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GRAVITY HARMONICS FROM A RESONANT TWO-HOUR SATELLITE

JAMES P. MURPHY
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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

The semimajor axis of an artificial satellite can undergo long term variations if its mean motion and the rotational speed of the Earth are nearly commensurable. A theoretical expression for this variation is obtained for satellites with a twelve to one such commensurability by considering twelfth order harmonics of degree twelve through fifteen. The TIROS IX satellite is in such a near resonant orbit. From this single satellite, only two pairs of resonant gravity harmonics of twelfth order can be estimated. One pair must be of even degree while the other is of odd degree. The values obtained were:

$$10^7 \times \overline{C}_{14,12} = 1.0 \pm .4$$

$$10^7 \times \overline{S}_{14,12} = 2.1 \pm .4$$

$$10^7 \times \overline{C}_{15,12} = -1.7 \pm .2$$

$$10^7 \times \overline{S}_{15,12} = 2.1 \pm .2$$

These values must be considered to be in some sense composite harmonics since all harmonics whose order is divisible by twelve are nearly resonant. These values will however serve as estimates with widest application to orbit prediction for satellites with orbit characteristics similar to those of TIROS IX.

LIST OF SYMBOLS

$\{a, e, I, M, \omega, \Omega\}$	Keplerian elements of the satellite orbit
a_e	mean radius of the earth
$(C_{n,m}, S_{n,m})$	unnormalized gravity harmonics of the earth
$(\bar{C}_{n,m}, \bar{S}_{n,m})$	normalized gravity harmonics of the earth
f	true anomaly
G	gravitational constant
$M_\oplus$	mass of the earth
n	mean motion of the satellite
n'	speed of rotation of the earth
$P_{n,m}(x)$	associated Legendre function
R	"resonant" part of U
(r, ϕ, λ)	spherical coordinates
U	potential function for the earth
$X_{q+n}^{-p,q}(e)$	Hansen coefficient
δa	perturbation in semimajor axis
θ_g	Greenwich Sidereal Time
μ	$GM_\oplus$
$\dot{\Omega}$	secular motion of the ascending node
$\dot{\omega}$	secular motion of the argument of perigee

GRAVITY HARMONICS FROM A RESONANT TWO-HOUR SATELLITE

INTRODUCTION

When the mean motion of a satellite orbit and the rotation rate of the Earth are nearly commensurate, the perturbations due to a particular pair or set of pairs of tesseral or sectorial harmonics $(\bar{C}_{n,m}, \bar{S}_{n,m})$ are magnified. In principle, this enables one to deduce these harmonics from the observed perturbations in the motion of such satellites. The difficulty arises in trying to separate the effects due to the various nearly resonant harmonics.

The TIROS IX satellite is in an orbit whose period is in a twelve to one commensurability with the rotational period of the Earth. In this report an analysis of the motion of TIROS IX is carried out and a determination of some "composite" twelfth order gravity harmonics is made from this two-hour satellite. The term "composite" has a special meaning. Since all harmonics of order twelve produce perturbations of the same or nearly the same periods, the problem of separating the effects of many such harmonics of degree twelve or larger on the motion of a single satellite is nearly impossible. However, the perturbations due to twelfth order gravity harmonics of even degree are of slightly different periods than those due to odd degree harmonics. It is therefore possible, at least in principle, to obtain values for two pairs of gravity harmonics from the observed perturbations in the semimajor axis of the TIROS IX satellite. The analysis will be similar to that followed in a previous paper, Murphy and Victor (1968), for satellites with two to one commensurabilities. The harmonics so obtained in this case must be considered to be "composite" harmonics.

THE DISTURBING FUNCTION

The recommended form for the potential of the Earth at an exterior point with spherical coordinates (r, ϕ, λ) as given by Hagihara (1962) is

$$U = \frac{\mu}{r}\left[1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n P_{n,m}(\sin\phi)\,(C_{n,m}\cos m\lambda + S_{n,m}\sin m\lambda)\right] \tag{1}$$

where μ is the product of the gravitational constant G and the mass of the Earth $M_\oplus$; a_e is the mean equatorial radius of the Earth, and $P_{n,m}$ is the associated Legendre function defined by

$$P_{n,m}(x) = \frac{1}{2^n n!}\,(1 - x^2)^{m/2}\,\frac{d^{n+m}(x^2 - 1)^n}{dx^{n+m}}\,. \tag{2}$$

The portion of the potential function of the Earth that is dependent upon spherical harmonics of order twelve $(m = 12)$ is considered to be the disturbing function, R, for this analysis. The formulation developed in this report will consider all the harmonics of order twelve and degree less than or equal to fifteen ($n = 15$). The disturbing function is then transformed from spherical coordinates to a function of the orbital parameters of the satellite by applying standard formulas, Equations (3), from elliptic motion. The geometry connected with this transformation is described in Figure 1.

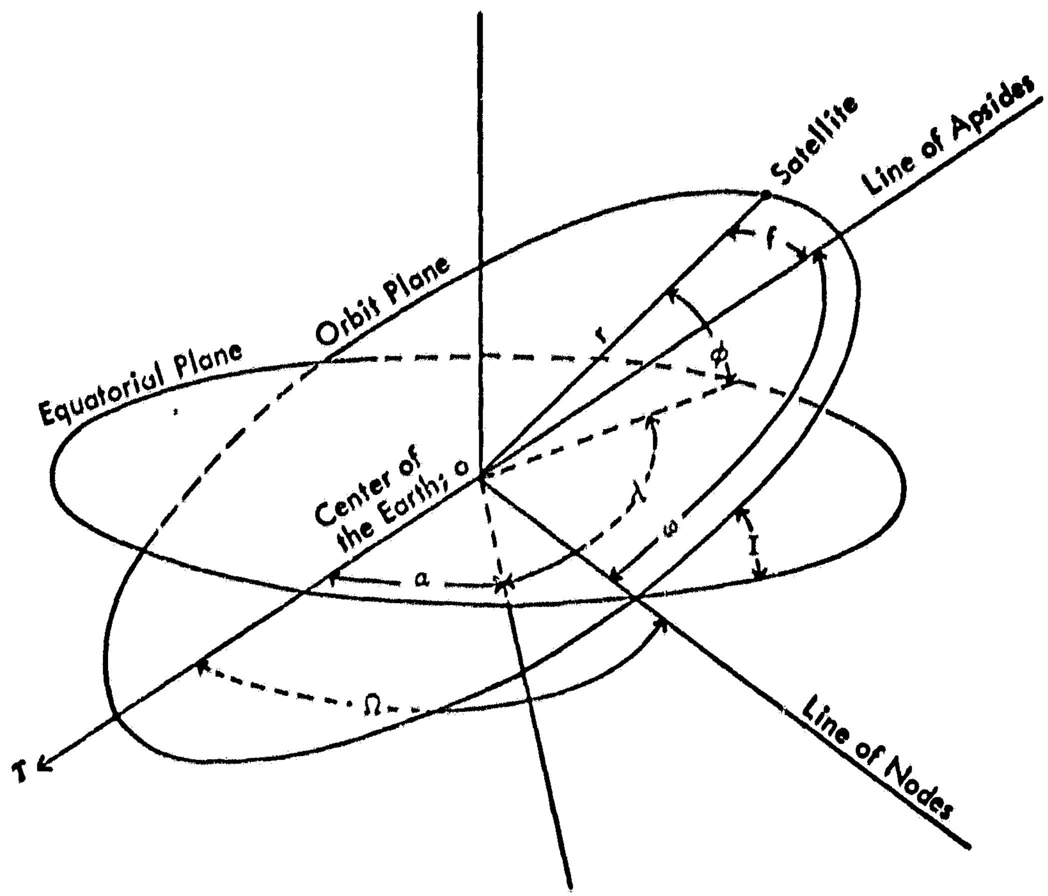

Figure 1–The Geometry of a Satellite Orbit

$$\sin \phi = \sin I \sin (f + \omega)$$

$$\cos \phi \cos \lambda = \frac{1}{2} [(1 + \cos I) \cos (f + \omega + \Omega - \theta_g) + (1 - \cos I) \cos (f + \omega - \Omega + \theta_g)] \tag{3}$$

$$\cos \phi \sin \lambda = \frac{1}{2} [(1 + \cos I) \sin (f + \omega + \Omega - \theta_g) - (1 - \cos I) \sin (f + \omega - \Omega + \theta_g)]$$

Next, the disturbing function is transformed to express terms involving the true anomaly, f, as a function of the mean anomaly. However, only the resonant terms of order e^2 or larger will be kept in this analysis. This transformation is accomplished by means of the following relationships given by Tisserand (1960):

$$\left(\frac{a}{r}\right)^p \frac{\sin}{\cos} qf = \sum_{i=-\infty}^{\infty} X_{q+i}^{-p,q}(e) \frac{\sin}{\cos} (q+i) M, \tag{4}$$

where M is the mean anomaly of the satellite and $X_{q+i}^{-p,q}(e)$ is the Hansen coefficient. Let $k = -p$, $m = q$, and $q + i = j$. Then, if $j > m$,

$$X_j^{k,m}(e) = (-1)^{j-m} (1+\beta^2)^{-k-1} \beta^{j-m} (P_{j-m} + P_{j-m+1} Q_1 \beta^2 + P_{j-m+2} Q_2 \beta^4 + \cdots) \tag{5}$$

and if $j < m$,

$$X_j^{k,m}(e) = (-1)^{m-j} (1+\beta^2)^{-k-1} \beta^{m-j} (Q_{m-j} + Q_{m-j+1} P_1 \beta^2 + Q_{m-j+2} P_2 \beta^4 + \cdots) \tag{6}$$

where

$$P_1 = \frac{k-m+1}{1} - \frac{\nu}{1}$$

$$P_2 = \frac{(k-m+1)(k-m)}{1.2} - \frac{k-m+1}{1}\frac{\nu}{1} + \frac{\nu^2}{1.2}$$

$$P_3 = \frac{(k-m+1)(k-m)(k-m-1)}{1.2.3} - \frac{(k-m+1)(k-m)}{1.2}\frac{\nu}{1} + \frac{k-m+1}{1}\frac{\nu^2}{1.2} - \frac{\nu^3}{1.2.3} \tag{7}$$

and so on; and

$$Q_1 = \frac{k+m+1}{1} + \frac{\nu}{1}$$

$$Q_2 = \frac{(k+m+1)(k+m)}{1.2} + \frac{k+m+1}{1}\frac{\nu}{1} + \frac{\nu^2}{1.2}$$

$$Q_3 = \frac{(k+m+1)(k+m)(k+m-1)}{1.2.3} + \frac{(k+m+1)(k+m)}{1.2}\frac{\nu}{1} + \frac{k+m+1}{1.2}\frac{\nu^2}{1.2} + \frac{\nu^3}{1.2.3} \tag{8}$$

and so on. Also,

$$\beta = \frac{e}{1+\sqrt{(1-e^2)}}$$

$$\nu = \frac{je}{2\beta}. \tag{9}$$

The disturbing function, R, then becomes

$$R = \frac{1}{a^{13}} \left\{ .54891633\, s^{10} (1+c)^2 X_1^{-13,2} \left[\bar{C}_{12}^{(12)} \cos [M + 2\omega + 12(\Omega - \theta_g)] + \bar{S}_{12}^{(12)} \sin [M + 2\omega + 12(\Omega - \theta_g)] \right] \right\}$$

$$+ .64040239\, s^{12} X_1^{-13,0} \left[\bar{C}_{12}^{(12)} \cos [M + 12(\Omega - \theta_g)] + \bar{S}_{12}^{(12)} \sin [M + 12(\Omega - \theta_g)] \right] \Big\}$$

$$+ \frac{1}{a^{14}} \Big\{ - .23768775\, s^{11} (1-c)(1+13c) X_1^{-14,-1}$$

$$\times \left[\bar{C}_{13}^{(12)} \sin [M - \omega + 12(\Omega - \theta_g)] - \bar{S}_{13}^{(12)} \cos [M - \omega + 12(\Omega - \theta_g)] \right] \Big\}$$

$$+ .23768775\, s^{11} (1+c)(1-13c) X_1^{-14,1}$$

$$\times \left[\bar{C}_{13}^{(12)} \sin [M + \omega + 12(\Omega - \theta_g)] - \bar{S}_{13}^{(12)} \cos [M + \omega + 12(\Omega - \theta_g)] \right]$$

$$+ .17826582\, s^9 (1+c)^3 (3-13c) X_1^{-14,3}$$

$$\times \left[\bar{C}_{13}^{(12)} \sin [M + 3\omega + 12(\Omega - \theta_g)] - \bar{S}_{13}^{(12)} \cos [M + 3\omega + 12(\Omega - \theta_g)] \right]$$

$$+ \frac{1}{a^{15}} \Big\{ .16653168\, s^{10} (1+c)^2 (1 + 18c - 63c^2) X_1^{-15,2}$$

$$\times \left[\bar{C}_{14}^{(12)} \cos [M + 2\omega + 12(\Omega - \theta_g)] + \bar{S}_{14}^{(12)} \sin [M + 2\omega + 12(\Omega - \theta_g)] \right]$$

$$+ .44408448\, s^{12} (1 - 27c^2) X_1^{-15,0} \left[\bar{C}_{14}^{(12)} \cos [M + 12(\Omega - \theta_g)] + \bar{S}_{14}^{(12)} \sin [M + 12(\Omega - \theta_g)] \right] \Big\}$$

$$+ \frac{1}{a^{16}} \Big\{ - .25826763\, s^{11} (1-c)(1 + 13c - 29c^2 - 145c^3) X_1^{-16,-1}$$

$$\times \left[\bar{C}_{15}^{(12)} \sin [M - \omega + 12(\Omega - \theta_g)] - \bar{S}_{15}^{(12)} \cos [M - \omega + 12(\Omega - \theta_g)] \right] \Big\}$$

$$+ .25826763\, s^{11} (1+c)(1 - 13c - 29c^2 + 145c^3) X_1^{-16,1}$$

$$\times \left[\bar{C}_{15}^{(12)} \sin [M + \omega + 12(\Omega - \theta_g)] - \bar{S}_{15}^{(12)} \cos [M + \omega + 12(\Omega - \theta_g)] \right]$$

$$+ .028696403\, s^9 (1+c)^3 (13 + 21c - 609c^2 + 1015c^3) X_1^{-16,3}$$

$$\times \left[\bar{C}_{15}^{(12)} \sin [M + 3\omega + 12(\Omega - \theta_g)] - \bar{S}_{15}^{(12)} \cos [M + 3\omega + 12(\Omega - \theta_g)] \right] \Big\}$$

where

$$s = \sin I$$
$$c = \cos I$$

It should be noted that unnormalized gravity harmonics $(C_{n,m}\ S_{n,m})$ appear in the expression for the potential function, Equation (1), while normalized gravity harmonics appear in the expression for the disturbing function, Equation (10). The relationship between normalized and unnormalized harmonics is

$$(\overline{C}_{n,m},\ \overline{S}_{n,m}) = N_{n,m}\,(C_{n,m},\ S_{n,m}) \tag{11}$$

where, for $m \neq 0$,

$$N_{n,m} = \left[\frac{2\,(2n+1)\,(n-m)!}{(n+m)!}\right]^{-1/2} \tag{12}$$

THE EQUATION FOR THE SEMIMAJOR AXIS

The semimajor axis of the orbit of an artificial satellite is usually free of long term perturbations (i.e. perturbations whose period is not of the same order as the orbital period but considerably longer). This type of perturbation can occur from two sources. First, if the orbit is not circular, an accumulative perturbation occurs due to solar radiation pressure for satellites in orbits that pass through the Earth's shadow. Second, the longitude dependent part of the potential function of the Earth can cause long term perturbations in the semimajor axis if the mean motion of the satellite's mean anomaly and the speed of the rotation of the Earth are very nearly commensurate. For the TIROS IX satellite, both of these effects are present. The first of these effects causes periodic perturbations that range in period from about one hundred days to about five hundred and fifty days while the second produces periodic perturbations with periods ranging from twelve days to eighteen days. The solar radiation pressure perturbation was eliminated from the observed variation in the semimajor axis by applying a method given by Murphy (1966). This method was used to compute the quantity, δa_{RP}, discussed in the section on numerical results.

The equation for the semimajor axis is now obtained to use in the determination of some of the gravity harmonics. The rate of change of the semimajor axis is

$$a = \frac{2}{na}\frac{\partial R}{\partial M} \tag{13}$$

After obtaining the disturbing function R, Equation (10), and substituting into Equation (13) taking the partial derivative and integrating, the equation for the semimajor axis is obtained. It is

$$a = a_0 + \delta a \tag{14}$$

where

$$
\begin{aligned}
\delta a = \tau_1\ &\{\overline{C}_{12,12}\cos(M+2\omega+12\Omega-12\theta_g)+\overline{S}_{12,12}\sin(M+2\omega+12\Omega-12\theta_g)\}\\
+\tau_2\ &\{\overline{C}_{12,12}\cos(M\qquad+12\Omega-12\theta_g)+\overline{S}_{12,12}\sin(M\qquad+12\Omega-12\theta_g)\}\\
+\tau_3\ &\{\overline{C}_{13,12}\sin(M-\omega+12\Omega-12\theta_g)-\overline{S}_{13,12}\cos(M-\omega+12\Omega-12\theta_g)\}\\
+\tau_4\ &\{\overline{C}_{13,12}\sin(M+\omega+12\Omega-12\theta_g)-\overline{S}_{13,12}\cos(M+\omega+12\Omega-12\theta_g)\}\\
+\tau_5\ &\{\overline{C}_{13,12}\sin(M+3\omega+12\Omega-12\theta_g)-\overline{S}_{13,12}\cos(M+3\omega+12\Omega-12\theta_g)\}\\
+\tau_6\ &\{\overline{C}_{14,12}\cos(M+2\omega+12\Omega-12\theta_g)+\overline{S}_{14,12}\sin(M+2\omega+12\Omega-12\theta_g)\}\\
+\tau_7\ &\{\overline{C}_{14,12}\cos(M\qquad+12\Omega-12\theta_g)+\overline{S}_{14,12}\sin(M\qquad+12\Omega-12\theta_g)\}\\
+\tau_8\ &\{\overline{C}_{15,12}\sin(M-\omega+12\Omega-12\theta_g)-\overline{S}_{15,12}\cos(M-\omega+12\Omega-12\theta_g)\}\\
+\tau_9\ &\{\overline{C}_{15,12}\sin(M+\omega+12\Omega-12\theta_g)-\overline{S}_{15,12}\cos(M+\omega+12\Omega-12\theta_g)\}\\
+\tau_{10}\ &\{\overline{C}_{15,12}\sin(M+3\omega+12\Omega-12\theta_g)-\overline{S}_{15,12}\cos(M+3\omega+12\Omega-12\theta_g)\}
\end{aligned}
\tag{15}
$$

and where

$$\tau_1 = 1.0978327\ \frac{n\, s^{10}\, a_e^{10}\,(1+c)^2\, X_1^{-13,2}(e)}{a^{11}\,(n+2\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_2 = 1.2808048\ \frac{n\, s^{12}\, a_e^{10}\, X_1^{-13,0}(e)}{a^{11}\,(n+12\dot{\Omega}-12n')}$$

$$\tau_3 = -.47537550\ \frac{n\, s^{11}\, a_e^{11}\,(1-c)\,(1+13c)\, X_1^{-14,-1}(e)}{a^{12}\,(n-\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_4 = .47537550\ \frac{n\, s^{11}\, a_e^{11}\,(1+c)\,(1-13c)\, X_1^{-14,1}(e)}{a^{12}\,(n+\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_5 = .35653164\ \frac{n\, s^{9}\, a_e^{11}\,(1+c)^3\,(3-13c)\, X_1^{-14,3}(e)}{a^{12}\,(n+3\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_6 = .33306336\ \frac{n\, s^{10}\, a_e^{12}\,(1+c)^2\,(1+18c-63c^2)\, X_1^{-15,2}(e)}{a^{13}\,(n+2\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_7 = .88816896\ \frac{n\, s^{12}\, a_e^{12}\,(1-27c^2)\, X_1^{-15,0}(e)}{a^{13}\,(n+12\dot{\Omega}-12n')}$$

$$\tau_8 = -.51653526 \frac{n\, s^{11}\, a_e^{13}\, (1-c)\, (1+13c-29c^2-145c^3)\, X_1^{-16,-1}(e)}{a^{14}\, (n-\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_9 = .51653526 \frac{n\, s^{11}\, a_e^{13}\, (1+c)\, (1-13c-29c^2+145c^3)\, X_1^{-16,1}(e)}{a^{14}\, (n+\dot{\omega}+12\dot{\Omega}-12n')}$$

$$\tau_{10} = .057392806 \frac{n\, s^{9}\, a_e^{13}\, (1+c)^3\, (13+21c-609c^2+1015c^3)\, X_1^{-16,3}(e)}{a^{14}\, (n+3\dot{\omega}+12\dot{\Omega}-12n')} \qquad (16)$$

NUMERICAL RESULTS

The orbit of Tiros IX was determined using a general perturbation theory, Brouwer (1959), from three day arcs of Minitrack observations in the following manner. Mean elements for a three day arc were obtained. Next the first day's data was deleted and an additional day's data was added and the epoch advanced one day. Then mean elements for this epoch were obtained. This process was repeated several times. From the mean semimajor axis, a'', a computed perturbation due to solar radiation pressure, δa_{RP}, was subtracted. If the zonal harmonics considered in Brouwer's theory and solar radiation pressure were the only forces acting on this satellite, then $a'' - \delta a_{RP}$ would be a constant to first order for all time. However, a periodic variation was observed. This observed variation has the same periods present as in the theoretical expression for δa presented above. Computed values of the near resonant periods appears in Table 1.

It should be noted that when elements were obtained for one day arcs with no overlap the same periods were observed. This data was not analyzed for gravity harmonics however, since the fewer number of observations causes some degree of scatter in the plot of $a'' - \delta a_{RP}$.

Table 1

Periods of Near Resonant Terms, $M + q\omega + 12(\Omega - \theta_g)$

	q	Period in Days
1	-1	12.291
2	0	13.265
3	1	14.407
4	2	15.764
5	3	17.402

The satellite orbit for TIROS IX has the following set of average values.

a = 8018.6 kilometers

e = .11697

I = 96.404 degrees

These average values were used in the evaluation of the τ's in δa over the time period considered here. Owing to the substantial value of the eccentricity of this satellite orbit, twelfth order gravity harmonics of both even and odd degree are present in the perturbation equation for the semimajor axis. Each harmonic of odd degree results in a perturbation with the same set of periods. The same is true of terms of even degree. Further, the periods present in these two sets of periods are close. This fact makes the task of trying to attribute the observed perturbation in the semimajor axis of a single satellite as being due to a specific harmonic or set of harmonics difficult. The narrow separation is clearly seen from Equation (15) and Table 1. All the possible solutions for the gravity parameters appearing in Equation (15) taken individually and taken pairwise with one even degree and one odd degree term were made by least squares fit to forty-three values of the semimajor axis. The best fit was obtained when the fourteenth and fifteenth degree harmonics were solved for. These values obtained were the following.

$$10^7 \times \bar{C}_{14,12} = 1.0 \pm .4$$

$$10^7 \times \bar{S}_{14,12} = 2.1 \pm .4$$

$$10^7 \times \bar{C}_{15,12} = -1.7 \pm .2$$

$$10^7 \times \bar{S}_{15,12} = 2.1 \pm .2.$$

In Figure 2 there appears a plot of the $a^* - \delta a_{R.P.}$ together with a plot of the residuals of the solution above added to the mean value of the semimajor axis, a_0. The solid curves in this figure are drawn in for purposes of illustration.

In Table 2 there appears a list of some previous determinations of twelfth order gravity harmonics (see list of references).

Table 2

Twelfth Order Gravity Harmonics*

Source	$\bar{C}_{12,12}$	$\bar{S}_{12,12}$	$\bar{C}_{13,12}$	$\bar{S}_{13,12}$	$\bar{C}_{14,12}$	$\bar{S}_{14,12}$	$\bar{C}_{15,12}$	$\bar{S}_{15,12}$
Gaposchkin (1966)	-.31	.008	-.59	.50	.94	-.28	-.619	.578
Kōhnlein (1967)	-.1	-.1	-.2	.6	.5	-.3	-.7	.5
Fischer (1968)	.25	.24	-.64	-.08				
Kaula (1968)	-1.1	-.1	-.8	.8	-.5	-.4	-.8	.1
Gaposchkin and Veis (1967)	-.31	.008	-.6769	.6245	.0261	-.2457	-.7473	-.1026

*Unit = 10^{-7}

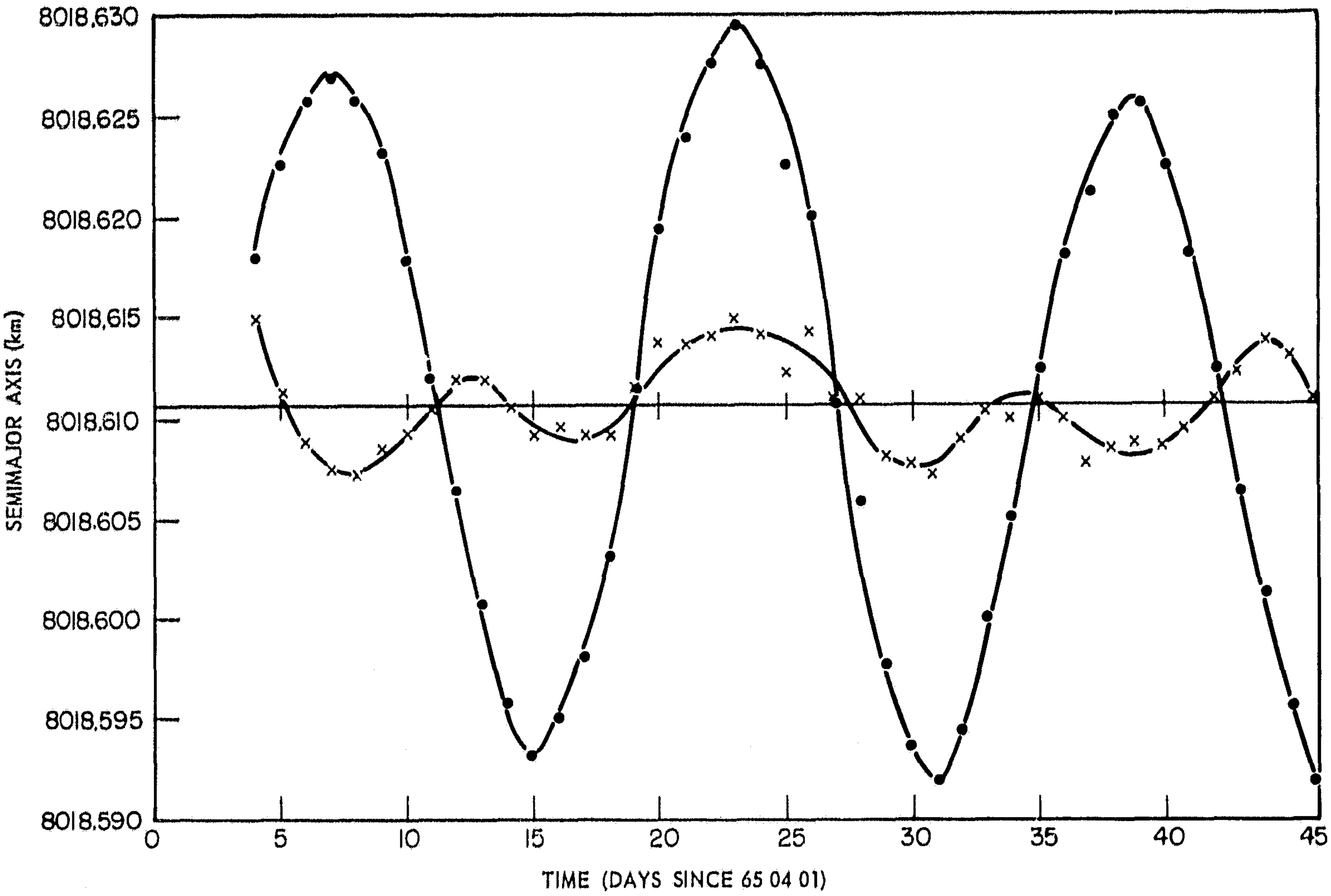

Figure 2–Semimajor Axis of TIROS IX

DISCUSSION

The expression for the perturbation in the semimajor axis, δa, of a two hour satellite due to nearly resonant gravity harmonics, Equation (15), includes twelfth order harmonics of degree twelve through fifteen. In actual fact twelfth order harmonics of even higher degree than fifteen may have some non-negligible effect. They are, however, neglected in this analysis. All of the harmonics present in the expression for δa could be determined if a solution were made involving several satellites of near two hour periods. Such a solution might be made from an analysis of the motion of TIROS IX, GEOS I, Alouette II and the Echo I Rocket in a combination solution since all of these satellites are in different orbits but with nearly two-hour periods.

The best fit to the mean semimajor axis was obtained when the twelfth order harmonics of degrees fourteen and fifteen were solved for. The results so obtained are given in the previous section of this report. The residuals from this fit appear in Figure 2 plotted about the mean value of the mean semimajor axis. It is interesting to note that these residuals appear to be periodic. This period appears to be approximately a little less than two-thirds the period of $a'' - \delta a_{R.P.}$ This would be very nearly the period one would expect if any of the resonant twenth-fourth order harmonics in the geopotential were significant contributors to the perturbation of the TIROS IX orbit. Notes of caution must be made. The original data reduction involved only a few zonal harmonics (Brouwer Theory), the radiation pressure reduction was very approximate, overlapping data arcs were used in reduction, and the programs used were single precision (eight decimal digits). If, however, this apparent effect is genuine, one would make a "rough" order of magnitude estimate of the size of say a composite twenty-fourth order sectorial harmonic. The conclusion would then be that $\bar{J}_{24,24} = 0\ (10^{-8})$. A more comprehensive reduction of the Minitrack observations using double precision programs will reveal, more clearly, the presence of any true twenth-fourth order harmonics.

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