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6—2 Gray Level Corner Detection

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Abstract

In this paper the analysis of gray level corner detections has been carried out. The performances of cornerness measures for some corner detection algorithms are discussed. This paper presents a new approach for corner detection called the Gradient-Direction corner detector which is developed from the popular Plessey corner detector. The Gradient-Direction corner detector is based on the measure of the gradient module of the image gradient direction and the constraints of false corner response suppression.

1 Introduction

In this paper, the term gray level corner denotes the gray-level transitions in a sufficiently large area around the prominent point of the gray level corner. This gray level corner point is usually defined as the point of maximal planar curvature in the line of the steepest gray-level slope (see [1]). Corner detection can simply mean to detect and localize this prominent point or can, in addition, include the determination of inherent attributes. In the scale space, each line pattern describing the corner location either persists, terminates or merges with a neighboring pattern. Moreover, new additional line patterns will not be introduced by smoothing (see [2]).

There are mainly two categories of gray lever corner detections, namely, template based corner detection and geometry based corner detection. Four performances of robustness must be addressed by all of the corner detections to compare their advantages and shortcomings.

Detection The corner detection should detect even the very subtle corners, with ignoring noise effects.

Localization The corners should be detected as close as possible to their true locations.

Stability The detected position of a corner should not move when multiple images are acquired of the same scene.

Complexity The reduced algorithm complexity contribute to more automation process and faster implementation.

We have defined several cornerness measure in the gray level images and implement corner detectors based on these measures.

2 Template Based Corner Detection

Template based corner detection involves determining the similarity, or correlation, between a given template size $n \times n$ and all sub-windows of size $n \times n$ in a given image (see [3]). We assume that the template is square, $n = 2k+1$. Let $P_{i,j}$ and $Q_{i,j}$ represent the (i,j) element of the template P and the image Q. In the template we define the mean α and the variance σ and in an window $n \times n$ centered at the (i,j) pixel of the image the mean β and the variance $\sigma_{i,j}$. Then a generalized correlation between the image and the template at the (i,j) pixel can be defined as:

$$\Delta_{i,j} = \frac{\frac{1}{n}\sum_{l=-k}^{k}\sum_{m=-k}^{k}(p_{l,m}-\alpha)(q_{i+l,j+m}-\beta)}{\sqrt{\sigma(p)\sigma(q)_{i,j}}} \tag{1}$$

Unfortunately it is impossible to design the templates which can cover all orientations and corner angles. Thus, the error is inevitable because of the complicated structure of the corners.

3 Geometry Based Corner Detection

Geometry based corner detection relies on measuring the differential geometry features of corner points. The methods can be divided into three ways: Edge-Related corner detection, Topology corner detection and Auto-Correlation corner detection.

Edge-Related corner detection considers the corner point as the junction of two or more edge lines. So that the corner point becomes the additional feature of edge points. The approaches can be done by two ways. One approach treats it as the two stage

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process of edge detection and edge grouping followed by digital curve representation. The other approach makes the use of the geometry of second differential operators from the standpoint of edge detector performance.

In 1982, Kitchen and Rosenfeld proposed a corner detector based on the change of gradient direction along an edge contour multiplied by the local gradient magnitude, that is:

$$\Delta_{Kitchen} = \frac{I_{xx}I_y^2 - 2I_{xy}I_xI_y + I_{yy}I_x^2}{I_x^2 + I_y^2} \qquad (2)$$

In 1995 Han Wang and Michael Brady (see [4]) proposed a corner detection algorithm based on the measurement of surface curvature, that is:

$$\kappa \approx \frac{\partial^2 I}{\partial t^2}/|\nabla I| \quad \text{where} \quad \nabla^2 I \gg 1$$

This cornerness measure means that the total curvature κ is proportional to the second derivative along the edge tangential t and is inversely proportional to the edge strength. Use the false response suppression, the cornerness measure becomes:

$$\Delta_{Wang} = \left(\frac{\partial^2 I}{\partial t^2}\right)^2 - S|\nabla I|^2 \qquad (3)$$

The Wang corner detector simplifies the cornerness measure so that it can be used in real-time corner detection for motion estimation.

Topology corner detection (see [5]) considers corner point as the interior geometric feature of image surface. So that these techniques work directly on a gray-level image. These are based on the measurement of the topological feature of differential geometry of corner in the image surface.

In 1978, Beaudet proposed a rotationally invariant corner detector which is derived from the Hessian Matrix H, second-order Taylor expression of intensity surface, that is:

$$\Delta_{Beaudet} = \begin{vmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{vmatrix} \qquad (4)$$

Auto-Correlation corner detection (see [6]) functions by considering a local window in the image, and determining the average changes of intensity which result from shifting the window by a small amount in various directions. Notes that when and only when all shifts result in a large change, the windowed image is a corner or isolated point.

Harris had performed an analytic expansion about the origin shift and thus derive the corner measure as follows:

$$\Delta_{Plessey} = \hat{I_x^2}\hat{I_y^2} - (\hat{I_xI_y})^2 - K(\hat{I_x^2} + \hat{I_y^2})^2 \qquad (5)$$

Where $\hat{I_x^2}$ $\hat{I_y^2}$ and $\hat{I_xI_y}$ are Gaussian scaled gradient-multiple images, that is,

$$\hat{I_x^2} = I_x^2 \otimes g \quad \hat{I_y^2} = I_y^2 \otimes g \quad \hat{I_xI_y} = (I_x \cdot I_y) \otimes g$$

This corner detector is also called the Plessey corner detector.

4 A New Approach

We define $g(x, y)$ as the Gaussian weighted coefficient at the location (x, y) within the image window:

$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{\frac{x^2+y^2}{2\sigma^2}}$$

We have mathematically analyzed the Plessey corner detector's cornerness measure which is defined as follows:

$$\Delta(x, y) = C_1(x, y) - KC_2(x, y)$$

$$\begin{aligned} \text{where} \quad C_1(x,y) &= \hat{I_x^2}\hat{I_y^2}(x,y) - (\hat{I_xI_y}(x,y))^2 \\ &= \frac{1}{2}\sum_{x_1,x_2,y_1,y_2} g(x_1,y_1)g(x_2,y_2) \\ &\quad [(I_x(x-x_1,y-y_1), I_y(x-x_1,y-y_1)) \\ &\quad \cdot(-I_y(x-x_2,y-y_2), I_x(x-x_2,y-y_2))]^2 \end{aligned}$$

$$C_2(x,y) = \hat{I_x^2}(x,y) + \hat{I_y^2}(x,y) \approx [I_x^2(x,y) + I_y^2(x,y)]^2$$

Note the following Taylor expressions:

$$I_x(x-u, y-v) \approx I_x(x,y) - uI_{xx}(x,y) - vI_{xy}(x,y)$$

$$I_y(x-u, y-v) \approx I_y(x,y) - uI_{xy}(x,y) - vI_{yy}(x,y)$$

Replace with these equalities,

$$C_1 \approx \sigma^2[(I_{xy}I_x - I_{xx}I_y)^2 + (I_{yy}I_x - I_{xy}I_y)^2]$$

Note that for most gradient operators I_x and I_y, the follows equalities are held:

$$\frac{\partial I_x}{\partial y} = 0 \quad \frac{\partial I_y}{\partial x} = 0$$

So that Taylor expressions simplify to be:

$$\begin{aligned} I_x(x-u, y-v) &\approx I_x(x,y) - uI_{xx}(x,y) \\ I_y(x-u, y-v) &\approx I_y(x,y) - vI_{yy}(x,y) \end{aligned}$$

Replace with these equalities,

$$C_1 \approx \sigma^2[I_{xx}^2I_y^2 + I_{yy}^2I_x^2]$$

The cornerness measure thus be simplified as follows:

$$\Delta = [I_{xx}I_y]^2 + [I_{yy}I_x]^2 - K[I_x^2 + I_y^2]^2 \qquad (6)$$

So that we can use it as the cornerness measure of the new corner detector. The most important improvement comparing with the Plessey corner detector is that it reduces the complexity yet achieves comparable results. Instead of calculating the three Gaussian smoothed gradient-multiple images, Only two second-order gradient-multiple images are required.

5 False Corner Response Suppression

The false corner response suppression is a common tool that widely be used in the image processing. Suppose that the cornerness measure exceeds a certain thresholding value K, that is:

$$\Delta_1(x,y) = \frac{C_1(x,y)}{C_2(x,y)} > K \qquad (7)$$

Multiplying both sides by $C_2(x,y)$, we find:

$$\Delta_2(x,y) = C_2(x,y)(\Delta_1 - K) > 0 \qquad (8)$$

We define the above inequality as the cornerness measure after the application of false corner response suppression. Usually we call the first term of the above equation $C_1(x,y)$ as the corner strength response. It is evident that both corners and edges can response it in the discrete case. The second term $\Delta_1 - K$ is the corner structure strength that responses well only at the corners. In common condition, the first term is a function of the intensity gradient $\nabla^2 I$ while the second term is a function of the corner angle θ, that is,

$$\Delta(x,y) = F(\nabla^2 I)(G(\theta) - K) > 0 \qquad (9)$$

Thus the aim of corner detection is reduced to look for the maximum wherever the above inequality is found.

In the previous applications (see [4]), they set K as a constant variable. Here we introduce a new kind of variable: partial variable $K(x,y)$ which is defined as the convolution of the Gaussian operator $G(\sigma,x,y)$ with the original cornerness measure $\Delta(x,y)$:

$$K(x,y) = g(\sigma,x,y) \otimes \Delta(x,y)$$

As we all know, constant variable K must vary with different differentiation masks and different Gaussian convolution which means it causes the corner detections to be instable and thus not suitable to the automation process; while partial variable $K(x,y)$ increase the application scope to satisfy changing image surface and removes the artificial effect of parameters to balance the performances of detection and localization. In general, the usage of $K(x,y)$ increase the performance of localization and detection but slightly adds the complexity of the corner detection.

6 The Geometry Property of Cornerness Measure

Let I(x,y) to be the image, $n = \frac{\nabla I}{|\nabla I|}$ the edge normal and t the edge tangential, we synthesize the Edge-Related corner detection as follows:

$$\Delta = \frac{\partial^2 I}{\partial t^2} / |\nabla I|^n \qquad (10)$$

Denote $\theta = \arctan\frac{I_y}{I_x}$ as the gradient direction, the cornerness measure of the new corner detector becomes:

$$\begin{aligned}\Delta(x,y) &= I_{xx}^2 I_y^2 + I_{yy}^2 I_x^2 - K[I_x^2 + I_y^2]^2 \\ &\approx [I_{xy}I_x - I_{xx}I_y]^2 + [I_{yy}I_x - I_{xy}I_y]^2 - K[I_x^2 + I_y^2]^2 \\ &= ||\nabla I(x,y)||^2[||\nabla\theta(x,y)||^2 - K]\end{aligned}$$

This inequality gives us a clue of how the new corner detector works. In fact, the first term of the above equation measures the gradient magnitude while the second term of equation provides a measure of the gradient module of the image gradient direction. Hence the aim of the corner detection is reduced to look for the maximum whenever the above inequality is found.

we exclude the constraint of false corner response suppression, the cornerness measure of the new corner detector can be expressed as follows:

$$\begin{aligned}\Delta(x,y) &= \frac{I_{xx}^2 I_y^2 + I_{yy}^2 I_x^2}{[I_x^2 + I_y^2]^2} \\ &\approx \frac{[I_{xy}I_x - I_{xx}I_y]^2 + [I_{yy}I_x - I_{xy}I_y]^2}{[I_x^2 + I_y^2]^2} \\ &= ||\nabla\theta(x,y)||^2\end{aligned}$$

Which means that the cornerness measure of the new Corner detector is simply the gradient module of the image gradient direction. So we call it the Gradient-Direction corner detector.

So that the Gradient-Direction cornerness measure is simply that: based on the measure of the gradient module of the image gradient direction, imposes the constraint of the false corner response suppression to obtain a clear corner response.

7 Experiments

In order to illustrate the efficiency of the Gradient-Direction corner detector on the synthetic Image, Figure 1,2,3, and 4 show examples of the promising results we have obtained by applying some corner detectors for noisy synthetic image. The Kitchen corner detector falsely detects some noisy edge points and misses the polygon vertexes. The Beaudet corner detector falsely detects some noisy points and detects some corners too far away from their true locations. The Plessey corner detector falsely detects some noisy edge points and fails to detect one polygon vertex. Only The Gradient-Direction corner detector has well detected and positioned all the features, which means that it keeps the best balance between the performance of detection and localization.

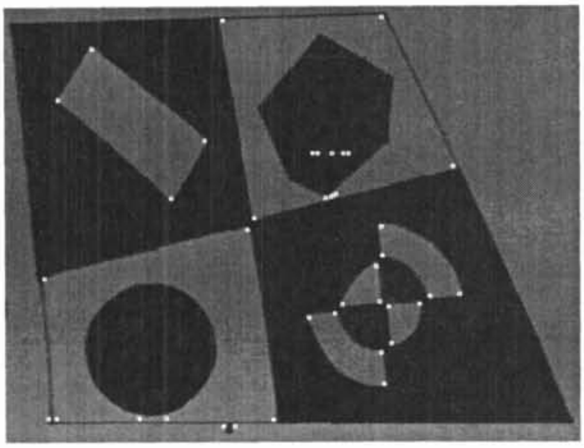

Figure 1: Kitchen corner detector's result

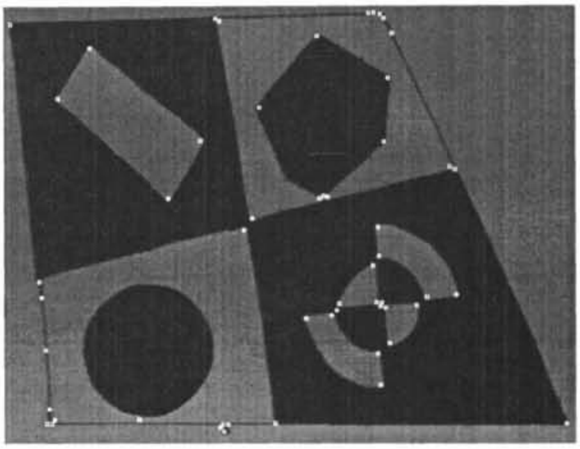

Figure 3: Plessey corner detector's result

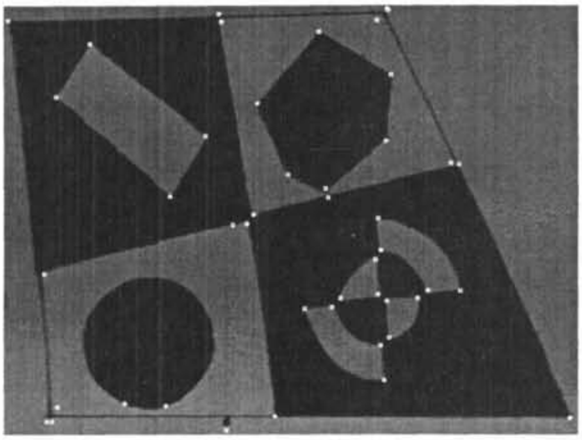

Figure 2: Beaudet corner detector's result

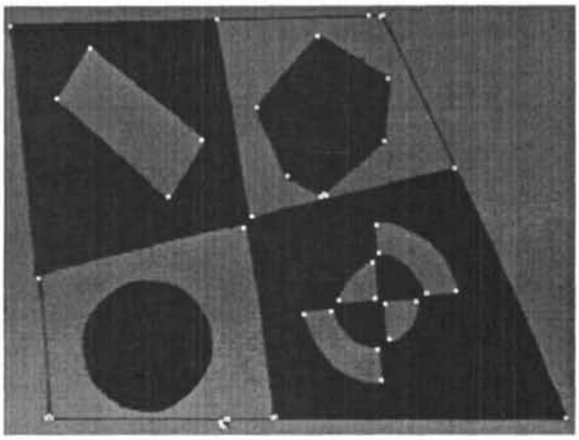

Figure 4: Gradient-Direction corner detector's result

8 Conclusion

We have proposed a new approach called the Gradient-Direction corner detector. Based on the measure of the gradient module of the image gradient direction, it simply imposes the constraint of the false corner response suppression to obtain a clear corner response. The proposed approach has been tested on real images in comparison with other corner detectors. We conclude that the Gradient-Direction corner detector is more practical for the corner detection than the Plessey corner detector with application in real-time image processing.

References

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