Gravity Models in **R**

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Abstract

Gravity models are used to explain bilateral flows related to the sizes of bilateral partners, a measure of distance between them and other influences on interaction costs. The underlying idea is rather simple. The greater the masses of two bodies and the smaller the distance between them, the stronger their attraction. Even though their basic idea is rather simple, gravity models can become complex regarding the choice of models and estimation methods. As especially for gravity beginners it is difficult to get an overview of the different methods and implement them in **R**, the package **gravity** provides a wrapper of different standard estimation methods. By considering the descriptions and codes of these methods, users can get a comprehensive and application-oriented access, see which method may be suitable for certain research questions or types of data, and extend the code available for their specific research projects.

Gravitationsmodelle werden verwendet, um bilaterale Ströme zu erklären. Dabei werden sowohl Größen und Distanz der jeweiligen Partner als auch weitere Einflussfaktoren zur Erklärung herangezogen. Die zugrundeliegende Idee dieser Modelle beruht auf dem Gravitationsprinzip. Je schwerer zwei Körper und je kleiner deren Distanz zueinander, desto stärker ist ihre gegenseitige Anziehungskraft. Auch wenn die zugrundeliegende Idee von Gravitationsmodellen zunächst intuitiv erscheint, kann deren Schätzung je nach theoretischem Hintergrund und verwendeter Schätzmethode komplex werden. Da es insbesondere für Nutzer, die sich neu mit Gravitationsmodellen beschäftigen, schwierig ist einen Überblick über die verschiedenen Methoden zu erhalten und diese direkt selbst anzuwenden, bietet das Paket **gravity** einen Wrapper verschiedenster Standard-Schätzmethoden für Gravitationsmodelle. Durch die Beschreibung sowie die direkte Anwendbar- und Vergleichbarkeit dieser anhand des beigefügten Datensatzes können Anwender einen anwendungsorientierten Überblick über die Methoden, ihre Anwendbarkeit für verschiedene Forschungsfragen und Voraussetzungen an Daten gewinnen und den zugrundeliegenden Code für ihre eigenen Forschungsfragen erweitern.

Keywords: gravity model, international trade, migration, **R**.

1. Introduction

Gravity models in their naive form trace back to the work of Tinbergen (1962) and Ravenstein (1889). They owe their popularity to high explanatory power and relatively robust results across many datasets and different scientific works (Anderson 2011). As Baier and Bergstrand (2010) argue, the topics addressed by gravity models, such as international trade, migration

flows or foreign direct investment, are of high political relevance. The most widespread application, however, is for international trade flows (Möhlmann, Ederveen, de Groot, and Linders 2010). In consequence of a sound theoretical basis (for trade mainly thanks to the contribution of Anderson and van Wincoop (2003)) the popularity of gravity models rose within the last twenty years and they can nowadays be found in many variations and applications.

Even though the basic idea of gravity models is rather simple, they can become complex when it comes to the choice of models or estimation methods. For the choice of a specific gravity model and its theoretical background, see the literature referred to in this article. Some estimation methods can be burdensome to understand and program for researchers starting to work with gravity models. In order to provide users of gravity models and R with a wrapper of some of the most common used methods for estimating gravity models, we implemented the package **gravity**. The functions included in the package are listed in Table 1.

Table 1: Functions of the package **gravity**

Algorithm	Function	Section
Ordinary Least Squares	OLS()	4.2
Fixed Effects	Fixed_Effects()	4.3
Double Demeaning	DDM()	4.4
Bonus vetus OLS with simple averages	BVU()	4.5
Bonus vetus OLS with GDP weights	BVW()	4.5
Structural Iterated Least Squares	SILS()	4.6
Tetrads	Tetrads()	4.7
Tobit	Tobit()	4.8
ET-Tobit, Eaton and Tamura (1994)	ET_Tobit()	4.9
EK-Tobit, Eaton and Kortum (2001)	EK_Tobit()	4.10
Poisson Pseudo Maximum Likelihood	PPML()	4.11
Gamma Pseudo Maximum Likelihood	GPML()	4.12
Negative Binomial Pseudo Maximum Likelihood	NBPML()	4.13
Nonlinear Least Squares	NLS()	4.14

They can be subdivided into two types of estimation methods: those estimating gravity models in their log-log form, such as Ordinary Least Squares (OLS), Fixed Effects, Double Demeaning (DDM), Bonus vetus OLS with simple averages (BVU) and with GDP-weights (BVW), Structural Iterated Least Squares (SILS), Tetrads, and different Tobit style methods (Tobit, ET-Tobit, and EK-Tobit). These methods are partly complex to understand and program and thus a comparison of them is not straightforward. Therefore the package aims at easing an overview of the different methods combined with a direct application. Furthermore, except for the Tobit style methods, these methods are not capable of handling zero values in the dependent variable just like that. A second type of estimation methods estimates gravity models in their multiplicative form via generalized linear models with a log-link using different distribution families. These methods are relatively easy to compute and added for completeness. They contain the methods Poisson Pseudo Maximum Likelihood (PPML), Gamma Pseudo Maximum Likelihood (GPML), Negative Binomial Pseudo Maximum Likelihood (NBPML), and Nonlinear Least Squares (NLS). As they estimate the gravity equation in its multiplicative form, all of them but GPML are capable of utilizing datasets with zero values in the dependent variable. The functions of the package all differ with respect to their underlying assumptions and prerequisites concerning data, their ability to handle missing values, as well as their capability of including different effects in the estimation. By considering the descriptions of the estimation methods, users can see which method and data may be suited for certain research questions. In order to have a straightforward application of all methods, the package comes with two example datasets (`Gravity_no_zeros` and `Gravity_zeros`, see Section

3). Examples utilizing these datasets are included in the descriptions of the methods. The user can for example see how the results of the estimation methods may differ even though they are applied to the same dataset. As the range of gravity models and their suitable estimation methods is huge and specific to data and research question at hand, the functions are kept simple such that researchers can get familiar with them and utilize them as a starting point of their research.

In the following we describe gravity models in general as well as the example datasets and present the different estimation methods in the R package **gravity**.

2. Gravity models

Gravity models in their traditional form are inspired by Newton's (1729) law of gravitation:

$$F_{ij} = G \frac{M_i M_j}{D_{ij}^2}. \tag{1}$$

The force F between two bodies i and j with $i \neq j$ is proportional to the masses M of these bodies and inversely proportional to the square of their geographical distance D. G is a constant and as such of no major concern.

The underlying idea of a traditional gravity model, shown for international trade, is equally simple (based on Anderson 2011):

$$X_{ij} = G \frac{Y_i^{\beta_1} Y_j^{\beta_2}}{D_{ij}^{\beta_3}}. \tag{2}$$

Let interaction in form of trade flows be X and let Y_i and Y_j be the masses of the exporting and importing country, usually approximated by a measure of GDP. Following equation (2), the greater the GDPs of two countries and the smaller the geographical distance D_{ij} between them, the more they trade with each other. Similar to Newton's model in equation (1), the coefficients for countries' incomes were often found to be close to unity (see for example Head and Mayer 2014). As unitary income elasticities are in line with theoretical foundations like Anderson and van Wincoop (2003), it is therefore sometimes assumed that the income elasticities are equal to unity (see for example Anderson 2010), meaning $\beta_1 = \beta_2 = 1$. Besides these basic parts of a gravity model all other variables determining the trade volume or interaction between two countries can be added to the model as proxies for transaction costs. Tinbergen (1962) mentions trade agreements, historical backgrounds, political situations, common languages, religion or cultural similarities between bilateral partners as potential dummy variables. We can define a term t in equation (3) catching the transaction costs between two countries. Dummy variables can be added in t, e.g. *contig* for common borders or *rta* for regional trade agreements.

$$t_{ij} = D_{ij} e^{contig_{ij}} e^{rta_{ij}} = \exp(\log D_{ij} + contig_{ij} + rta_{ij}) \tag{3}$$

After estimating the coefficients of the transaction costs, one can for example compare theoretical and real trade flows and thereby reveal trade barriers. Gravity equations can be easily transformed from a multiplicative to an additive form by taking the logarithm. The gravity model in equation (2) added by the transaction costs in equation (3) can therefore be written in an additive form as:

$$\log X_{ij} = \beta_0 \log G + \beta_1 \log Y_i + \beta_2 \log Y_j + \beta_3 \log D_{ij} + \beta_4 contig_{ij} + \beta_5 rta_{ij}. \tag{4}$$

A gravity equation in its additive form can be estimated by standard estimation methods such as OLS. For inference problems arising when estimating a logged variable, see Manning

(1998). He presents potential retransformations when a variable of interest is estimated on its logged-scale even though the real interest is in its unlogged-scale with focus on the case of heteroscedasticity.

Baier and Bergstrand (2010) state that the main disadvantage of traditional gravity models, such as (2) and (4), is the negation of the rest of the world. Baldwin (2007) calls this negation the *gold medal mistake*. The traditional model only captures transaction costs between bilateral partners and ignores transaction costs to the rest of the world and can therefore lead to implausible results and does not allow for comparative statics (see Anderson and van Wincoop (2003) for further information). In order to incorporate the effects other countries can have on the trade volume of bilateral partners, a measure of *multilateral resistance* (MR) is usually included in gravity models.

Anderson (1979) includes the rest of the world in the theoretical basis of gravity models to allow the exports from i to j to be influenced by other demands for the goods of i and other suppliers for the goods of j. The term MR is used in later work of Anderson and van Wincoop (2003). They describe how the trade volume between two countries can decrease as a consequence of changing relative trade barriers of the two countries to their other trading partners. Anderson and van Wincoop (2003) define MR as the average trade barrier for the importing and exporting country. They furthermore find that small countries, which trade much with the rest of the world, react stronger to changes in trade barriers than bigger countries.

For the theoretical basis of a gravity model including MR terms, also called *structural gravity*, see Anderson and van Wincoop (2003) and Anderson (1979). Anderson and van Wincoop (2003) set up a gravity model with MR terms as:

$$X_{ij} = \frac{Y_i Y_j}{Y} \frac{t_{ij}^{1-\sigma}}{P_j^{1-\sigma} \Pi_i^{1-\sigma}} \tag{5}$$

with

$$\Pi_i^{1-\sigma} = \sum_j \frac{t_{ij}^{1-\sigma}}{P_j^{1-\sigma}} \frac{Y_j}{Y} \tag{6}$$

and

$$P_j^{1-\sigma} = \sum_i \frac{t_{ij}^{1-\sigma}}{\Pi_i^{1-\sigma}} \frac{Y_i}{Y}. \tag{7}$$

Just like in the traditional gravity models, the exports X_{ij} from i to j are determined by the supply factors in i, Y_i, the demand factors in j, Y_j, as well as the transaction costs t_{ij} from i to j. Next to information on bilateral partners i and j, information on the rest of the world is included in the gravity model (5). Y with $Y = \sum_i Y_i = \sum_j Y_j$ represents the worldwide sum of incomes. σ represents the elasticity of substitution between all goods with $\sigma > 1$ in order to account for the preference for a variation of goods. The MR terms are included via the terms P, inward MR, and Π, outward MR. The outward MR Π_i is a function of the transaction costs of i to all trade partners j and their demand, P_j vice versa. The MR terms (6) and (7) dependent on each other.

Under the assumption of symmetrical trade costs, $t_{ij} = t_{ji}$, and thereby $P_i = \Pi_i$ one obtains a symmetric gravity model (Anderson and van Wincoop 2003):

$$X_{ij} = \frac{Y_i Y_j}{Y} \frac{t_{ij}^{1-\sigma}}{P_j^{1-\sigma} P_i^{1-\sigma}}. \tag{8}$$

Due to the MR terms, the estimation of structural gravity models is more complex. See for example Head and Mayer (2014) or Gómez-Herrera (2013) for information on the development of gravity models for trade data.

Apart from the application to international trade, gravity models can be used for many purposes such as migration (see for example Orlova and Jost 2006), traffic (see for example Wang 2011), Foreign Direct Investment (FDI) (see for example Leibrecht and Riedl 2014), or the estimation of political action such as trade unions (see for example Martínez-Zarzoso, Nowak-Lehmann D., and Vollmer 2007). Head and Mayer (2014) give an overview about these different applications. Anderson (1979) describes gravity models as flexible concerning data used and Bergeijk and Brakman (2010) highlight the advantages of the possibility to use different levels of aggregation.

Datasets for gravity models often contain a huge amount of trade flows equal to zero. In data for international trade flows, these zeros can occur if two countries do not trade with each other, the trade flow between countries is so small that it rounds to zero or the trade flows were not observed. As Metulini, Patuelli, and Griffith (2016) highlight, it is nowadays - especially considering sector disaggregated flows - widely recognized that the level of trade between two entities can in fact be zero. The econometric models behind the gravity equation used should therefore reflect the possibility of zero trade flows (see e.g. the discussion about zeros in Anderson (2010)). Next to the theoretical background, also the estimation method used should be capable of handling zero value. Estimation methods utilizing gravity models in their logged form, such as OLS, Fixed Effects, DDM, BVU, BVW, SILS, and Tetrads cannot handle zero trade flows without further ado as the log of zero is not defined. In addition, also GPML is not capable of handling zeros flows as such. One simple solution would be to add a small constant to the flow values such that the zeros disappear and all flows can be logged. This, however, is criticized by many authors (see e.g. the discussion by Head and Mayer (2014)) as not only the theoretical foundation of this transformation is missing, but results can be highly volatile with respect to the magnitude of the constant. In order to see how volatile the estimation outcomes are with respect to the chosen constant, a possibility would be to estimate a chosen model with differently scaled constants and see whether the results change much by choice of the constant.

Another type of methods estimating the gravity equation in its additive form, but explicitly designed for a handling flows equal to zero are Tobit style models. Tobit models are used for censored data, where instead of the observed data a latent variable is modeled. The function `Tobit` adds a constant (default constant is 1) to the dependent variable and utilizes a Tobit regression with the censoring value equal to $\log(constant)$ for the logged dependent variable, which in case of $constant = 1$ is 0. For the threshold Tobit model by Eaton and Tamura (1994), called ET-Tobit, on the other hand, this constant, or threshold, is estimated from the data. For the estimation of the threshold we follow Carson and Sun (2007), who show that taking the minimum positive flow value as the threshold is super-consistent and that using this threshold estimate ensures that the parameter maximum likelihood estimates are asymptotically normal with the asymptotic variance identical to the variance achieved when the threshold is known. In addition, there is the Eaton and Kortum (2001) Tobit model, called EK-Tobit. In EK-Tobit, all observations of the dependent variable are redefined as intervals. The positive observations have both interval bounds equal to their original value. For zero flows the interval is left open. The right border of the interval is set to the log of the minimum trade flow of the respective importing country. Then, an interval regression is applied. For a short comparison of the three methods, see e.g. Head and Mayer (2014).

The estimation methods estimating the gravity equation in its multiplicative form (except for GPML), i.e. PPML, NBPML, and NLS, on the other hand, can utilize zero trade values. Nevertheless, these estimation methods can often not explain the great amount of zeros in data, but only part of it. Possible solutions for both estimation methods utilizing the multiplicative or logged form of the gravity equation when zero trade flows are frequent, e.g. censored, truncated, and zero-inflated methods, are proposed by many authors. See e.g. Mar-

tin and Pham (2015), Burger, Van Oort, and Linders (2009), Kareem, Martinez-Zarzoso, and Brümmer (2016), Anderson (2011), Gómez-Herrera (2013), Linders and De Groot (2006), and Head and Mayer (2014) for further information.

We refer the user to the Gravity Cookbook website[1] for more information on gravity models in general. Head and Mayer (2014) provide a comprehensive and accessible overview of the theoretical and empirical development of the gravity literature as well as the use of gravity models and the various estimation methods, especially their merits and potential problems regarding applicability as well as different gravity datasets.

3. Example datasets

In order to illustrate the different estimation methods, the package **gravity** includes two dataset called `Gravity_zeros` and `Gravity_no_zeros`. These datasets on international trade are obtained by editing the gravity dataset "`col_regfile09.dta`" available at the Gravity Cookbook website[2]. In order to have dataset suited for most functions, the datasets are limited to cross-sectional data. All incomplete rows and observations with missing trade flows were excluded from the dataset. In addition, for `Gravity_no_zeros` observations with trade flows equal to zero were deleted.

The dataset was downloaded and transformed in the following way:

```
 1   > # Reading in the dataset
 2   > library(foreign)
 3   > col_regfile09 <- read.dta("col_regfile09.dta")
 4   > # Isolation of one year
 5   > data06 <- col_regfile09[col_regfile09$year == 2006,]
 6   > # Choosing variables (select columns)
 7   > data06 <- data06[,c(2, 3, 6, 8, 12, 27, 34, 4, 5, 29)]
 8   > # Transforming data
 9   > # Isolation of complete cases
10   > data06 <- data06[complete.cases(data06) == TRUE,]
11   > # Exclusion of trade flows equal to 0
12   > Gravity_zeros <- data06[data06$flow != 0,]
13   > row.names(Gravity_zeros) <- 1:length(row.names(Gravity_zeros))
14   > # Divide GDPs by 1,000,000 for scaling purposes
15   > Gravity_zeros$gdp_o <- Gravity_zeros$gdp_o / 1000000
16   > Gravity_zeros$gdp_d <- Gravity_zeros$gdp_d / 1000000
```

For `Gravity_no_zeros` the following line is added

```
 1   > # Exclusion of trade flows equal to 0
 2   > Gravity_no_zeros <- data06[data06$flow != 0,]
```

The resulting datasets consist of 22.588 (`Gravity_zeros`) and 17.088 (`Gravity_no_zeros`) observations respectively. Both include the following variables, exemplary shown for `Gravity_zeros`:

```
 1   library(gravity)
 2   data(Gravity_zeros)
 3   str(Gravity_zeros)
```

```
 1   'data.frame': 22588 obs. of  10 variables:
 2   $ iso_o      : chr  "AFG" "AFG" "AFG" "AFG" ...
 3   $ iso_d      : chr  "ARG" "AUS" "AUT" "AZE" ...
 4   $ distw      : num  15341 11086 4567 1822 5849 ...
 5   $ gdp_o      : num  0.0084 0.0084 0.0084 0.0084 0.0084 ...
 6   $ gdp_d      : num  0.214058 0.768178 0.322444 0.020122 0.000807 ...
 7   $ rta        : int  0 0 0 0 0 0 0 0 0 0 ...
 8   $ flow       : num  0.061 0.4055 0.1649 0.0016 0 ...
 9   $ contig     : int  0 0 0 0 0 0 0 0 0 0 ...
10   $ comlang_off: int  0 0 0 0 0 0 0 0 0 0 ...
11   $ comcur     : int  0 0 0 0 0 0 0 0 0 0 ...
```

The descriptions of the variables are provided in Table 2.

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[1]https://sites.google.com/site/hiegravity/

[2]http://econ.sciences-po.fr/sites/default/files/file/tmayer/data/col_regfile09.zip

Table 2: Description of the variables in the `Gravity` dataset

name	description
iso_o	ISO-Code of country of origin
iso_d	ISO-Code of country of destination
distw	weighted distance (see Mayer and Zignago 2011)
gdp_o	GDP of country of origin in trillion dollars
gdp_d	GDP of country of destination in trillion dollars
rta	dummy for a regional trade agreement
flow	trade flow in million dollars
contig	dummy for regional contiguity
comlang_off	dummy for a common official language
comcur	dummy for a common currency

4. Gravity functions

4.1. Structure of the functions

On the Gravity Cookbook website[3] Keith Head and Thierry Mayer provide `Stata` code the most common estimation methods for gravity models. Where possible the functions were designed and tested to lead similar results when choosing the option of robust variance estimation (default). Compared to the `Stata` code available, the functions presented in this package provide users with more flexibility regarding the type of estimation (robust or not robust), the number and type of independent variables as well as the possible data and are directly applicable to the dataset included in the package.

As the functions were - where possible - tested to lead the same results as the provided `Stata` code for cross-sectional data, it is up to the user to ensure that the functions can be applied to panel data. In order to be comparable, the functions of the package include distance as an independent variable. If panel data are used, time-invariant effects, such as geographical distance, may have to be excluded. The most common challenges when using panel data with gravity models arise because of missing trade flows, unbalanced panels, and computational difficulties. For a comprehensive overview see Egger and Pfaffermayr (2003), Gómez-Herrera (2013) and Head, Mayer, and Ries (2010) as well as the references therein. Egger and Pfaffermayr (2003) provide some comparison of cross-sectional and panel models and show differences when using different variables. Depending on the panel dataset and the variables - specifically the type of fixed or random effects - included in the model, it may easily occur that the model is not computable. Also, note that by including bilateral fixed effects such as country-pair effects, the coefficients of time-invariant observables such as distance can no longer be estimated. Depending on the specific model, the code of the respective function may has to be changed by hand in order to exclude the distance variable from the estimation. When using panel data, the parameter and variance estimation of the models may have to be adjusted accordingly. To our knowledge at the moment, there is no explicit literature covering the estimation of a gravity equation by Double Demeaning (DDM), Structural Iterated Least Squares (SILS) or Bonus vetus OLS (BVU, BVW) using panel data. Therefore, we do not recommend to apply these methods in this case.

The functions in **gravity** all estimate gravity models, but they do so in different ways. They differ in whether they estimate gravity equations in their additive or multiplicative form, their requirements with respect to data, their handling of Multilateral Resistance (MR) terms as well as their possibilities concerning the inclusion of unilateral independent variables, such as GDPs or population sizes. Therefore, they usually lead to different estimation results.

To execute the functions in **gravity**, a square gravity dataset with all pairs of countries, ISO-

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[3]https://sites.google.com/site/hiegravity/

codes for the country of origin and destination, a measure of distance between the bilateral partners as well as all information that should be considered as dependent and independent variables is needed. Missing bilateral flows as well as incomplete rows should be excluded from the dataset. Depending on the method used, zero values in the dependent variable may have to be excluded.

The functions listed in Table 1 generally have the same structure. Table 3 lists variables common to all functions. Additional arguments for specific functions are listed in the section describing the respective function.

Table 3: Function variables

Variable	Usage
y	name (type: character) of the dependent variable in the dataset `data`, e.g. trade flows. It should not yet be logged. For functions estimating gravity equations in their additive form, `y` is logged automatically.
dist	name (type: character) of the distance variable in the dataset `data` containing a measure of distance between all pairs of bilateral partners. It should not yet be logged. For functions estimating gravity equations in their additive form `dist` is logged automatically.
x	vector of names (type: character) of bilateral variables in the dataset `data` that should be taken as the independent variables in the estimation. If an independent variable is a dummy variable, it should be of type numeric in the dataset. If an independent variable is defined as a ratio, it should be logged.
inc_o	variable name (type: character) of the income of the country of origin in the dataset `data`.
inc_d	variable name (type: character) of the income of the country of destination in the dataset `data`.
vce_robust	robust (type: logic) determines whether a robust variance-covariance matrix should be used for the standard errors. The default is set to `TRUE`. If set `TRUE` the estimation results are similar to the `Stata` results for robust estimation.
data	name of the dataset to be used (type: character). To estimate gravity equations, a square gravity dataset including bilateral flows defined by the argument `y`, ISO-codes (called `iso_o` for the country of origin and `iso_d` for the destination country), a distance measure defined by the argument `dist` and other potential influences given as a vector in `x` are required. All dummy variables should be of type numeric. Missing trade flows as well as incomplete rows should be excluded from the dataset. Note that for some functions, zero flow values have to be excluded or transformed, see the descriptions of the respective functions. When using panel data, a variable for the time may be included in the dataset. Note that the variable for the time dimension should be of type: factor.

In order to show users how the functions can be applied, each description of a function comes with an example of how to apply it using an example dataset in **gravity** including the resulting output. As our main aim is to provide R users with a wrapper of different estimation methods for gravity models and we would like to illustrate all functions using simple examples, the variables chosen to be included in `x` as well as the choice of setting `vce_robust` to `TRUE` or `FALSE` are arbitrary. In order to execute the functions, the package and the example datasets have to be called first:

```
1  > if (!"gravity" %in% installed.packages()) install.packages(gravity)
```

```
2   > library(gravity)
3   > data(Gravity_zeros)
4   > data(Gravity_no_zeros)
```

4.2. OLS

The function `OLS` estimates gravity models in their traditional form via Ordinary Least Squares
using the `lm` function implemented in R. MR terms are not considered by this function;
see the *gold medal mistake* described in Baldwin (2007). As OLS is limited to (log-)linear
relationships, gravity equations are estimated in their additive form. The user has to pay
attention that no zero trade flows are allowed in the dataset. Simple OLS is implemented
in **gravity** as it is the traditional way of estimating gravity equations and was used by e.g.
Tinbergen (1962). Baier and Bergstrand (2010) argue that the possibility to estimate gravity
equations by simple OLS was a major contribution to the popularity of gravity models. As
the estimated coefficients for the country's incomes were often found to be close to unity (see
for example Head and Mayer 2014) and unitary income elasticities are in line with theoretical
foundations like Anderson and van Wincoop (2003), it is sometimes assumed that the income
elasticities are equal to unity (see for example Anderson 2010). In order to allow for the
estimation with and without the assumption of unitary income elasticities, the option `uie` is
built into the function `OLS` with the default set to `FALSE`; see Table 4.

Table 4: Additional variables for `OLS`

Variable	Usage
uie	Unitary Income Elasticities (type: logic) determines whether the parameters are to be estimated assuming unitary income elasticities. The default value is set to `FALSE`. If `uie` is set `TRUE`, the flows in the dependent variable y are divided by the product of the country-pairs' incomes before the estimation. If `uie` is set to `FALSE`, the income variables are logged and taken as independent variables in the estimation. The variable names for the incomes should be inserted into `inc_o` for the country of origin and into `inc_d` for the destination country.

An example of how to apply the function `OLS` to an example dataset in **gravity** and the
resulting output is shown in the following:

```
1   > OLS(y="flow", dist="distw", x=c("comlang_off", "rta", "comcur", "contig"),
2   +       inc_o="gdp_o", inc_d="gdp_d", uie=FALSE, vce_robust=TRUE,
3   +       data=Gravity_no_zeros)
4
5   Call:
6   y_log ~ dist_log + comlang_off + rta + comcur + contig + inc_o_log +
7   inc_d_log
8
9   Residuals:
10  Min       1Q   Median      3Q      Max
11  −20.265   −1.226   0.182    1.521    9.976
12
13  Coefficients:
14  Estimate Std. Error t value Pr(>|t|)
15  (Intercept) 17.700041    0.262079   67.537     <2e−16 ***
16  dist_log    −1.171616    0.028801  −40.679     <2e−16 ***
17  comlang_off  1.194314    0.054077   22.085     <2e−16 ***
18  rta          0.885798    0.060390   14.668     <2e−16 ***
19  comcur      −0.003384    0.151723   −0.022      0.982
20  contig       1.118794    0.106636   10.492     <2e−16 ***
21  inc_o_log    1.232566    0.008466  145.596     <2e−16 ***
22  inc_d_log    0.910198    0.008192  111.103     <2e−16 ***
23  −−−
24  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
25
```

```
26    Residual standard error: 2.451 on 17080 degrees of freedom
27    Multiple R-squared:  0.6461,  Adjusted R-squared:  0.646
28    F-statistic:  4875 on 7 and 17080 DF,  p-value: < 2.2e-16
```

4.3. Fixed effects

To account for MR terms, Feenstra (2002) and Feenstra (2004) propose to use importer and exporter fixed effects. Due to these effects, all unilateral influences such as GDPs can no longer be estimated. A disadvantage of the use of fixed effects is that, when applied to panel data, the number of country-year or country-pair fixed effects can be computationally too high for estimation. In addition, no comparative statics are possible with fixed effects as the MR terms are not estimated explicitly. Nevertheless, Head and Mayer (2014) highlight the importance of the use of fixed effects. Country specific fixed effects are considered by incorporating `"iso_o"` and `"iso_d"` in `fe` (see Table 5) next to other possible bilateral variables. When applying the function `Fixed_Effects` to panel data, country-pair fixed effects or interaction effects with the time variable may be applied, but the function and especially the inclusion of distance as an independent variable may have to be changed depending on the effects used. See Section 1's part on panel data for further information. The function `Fixed_Effects` estimates gravity models in their additive form.

Table 5: Additional variables for `Fixed_Effects`

Variable	Usage
fe	vector of names (type: character) of fixed effects. The default is set to the unilateral identifiers `"iso_o"` and `"iso_d"` for cross-sectional data. When using panel data, interaction terms of the iso-codes and time may be added in either `fe` or `x`.

An example of how to apply the function `Fixed_Effects` to an example dataset in **gravity** and the resulting output is shown in the following:

```
1    > Fixed_Effects(y="flow", dist="distw", fe=c("iso_o", "iso_d"),
2    +                x=c("comcur"), vce_robust=TRUE, data=Gravity_no_zeros)
3
4    Call:
5    y_log ~ dist_log + comcur + iso_o + iso_d
6
7    Residuals:
8    Min       1Q     Median      3Q        Max
9    -20.2169  -1.0732   0.1016   1.2322    9.7312
10
11   Coefficients:
12   Estimate Std. Error t value Pr(>|t|)
13   (Intercept) 12.419632    0.411761   30.162  < 2e-16 ***
14   dist_log     -1.937231   0.024921  -77.735  < 2e-16 ***
15   comcur        0.407666   0.153327    2.659 0.007850 **
16   iso_oAGO      1.425700   0.537372    2.653 0.007983 **
17   iso_oALB     -0.851948   0.429502   -1.984 0.047320 *
18   .
19   .
20   .
21   iso_dCOL      2.161471   0.315343    6.854 7.41e-12 ***
22   iso_dCOM     -1.981948   0.397469   -4.986 6.21e-07 ***
23   iso_dCPV     -2.306427   0.393837   -5.856 4.82e-09 ***
24   [ reached getOption("max.print") -- omitted 133 rows ]
25   ---
26   Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
27
28   Residual standard error: 2.153 on 16755 degrees of freedom
29   Multiple R-squared:  0.7321,  Adjusted R-squared:  0.7268
30   F-statistic: 162.3 on 332 and 16755 DF,  p-value: < 2.2e-16
```

4.4. Double demeaning (DDM)

A further method to estimate gravity models in their additive form is the usage of Double Demeaning, DDM, as introduced by Head and Mayer (2014). By subtracting importer and exporter averages on the left and right hand side of the respective gravity equation, all unilateral influences including the MR terms vanish. Therefore, no unilateral variables may be added as independent variables for the estimation. The function DDM first logs the dependent variable and the distance variable. Afterwards, the dependent and independent variables are transformed in the following way (exemplary shown for trade flows, X_{ij}):

$$
\begin{aligned}
(\log X_{ij}).DDM = \; &(\log X_{ij}) - (\log X_{ij}).origin.mean \\
&- (\log X_{ij}).destination.mean + (\log X_{ij}).mean.
\end{aligned}
\tag{9}
$$

One subtracts the mean value for the origin country and the mean value for the destination country and adds the overall mean value to the logged trade flows. This procedure is repeated for all dependent and independent variables. The transformed variables are then used for the estimation. DDM is easily applied, but, as shown in Head and Mayer (2014), very sensitive to missing data.

An example of how to apply the function DDM to an example dataset in **gravity** and the resulting output is shown in the following:

```
> DDM(y="flow", dist="distw", x=c("rta","comcur","contig"),
+       vce_robust=TRUE, data=Gravity_no_zeros)

Call:
y_log_dd ~ dist_log_dd + rta_dd + comcur_dd + contig_dd − 1

Residuals:
Min        1Q     Median       3Q        Max
−20.9612  −1.2462    0.2849    1.5018    8.4425

Coefficients:
Estimate Std. Error t value Pr(>|t|)
dist_log_dd  −1.60215     0.03217  −49.799    <2e−16 ***
rta_dd        0.82253     0.06390   12.872    <2e−16 ***
comcur_dd     0.11431     0.13275    0.861     0.389
contig_dd     0.97793     0.10620    9.209    <2e−16 ***
−−−
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.318 on 17084 degrees of freedom
Multiple R−squared:  0.2542,   Adjusted R−squared:  0.254
F−statistic:  1772 on 4 and 17084 DF,   p−value: < 2.2e−16
```

4.5. Bonus vetus OLS (BVU, BVW)

Baier and Bergstrand (2009) and Baier and Bergstrand (2010) argue that on the one hand, the non-linear least squares (NLS) approach in Anderson and van Wincoop (2003) is in line with the theory and capable of conducting comparative statics, but on the other hand is too computationally costly and complex and therefore received little attention in the literature. On the other hand, fixed effects estimation, while incorporating MR terms and being easily set up, does not allow for comparative statics. Therefore, Baier and Bergstrand (2010) suggest a modification of the simple OLS that is easily implemented, allows for comparative statics and yields results close to those of NLS, called Bonus vetus OLS (BVU and BVW). They estimate gravity models in their additive form. For the theoretical foundation of their estimation method see Anderson and van Wincoop (2003) and Anderson and van Wincoop (2004). As unilateral income elasticities are assumed, flows are divided by the product of unilateral incomes. The dependent variable for the estimation is therefore $\log(y/(inc_o * inc_d))$. By applying a Taylor-series expansion and the assumption of symmetrical, bilateral trade costs, they develop a reduced gravity model in which multilateral and worldwide resistance enter exogenously. Baier and Bergstrand (2010) distinguish two types of Bonus vetus estimations depending on how the Taylor-series is centered. One method, called BVU, uses simple averages

while the other, called BVW, uses GDP weights. Depending on which method is used, the transaction costs are weighted differently. For advantages and disadvantages of both methods see Baier and Bergstrand (2009) and Baier and Bergstrand (2010).

To give an example with simple averages (BVU), distance would be transformed to Multilateral and World Resistance, MWR, of distance in the following way:

$$MWR.D_{ij} = \frac{1}{N}\left(\sum_{i=1}^{N} \log D_{ij}\right) + \frac{1}{N}\left(\sum_{j=1}^{N} \log D_{ij}\right) - \frac{1}{N^2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} \log D_{ij}\right) \quad (10)$$

with D_{ij} denoting the bilateral distance, N the number of countries and $MWR.D_{ij}$ the transformed variable adjusted for multilateral resistances.

When using BVW, the simple averages are replaced by GDP weights. The transformed variables are included as independent variables in the estimation. The resulting equation can be estimated by simple OLS. For an application of a method close to Bonus vetus on panel data, see Kareem *et al.* (2016).

An example of how to apply the functions BVU and BVW to an example dataset in **gravity** and the resulting output is shown in the following:

```
> BVU(y="flow", dist="distw", x=c("rta", "comlang_off"), inc_o="gdp_o",
+      inc_d="gdp_d", vce_robust=TRUE, data=Gravity_no_zeros)

Call:
y_inc_log ~ dist_log_mr + rta_mr + comlang_off_mr

Residuals:
Min        1Q     Median       3Q        Max
-20.2613   -1.2170   0.2941    1.5725    9.4282

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept)        7.47331     0.01910  391.192    <2e-16 ***
dist_log_mr       -1.66196     0.03431  -48.436    <2e-16 ***
rta_mr             0.64216     0.07011    9.159    <2e-16 ***
comlang_off_mr     0.95722     0.06749   14.183    <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.497 on 17084 degrees of freedom
Multiple R-squared:  0.236, Adjusted R-squared:  0.2359
F-statistic:  2053 on 3 and 17084 DF,  p-value: < 2.2e-16
```

```
> BVW(y="flow", dist="distw", x=c("rta", "comlang_off", "contig"), inc_o="gdp_o",
+      inc_d="gdp_d", vce_robust=TRUE, data=Gravity_no_zeros)

Call:
y_inc_log ~ dist_log_mr + rta_mr + comlang_off_mr + contig_mr

Residuals:
Min        1Q     Median       3Q        Max
-19.4113   -1.3282   0.4197    1.7021   10.5317

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept)        7.50636     0.02150  349.11    <2e-16 ***
dist_log_mr       -0.56606     0.02838  -19.94    <2e-16 ***
rta_mr             1.27323     0.06559   19.41    <2e-16 ***
comlang_off_mr     0.93975     0.05901   15.93    <2e-16 ***
contig_mr          1.13429     0.11159   10.16    <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.667 on 17083 degrees of freedom
Multiple R-squared:  0.129, Adjusted R-squared:  0.1288
F-statistic: 682.1 on 4 and 17083 DF,  p-value: < 2.2e-16
```

4.6. Structural iterated least squares (SILS)

The function `SILS`, structural iterated least squares, aims at estimating the MR terms (6) and (7) explicitly. The procedure was introduced by Anderson and van Wincoop (2003) using non-linear least squares and adapted by Head and Mayer (2014). See Egger and Staub (2016) for a comparison of iterative estimation methods and fixed effects methods. The function utilizes the dependencies between the MR terms and the transaction costs. The transaction costs influence the transactions between two countries in a direct and an indirect way by their effect on the MR terms. It estimates gravity equations in their additive form. SILS performs the following steps (Head and Mayer 2014):

- The unknown terms Π_i and P_j are initially set equal to 0.

- The particular gravity equation is estimated in its additive form via OLS to obtain estimates for the parameters determining the transaction costs. The dependent variable for the estimation equals $\log(X_{ij}/(Y_i * Y_j * \Pi_i * P_j))$.

- Given the newly estimated parameters, a contraction mapping algorithm is used to update Π_i and P_j.

- These steps are repeated until the estimated parameters stop changing significantly.

Table 6 contains the additional variables for the function `SILS`. They are used to restrict the maximum iterations in the inner and outer loop of SILS as well as the stopping rule. Compared to the other estimation methods, SILS can be very time consuming. When using it, the updated coefficients for every iteration are shown. When the iteration stops, summary statistics for the last iteration are shown.

Table 6: Additional variables for `SILS`

Variable	Usage
`maxloop`	maximum number of iterations. The default is set to 50.
`maxloop2`	maximum number of inner loop iterations. The default is set to 50.
`dec_places`	number of decimal precision. The default is set to 4.
`verbose`	verbose (type: logic) determines whether the estimated coefficients of each iteration should be printed in the console. The default is set to `FALSE`.

An example of how to apply the function `SILS` to an example dataset in **gravity** and the resulting output is shown in the following:

```
 1   > SILS(y="flow", dist="distw", x=c("rta","contig"),
 2   +        inc_o="gdp_o", inc_d="gdp_d", maxloop=50, maxloop2=50, dec_places=4,
 3   +        vce_robust=TRUE, verbose = TRUE, data=Gravity_no_zeros)
 4   This is round 0
 5   The coefficients are −0.9228275 1.116234 1.6975
 6   This is round 1
 7   The coefficients are −1.163416 1.761599 0.8989856
 8   This is round 2
 9   The coefficients are −1.150725 2.009707 0.7149999
10   This is round 3
11   The coefficients are −1.137014 2.08095 0.6847332
12   This is round 4
13   The coefficients are −1.131941 2.099618 0.6807216
14   This is round 5
15   The coefficients are −1.130408 2.104251 0.6804536
16   This is round 6
17   The coefficients are −1.129989 2.105352 0.6805352
18
19   Call:
```

```
20  log(d[y][, 1]) - log((d[inc_o][, 1] * d[inc_d][, 1])/(d$P_i *
21  d$P_j)) ~ dist_log + rta + contig
22
23  Residuals:
24  Min       1Q    Median      3Q       Max
25  -21.1974  -1.5555  0.3106   1.8511  10.2038
26
27  Coefficients:
28  Estimate Std. Error t value Pr(>|t|)
29  (Intercept) 12.59105    0.29222   43.087  < 2e-16 ***
30  dist_log    -1.12999    0.03302  -34.217  < 2e-16 ***
31  rta          2.10535    0.06841   30.776  < 2e-16 ***
32  contig       0.68054    0.11854    5.741  9.58e-09 ***
33  ---
34  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
35
36  Residual standard error: 2.858 on 17084 degrees of freedom
37  Multiple R-squared:  0.211, Adjusted R-squared:  0.2108
38  F-statistic:  2144 on 3 and 17084 DF,  p-value: < 2.2e-16
```

4.7. Tetrads

In order to use the fixed effects method with panel data, a huge number of dummy variables has to be included into the estimation. Thus, estimating these models can lead to significant computational difficulties. Head *et al.* (2010) present Tetrads as an estimation method circumventing this problem. They exploit the multiplicative form of the gravity equation to form the ratio of ratios. In doing so, both MR terms drop out of the equation. Table 7 shows the additional arguments for the function `Tetrads`.

Table 7: Additional variables for `Tetrads`

Variable	Usage
k	reference importing country, default is set to "USA".
l	reference exporting country, default is set to "JPN".
multiway_vcov	(type: logic) optional; determines whether a function implementing the multi-way clustering of variance-covariance matrices of Cameron, Gelbach, and Miller (2011) in the package **multiway_vcov** is used for the estimation. In case `multiway_vcov=TRUE`, the `cluster.vcov` function is used. The default value is set to `TRUE`.

A reference importer and a reference exporter have to be chosen. Preferably, these reference countries should have very few (if any) zero trade flows. The default importing country k is the United States, while the default exporting country l is Japan. Only those exporters trading with the reference importer and importers trading with the reference exporter will be included in the estimation. Tetrads can be used for both, cross-sectional and panel data. Here, the notation for panel data as shown in Head *et al.* (2010) is used.

The two ratios (R) are calculated as:

$$R_{i\{jk\}t} = \frac{X_{ijt}}{X_{ikt}} = \frac{(Y_{jt}/P_{jt})t_{ijt}}{(Y_{kt}/P_{kt})t_{ikt}} \tag{11}$$

and

$$R_{l\{jk\}t} = \frac{X_{ljt}}{X_{lkt}} = \frac{(Y_{jt}/P_{jt})t_{ljt}}{(Y_{kt}/P_{kt})t_{lkt}}. \tag{12}$$

In a second step, the ratio of ratios is computed as:

$$r_{\{il\}\{jk\}t} = \frac{R_{i\{jk\}t}}{R_{l\{jk\}t}} = \frac{X_{ijt}/X_{ikt}}{X_{ljt}/X_{lkt}} = \frac{t_{ijt}/t_{ikt}}{t_{ljt}/t_{lkt}}. \tag{13}$$

The equation is then estimated in its additive form. To account for a possible correlation between the error terms, a three-way-clustering - it, jt, and ij - can be applied using the method of Cameron *et al.* (2011) with `multiwayvcov=TRUE`. Due to the transformation of data, it is not possible to estimate the influences of unilateral effects. If `Tetrads` is used for panel data, the user has to exclude time-invariant effects, such as geographical distance.

An example of how to apply the function `Tetrads` to an example dataset in **gravity** and the resulting output is shown in the following:

```
1   > Tetrads(y="flow", dist="distw", x=c("rta","comcur","contig"),
2   +          k="USA", ell="JPN", multiway_vcov=TRUE, data=Gravity_no_zeros)
3
4   Call:
5   y_log_rat ~ dist_log_rat + rta_rat + comcur_rat + contig_rat
6
7   Residuals:
8   Min      1Q      Median    3Q       Max
9   -18.3732 -1.2976  0.2886    1.6030   11.8736
10
11  Coefficients:
12  Estimate Std. Error t value Pr(>|t|)
13  (Intercept)   -0.2988    0.1381  -2.164   0.03051 *
14  dist_log_rat  -1.4974    0.1133 -13.215  < 2e-16 ***
15  rta_rat        0.5749    0.2102   2.736   0.00623 **
16  comcur_rat     0.6579    0.6026   1.092   0.27497
17  contig_rat     0.7539    0.4041   1.866   0.06210 .
18  ---
19  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
20
21  Residual standard error: 2.785 on 16466 degrees of freedom
22  Multiple R-squared:  0.2941,   Adjusted R-squared:  0.2939
23  F-statistic:  1791 on 4 and 16466 DF,  p-value: < 2.2e-16
```

4.8. Tobit

In many cases, data values underlie natural boundaries. Tobin (1958) analyzes how to properly model this kind of data where on the one hand a substantial amount of observations equals the boundary value and on the other hand the observations exceeding the boundary do so to a large extend. As an example, he uses household expenditures and the Engel curve. As a binary regression model would be suitable for modeling whether an observation equals or exceeds the boundary and linear regression would be suitable for those values exceeding the boundary, the two approaches are combined in a so called Tobit model. In a Tobit model where the dependent variable is bounded by 0, a latent variable y_i^* is modeled and the observed values of the dependent variable y_i are assumed to either equal this variable when they exceed threshold 0 or 0 elsewise, see (14). The model is therefore also known as censored regression model.

$$y_i = \begin{cases} y_i^* & \text{if} \quad y_i^* > 0 \\ 0 & \text{else} \end{cases} \tag{14}$$

Tobit models exist in various forms with possible upper or lower boundaries, which not necessarily equal 0. As a combination of a binary and linear regression is modeled, the interpretation of the estimated coefficients has to take this into account. Therefore, as stated in e.g. Baldwin and Nino (2006), the marginal effects of an explanatory variable on the expected value of the dependent variable, i.e. the estimated coefficient, equals the product of both the probability of the latent variable exceeding the threshold and the marginal effect of the explanatory variable on the expected value of the latent variable. As gravity datasets for trade flows often contain a great number of values equal to zero, the Tobit approach can be suitable for this kind of data, especially if zeros arise from rounding small trade volumes. This approach of adding an arbitrary constant to the data is criticized, e.g. by Head and Mayer (2014). They argue that an underlying economic theory is missing and the estimates are volatile with respect to the choice of the constant. For more information about Tobit models in the context of gravity models for trade, see e.g. Kareem *et al.* (2016).

The `Tobit` function implemented in the package **gravity** represents a left-censored Tobit model with a censoring threshold assumed to be known. In order to provide users with the possibility to set the threshold to a certain number, the function comes with the argument `added_constant` described in Table 8.

Table 8: Additional variables for `Tobit`

Variable	Usage
`added_constant`	scalar (type: numeric); represents the constant to be added to the dependent variable. The default value is 1. The minimum of $log(y + added\_constant)$ is taken as the left boundary in the Tobit model. In the often used case of `added_constant=1`, the dependent variable is left censored at value 0 as $log(1) = 0$.

An example of how to apply the function `Tobit` to an example dataset in **gravity** and the resulting output is shown in the following:

```
1   > Gravity_zeros$lgdp_o <- log(Gravity_zeros$gdp_o)
2   > Gravity_zeros$lgdp_d <- log(Gravity_zeros$gdp_d)
3   >
4   > Tobit(y="flow", dist="distw", x=c("rta","lgdp_o","lgdp_d"),
5   +       added_constant = 1, data=Gravity_zeros)
6
7   Call:
8   y_plus1_log ~ dist_log + rta + lgdp_o + lgdp_d
9
10  Observations:
11  Total   Left-censored      Uncensored  Right-censored
12  22588            5500          17088               0
13
14  Coefficients:
15  Estimate Std. error t value Pr(> t)
16  (Intercept) 14.044188    0.153214     91.66   <2e-16 ***
17  dist_log    -0.852024    0.016592    -51.35   <2e-16 ***
18  rta          1.002807    0.042651     23.51   <2e-16 ***
19  lgdp_o       0.814971    0.004956    164.43   <2e-16 ***
20  lgdp_d       0.667020    0.004893    136.34   <2e-16 ***
21  logSigma     0.437644    0.005428     80.62   <2e-16 ***
22  ---
23  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
24
25  Newton-Raphson maximisation, 4 iterations
26  Return code 1: gradient close to zero
27  Log-likelihood: -34603.75 on 6 Df
```

4.9. ET-Tobit

The main critic of the Tobit model described in Section 4.8 is that the censoring value is arbitrarily chosen and this choice can have a great influence on the parameter estimates. In contrast to that approach, in the Eaton and Tamura (1994) Tobit model, also called threshold Tobit model, the censoring value or threshold is assumed to be unknown and therefore has to be estimated from the data. It is for example used in Martin and Pham (2015) and Santos Silva and Tenreyro (2011). Compared to the usual ET-Tobit approaches, in this package, the estimation of this threshold is done before the other parameters are estimated. We follow the approach in Carson and Sun (2007), who show that taking the minimum positive flow value as an estimate of the threshold is super-consistent and that using this threshold estimate ensures that the parameter maximum likelihood estimates are asymptotically normal with the asymptotic variance identical to the variance achieved when the threshold is known. Hence, in the function `ET_Tobit` first the threshold is estimated as the minimum positive flow. This threshold is added to the flow variable. It is logged afterwards and taken as the dependent variable in a left-censored regression with the left boundary equal to the smallest logged value. Even though this approach is more elaborate than the Tobit approach with an

arbitrarily set threshold, Head and Mayer (2014) criticize it for a lacking compelling structural interpretation. Furthermore, in their study Santos Silva and Tenreyro (2011) find very large bias for ET-Tobit.

An example of how to apply the function `ET_Tobit` to an example dataset in **gravity** and the resulting output is shown in the following:

```
1   > Gravity_zeros$lgdp_o <- log(Gravity_zeros$gdp_o)
2   > Gravity_zeros$lgdp_d <- log(Gravity_zeros$gdp_d)
3   >
4   > ET_Tobit(y="flow", dist="distw", x=c("rta","lgdp_o","lgdp_d"), data=Gravity_zeros)
5
6   Call:
7   y_plus_flow_min_log ~ dist_log + rta + lgdp_o + lgdp_d
8
9   Observations:
10  Total   Left-censored      Uncensored  Right-censored
11  22588          5500            17088               0
12
13  Coefficients:
14  Estimate Std. error t value  Pr(> t)
15  (Intercept) 36.381290   0.774607   46.97  < 2e-16 ***
16  dist_log    -2.842963   0.084288  -33.73  < 2e-16 ***
17  rta          1.098953   0.221564    4.96 7.05e-07 ***
18  lgdp_o       2.609596   0.024256  107.58  < 2e-16 ***
19  lgdp_d       2.009061   0.024310   82.64  < 2e-16 ***
20  logSigma     2.096474   0.005822  360.11  < 2e-16 ***
21  ---
22  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
23
24  Newton-Raphson maximisation, 6 iterations
25  Return code 2: successive function values within tolerance limit
26  Log-likelihood: -65571.15 on 6 Df
```

4.10. EK-Tobit

In addition to the two Tobit approaches presented in Section 4.8 and 4.9, the Eaton and Kortum (2001) Tobit approach utilizes import country specific censoring values. Compared to ET-Tobit, instead of taking the minimum positive flow as an estimate of the threshold for all countries, for each importing country the minimum positive threshold is taken as an estimate of its country specific threshold. Using the function `EK_Tobit`, all values of the dependent variable are redefined as intervals. The positive observations have both interval bounds equal to their original value. For zero flows the interval is left open. The right border of the interval is set to the log of the minimum positive trade flow of the respective importing country. The transformed data is then used in an interval regression with observation specific boundaries. Head and Mayer (2014) highlight this method in comparison to the other two Tobit approaches due to its applicability and interpretability.

An example of how to apply the function `EK_Tobit` to an example dataset in **gravity** and the resulting output is shown in the following:

```
1   > Gravity_zeros$lgdp_o <- log(Gravity_zeros$gdp_o)
2   > Gravity_zeros$lgdp_d <- log(Gravity_zeros$gdp_d)
3   >
4   > EK_Tobit(y="flow", dist="distw", x=c("rta","lgdp_o","lgdp_d"),
5   +           vce_robust=TRUE, data=Gravity_zeros)
6
7   Call:
8   dta_int ~ dist_log + rta + lgdp_o + lgdp_d
9   Value Std. Err (Naive SE)      z          p
10  (Intercept) 25.98  0.48553     0.45021  53.5  0.00e+00
11  dist_log    -1.92  0.05304     0.04893 -36.2 1.63e-287
12  rta          1.17  0.11649     0.12776  10.1  8.87e-24
13  lgdp_o       1.82  0.01408     0.01422 129.6  0.00e+00
14  lgdp_d       1.41  0.01435     0.01420  98.1  0.00e+00
15  Log(scale)   1.54  0.00906     0.00568 170.3  0.00e+00
16
17  Scale= 4.68
18
19  Gaussian distribution
```

```
20   Loglik(model)= −55298.3      Loglik(intercept only)= −64459.4
21   Chisq= 18322.09 on 4 degrees of freedom, p= 0
22   (Loglikelihood assumes independent observations)
23   Number of Newton−Raphson Iterations: 6
24   n= 22588
```

4.11. Poisson pseudo maximum likelihood (PPML)

Poisson Pseudo Maximum Likelihood, PPML, is recommended in the influential paper of Santos-Silva and Tenreyro (2006). They argue that estimating gravity equations in their additive form by OLS leads to inconsistency in the presence of heteroscedasticity. Due to Jensen's inequality $E(\log Y) \neq \log E(Y)$, Santos-Silva and Tenreyro (2006) advise researchers to estimate gravity models in their multiplicative form. Furthermore, as trade flows are not logged, one major drawback of many other estimation methods diminishes, namely the handling of trade flows equal to zero. They can remain in the dataset. Following Santos-Silva and Tenreyro (2006), heteroscedasticity likely occurs in trade data, with $0 \leq X_{ij}$, as the variance $var(X_{ij})$ is decreasing the closer trade flows are to zero. On the other hand, the greater the trade flows become, the higher the variance becomes as well. In order to utilize the pseudo ML estimates, the option vce_robust=TRUE by default in analogy to the vce(robust) in Stata. When choosing PPML the Poisson family is used with log-link. Similar methods utilizing the log-link, but different underlying distributions are Gamma PML (GPML), Negative Binomial PML (NBPML), and Nonlinear Least Squares (NLS), see Section 4.12, 4.13, and 4.14. See Manning and Mullahy (2001) or Gómez-Herrera (2013) who compare the performance of these methods under different data generating processes and their usage in the literature. Under consideration of zeros and heteroscedasticity, Santos-Silva and Tenreyro (2006) argue that the PPML estimator would be preferable over GPML or NLS. See also Santos Silva and Tenreyro (2011) for an extension of their simulation study in Santos-Silva and Tenreyro (2006). Martin and Pham (2015) tests the PPML estimator among others under different economically-based data generating processes, also with a focus on heterogenous data containing many zero values, and also finds it to have a good performance. For extensions of PPML, see for example Burger *et al.* (2009).

In order to illustrate all estimation methods using the same example dataset, the function PPML is also applied to data restricted to positive trade flows even though it would allow trade flows to be zero. An exemplary output is shown in the following:

```
1   > Gravity_zeros$lgdp_o <− log(Gravity_zeros$gdp_o)
2   > Gravity_zeros$lgdp_d <− log(Gravity_zeros$gdp_d)
3   >
4   > PPML(y="flow", dist="distw", x=c("rta","lgdp_o","lgdp_d"),
5   +       vce_robust=TRUE, data=Gravity_zeros)
6
7   Call:
8   y ~ dist_log + rta + lgdp_o + lgdp_d
9
10  Weighted Residuals:
11  Min       1Q    Median      3Q      Max
12  −221.53   −5.14   −2.02    −0.63  1738.13
13
14  Coefficients:
15  Estimate Std. Error z value Pr(>|z|)
16  (Intercept) 16.02083    0.65478   24.47    <2e−16 ***
17  dist_log     −0.85450    0.07278  −11.74    <2e−16 ***
18  rta          −0.10610    0.18300   −0.58     0.562
19  lgdp_o        0.80813    0.02179   37.09    <2e−16 ***
20  lgdp_d        0.86062    0.03296   26.11    <2e−16 ***
21  −−−
22  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
23
24  Residual standard error: 35.89 on 22583 degrees of freedom
```

4.12. Gamma pseudo maximum likelihood (GPML)

For the main description of the kind of estimation method used for Gamma Pseudo Maximum Likelihood, GPML, see Section 4.11. The estimation method is similar to PPML, but utilizes the gamma instead of the poisson distribution, thereby implies different assumptions to the data structure and does not allow for zero trade values. Even though Santos-Silva and Tenreyro (2006) argue in favor of PPML instead of GPML, especially in case of heteroscedasticity, Head and Mayer (2014) highlight that depending on data structure there exist cases where GPML is preferable to PPML.

An example of how to apply the function GPML to an example dataset in **gravity** and the resulting output is shown in the following:

```
1   > Gravity_no_zeros$lgdp_o <- log(Gravity_no_zeros$gdp_o)
2   > Gravity_no_zeros$lgdp_d <- log(Gravity_no_zeros$gdp_d)
3   >
4   > GPML(y="flow", dist="distw", x=c("rta", "lgdp_o", "lgdp_d"),
5   +       vce_robust=TRUE, data=Gravity_no_zeros)
6
7   Call:
8   y ~ dist_log + rta + lgdp_o + lgdp_d
9
10  Residuals:
11  Min      1Q Median      3Q      Max
12  −1.00   −0.98  −0.83   −0.35  505.68
13
14  Coefficients:
15  Estimate Std. Error z value Pr(>|z|)
16  (Intercept) 16.14322    0.75713   21.322  < 2e−16 ***
17  dist_log      −0.94013    0.08387  −11.210  < 2e−16 ***
18  rta            0.47097    0.10676    4.412 1.03e−05 ***
19  lgdp_o         0.80919    0.01965   41.188  < 2e−16 ***
20  lgdp_d         0.66129    0.03620   18.269  < 2e−16 ***
21  −−−
22  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
23
24  Residual standard error: 6.405 on 17083 degrees of freedom
```

4.13. Negative binomial pseudo maximum likelihood (NBPML)

For the main description of the kind of estimation method used for Negative Binomial Pseudo Maximum Likelihood, NBPML, see Section 4.11. The estimation method is similar to PPML, but utilizes the negative binomial instead of the poisson distribution and thereby implies different assumptions to the data structure. It also allows for zero values in the dependent variable. See e.g. De Benedictis and Taglioni (2011) for an application of this method and Head and Mayer (2014) for critics.

In order to illustrate all estimation methods using the same example dataset, the function NBPML is also applied to data restricted to positive trade flows even though it would allow trade flows to be zero. An exemplary output is shown in the following:

```
1   > Gravity_zeros$lgdp_o <- log(Gravity_zeros$gdp_o)
2   > Gravity_zeros$lgdp_d <- log(Gravity_zeros$gdp_d)
3   >
4   > NBPML(y="flow", dist="distw", x=c("rta", "lgdp_o", "lgdp_d"),
5   +                     vce_robust=TRUE, data=Gravity_zeros)
6
7   Call:
8   y ~ dist_log + rta + lgdp_o + lgdp_d
9
10  Weighted Residuals:
11  Min      1Q Median      3Q      Max
12  −0.53   −0.50  −0.42   −0.23  421.63
13
14  Coefficients:
15  Estimate Std. Error z value Pr(>|z|)
16  (Intercept) 17.12430    0.86571   19.781  < 2e−16 ***
17  dist_log      −1.01850    0.09513  −10.706  < 2e−16 ***
18  rta            0.60926    0.11726    5.196 2.04e−07 ***
19  lgdp_o         0.91558    0.01783   51.356  < 2e−16 ***
```

```
20    lgdp_d          0.73991      0.03693    20.037  < 2e−16 ∗∗∗
21    —––
22    Signif. codes:  0 '∗∗∗' 0.001 '∗∗' 0.01 '∗' 0.05 '.' 0.1 ' ' 1
23
24    Residual standard error: 4.23 on 22583 degrees of freedom
```

4.14. Nonlinear least squares (NLS)

For the main description of the kind of estimation method used for Nonlinear Least Squares, NLS, see Section 4.11. The estimation method is similar to PPML, but utilizes the negative binomial instead of the poisson distribution and thereby implies different assumptions to the data structure. It also allows for zero values in the dependent variable. As the method may not lead to convergence when poor starting values are used, the output of the PPML function is used for the starting values. For a comparison of different types of NLS to other methods, see e.g. Martin and Pham (2015) or Manning and Mullahy (2001). For critics see e.g. Head and Mayer (2014).

In order to illustrate all estimation methods using the same example dataset, the function NLS is also applied to data restricted to positive trade flows even though it would allow trade flows to be zero. An exemplary output is shown in the following:

```
1    > Gravity_zeros$lgdp_o <− log(Gravity_zeros$gdp_o)
2    > Gravity_zeros$lgdp_d <− log(Gravity_zeros$gdp_d)
3    >
4    > NLS(y="flow", dist="distw", x=c("rta","lgdp_o","lgdp_d"),
5    +                       vce_robust=TRUE, data=Gravity_zeros)
6
7    Call:
8    y ~ dist_log + rta + lgdp_o + lgdp_d
9
10   Weighted Residuals:
11   Min      1Q Median      3Q      Max
12   −83262      −1      0      4 210893
13
14   Coefficients:
15   Estimate Std. Error z value Pr(>|z|)
16   (Intercept) 11.79801     1.26431     9.332  < 2e−16 ∗∗∗
17   dist_log     −0.45502     0.14666    −3.103 0.001919 ∗∗
18   rta           1.04830     0.30710     3.414 0.000641 ∗∗∗
19   lgdp_o        0.90474     0.08423    10.742  < 2e−16 ∗∗∗
20   lgdp_d        1.16900     0.07440    15.713  < 2e−16 ∗∗∗
21   —––
22   Signif. codes:  0 '∗∗∗' 0.001 '∗∗' 0.01 '∗' 0.05 '.' 0.1 ' ' 1
23
24   Residual standard error: 3328 on 22583 degrees of freedom
```

5. Conclusion

As Head and Mayer (2014) or Martínez-Zarzoso (2013) point out there is no best estimation method for gravity models. Depending on the dataset at hand, the research question as well as robustness checks, it is up to the researcher to decide which method(s) to use. As especially for gravity beginners it is difficult to get an overview of the different possible estimation methods as well as to implement the methods in R, the package **gravity** provides a wrapper of different standard estimation methods for gravity models. Next to the more complicated methods DDM, BVU, BVW, SILC, Tetrads, and EK-Tobit which were programmed to lead identical results to the Stata code provided on Gravity Cookbook website[4] by Keith Head and Thierry Mayer, other common estimation methods for gravity models are added, namely OLS, Fixed Effects, PPML, GPML, NBPML, NLS, ET-Tobit, and Tobit. By considering the descriptions and codes of the estimation methods, users can get a comprehensive and application-oriented overview of the different methods, see which method may be suitable for certain research questions or type of data, and extend the code available in order to fit

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[4]https://sites.google.com/site/hiegravity/

their research projects. Furthermore, users can directly apply all methods to the example datasets `Gravity_no_zeros` or `Gravity_zeros`. For more advanced gravity models, e.g. the use of panel data, see the literature referred to in sections 1 and 2. Future work is to expand the functions to the needs of panel data, i.e. implementing grouped fixed effects as shown in Guimarães and Portugal (2009) in order to circumvent high dimensional fixed effects in large scale longitudinal applications. Contributions, extensions and error corrections are very welcome. Please do not hesitate to contact us.

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