\left.\frac{\partial f_\gamma(\epsilon'_\gamma)}{\partial t}\right|_{\rm CS} = \bar{n}_e \int_{\epsilon'_\gamma}^{\infty} d\epsilon_\gamma f_\gamma(\epsilon_\gamma) \frac{d\sigma_{\rm CS}(\epsilon'_\gamma, \epsilon_\gamma)}{d\epsilon'_\gamma} - \bar{n}_e \sigma_{\rm CS} f_\gamma(\epsilon'_\gamma)\,, \tag{A12}$$

$$\left.\frac{\partial f_e(E'_e)}{\partial t}\right|_{\rm CS} = \bar{n}_e \int_{E'_e}^{\infty} d\epsilon_\gamma f_\gamma(\epsilon_\gamma) \frac{d\sigma_{\rm CS}(\epsilon_\gamma + m_e - E'_e, \epsilon_\gamma)}{d\epsilon'_\gamma}\,, \tag{A13}$$

where $\bar{n}_e$ is the number density of thermal electron.

(V) Inverse compton scattering $[e^\pm + \gamma \to e^\pm + \gamma]$

Contribution from the inverse Compton process is given by Jones,[33] and $(\partial f/\partial t)|_{\rm IC}$ is given by

$$\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm IC} = 2\pi r_e^2 m_e^2 \int_{\epsilon_\gamma+m_e}^{\infty} dE_e \frac{2 f_e(E_e)}{E_e^2} \int_0^{\infty} d\bar{\epsilon}_\gamma \frac{\bar{f}(\bar{\epsilon}_\gamma)}{\bar{\epsilon}_\gamma} F(\epsilon_\gamma, E_e, \bar{\epsilon}_\gamma)\,, \tag{A14}$$

$$\begin{aligned}\left.\frac{\partial f_e(E'_e)}{\partial t}\right|_{\rm IC} =& 2\pi r_e^2 m_e^2 \int_{E'_e}^{\infty} dE_e \frac{f_e(E_e)}{E_e^2} \int_0^{\infty} d\bar{\epsilon}_\gamma \frac{\bar{f}_\gamma(\bar{\epsilon}_\gamma)}{\bar{\epsilon}_\gamma} F(E_e + \bar{\epsilon}_\gamma - E'_e, E_e, \bar{\epsilon}_\gamma) \\ & - 2\pi r_e^2 m_e^2 \frac{f_e(E'_e)}{E'^2_e} \int_{E'_e}^{\infty} d\epsilon_\gamma \int_0^{\infty} d\bar{\epsilon}_\gamma \frac{\bar{f}_\gamma(\bar{\epsilon}_\gamma)}{\bar{\epsilon}_\gamma} F(\epsilon_\gamma, E'_e, \bar{\epsilon}_\gamma)\,,\end{aligned} \tag{A15}$$

where function $F(\epsilon_\gamma, E_e, \bar{\epsilon}_\gamma)$ is given by

$$F(\epsilon_\gamma, E_e, \bar{\epsilon}_\gamma)=\begin{cases} 2q\log q+(1+2q)(1-q)+\dfrac{(\Gamma_\epsilon q)^2}{2(1-\Gamma_\epsilon q)}(1-q): & \text{for} \quad 0\le q\le 1\,, \\ 0: \quad \text{otherwise} & \end{cases} \tag{A16}$$

with

$$\Gamma_\epsilon=\frac{4\,\bar{\epsilon}_\gamma E_e}{m_e^2}\,, \quad q=\frac{\epsilon_\gamma}{\Gamma_\epsilon(E_e-\epsilon_\gamma)}\,.$$

(VI) Gravitino radiative decay [$\psi_\mu \to \gamma+\tilde{\gamma}$]

Source of the non-thermal photon and electron spectra is radiative decay of gravitino. Since gravitinos are almost at rest when they decay and we only consider two body decay process, incoming high energy photons have fixed energy $\epsilon_{\gamma 0}$, which is given by

$$\epsilon_{\gamma 0}=\frac{m_{3/2}^2-m_{\tilde{\gamma}}{}^2}{2m_{3/2}}\,. \tag{A17}$$

Therefore, $(\partial f_\gamma/\partial t)|_{\rm DE}$ can be written as

$$\left.\frac{\partial f_\gamma(\epsilon_\gamma)}{\partial t}\right|_{\rm DE}=\delta(\epsilon_\gamma-\epsilon_{\gamma 0})\frac{n_{3/2}}{\tau_{3/2}}\,. \tag{A18}$$

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