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Gravity and Electromagnetism as Collective Phenomena of Fermion-Antifermion Pairs

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A generally covariant formulation is made for the previously proposed unified model of the Nambu-Jona-Lasinio type for gravity and electromagnetism. The gravitational and electromagnetic fields are generated as collective excitations of fermion-antifermion pairs. The model is shown to be effectively equivalent to the Einstein-Weyl theory of general relativity. The G-α relation, the relation between the fine structure constant and the Newtonian gravitational constant is re-derived.

§ 1. Introduction

In 1963, Bjorken proposed a model of Nambu-Jona-Lasinio[1] type for electromagnetism, in which photons are collective excitations of fermion-antifermion pairs generated via four Fermi interactions.[2] Phillips attempted to extend this idea to gravity.[3] Recently we proposed a unified model of this type for all elementary particle forces including gravity.[4],[5] In the model, leptons and quarks are the only fundamental particles, while the intermediate bosons such as photons, weak vector bosons, Higgs scalars and gravitons are generated as collective excitation modes of the fundamental-fermion-antifermion pairs. The compositeness conditions lead to many interesting relations between various coupling constants and among various masses. Especially in unifying gravity and electromagnetism in this picture, we obtained a simple relation between the fine structure and gravitational constants which was first conjectured by Landau in 1955, historically.[6]

There are two important questions left unanswered in the gravitational sector of our model.[5] The first one is whether our model is equivalent to Einstein's gravity and the other is whether our model is consistent with the presently existing data on gravity. In this paper, we shall answer these questions both affirmatively. To this end, we recall Sakharov's idea[7] that the action of Einstein's gravitation is identified with that of quantum fluctuations of the vacuum. Adopting his general idea, we shall reformulate the gravitational and electromagnetic sectors of our previous unified model. In § 2, we shall first rewrite the model in a generally covariant way, and then analyze it by adopting the Pauli-Villars regulator method which is invariant under the general coordinate transformation as well as the local Lorentz and gauge transformations.[8] It will be shown that the reformulated model is effectively equivalent to Einstein's theory of gravity. In § 3, we shall include electromagnetism and show that the interaction between photons and gravitons is also governed by the general relativity. Thus, combining the results of §§ 2 and 3, we shall be able to answer the above-mentioned two questions affirmatively. Finally, § 4 is devoted to conclusion and discussions.

§ 2. Model of gravity

We start with the Lagrangian for a fundamental fermion field ψ with the mass m moving on a curved space,[9]

$$L=e^{k\mu}\bar{\psi}\frac{i}{2}(\gamma_k\overrightarrow{D}_\mu-\overleftarrow{D}_\mu\gamma_k)\psi-m\bar{\psi}\psi+C\,, \tag{2·1}$$

where $e^{k\mu}$ is the vierbein and D_μ is the covariant differentiation defined by

$$D_\mu=\partial_\mu-\frac{i}{2}\gamma_{mn\mu}S^{mn}\,, \tag{2·2}$$

$$\gamma_{mn\mu}=\frac{1}{2}(c_{mn\mu}-c_{nm\mu}-c_{\mu mn})\,, \tag{2·3}$$

$$c_{m\mu\nu}=\partial_\mu e_{m\nu}-\partial_\nu e_{m\mu} \tag{2·4}$$

and

$$S^{mn}=\frac{i}{4}[\gamma^m,\gamma^n]. \tag{2·5}$$

The constant C is a counter term which will later partially cancel the quartically divergent cosmological term. The Lagrangian L is not only invariant under the general coordinate transformation (GCT),

$$\delta x^\mu=\xi^\mu \quad\text{and}\quad \delta e^{k\mu}=e^{k\nu}\partial_\nu\xi^\mu\,, \tag{2·6}$$

but under the local Lorentz transformation (LLT),

$$\delta\psi=-\frac{i}{2}\omega_{mn}S^{mn}\psi \quad\text{and}\quad \delta e^{k\mu}=\omega^k{}_l e^{l\mu} \tag{2·7}$$

with

$$\omega_{mn}=-\omega_{nm}\,.$$

Apparently, L has no kinetic term for the vierbein field $e^{k\mu}$. As will be seen immediately, the divergent parts of the fermion loop integrals for quantum corrections play a role of the kinetic term, so that the field $e^{k\mu}$ becomes the "genuine" gravitational field as a collective excitation of fermion-antifermion pairs.

The invariant action on the curved space is given by

$$\int d^4x\sqrt{-g}L\,, \tag{2·8}$$

where $g=\det g_{\mu\nu}$ and $g_{\mu\nu}=e^k{}_\mu e_{k\nu}$. We now define the effective Lagrangian $L_{\rm eff}$ for the field $e^{k\mu}$ by the path integral over the fermion field:

$$\exp\left(i\int d^4x\sqrt{-g}\,L_{\rm eff}\right)=\int(d\psi)\,(d\bar{\psi})\exp\left(i\int d^4x\sqrt{-g}L\right). \tag{2·9}$$

Let us write $\sqrt{-g}L$ in the form

$$\sqrt{-g}L=\bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/}-m+\Gamma\right)\psi+C\sqrt{-g}\,, \tag{2·10}$$

where $\partial\!\!\!/=\eta^{k\mu}\gamma_k\partial_\mu$ with $\eta^{k\mu}=\mathrm{diag}(1,\,-1,\,-1,\,-1)$ and

$$\Gamma=\sqrt{-g}\;e^{k\mu}\frac{i}{2}(\gamma_k\overrightarrow{D}_\mu-\overleftarrow{D}_\mu\gamma_k)-\frac{i}{2}\overleftrightarrow{\partial\!\!\!/}-(\sqrt{-g}-1)\,m\,. \tag{2·11}$$

Performing the path integration formally, we then obtain

$$\int d^4x\sqrt{-g}\,L_{\rm eff}=\int d^4x\,C\sqrt{-g}-i\,\mathrm{Tr}\ln\left(1+\frac{1}{i\partial\!\!\!/-m}\Gamma\right), \tag{2·12}$$

where Tr denotes the trace operation with respect to the space-time points and the γ matrices. The second term in (2·12) corresponds to a series of one-fermion-loop diagrams if it is expanded into a Taylor series in Γ. Since Γ contains one differentiation, all the loop integrals are quartically divergent. We believe, however, that there exists a realistic momentum cutoff at around the Planck mass ($G^{-1/2}\sim10^{19}$ GeV where G is the Newtonian gravitational constant). Cutoff around the Planck mass was introduced by Landau, based on the idea that the effects of gravitational interaction may exceed the electromagnetic effects at such high energy.[6] Later, Isham, Salam and Strathdee demonstrated in a model that gravity realistically regularizes all infinities including its own.[10] Our assumption is, however, different from theirs at the point that, in our picture, all the basic nonlinear fermion interactions are cutoff universally at a certain short length, not due to gravitation. We further assume in this paper that this cutoff is invariant under GCT and LLT. For this reason, we adopt the cutoff by Pauli-Villars regulators, which is invariant under GCT and LLT and which may offer a good approximation to the real cutoff as far as the external momentum is much smaller than the cutoff momentum. We introduce three regulators with the masses M_i $(i=1,2,3)$ and the weight coefficient c_i which must satisfy

$$\sum_{i=1}^{3}c_iM_i^k+m^k=0\ \text{ for }\ k=0,\ 2,\ 4\,. \tag{2·13}$$

Let us first proceed in the weak field approximation, writing the vierbein as

$$e^{k\mu}=\eta^{k\mu}+H^{k\mu}. \qquad (2\cdot14)$$

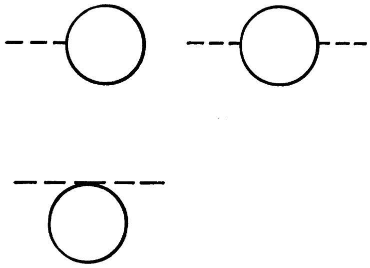

Fig. 1. The diagrams which contribute to $L_{\rm div}$ up to second order $H^{k\mu}$. The solid and dashed lines denote ψ and $H^{k\mu}$, respectively.

For the purpose of this section, $H^{k\mu}$ can be assumed to be symmetric. Then, Γ in Eq. (2·11) is given by

$$\begin{aligned}\Gamma=&H^{k\mu}\left[\frac{i}{2}\gamma_k\overleftrightarrow{\partial}_\mu-\eta_{k\mu}\left(\frac{i}{2}\overleftrightarrow{\not\partial}-m\right)\right]\\&+H^{k\mu}H^{l\nu}\left[-\eta_{k\mu}\frac{i}{2}\gamma_l\overleftrightarrow{\partial}_\nu\right.\\&\left.+\frac{1}{2}(\eta_{k\mu}\eta_{l\nu}+\eta_{k\nu}\eta_{l\mu})\left(\frac{i}{2}\overleftrightarrow{\not\partial}-m\right)\right]\\&+\frac{1}{4}\varepsilon_{mnkl}\gamma_5\gamma^l H^{m\mu}\partial^k H^{n\nu}\eta_{\mu\nu}\\&+O(H^3), \qquad (2\cdot15)\end{aligned}$$

where the differentiation ∂_μ in the first two terms does not operate on $H^{k\mu}$ but on the fermions outside. Up to second order in $H^{k\mu}$, only the three diagrams shown in Fig. 1 contribute to $L_{\rm eff}$. Notice that the spinor-connection term [the third term in (2·15)] does not contribute to $L_{\rm eff}$ to this order. After somewhat lengthy calculations, we find the following expression for the divergent part of $L_{\rm eff}$, which we call $L_{\rm div}$:

$$\begin{aligned}\sqrt{-g}L_{\rm div}=&J_4\left[-H^\lambda{}_\lambda+\frac{1}{2}(H^{\mu\nu})^2+\frac{1}{2}(H^\lambda{}_\lambda)^2\right]\\&+\frac{1}{6}J_2[(\partial_\lambda H^{\mu\nu})^2-2(\partial_\mu H^{\mu\nu})^2+2\partial_\mu H^{\mu\nu}\partial_\nu H^\lambda{}_\lambda-(\partial_\mu H^\lambda{}_\lambda)^2]\\&+\frac{1}{60}J_0[(\Box H^\lambda{}_\lambda)^2-2\Box H^\lambda{}_\lambda\partial_\mu\partial_\nu H^{\mu\nu}-2(\partial_\mu\partial_\nu H^{\mu\nu})^2\\&-3(\Box H^{\mu\nu})^2+6(\partial_\mu\partial_\nu H^{\nu\lambda})^2]+O(H^3), \qquad (2\cdot16)\end{aligned}$$

where $\Box=\eta^{\mu\nu}\partial_\mu\partial_\nu$ and the suffices of $H^{k\mu}$ are raised or lowered by multiplying $\eta^{\mu\nu}$. The J_k's are divergent factors defined by

$$J_k=\frac{1}{(4\pi)^2}\sum_{i=1}^{3}c_iM_i^k\ln(M_i^2/m^2) \quad \text{for} \quad k=0,\ 2,\ 4. \qquad (2\cdot17)$$

The form in the square bracket of the first term in (2·16) coincides with $(\sqrt{-g}-1)$ up to second order in $H^{k\mu}$. Also, the form in the square bracket of the second term in (2·16) is nothing but the weak field approximation of $\sqrt{-g}R$, where R is the scalar curvature defined by

$$R=g^{\mu\nu}R_{\mu\nu}, \qquad (2\cdot18)$$

$$R_{\mu\nu}=\partial_\nu\Gamma^\alpha_{\mu\alpha}-\partial_\alpha\Gamma^\alpha_{\mu\nu}+\Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha}-\Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{\mu\nu} \qquad (2\cdot19)$$

and

$$\Gamma^{\alpha}_{\mu\nu}=\frac{1}{2}g^{\alpha\beta}(\partial_{\mu}g_{\beta\nu}+\partial_{\nu}g_{\beta\mu}-\partial_{\beta}g_{\mu\nu}). \tag{2·20}$$

Furthermore, the form in the square bracket of the third term in (2·16) can be identified with $R^2-3R_{\mu\nu}R^{\mu\nu}$ in the weak field approximation. Disregarding the trivial constant term, we, therefore, obtain up to second order in $H^{k\mu}$

$$\sqrt{-g}L_{\text{div}}=\sqrt{-g}\left[J_4+\frac{1}{6}J_2R+\frac{1}{60}J_0(R^2-3R_{\mu\nu}R^{\mu\nu})\right]. \tag{2·21}$$

The third term in the square bracket in (2·21) is not only practically negligible (smaller by the order of G than the second term) but taken as the divergent part of radiative corrections in the usual quantum gravity.[11] We, therefore, ignore it hereafter (, which means that we do not take it as a large amplitude).

Let us now consider what L_{eff} looks like to all orders in $H^{k\mu}$. Since the original Lagrangian L and the method of momentum cutoff are both invariant under GCT and LLT, L_{eff} is also invariant. Furthermore, since the cutoff momenta M_i are arbitrary, L_{eff} should be invariant separately in each order of divergence. From dimensional analysis, the quartically (quadratically) divergent terms in L_{div} involve no (two) differentiations. It is known that the only GCT and LLT invariant scalar made of the vierbeins with no (two) differentiations for each term is a constant (the scalar curvature R). Therefore, Eq. (2·21) is proved to all orders in $H^{k\mu}$ without any approximation.

We construct the new Lagrangian

$$L''=L+L_{\text{div}}, \tag{2·22}$$

where the vierbein $e^{k\mu}$ aquires the kinetic term. The original Lagrangian is then written as

$$L=L''-L_{\text{div}}, \tag{2·23}$$

where L_{div} becomes the counter term which subtracts divergent parts arising from the fermion loop integration due to L''. Of course, perturbation theory of L'' involves infinite series of divergent loops with internal graviton propagators. This difficulty is not proper to our model but to any theory of quantum gravity.[12] How to avoid these divergences by renormalization is beyond the scope of this paper. Introducing G and λ by

$$\frac{1}{6}J_2=\frac{1}{16\pi G} \tag{2·24}$$

and

$$C+J_4=\lambda, \tag{2·25}$$

we finally obtain

$$L''=e^{k\mu}\bar{\psi}\frac{i}{2}(\gamma_k\overrightarrow{D}_\mu-\overleftarrow{D}_\mu\gamma_k)\psi-m\bar{\psi}\psi+\frac{1}{16\pi G}R+\lambda\,. \tag{2·26}$$

This is precisely the Lagrangian for the theory of general relativity for a fermion field, with the Newtonian gravitational constant G and the cosmological constant λ. In the Lagrangian L'', the vierbein has become the genuine gravitational field. Eq. (2·24) shows that the cutoff momentum is indeed determined to be around the Planck mass.

§ 3. Photon-graviton interactions

Bjorken showed that the photon can be considered as a collective excitation of fermion-antifermion pairs.[2] In the last section, we have shown that the graviton in Einstein's general relativity can also be such a collective mode. In this section, we investigate how such photons and gravitons interact with each other and show that the Lagrangian describing them is again that of general relativity.

Let us start again with the nonlinear Lagrangian $\tilde{L}$ for a fermion ψ with the mass m on a curved space,

$$\tilde{L}=L+f_1\eta^{kl}(\bar{\psi}\gamma_k\psi)(\bar{\psi}\gamma_l\psi) \tag{3·1}$$

where f_1 is a coupling constant. This $\tilde{L}$ is equivalent to the following Lagrangian including the auxiliary field V_μ

$$\tilde{L}'=L+e^{k\mu}V_\mu\bar{\psi}\gamma_k\psi+C_1 g^{\mu\nu}V_\mu V_\nu \quad \text{with} \quad C_1=-\frac{1}{4f_1} \tag{3·2}$$

if the "equation of motion" for V_μ is taken into account. These Lagrangians $\tilde{L}$ and $\tilde{L}'$ are invariant under GCT and LLT. Furthermore, the sum of the first two terms in $\tilde{L}'$ is invariant under the local gauge transformation (GT):

$$\delta V_\mu=\partial_\mu\Lambda \quad \text{and} \quad \delta\psi=i\Lambda\psi\,. \tag{3·3}$$

Define the effective Lagrangian $\tilde{L}_{\text{eff}}$ by

$$\exp\left(i\int d^4x\sqrt{-g}\,\tilde{L}_{\text{eff}}\right)=\int(d\psi)(d\bar{\psi})\exp\left(i\int d^4x\sqrt{-g}\tilde{L}'\right) \tag{3·4}$$

and write $\sqrt{-g}\tilde{L}'$ in the form

$$\sqrt{-g}L'=\bar{\psi}\left(\frac{i}{2}\overleftrightarrow{\partial\!\!\!/}-m+\tilde{\Gamma}\right)\psi+\sqrt{-g}(C+C_1 g^{\mu\nu}V_\mu V_\nu), \tag{3·5}$$

where

$$\tilde{\Gamma}=\Gamma+\sqrt{-g}e^{k\mu}\gamma_k V_\mu\,. \tag{3·6}$$

Performing the path integration, we obtain

$$\int d^4x\sqrt{-g}\,\tilde{L}_{\text{eff}}=\int d^4x\sqrt{-g}(C+C_1 g^{\mu\nu}V_\mu V_\nu)-i\,\mathrm{Tr}\left(1+\frac{1}{i\partial\!\!\!/-m}\tilde{\Gamma}\right). \tag{3·7}$$

The last term corresponds to a series of one-fermion-loop diagrams. Among them, loop diagrams to which an arbitrary number of external $e^{k\mu}$'s and no or two external V_μ's are attached involve divergent integrals. We adopt the same cutoff procedure as in § 2, which is also invariant under GT. We call the divergent part of $\tilde{L}_{\rm eff}$ as $\tilde{L}_{\rm div}$. Obviously, the loops without any external V_μ give the same result to $\tilde{L}_{\rm div}$ as that to $L_{\rm div}$ in § 2. The additional contributions come out of the loop with two external V_μ.

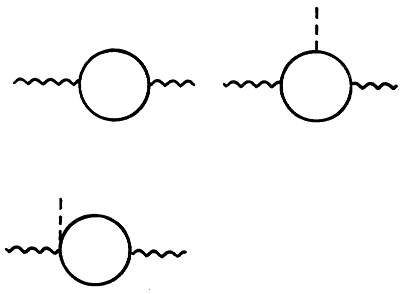

Fig. 2. The diagrams which contribute to the terms with two V_μ's in $\tilde{L}_{\rm div}$ up to first order in $H^{k\mu}$. The solid, wavy and dashed lines denote ψ, V_μ and $H^{k\mu}$, respectively.

In the weak field approximation, $\tilde{\Gamma}$ becomes

$$\tilde{\Gamma}=\eta^{k\mu}V_\mu\gamma_k+H^{k\mu}\left[\frac{i}{2}\gamma_k\overleftrightarrow{\partial}_\mu-\eta_{k\mu}\left(\frac{i}{2}\overleftrightarrow{\partial}-m\right)\right]$$
$$+H^{k\mu}V_\nu[\delta_\mu{}^\nu\gamma_k-\eta_{k\mu}\eta^{l\nu}\gamma_l]+0(H^2). \qquad (3\cdot 8)$$

Up to first order in $H^{k\mu}$, the three diagrams shown in Fig. 2 contribute to the terms with two V_μ's in $\tilde{L}_{\rm div}$. Some lengthy calculation leads to the result

$$\sqrt{-g}\tilde{L}_{\rm div}=\sqrt{-g}L_{\rm div}+\frac{1}{3}J_0\eta^{\mu\rho}\eta^{\nu\sigma}V_{\mu\nu}V_{\rho\sigma}$$
$$+\frac{4}{3}J_0H^{k\lambda}\left[\eta^{\mu\nu}V_{k\mu}V_{\lambda\nu}-\frac{1}{4}\eta_{k\lambda}\eta^{\mu\rho}\eta^{\nu\sigma}V_{\mu\nu}V_{\rho\sigma}\right]+0(H^2), \quad (3\cdot 9)$$

where

$$V_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu\,.$$

Again up to first order in $H^{k\mu}$, this can be written as

$$\sqrt{-g}\tilde{L}_{\rm div}=\sqrt{-g}\left(\tilde{L}_{\rm div}+\frac{1}{3}J_0g^{\mu\nu}g^{\rho\sigma}V_{\mu\rho}V_{\nu\sigma}\right). \qquad (3\cdot 10)$$

The second term in the bracket in (3·10) is the only possible form that is made of two V_μ's and an arbitrary number of $e^{k\mu}$'s and that is invariant under GCT, LLT and GT. Therefore, the expression in (3·10) is correct to all orders in $H^{k\mu}$ without any approximation.

Let us construct the Lagrangian

$$\tilde{L}''=\tilde{L}'+\tilde{L}_{\rm div}\,, \qquad (3\cdot 11)$$

where the auxiliary field V_μ aquires the kinetic term on the curved space, which contains interactions between V_μ and $e^{k\mu}$. If one writes

$$\tilde{L}'=\tilde{L}''-\tilde{L}_{\rm div}\,, \qquad (3\cdot 12)$$

one then finds that the $\tilde{L}_{\rm div}$ becomes the counter term for the divergent fermion loops coming from $\tilde{L}''$. $\tilde{L}''$ is invariant under GT if we require the massless condition $C_1=0$, which implies $f_1=\infty$. Therefore, GT-invariance of $\tilde{L}''$ holds only in the strong coupling limit in the original nonlinear Lagrangian $\tilde{L}$. This situation is somewhat different from that in our previous paper,[4] where the coupling constant f_1 is rather small as $1/J_0$. This is because in the present paper we have adopted the gauge invariant cutoff so that the loop diagrams give no term to cancel the $g^{\mu\nu}V_\mu V_\nu$ term in $\tilde{L}'$. One can, however, show by modifying the Pauli-Villars regulators that this strong coupling limit can be taken smoothly with the conventional massless condition kept tight.

Rescaling the field by

$$V_\mu = eA_\mu \tag{3·13}$$

with

$$e^2 = \frac{3}{-4J_0}, \tag{3·14}$$

we finally obtain

$$\tilde{L}'' = L'' + e\bar{\psi} e^{k\mu}\gamma_k A_\mu \psi - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma}, \tag{3·15}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

This is precisely the Lagrangian for the general relativity of a photon A_μ and a fermion ψ with the electric charge e.

§ 4. Conclusion and discussion

Starting with the Lagrangian for a fundamental fermion field moving on a curved space, we have shown that it implies, as vacuum fluctuation, the Lagrangian for the general relativistic theory of photon and fermion. The gravitational and electromagnetic fields are generated as collective excitation modes of fermion-anti-fermion pairs. We have assumed a realistic and GCT, LLT and GT-invariant momentum cutoff at around the Planck mass, adopting the Pauli-Villars regulators.

The compositeness of the photon and graviton implies interesting relations between the coupling constants and the momentum cutoff. If we take the limit of the equal regulator masses ($M_i \to M$) for simplicity, we obtain from (2·24) and (3·14)

$$G = \frac{12\pi}{M^2} \quad \text{and} \quad \alpha \equiv \frac{e^2}{4\pi} = \frac{3\pi}{\ln(M^2/m^2)}. \tag{4·1}$$

In the more realistic case where there exist a number (the total number N_0) of fundamental fermions (the charge Q_i and the mass m_i ($i=1, \cdots, N_0$)) as in our previous unified model,[5] these equations should be replaced by

$$G=\frac{12\pi}{N_0 M^2} \quad \text{and} \quad \alpha=\frac{3\pi}{\sum_i Q_i^2 \ln(M^2/m_i^2)}. \tag{4·2}$$

Eliminating M^2 from these relations, we again obtain the G-α relation between the fine structure and Newtonian gravitational constants:[6]

$$\alpha=\frac{3\pi}{\sum_i Q_i^2 \ln(12\pi/N_0 G m_i^2)}. \tag{4·3}$$

Comparing this with the previous result[5]

$$\alpha=\frac{3\pi}{\sum_i Q_i^2 \ln(4\pi/\kappa_0 N_0 G m_i^2)}, \tag{4·4}$$

where $\kappa_0=5/9$ and $2/3$ depending on the invariant cutoffs with and without introducing the Feynman integration, respectively, we find that the G-α relation is rather insensitive to cutoff procedures. Different cutoffs may only change the argument of logarithm in the relation.

In conclusion, since our model is effectively equivalent to the Einstein-Weyl theory of general relativity, it is consistent with the presently existing data on gravity.

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