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GREEN NATIONAL ACCOUNTING:
WHY AND HOW?

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Abstract

The present paper gives an overview of the *theory* of green national accounting. Three purposes of green national accounting (measurement of sustainable income, social welfare, or net social profit) and two measures (*Green NNP* and *Hicksian income*) are considered. It is argued that sustainable income and social welfare correspond to different purposes. Under the assumption of no exogenous technological progress, Green NNP is shown to equal Hicksian income if there is a constant interest rate or if consumption is constant. It is established as a general result that sustainable income ≤ Hicksian income ≤ social welfare, while Green NNP ≤ social welfare under no exogenous technological progress and a constant utility discount rate. Green NNP is shown to measure *gross* social profit rather than *net* social profit.

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1. INTRODUCTION

During the last decades concern has been expressed for the long-term effects of natural resource depletion and environmental deterioration. This concern has spilled over into an interest in the question of whether national accounting can be "greened" by taking into account the changes in the stocks of natural and environmental resources. In particular, would such an expanded concept of Net National Product ("Green NNP") serve as a welfare measure? Furthermore, would Green NNP be able to indicate whether the actual development is sustainable?

The present paper seeks to give an overview of the *theory* of green national accounting. In particular, I will pose the following two questions:

- What purposes should green national accounting serve (*Why* do green national accounting)?
- What measures are available for these purposes (*How* to do green national accounting)?

The immense practical problems associated with obtaining data to estimate the suggested measures are not discussed here. I abstract from such problems since any practical method of green national accounting needs to be based on theoretical results. The purpose of this paper is to present a survey of such theoretical results.

Section 2 — which uses the elegant analysis of Dixit et al. (1980) as its point of departure — gives an interpretation of the main theorems of welfare economics in the present intertemporal (or rather, intergenerational) setting. Section 3 provides an overview of various purposes (measurement of sustainable income, social welfare, or net social profit) for doing green national accounting. Section 4 presents two measures that have been discussed in the literature – *Green NNP* (i.e. consumption plus the value of net investments) and *Hicksian income* (Hicks, 1946, Ch. 14) – and reviews, by comparing these measures, Weitzman's (1976) fundamental result on national accounting as well as extending results on green national accounting and sustainability that I have presented in previous work (Asheim, 1994, 1997). Section 5 then investigates to what extent these measures can serve the purposes of green national accounting discussed in Section 3. The objective is to give a comprehensive presentation of such results. Some of these results are novel, in particular the analysis of Green NNP and net social profit.

The present paper is a review of results, not a survey of literature. Hence, I have not attempted to include all relevant references; for an extensive bibliography as well as an interesting treatment of green national accounting, see Aronsson et al. (1997). As a consequence, some results may be presented without reference to the contributions where the results first appeared. In the mathematical analysis, I have put an emphasis on generality and elegance as well as economic interpretation at the expense of formal stringency. In particular, I have invoked an assumption of differentiability whenever needed. Some proofs have been relegated to an appendix.

2. Setting

To concentrate on issues that are central to this paper (and to the debate on green national accounting), I will make the following simplifying assumptions:

- *Constant population.* I will assume that each generation lives for one instance; i.e., generations are not overlapping nor infinitely lived. Distributional issues within each generation will not be discussed.
- *Constant and inelastic supply of "raw" labor.* This means that, by assumption, there is no trade-off between labor and leisure.
- *One consumption good* which is an indicator of instantaneous well-being derived from the situation that people live in. This indicator depends not only on material goods; rather, it is assumed to be increasing in the availability of environmental amenities, etc.

However, the analysis will allow for *multiple capital goods*. This is needed since the background for the interest in sustainability and green accounting is that human economic activity leads to depletion of natural capital. It is evident that a question like "is our accumulation of man-made capital sufficient to make up for the decreased availability of natural capital?" cannot be posed in a one-capital good setting.

In the real world environmental externalities are not always internalized. This is one of many causes that prevents market economies from being fully efficient. Furthermore, for many capital stocks (e.g., stocks of natural and environmental resources or stocks of accumulated knowledge) it is hard to find market prices (or to calculate shadow prices) that can be used to estimate the value of such stocks. In the present setting, I will abstract from these problems. I will assume the existence of an intertemporal competitive equilibrium that leads to efficiency and that provides market prices for all capital goods. Such an unrealistic assumption would undermine the relevance of the analysis if I would show that, with this assumption, all problems of green national accounting would be solved. On the contrary, I will try to convince the reader that, even with the existence of an intertemporal competitive equilibrium, there are hard challenges that remain.

In my setting — where generations follow in sequence, each living only an instance — dynamic efficiency is equivalent to Pareto-efficiency. This entails that there is an interesting relationship between techniques of dynamic optimization and the main theorems of welfare economics. Let me first indicate how the Second Welfare Theorem applies.

To illustrate, consider the case with only two generations. Assume that the technology is such that the set of feasible utility paths is convex. Then, for any efficient utility path (u_1^*, u_2^*) there exist utility discount factors (λ_1, λ_2) such that (u_1^*, u_2^*) maximizes $\lambda_1 u_1 + \lambda_2 u_2$ subject to (u_1, u_2) being feasible. This is illustrated by Figure 1. Say that (λ_1, λ_2) *supports* (u_1^*, u_2^*).

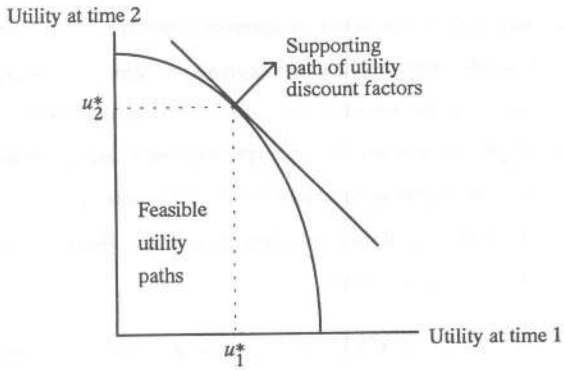

FIGURE 1.

Turn now to the case with an infinite number of generations. Assume that any efficient utility path is supported by a path of positive discount factors. In this context, the *Second Welfare*

Theorem can be restated as follows: Any utility path that is supported by utility discount factors can be implemented as a *competitive path* (where at each point in time consumers maximize utility and producers maximize profit) provided that each generation is given an endowment that enables it to achieve the utility level at its point in time. Hence, the intergenerational distribution is taken as given, and it is shown that there exist prices to which agents maximize.

The precise statement of this result requires that the general model – to be used throughout this paper – is presented. Following Dixit et al. (1980), I assume that consumption at time t, $c(t)$, the vector of capital stocks at time t, $\mathbf{k}(t)$, and the vector of investments at time t, $\dot{\mathbf{k}}(t)$, is feasible if $(c(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is in the set of feasible triples $F(t)$. Here, $c(t)$ is the indicator of well-being at time t, while $\mathbf{k}(t)$ comprises not only different kinds of manmade capital, but also stocks of natural capital and stocks of accumulated knowledge (thereby capturing *endogenous technological progress*). In contrast to Dixit et al. (1980) I also allow for *exogenous technological progress* by permitting the set of feasible triples to be time-dependent. I will assume that $F(t)$ is a closed and convex set that satisfies: (a) Capital stocks are non-negative ($(c, \mathbf{k}, \dot{\mathbf{k}}) \in F(t)$ implies $\mathbf{k} \geq 0$) and (b) free disposal of investment flows ($(c, \mathbf{k}, \dot{\mathbf{k}}) \in F(t)$ implies $(c, \mathbf{k}, \dot{\mathbf{k}}') \in F(t)$ if $\mathbf{k} \geq 0$ and $\dot{\mathbf{k}}' \leq \dot{\mathbf{k}}$). The latter assumption means e.g. that stocks of environmental resources are considered instead of stocks of pollutants. Lastly, consumption is non-negative and generates utility, $u(t) = u(c(t))$, where u is a time-invariant strictly increasing, concave, and differentiable function.

Let $p(t)$ denote the present value price of consumption at time t, and let $\mathbf{q}(t)$ denote the vector of present value prices of the capital stocks at time t. The term *present value* reflects that prices at different points in time are in terms of the same numeraire at time 0. Hence, these prices need not be discounted. State the following definition:

DEFINITION 1. The path $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is *competitive* at present value prices $(p(t), \mathbf{q}(t))_{t=0}^{\infty}$ and utility discount factors $(\lambda(t))_{t=0}^{\infty}$ if, at each t,

C1 instantaneous *utility* is maximized (i.e. $c^*(t)$ maximizes $\lambda(t)u(c) - p(t)c$)

C2 instantaneous *profit* is maximized (i.e. $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))$ maximizes $p(t)c + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ subject to $(c, \mathbf{k}, \dot{\mathbf{k}}) \in F(t)$).

Why is $p(t)c + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ instantaneous *profit*? By writing $\mathbf{Q}(t) = \mathbf{q}(t)/p(t)$ for the vector of current value prices of capital, we have that $\dot{\mathbf{Q}}(t) = \frac{d}{dt}\left(\frac{\mathbf{q}(t)}{p(t)}\right) = \frac{\dot{\mathbf{q}}(t)}{p(t)} - \frac{\dot{p}(t)}{p(t)}\frac{\mathbf{q}(t)}{p(t)} = \frac{\dot{\mathbf{q}}(t)}{p(t)} + r_0(t)\mathbf{Q}(t)$, where $r_0(t)$ is the instantaneous (consumption) interest rate at time t. Hence, $c + \frac{\mathbf{q}(t)}{p(t)}\dot{\mathbf{k}} + \frac{\dot{\mathbf{q}}(t)}{p(t)}\mathbf{k}$ = $c + \mathbf{Q}(t)\dot{\mathbf{k}} - \left(r_0(t)\mathbf{Q}(t) - \dot{\mathbf{Q}}(t)\right)\mathbf{k}$, where $c + \mathbf{Q}(t)\dot{\mathbf{k}}$ is the current value of production and $\left(r_0(t)\mathbf{Q}(t) - \dot{\mathbf{Q}}(t)\right)\mathbf{k}$ is the current cost of holding capital.

PROPOSITION 1. (*Second welfare theorem*) *If* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *maximizes* $\int_0^{\infty} \lambda(t)u(c(t))dt$ *subject to* $(c(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)) \in F(t)$ *for all t and* $\mathbf{k}(0) = \mathbf{k}^0$, *then there exist present value prices* $(p(t), \mathbf{q}(t))_{t=0}^{\infty}$ *such that* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is competitive at* $(p(t), \mathbf{q}(t))_{t=0}^{\infty}$ *and* $(\lambda(t))_{t=0}^{\infty}$.

The proof of this proposition derives the vector of capital prices though optimal control theory, while the consumption price simply measures the discounted value of marginal utility. The conclusion is that any utility path that is supported by utility discount factors is supported by present value prices of consumption and capital stocks. Hence, any Pareto-efficient path can be seen to be the outcome an intertemporal competitive equilibrium, provided that the intergenerational distribution is given by the consumption path $(c^*(t))_{t=0}^{\infty}$.

Turn now to the *First Welfare Theorem*, which in the present context can be restated as follows: Any competitive path (where consumers maximize utility and producers maximize profit) is efficient. However, this result requires certain regularity conditions.

DEFINITION 2. The competitive path $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is *regular* at present value prices $(p(t), \mathbf{q}(t))_{t=0}^{\infty}$ and utility discount factors $(\lambda(t))_{t=0}^{\infty}$ if,

R1 $\int_0^{\infty} \lambda(t)u(c^*(t))dt$ exists (and is finite)

R2 $\mathbf{q}(t)\mathbf{k}^*(t) \to 0$ as $t \to \infty$.

PROPOSITION 2. (*First welfare theorem*) *If* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is regular at present value prices* $(p(t), \mathbf{q}(t))_{t=0}^{\infty}$ *and utility discount factors* $(\lambda(t))_{t=0}^{\infty}$, *then* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *maximizes* $\int_0^{\infty} \lambda(t)u(c(t))dt$ *subject to* $(c(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)) \in F(t)$ *for all t and* $\mathbf{k}(0) = \mathbf{k}^0$.

Provided that the utility discount factors are positive, this means that any competitive path satisfying the regularity conditions R1 and R2 is efficient. Hence, with these qualifications, any intertemporal competitive equilibrium is Pareto-efficient.

I end this section with the following useful lemma.

LEMMA 1. *(i) If* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is a competitive path with* $c^*(t) > 0$, *then* $\lambda(t)u'(c^*(t)) = p(t)$. *(ii) (Dixit et al., 1980) If F(t) is smooth and time-invariant (i.e., no exogenous technological progress) and* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is a competitive path, then* $p(t)\dot{c}^*(t) + \frac{d}{dt}\left(\mathbf{q}(t)\dot{\mathbf{k}}^*(t)\right) = 0$.

Hence, if there is no exogenous technological progress and $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is a competitive path satisfying that $\mathbf{q}(t)\dot{\mathbf{k}}^*(t) \to 0$ as $t \to \infty$, then $\mathbf{q}(0)\dot{\mathbf{k}}^*(0) = \int_0^{\infty} p(t)\dot{c}^*(t)dt$. In particular, if $\dot{c}^*(t) = 0$ for all t, then $\mathbf{q}(0)\dot{\mathbf{k}}^*(0) = 0$. This is the converse of Hartwick's rule (Hartwick (1977), Dixit et al. (1980), Withagen and Asheim (1997)).

3. PURPOSE

What purpose should green national accounting serve? There are at least three different purposes that have been mentioned in the literature.

3a Sustainable income

Let $y(0)$ be the maximum consumption that can be sustained from time 0 on, given the capital stocks $\mathbf{k}^0$ that generation 0 has inherited. Refer to $y(0)$ as *sustainable income* at time 0. In the wake of the World Commission on Environment and Development — which popularized the term 'sustainable development' through its report (WCED, 1987) — the following question has gained attention: Is the present management of natural and environmental resources compatible with sustainable development? If $y(0)$ could be measured, it would in fact be possible to answer this question by comparing actual consumption to the sustainable income.

The normative relevance of sustainable income was discussed by Solow (1974) in his application of the Rawlsian maximin principle in an intertemporal setting. More recently, Buchholz (1997) has provided a normative foundation for sustainability by imposing the following two axioms on any order on the set of feasible consumption paths (the statement of these axioms requires that time is discrete):

- WEAK ANONYMITY (**WA**): Indifference between two paths if the one can be obtained from the other by changing the sequence of a *finite* number of items.
- STRONG PARETO (**SP**): One path is strictly preferred to another path if it has higher consumption at some date, without having lower consumption at any other date.

Some degree of intergenerational equity is ensured by **WA**, which seems compelling if there is a negligible probability that the economy will cease to exist. Some responsiveness to the interest of a single generation is ensured by **SP**, which seems quite uncontroversial. The partial order induced by the axioms is illustrated by Figure 2 in the two generation case. Since A and B are symmetrical around the 45° line, **WA** deems the paths A and B as indifferent since one can be obtained from the other by permuting consumption at times 1 and 2. By **SP** and transitivity, a path like C in the shaded area is strictly preferred to A.

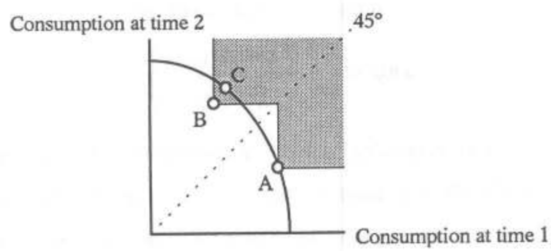

FIGURE 2.

Consider now a *productive* economy, entailing that if $s < t$ and $c_s > c_t$, then a permutation of s and t is feasible, and moreover, there will be some consumption left-over after such a permutation. In the two-generation case, an assumption of a productive economy means that the set of feasible consumption path looks like the set inside the curved line in Figure 2. It follows that an efficient, but decreasing path like A cannot be Buchholz-maximal, since a path like C is feasible and strictly preferred to A. This argument carries over to the case with an infinite number of generations: Any Buchholz-maximal path is efficient and non-decreasing.

One should think of **WA** and **SP** as agreed on by generations in the original position (behind a veil of ignorance). Generations in the original position can only agree on widely shared and weak conditions. The resulting incompleteness means that generations are granted some leeway when determining their capital management, even if **WA** and **SP** are imposed. In productive technologies, imposing **WA** and **SP** is equivalent to imposing sustainability as a side constraint. Thus, the Buchholz criterion yields a solid foundation for sustainability (as often interpreted) in productive technologies, while being consistent with the view that talk of sustainability in a non-productive economy (e.g. a cake-eating economy) makes little sense.

3b Social welfare

Assume that generation 0 seeks to maximize $\int_0^\infty \lambda(t)u(c(t))dt$ over all feasible paths given the capital stocks $\mathbf{k}^0$ that generation 0 has inherited. Following Weitzman (1970), consider the level of consumption $w(0)$ that if held constant will yield the same welfare as the welfare maximizing path $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^\infty$. I.e., $\int_0^\infty \lambda(t)u(w(0))dt = \int_0^\infty \lambda(t)u(c^*(t))dt$, or

$$w(0) = u^{-1}\left(\frac{\int_0^\infty \lambda(t)u(c^*(t))dt}{\int_0^\infty \lambda(t)dt}\right).$$

Refer to $w(0)$ — which is a consumption index of $\int_0^\infty \lambda(t)u(c^*(t))dt$ — as *social welfare* at time 0.

A measure of social welfare expressed in terms of consumption can be of interest for different reasons. One reason is that one wants to measure whether an economy grows. If $\lambda(t)$ is an exponentially decreasing function — i.e., $\lambda(t) = \lambda(0)e^{-\delta t}$, meaning that there is a constant utility discount rate $\delta = -\frac{\dot{\lambda}(t)}{\lambda(t)}$, and implying that the mapping from $(c^*(s))_{s=t}^\infty$ to $w(t)$ is time-invariant — then an increase in social welfare can be interpreted as growth. A foundation for the case where $\lambda(t)$ is an exponentially decreasing function has been provided by Koopmans (1960). In discrete time this case corresponds to each generation's welfare w_t being a stationary function of its own consumption c_t and the welfare of the immediate descendants w_{t+1}:

$$u(w_t) = (1-\beta)u(c_t) + \beta u(w_{t+1}), \quad \text{where } \beta = \exp(-\delta\Delta) \text{ with } \Delta \text{ denoting the period length.}$$

In my definition of $w(0)$ I have not imposed that $\lambda(t)$ be an exponentially decreasing function. The non-exponential case is of interest e.g. when — following Solow (1974) — the maximin principle is applied in the Dasgupta-Heal-Solow model (see Dasgupta and Heal (1974) and Solow (1974)). Then the economy behaves *as if* it maximizes $\int_0^{\infty} \lambda(t)u(c(t))dt$ with the instantaneous discount rate $-\frac{\dot{\lambda}(t)}{\lambda(t)}$ decreasing over time. Since $\lambda(t)$ depends on absolute time only, this does not lead to a time-inconsistency problem.

Another reason for being interested in measures of social welfare is that one wants to pose the question: Which of two economies is better off at the same point in time? If $(\lambda(t))_{t=0}^{\infty}$ is proportional in the two different economies, a comparison of social welfare will answer this question. Note that a comparison of sustainable income will not necessarily give the correct answer. The following example illustrates this. Let two economies 1 and 2 have the same constant utility discount rate δ, and the same capital stock $k^1(0) = k^2(0) = 1$ at time 0 (capital is here assumed to be a scalar). Furthermore, assume that the stationary set of feasible triples $(c^i, k^i, \dot{k}^i)$ in economy i is given by $c^i + \dot{k}^i \leq (k^i)^{\alpha^i}$, where $\delta = \alpha^1$ and $\delta < \alpha^2 < 1$. Hence, both countries are endowed with a Ramsey technology, entailing that sustainable income is the level of consumption that results if capital is held constant (see Section 5a). Standard calculations yield that economy 1 will wish to keep its capital stock constant since $\alpha^1(k^1(0))^{\alpha^1-1} = \alpha^1 = \delta$, while economy 2 will wish to accumulate capital since $\alpha^2(k^2(0))^{\alpha^2-1} = \alpha^2 > \delta$. Hence, for economy 2, the level of consumption that if held constant will yield the same welfare as the welfare maximizing path, is greater than the level of consumption that results if capital is held constant. Thus, $1 = y^1(0) = w^1(0) = y^2(0) < w^2(0)$. In this example is correct to say that economy 2 is better off (since the optimal path of economy 1 is feasible but not optimal for economy 2). This conclusion would be obtained if social welfare was measured, but not if sustainable income was measured.

In the example above, economy 2 accumulates more capital than economy 1 because its marginal productivity of capital is higher. Another situation would result if economy 2 has accumulated more capital, not because its technology is more productive, but rather, because it puts more weight on future utility through a lower utility discount rate. In this case, social welfare cannot be used for the comparison between the economies, while sustainable income — being dependent only on the technological constraints — might provide some comparative measure of the degree of development of the two economies.

3c Net social profit

A third purpose of green national accounting is to develop a criterion function for social cost-benefit analysis. Such a criterion function is an index having the property that the acceptance of a "small" policy change increases the discounted intertemporal sum of the index if and only the policy change leads to a potential Pareto improvement. This approach is based on the theory of social cost-benefit analysis (see Dasgupta, Marglin and Sen (1972) and Little and Mirrlees (1974)) and has been promoted in a series of papers by Dasgupta, Kriström and Mäler (1995, 1997). In the remainder of this paper, the value of such an index at time 0 will be denoted $\pi(0)$ and referred to as *net social profit*.

4. Measures

Green national accounting seeks to serve one or more of the purposes described in Section 3 by calculating a measure based on current prices and quantities. Two such measures, *Green NNP* and *Hicksian income*, are of special interest.

4a Green NNP

Green NNP is the sum of consumption and the value of net investments:

$$g(0) = c(0) + \mathbf{Q}(0)\dot{\mathbf{k}}(0).$$

I assume that the vector of capital goods, $\mathbf{k}(0)$, comprises all kinds of manmade capital (including stocks of accumulated knowledge) and all kinds of natural capital (including stocks of environmental resources). As in Section 2, $\mathbf{Q}(0) = \frac{\mathbf{q}(0)}{p(0)}$ denotes the vector of capital prices in terms of current consumption.

4b Hicksian income

The term 'Hicksian income' arises from Hicks' (1946) treatment of the notion of income in Chapter 14 of the second edition of *Value and Capital*. After considering the possibilities of changing interest rates and changing prices, Hicks writes on p. 174:

> "Income No.3 must be defined as the maximum amount of money which the individual can spend this week, and still expect to be able to spend the same amount *in real terms* in each ensuing week".

Then on p. 184:

> "The standard stream corresponding to Income No.3 is constant in real terms ...". "We ask ... how much he would be receiving if he were getting a standard stream of the same present value as his actual expected receipts. This amount is his income."

Whether income is associated with sustainable income (as on p. 174) or with the consumption that, if kept constant, would yield the same present value as the person's actual future receipts (as on p. 184) does not make any difference at a personal level since a price taker can turn the actual consumption path into a constant consumption path with the same present value. At a national level, however, these notions need not coincide (see Section 5a below). Since the latter notion in principle can be calculated in a marked economy, I choose to associate the term 'Hicksian income' at a national level with the consumption that, if kept constant, would yield the same present value as the actual future consumption path (see also Weitzman (1976)). If the regular path $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ is followed, and $h(0)$ denotes Hicksian income at time 0, we have that $\int_0^\infty p(t)h(0)dt = \int_0^\infty p(t)c^*(t)dt$, or

$$h(0) = \frac{\int_0^\infty p(t)c^*(t)dt}{\int_0^\infty p(t)dt}.$$

In this form, Hicksian income is not based on current prices and quantities only. In Section 4d I will show that there are circumstances under which Hicksian income can be expressed through current prices and quantities only.

4c Comparison of Green NNP and Hicksian income

If the present value price associated with consumption at time t, $p(t)$, is an exponentially decreasing function — i.e., $p(t) = p(0)e^{-rt}$ — then there is one constant (consumption) interest rate: $r = -\frac{\dot{p}(t)}{p(t)} = \frac{p(t)}{\int_t^\infty p(s)ds}$. If not, there is a term structure of interest rates. The *instantaneous interest rate* is $r_0(t) = -\frac{\dot{p}(t)}{p(t)}$, while the *infinitely long-term interest rate* is $r_\infty(t) = \frac{p(t)}{\int_t^\infty p(s)ds}$. Note that even if the utility discount factor $\lambda(t)$ is an exponentially decreasing function, so that there is a constant utility discount rate, there need not be a constant interest rate. E.g. in the Ramsey model, there is a constant interest rate only if consumption is constant. In certain resource models, like the Dasgupta-Heal-Solow model, not even a constant consumption path leads to a constant interest rate.

Using the expression for $r_\infty(t)$, Hicksian income can be written as

$$h(0) = r_\infty(0)\int_0^\infty \tfrac{p(t)}{p(0)} c^*(t)dt.$$

Since Lemma 1(ii) implies that $\mathbf{q}(0)\dot{\mathbf{k}}^*(0) = \int_0^\infty p(t)\dot{c}^*(t)dt$ under the assumption of no exogenous technological progress, it follows that $-p(0)c^*(0) = \int_0^\infty \big(p(t)\dot{c}^*(t) + \dot{p}(t)c^*(t)\big)dt = \mathbf{q}(0)\dot{\mathbf{k}}^*(0) + \int_0^\infty \dot{p}(t)c^*(t)dt$. Hence, as shown by Sefton and Weale (1996), Green NNP can be written as $g(0) = c^*(0) + \mathbf{Q}(0)\dot{\mathbf{k}}(0) = \frac{1}{p(0)}\big(p(0)c^*(0) + \mathbf{q}(0)\dot{\mathbf{k}}^*(0)\big) = -\int_0^\infty \frac{\dot{p}(t)}{p(0)} c^*(t)dt$, or, using the expression for $r_0(t)$:

$$g(0) = \int_0^\infty r_0(t)\tfrac{p(t)}{p(0)} c^*(t)dt.$$

This yields the following result.

PROPOSITION 3. *If there is no exogenous technological progress, then Green NNP is greater than Hicksian income if and only if* $\int_0^\infty r_0(t)\frac{p(t)}{p(0)} c^*(t)dt$ *is greater than* $r_\infty(0)\int_0^\infty \frac{p(t)}{p(0)} c^*(t)dt$.

Given the assumption of no exogenous technological progress, there are two special cases for which Green NNP and Hicksian income coincide: (*i*) If there is a constant interest rate (i.e., $r_0(t) = r_\infty(t) = r$, all t), then $g(0) = h(0) = r\int_0^\infty \frac{p(t)}{p(0)} c^*(t)dt$. This is Weitzman's (1976) fundamental result on green national accounting in his seminal contribution: If there is a constant interest rate under no exogenous technological progress, then Green NNP equals Hicksian income. Hence,

$$\int_0^\infty p(t)\big(c^*(0) + \tfrac{\mathbf{q}(0)}{p(0)}\dot{\mathbf{k}}^*(0)\big)dt = \int_0^\infty p(t)c^*(t)dt.$$

(*ii*) If consumption is constant (i.e., $c^*(t) = c^*$, all t), then $g(0) = h(0) = c^*$, since it follows from the definitions of $r_0(t)$ and $r_\infty(t)$ that $\int_0^\infty r_0(t)\frac{p(t)}{p(0)}dt = r_\infty(0)\int_0^\infty \frac{p(t)}{p(0)}dt = 1$.

If these two special cases do not apply, but the assumption of no exogenous technological progress holds, then it follows from Proposition 3 that Hicksian income exceeds Green NNP whenever consumption tends to increase (decrease) and interest rates tend to decrease (increase).

This is indeed the case in the Ramsey model when there is a constant utility discount rate and the initial capital stock is smaller (larger) than the size for which the marginal productivity of capital equals the utility discount rate. However, Green NNP exceeds Hicksian income whenever both consumption and interest rates tend to decrease. This can occur in the Dasgupta-Heal-Solow model; see the example of Asheim (1994, Section IV).

4d An expression for Hicksian income

The purpose of this subsection is to express Hicksian income, $h(0) = r_{\infty}(0)\int_0^{\infty} \frac{p(t)}{p(0)} c^*(t)dt$, in terms of current prices and quantities. Since $\frac{d}{dt}\left(\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds\right) = -c^*(t) + r_0(t)\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds$, it follows that $\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds = \frac{1}{r_0(t)}\left(c^*(t) + \frac{d}{dt}\left(\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds\right)\right)$. Hence,

$$h(t) = \frac{r_{\infty}(t)}{r_0(t)}\left(c^*(t) + \frac{d}{dt}\left(\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds\right)\right).$$

Assume now that the technology exhibits constant returns to scale (CRS). This assumption is in the spirit of Lindahl (1933, pp. 401–2) and implies that all factors of production, including labor, are dealt with as capital that is evaluated by the present value of future earnings. It amounts to assuming that all flows of future earnings can be treated as currently existing capital. CRS means that in the hypothetical case where all capital stocks were a given percentage larger, consumption and investments could be increased by the same percentage. This clearly allows for stocks in fixed supply — like "raw labor" and land — that cannot actually be accumulated. Although being informational demanding, the assumption of CRS is *not* technologically restrictive since the existence of an intertemporal competitive equilibrium entails that returns to scale are nondecreasing. Hence, CRS can be obtained by adding an additional fixed capital stock with which returns to scale become constant.

The importance of the assumption of CRS is revealed through the following lemma.

LEMMA 2. *If, for each t, F(t) is a convex cone (i.e., the technology exhibits CRS), and* $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ *is a regular path, then* $\mathbf{q}(t)\mathbf{k}^*(t) = \int_t^{\infty} p(s)c^*(s)ds$.

Hence, under CRS, the value of future consumption in terms of current consumption is equal to the value of current capital stocks: $\int_t^{\infty} \frac{p(s)}{p(t)} c^*(s)ds = \frac{\mathbf{q}(t)}{p(t)}\mathbf{k}^*(t) = \mathbf{Q}(t)\mathbf{k}^*(t)$. This implies that

$$h(t) = \frac{r_\infty(t)}{r_0(t)}\left(c^*(t) + \frac{d}{dt}\left(\mathbf{Q}(t)\mathbf{k}^*(t)\right)\right) = \frac{r_\infty(t)}{r_0(t)}\left(c^*(t) + \mathbf{Q}(t)\dot{\mathbf{k}}^*(t) + \dot{\mathbf{Q}}(t)\mathbf{k}^*(t)\right).$$

Thus, we have established the following result.

PROPOSITION 4. *If the technology exhibits constant returns to scale, then*

$$h(0) = \frac{r_\infty(0)}{r_0(0)}\left(g(0) + \dot{\mathbf{Q}}(0)\mathbf{k}^*(0)\right).$$

This result means that in order to arrive at Hicksian income ($h(0)$),

- *anticipated capital gains* ($\dot{\mathbf{Q}}(0)\mathbf{k}^*(0)$) must be added to Green NNP ($g(0)$),
- the sum $g(0) + \dot{\mathbf{Q}}(0)\mathbf{k}^*(0)$ must be adjusted for *interest rate effects* if there is not a constant interest rate, in which case $\frac{r_\infty(0)}{r_0(0)}$ need not equal 1.

*Un*anticipated capital gains (which fall outside the deterministic framework of the present paper) cannot be fully added to Green NNP. Only the interest on such windfall gains constitute current income (see e.g. Hicks (1946, p. 179)). The infinitely long-term interest rate, $r_\infty(0)$, is available as a market price at time 0 if there is a market for bonds with perpetual yield. To see this, observe that $\frac{1}{r_\infty(0)} = \frac{\int_0^\infty p(t)dt}{p(0)}$ is the price in terms of current consumption of a bond that pays one unit of consumption in perpetuity (in other words, $\frac{1}{r_\infty(0)}$ is the price of a consumption annuity).

One may discuss whether Proposition 4 is a useful result for practical estimation given the informational burden of the CRS case. The result is, however, informative since it shows how, in principle, exogenous technological progress can be measured through its effect on capital gains. Such exogenous technological progress is relevant (i) for an economy where accumulated knowledge cannot be represented by augmented capital stocks (see e.g. Aronsson and Löfgren (1995), Kemp and Long (1982), Weitzman (1997)), and (ii) for open economies whose "technology" is changing exogenously when resource prices influence their terms of trade (see e.g. Asheim (1996), Sefton and Weale (1996), Vincent et al. (1997)).

5. ABILITY TO MEASURE

The present section investigates to what extent Green NNP, $g(0)$, and Hicksian income, $h(0)$, can serve as measures of sustainable income, $y(0)$, social welfare, $w(0)$, or net social profit, $\pi(0)$.

5a Measuring sustainable income

In the Ramsey model — which one capital good technology is described by $c + \dot{k} \leq f(k)$, with f being a concave and strictly concave function — *Green NNP* measures sustainable income. To see this, note that $g(0) = c^*(0) + \dot{k}^*(0)$, while $f(k^*(0))$ is the sustainable income given that generation 0 has inherited the capital stock $k^*(0) = k^0$. Hence, since efficiency implies that $c^*(0) + \dot{k}^*(0) = f(k^*(0))$, it follows that $g(0) = c^*(0) + \dot{k}^*(0) = f(k^*(0)) = y(0)$.

This result seems not to generalize. Since Lemma 1(ii) implies that $\mathbf{q}(0)\dot{\mathbf{k}}^*(0) = 0$ if $c^*(t) = c^*$ for all t under the assumption of no exogenous technological progress, it follows that $g(0) = c^* = y(0)$ for a constant consumption path in a stationary technology. In the case of a constant interest rate under no exogenous technological progress, Green NNP equals Hicksian income and thus overestimates sustainable income (see Proposition 5 below). No general result seems, however, to be available on the relation between $g(0)$ and $y(0)$ when neither consumption nor the interest rate is constant, not even under the assumption of no exogenous technological progress. In Asheim (1994, Section IV) I show by way of an example in the Dasgupta-Heal-Solow model that it is possible construct situations in which $g(0) > c^*(0) > y(0)$. This can occur in their model under discounted utilitarianism since the scarcity of the non-renewable resource leads to an inverted-U shaped consumption path (see Pezzey and Withagen (1997)) and an decreasing interest rate. This means that a consumption path can be constructed where initial consumption $c^*(0)$ is slightly above $y(0)$, while $g(0) > h(0)$ (by Proposition 3) and $h(0) > y(0)$ (by Proposition 5 below). Hence, even when Green NNP exceeds consumption (implying that the value of net investments $\mathbf{Q}(0)\dot{\mathbf{k}}^*(0)$ is positive), consumption may be at an unsustainable level.

At the level of a small open economy faced with given international prices, *Hicksian income* measures sustainable income. The reason is (as pointed out by e.g. Brekke (1997) and Vincent et al. (1997)) that the actual consumption path can be turned into a constant consumption path with the same present value. However, at the level of a closed economy with a non-linear technology, and at the level of a large open economy that influences international prices, Hicksian income overestimates sustainable income, since turning the actual consumption path into a constant consumption path will lead to a loss of present value. This is illustrated by Figure 3.

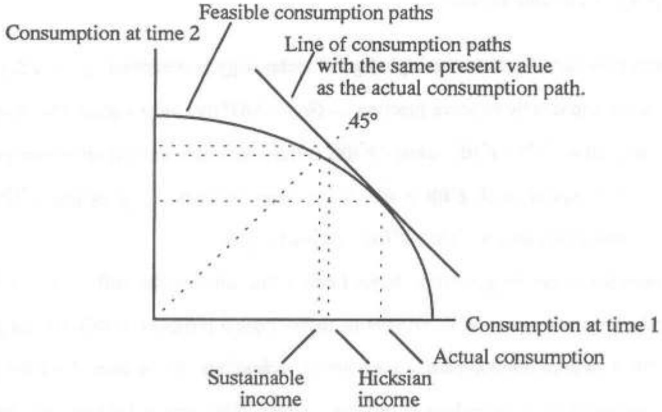

FIGURE 3.

PROPOSITION 5. *Hicksian income $h(0)$ is greater than or equal to sustainable income $y(0)$.*

It follows directly from the definition of $h(0)$ that $h(0) = c^* = y(0)$ if consumption is constant along the actual path. Note that an assumption of no exogenous technological progress is *not* necessary for the general result of Proposition 5 nor for the result in the special case of a constant consumption path.

5b Measuring social welfare

If there is a constant utility discount rate under no exogenous technological progress, then it follows from a generalization of Weitzman's (1976) result that

$$\int_0^\infty \lambda(t)\left(u(c^*(0)) + \tfrac{\mathbf{q}(0)}{\lambda(0)}\dot{\mathbf{k}}^*(0)\right)dt = \int_0^\infty \lambda(t)u(c^*(t))dt$$

(see Weitzman (1970) and Kemp and Long (1982)). This means that "Green NNP in terms of utility", $u(c^*(0)) + \frac{\mathbf{q}(0)}{\lambda(0)}\dot{\mathbf{k}}^*(0)$, is equal to the utility derived from social welfare, $u(w(0))$:

$$u(c^*(0)) + \tfrac{\mathbf{q}(0)}{\lambda(0)}\dot{\mathbf{k}}^*(0) = \frac{\int_0^\infty \lambda(t)u(c^*(t))dt}{\int_0^\infty \lambda(t)dt} = u(w(0)).$$

As a foundation for using Green NNP, $g(0)$, Hartwick (1990) observed that $u'(c^*(0))\cdot g(0)$ is a linear approximation of "Green NNP in terms of utility" and thus of $u(w(0))$:

$$u'(c^*(0))\cdot g(0) = u'(c^*(0))\cdot c^*(0) + u'(c^*(0))\cdot \tfrac{q(0)}{p(0)}\dot{\mathbf{k}}^*(0) = u'(c^*(0))\cdot c^*(0) + \tfrac{q(0)}{\lambda(0)}\dot{\mathbf{k}}^*(0),$$

since $\lambda(0)u'(c^*(0)) = p(0)$ by Lemma 1(i). It turns out that this approximation is biased since the concavity of u implies that $u'(c^*(0))\cdot(w(0) - c^*(0)) \geq u(w(0)) - u(c^*(0)) = \tfrac{q(0)}{\lambda(0)}\dot{\mathbf{k}}^*(0) = \cdot\tfrac{q(0)}{p(0)}\dot{\mathbf{k}}^*(0) = u'(c^*(0))\cdot(g(0) - c^*(0))$. Hence, $w(0) \geq g(0)$.

PROPOSITION 6. *If there is no exogenous technological progress and the utility discount factor $\lambda(t)$ is an expontentially decreasing function, then Green NNP $g(0)$ is smaller than or equal to social welfare $w(0)$.*

Given the assumptions of no exogenous technological progress and a constant utility discount rate, there are two special cases for which Green NNP and social welfare coincide: (*i*) If u is linear, so that $u'(c^*(0))\cdot(w(0) - c^*(0)) = u(w(0)) - u(c^*(0))$, then $g(0) = w(0)$. (*ii*) If the value of net investments $\mathbf{Q}(0)\dot{\mathbf{k}}^*(0)$ is equal to zero, then $g(0) = c^*(0) = w(0)$. By Lemma 1(ii), this latter case is equivalent to the present value of future changes in consumption $\int_0^\infty \frac{p(t)}{p(0)}\dot{c}^*(t)dt$ being equal to zero. Hence, consumption being constant ($c^*(t) = c^*$, all t) is sufficient, but not necessary for $\mathbf{Q}(0)\dot{\mathbf{k}}^*(0)$ being equal to zero.

Also *Hicksian income* tends to underestimate social welfare. However, this is a perfectly general result which is valid also if there *is* exogenous technological progress and if there is *not* a constant utility discount rate.

PROPOSITION 7. *Hicksian income $h(0)$ is smaller than or equal to social welfare $w(0)$.*

To see this result, note that the definition of $h(0)$ implies that $\int_0^\infty \lambda(t)u'(c^*(t))\big(h(0) - c^*(t)\big)dt = 0$, since $\lambda(t)u'(c^*(t)) = p(t)$ by Lemma 1(i). Hence, since $\int_0^\infty \lambda(t)u(w(0))dt = \int_0^\infty \lambda(t)u(c^*(t))dt$ by the definition of $w(0)$, it follows from the concavity of u that

$$\int_0^\infty \lambda(t)(u(w(0)) - u(h(0)))dt = \int_0^\infty \lambda(t)\Big(u(c^*(t)) - u(h(0)) + u'(c^*(t))\big(h(0) - c^*(t)\big)\Big)dt \geq 0.$$

Note that $h(0) = w(0)$ if u is linear. Moreover, it follows directly from the definitions of $h(0)$ and $w(0)$ that $h(0) = c^* = w(0)$ if consumption is constant.

If $u(c) = \ln c$, then $w(0)$ is a Cobb-Douglas functional of $(c^*(t))_{t=0}^\infty$:

$$w(0) = \exp\left(\frac{\int_0^\infty \lambda(t)\ln c^*(t)dt}{\int_0^\infty \lambda(t)dt}\right).$$

Likewise, if $u(c) = \frac{c^\varrho}{\varrho}$, $\varrho \leq 1$, $\varrho \neq 0$, then $w(0)$ is a CES functional of $(c^*(t))_{t=0}^\infty$:

$$w(0) = \left(\frac{\int_0^\infty \lambda(t)\left(c^*(t)\right)^\varrho dt}{\int_0^\infty \lambda(t)dt}\right)^{\frac{1}{\varrho}}.$$

Using the theory of expenditure functions for Cobb-Douglas and CES functions, explicit expressions for social welfare can be found. For the statements of these expressions, it is useful to define $\tilde{r}_\infty(t)$ as follows:

$$\tilde{r}_\infty(t) = \begin{cases} \dfrac{p(t)}{\left(\int_t^\infty \lambda(s)ds\right)\cdot \exp\left(\frac{\int_t^\infty \lambda(s)\ln(p(s)/\lambda(s))ds}{\int_t^\infty \lambda(s)ds}\right)} & \text{if } u(c) = \ln c \\ \dfrac{p(t)}{\left(\int_t^\infty \lambda(s)ds\right)\cdot\left(\frac{\int_t^\infty \lambda(s)(p(s)/\lambda(s))^{\varrho/(\varrho-1)}ds}{\int_t^\infty \lambda(s)ds}\right)^{(\varrho-1)/\varrho}} & \text{if } u(c) = \frac{c^\varrho}{\varrho} \end{cases}$$

PROPOSITION 8. *If* $u(c) = \ln c$ *or if* $u(c) = \frac{c^\varrho}{\varrho}$, $\varrho < 1$, $\varrho \neq 0$, *then*

$$w(0) = \tilde{r}_\infty(0)\int_0^\infty \tfrac{p(t)}{p(0)} c^*(t)dt = \tfrac{\tilde{r}_\infty(0)}{r_\infty(0)} h(0).$$

Furthermore, if the technology exhibits constant returns to scale, then

$$w(0) = \tfrac{\tilde{r}_\infty(0)}{r_0(0)}\left(g(0) + \dot{\mathbf{Q}}(0)\mathbf{k}^*(0)\right).$$

Since, by Proposition 7, $h(0) \leq w(0)$, it follows that $r_\infty(t) \leq \tilde{r}_\infty(t)$. If consumption is constant, then Lemma 1(i) implies that $(p(t))_{t=0}^\infty$ and $(\lambda(t))_{t=0}^\infty$ are proportional, leading to $r_\infty(t)$ and $\tilde{r}_\infty(t)$ being equal. This is consistent with the observation subsequent to Proposition 7, namely that $h(0) = c^* = w(0)$ in this case. The second part of Proposition 8 supports the argument (made e.g. by Brekke (1997, Section 6.5)) that anticipated capital gains must be taken into account when welfare comparisons over time and across economies are made.

5c Measuring net social profit

The welfare economic results of Section 2 facilitate the discussion of social cost-benefit analysis in the present intertemporal setting since the analogy to ordinary cost-benefit analysis is straightforward. Let the path $(c^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=0}^{\infty}$ be a regular path at present value prices $(p(s), \mathbf{q}(s))_{s=0}^{\infty}$ and utility discount factors $(\lambda(s))_{s=0}^{\infty}$ given the capital stocks $\mathbf{k}^0$ that generation 0 has inherited. Let $(c^*(s;\mathbf{k},t))_{s=t}^{\infty}$ be a consumption path maximizing $\int_t^{\infty} \lambda(s)u(c(s))ds$ subject to feasibility if generation t inherits $\mathbf{k}$. By Proposition 2, we can set $(c^*(s;\mathbf{k}^0,0))_{s=0}^{\infty} = (c^*(s))_{s=0}^{\infty}$.

Say that a policy change from $(c^*(s), \mathbf{k}^*(s), \dot{\mathbf{k}}^*(s))_{s=0}^{t}$ to $(c(s), \mathbf{k}(s), \dot{\mathbf{k}}(s))_{s=0}^{t}$ is *small* if

the policy change being a potential Pareto-improvement

is equivalent to

$$\int_0^t \lambda(s)\big(u(c(s)) - u(c^*(s))\big)ds + \int_t^{\infty} \lambda(s)\big(u(c^*(s;\mathbf{k}(t),t)) - u(c^*(s))\big)ds > 0,$$

and that this in turn is equivalent to

$$\int_0^t p(s)\big(c(s) - c^*(s)\big)ds + \mathbf{q}(t)(\mathbf{k}(t) - \mathbf{k}^*(t)) > 0,$$

keeping in mind that $\lambda(s)u'(c^*(s)) = p(s)$ (see Lemma 1(i)) and $q_i(t) = \frac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}$, where $V(\mathbf{k}, t) = \int_t^{\infty} \lambda(s)u(c^*(s;\mathbf{k},t))ds$ (see the proof of Proposition 1). By rewriting the latter inequality as

$$\int_0^t \Big(\big(p(s)c(s) + \mathbf{q}(s)\dot{\mathbf{k}}(s) + \dot{\mathbf{q}}(s)\mathbf{k}(s)\big) - \big(p(s)c^*(s) + \mathbf{q}(s)\dot{\mathbf{k}}^*(s) + \dot{\mathbf{q}}(s)\mathbf{k}^*(s)\big)\Big)ds > 0,$$

it follows that a small policy change can be evaluated by the discounted intertemporal sum of

$$\pi(s) = c^*(s) + \tfrac{\mathbf{q}(s)}{p(s)}\dot{\mathbf{k}}^*(s) + \tfrac{\dot{\mathbf{q}}(s)}{p(s)}\mathbf{k}^*(s),$$

where $\pi(s)$ is discounted by $p(s)$. Since $\dot{\mathbf{Q}}(s) = \frac{d}{ds}\big(\frac{\mathbf{q}(s)}{p(s)}\big) = \frac{\dot{\mathbf{q}}(s)}{p(s)} - \frac{\dot{p}(s)}{p(s)}\frac{\mathbf{q}(s)}{p(s)} = \frac{\dot{\mathbf{q}}(s)}{p(s)} + r_0(s)\mathbf{Q}(s)$,

$$\pi(s) = c^*(s) + \mathbf{Q}(s)\dot{\mathbf{k}}^*(s) - \big(r_0(s)\mathbf{Q}(s) - \dot{\mathbf{Q}}(s)\big)\mathbf{k}^*(s).$$

PROPOSITION 9. *Net social profit $\pi(0)$ is an index for the evaluation of small policy changes if*

$$\pi(0) = g(0) - \big(r_0(0)\mathbf{Q}(0) - \dot{\mathbf{Q}}(0)\big)\mathbf{k}^*(0).$$

Hence, net social profit is not equal to Green NNP, $g(0) = c^*(0) + \mathbf{Q}(0)\dot{\mathbf{k}}^*(0)$; rather, it is equal to Green NNP *minus* the cost of holding capital, $\big(r_0(0)\mathbf{Q}(0) - \dot{\mathbf{Q}}(0)\big)\mathbf{k}^*(0)$.

The difference between $g(0)$ and $\pi(0)$ is that $g(0)$ measures *gross* social profit while $\pi(0)$ measures *net* social profit. Using — in the Ramsey model — a constant consumption path with its supporting prices as reference, it is possible to increase the discounted intertemporal sum of $g(s) = c^*(s) + Q(s)\dot{k}^*(s) = \frac{1}{p(s)}\big(p(s)c^*(s) + q(s)\dot{k}^*(s)\big)$ between 0 and t,

$$\int_0^t p(s)g(s)ds = \int_0^t \big(p(s)c^*(s) + q(s)\dot{k}^*(s)\big)ds,$$

by accumulating capital between 0 and $\frac{t}{2}$ and decumulating capital between $\frac{t}{2}$ and t. The reason is that $\int_0^t p(s)\big(c(s)-c^*(s)\big)ds$ is of second order, while $\int_0^t q(s)\big(\dot{k}(s)-\dot{k}^*(s)\big)ds$ is of first-order and positive since $q(s)$ is decreasing. Hence, *the discounted intertemporal sum of $g(s)$ is not a cost-benefit index for a small policy change that lasts for a non-trivial interval of time* unless the policy change does not influence the aggregate path of the vector of investments.

As observed by e.g. Vellinga and Withagen (1996, Section 5), it *does* hold that $g(0) = c^*(0)+\mathbf{Q}(0)\dot{\mathbf{k}}^*(0) = \frac{1}{p(0)}\big(p(0)c^*(0)+\mathbf{q}(0)\dot{\mathbf{k}}^*(0)\big)$ is a cost-benefit index for a small policy change lasting only an instance. To see this in the present setting, note that, for a small policy change,

$$\begin{aligned}\lambda(0)\,(u(c(0))-u(c^*(0))) &+ \tfrac{d}{dt}\Big(\int_t^\infty \lambda(s)\big(u(c^*(s;\mathbf{k}(t),t))-u(c^*(s))\big)ds\Big)\Big|_{t=0}\\ &\approx p(0)\,(c(0)-c^*(0)) + \tfrac{d}{dt}\big(\mathbf{q}(t)\big(\mathbf{k}(t)-\mathbf{k}^*(t)\big)\big)\Big|_{t=0}\\ &= p(0)\,(c(0)-c^*(0)) + \mathbf{q}(0)\big(\dot{\mathbf{k}}(0)-\dot{\mathbf{k}}^*(0)\big)\end{aligned}$$

since $\mathbf{k}(0) = \mathbf{k}^*(0) = \mathbf{k}^0$. Hence, the change in the value of consumption measures the current change in utility, while the change in the value of investments measures the time derivative of the discounted intertemporal sum of future changes in utility. However, as I have shown above, it is not correct to use the discounted intertemporal sum of Green NNP to evaluate a small policy change that influences aggregate investments if its duration is longer than one instance.

Note that these results on the measurement of net social profit are general; i.e., they do *not* depend on assumptions like there being a constant utility discount rate or there being no exogenous technological progress. In particular, $(\lambda(s))_{s=0}^{\infty}$ may be interpreted as a path of supporting utility discount factors as discussed in Section 2.

6. Conclusions

In the present paper I have given an overview of the theory of green national accounting by investigating three purposes that such accounting can be used for:

1. Measurement of *sustainable income*.
2. Measurement of *social welfare*.
3. Measurement of *net social profit*.

It has been shown that social welfare must be used for the purpose of welfare comparisons across economies; sustainable income will not give a correct result.

Furthermore, I have considered two measures that may potentially serve these purposes:

(i) *Green NNP* (being equal to consumption + value of net investments).

(ii) *Hicksian income* (being the level of consumption with the same present value as the actual future consumption path).

It has been established as a general result that Hicksian income overestimates sustainable income and underestimates social welfare. It has also been demonstrated how, in principle, Hicksian income can be expressed by current prices and quantities only. Practical estimation of such an expression is, however, likely to be informational demanding.

To establish results concerning Green NNP it is necessary to assume no exogenous technological progress. This is restrictive since (i) it requires that accumulated knowledge is represented by augmented capital stocks, and (ii) it excludes open economies whose "technology" is changing exogenously due to changing terms of trade. If, in addition, the utility discount rate is constant, it has been shown that Green NNP underestimates social welfare. No general result appears to be available concerning the relation between Green NNP and sustainable income except that Green NNP equals Hicksian income – and thus overestimates sustainable income – when the interest rate is constant under no exogenous technological progress.

It has been shown that it is *not* justified to associate Green NNP with net social profit. Rather, Green NNP measures *gross* social profit, from which it is necessary to subtract the cost of holding capital in order to arrive at *net* social profit. As a consequence, it is *not* correct to use the discounted intertemporal sum of Green NNP as a cost-benefit index for (even) a small policy change provided that the policy change lasts for a non-trivial interval of time. Green NNP is, however, a cost-benefit index for a small policy change lasting only an instance.

Throughout I have assumed that all externalities are internalized, and that technological progress is captured by current investments or capital gains. If such assumptions cannot be made, green national accounting must include forward-looking terms of the kind discussed by Aronsson et al. (1997, Ch. 4) or adjustments like the one suggested by Weitzman (1997).

APPENDIX: PROOFS

Proof of Proposition 1. Let $(c^*(t),\mathbf{k}^*(t),\dot{\mathbf{k}}^*(t))_{t=0}^{\infty}$ maximize $\int_0^{\infty}\lambda(t)u(c(t))dt$ subject to $(c(t),\mathbf{k}(t),\dot{\mathbf{k}}(t))\in F(t)$ for all t and $\mathbf{k}(0)=\mathbf{k}^0$. Let $V(\mathbf{k}^0,0) := \int_0^{\infty}\lambda(t)u(c^*(t))dt$ and define $V(\mathbf{k},t)$ likewise. Then $V(\mathbf{k}^*(t),t) \geq \int_t^{t+\Delta t}\lambda(s)u(c(s))ds + V(\mathbf{k}(t+\Delta t), t+\Delta t)$ if $(c(s),\mathbf{k}(s),\dot{\mathbf{k}}(s))_{s=t}^{t+\Delta t}$ satisfies $(c(s),\mathbf{k}(s),\dot{\mathbf{k}}(s))\in F(s)$ for all $s \in [t, t+\Delta t]$ and $\mathbf{k}(t) = \mathbf{k}^*(t)$, with equality for $(c^*(s),\mathbf{k}^*(s),\dot{\mathbf{k}}^*(s))_{s=t}^{t+\Delta t}$. Assuming differentiability, we obtain

$$dV/dt = \sum \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}\dot{k}_i(t) + \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial t} \leq -\lambda(t)u(c(t))$$
$$dV/dt = \sum \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}\dot{k}_i^*(t) + \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial t} = -\lambda(t)u(c^*(t)).$$

Hence, for each t, $(c^*(t),\dot{\mathbf{k}}^*(t))$ maximizes $\lambda(t)u(c(t)) + \sum \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}\dot{k}(t)$ subject to $(c(t),\mathbf{k}^*(t),\dot{\mathbf{k}}(t))\in F(t)$. Let $q_i(t) := \tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}$ denote the present value *price* of capital good i. Let $H(t,c,\dot{\mathbf{k}},\mathbf{q}) := \lambda(t)u(c) + \mathbf{q}\dot{\mathbf{k}}$ denote the present value *Hamiltonian*. Given our assumption of differentiability, we have shown the *Maximum principle*: For each t, $(c^*(t),\dot{\mathbf{k}}^*(t))$ maximizes $H(t,c,\dot{\mathbf{k}},\mathbf{q}(t))$ subject to $(c,\mathbf{k}^*(t),\dot{\mathbf{k}})\in F(t)$. Let $H^*(t,\mathbf{k},\mathbf{q}) := \max_{(c,\mathbf{k},\dot{\mathbf{k}})\in F(t)} H(t,c,\dot{\mathbf{k}},\mathbf{q})$. Assuming differentiability, then for each capital good i, $\dot{q}_i(t) = \partial\left(\tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial k_i}\right)/\partial t = \partial\left(\tfrac{\partial V(\mathbf{k}^*(t),t)}{\partial t}\right)/\partial k_i = -\partial\left(\lambda(t)u(c^*(t)) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t)\right)/\partial k_i = -\partial\left(H^*(t,\mathbf{k}^*(t),\mathbf{q}(t))\right)/\partial k_i$.

By the convexity of $F(t)$, it follows that $H^*(t,\mathbf{k},\mathbf{q}(t))$ is a concave function of $\mathbf{k}$. Assume that $(c,\mathbf{k},\dot{\mathbf{k}})\in F(t)$. Then $\lambda(t)u(c) + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k} \leq H^*(t,\mathbf{k},\mathbf{q}(t)) + \dot{\mathbf{q}}(t)\mathbf{k} \leq H^*(t,\mathbf{k}^*(t),\mathbf{q}(t)) + \sum\left(\partial\left(H^*(t,\mathbf{k}^*(t),\mathbf{q}(t))\right)/\partial k_i\right)\left(k_i - k_i^*(t)\right) + \dot{\mathbf{q}}(t)\mathbf{k} = \lambda(t)u(c^*(t)) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t)$ since, for each capital good i, $\dot{q}_i(t) = -\partial\left(H^*(t,\mathbf{k}^*(t),\mathbf{q}(t))\right)/\partial k_i$. Hence, $(c^*(t),\mathbf{k}^*(t),\dot{\mathbf{k}}^*(t))$ maximizes $\lambda(t)u(c) + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ subject to $(c,\mathbf{k},\dot{\mathbf{k}})\in F(t)$. By the convexity of $F(t)$ and the concavity of u, it follows that $c^*(t)$ maximizes $\lambda(t)u(c) - p(t)c$, and $(c^*(t),\mathbf{k}^*(t),\dot{\mathbf{k}}^*(t))$ maximizes $p(t)c + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ subject to $(c,\mathbf{k},\dot{\mathbf{k}})\in F(t)$, where $p(t) := \lambda(t)u'(c^*(t))$ denotes the present value *price* of the consumption good. □

Proof of Proposition 2. Assume $(c(t),\mathbf{k}(t),\dot{\mathbf{k}}(t))\in F(t)$ for all t and $\mathbf{k}(0)=\mathbf{k}^0$. Then

$$\int_0^T \lambda(t)\left(u(c(t)) - u(c^*(t))\right)dt \leq \int_0^T p(t)\left(c(t) - c^*(t)\right)dt \text{ (by C1)}$$
$$\leq \int_0^T\left[\mathbf{q}(t)\left(\dot{\mathbf{k}}^*(t) - \dot{\mathbf{k}}(t)\right) + \dot{\mathbf{q}}(t)\left(\mathbf{k}^*(t) - \mathbf{k}(t)\right)\right]dt \text{ (by C2)}$$
$$= \int_0^T\left[\tfrac{d}{dt}\left(\mathbf{q}(t)\left(\mathbf{k}^*(t) - \mathbf{k}(t)\right)\right)\right]dt = \mathbf{q}(T)\,(\mathbf{k}^*(T) - \mathbf{k}(T)) - \mathbf{q}(0)\,(\mathbf{k}^*(0) - \mathbf{k}(0))$$

$\leq \mathbf{q}(T)\mathbf{k}^*(T)$

since $\mathbf{k}^*(0) = \mathbf{k}(0) = \mathbf{k}^0$, $\mathbf{q}(T) \geq 0$ (by free disposal of investment flows) and $\mathbf{k}(T) \geq 0$. By R1 and R2 the result follows. □

Proof of Lemma 1. (i) follows directly from C1. (ii) Since $F(t)$ is time-invariant, C2 implies that

$$p(t)c^*(t+\Delta t) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t+\Delta t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t+\Delta t) \leq p(t)c^*(t) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t).$$

Divide by Δt, and let Δt go to zero both from the right and from the left. This yields $0 = p(t)c^*(t) + \mathbf{q}(t)\ddot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\dot{\mathbf{k}}^*(t) = p(t)\dot{c}^*(t) + \frac{d}{dt}\left(\mathbf{q}(t)\dot{\mathbf{k}}^*(t)\right)$, where differentiability follows since $F(t)$ is smooth. □

Proof of Proposition 3 is provided in the main text.

Proof of Lemma 2. If $F(t)$ is a convex cone, then $(c^*(t), \mathbf{k}^*(t), \dot{\mathbf{k}}^*(t))$ maximizes $p(t)c + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k}$ subject to $(c, \mathbf{k}, \dot{\mathbf{k}}) \in F(t)$ only if $p(t)c^*(t) + \mathbf{q}(t)\dot{\mathbf{k}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{k}^*(t) = 0$. Hence, $p(s)c^*(s) + \frac{d}{ds}\left(\mathbf{q}(s)\mathbf{k}^*(s)\right) = 0$ for all s, such that, by R2, $\mathbf{q}(t)\mathbf{k}^*(t) = \int_t^\infty p(s)c^*(s)ds$. □

Proof of Proposition 4 is provided in the main text.

Proof of Proposition 5. Suppose $y(0) > h(0)$. Then $(c(t))_{t=0}^\infty$ with $c(t) = y(0)$ for all t is feasible, and $\int_0^\infty p(t)c(t)dt > \int_0^\infty p(t)h(0)dt = \int_0^\infty p(t)c^*(t)dt$, where the last equality follows from the definition of $h(0)$. This yields a contradiction since C2 and R2 imply that $(c^*(t))_{t=0}^\infty$ maximizes $\int_0^\infty p(t)c(t)dt$ over all feasible consumption paths. □

Proof of Proposition 6 is provided in the main text.

Proof of Proposition 7 is provided in the main text.

Proof of Proposition 8. Case (i): $u(c) = \ln c$. Since, by Lemma 1(i), $\frac{\lambda(t)}{c(t)} = p(t)$ for all t,

$$\begin{aligned}\ln\left(\tilde{r}_\infty(0)\int_0^\infty \tfrac{p(t)}{p(0)}c^*(t)dt\right) &= -\ln\left(\int_0^\infty \lambda(t)dt\right) - \frac{\int_0^\infty \lambda(t)\ln(p(t)/\lambda(t))dt}{\int_0^\infty \lambda(t)dt} + \ln\left(\int_0^\infty p(t)c^*(t)dt\right) \\ &= -\ln\left(\int_0^\infty \lambda(t)dt\right) + \frac{\int_0^\infty \lambda(t)\ln c^*(t)dt}{\int_0^\infty \lambda(t)dt} + \ln\left(\int_0^\infty \lambda(t)dt\right) = \ln w(0).\end{aligned}$$

Hence, $\tilde{r}_\infty(0)\int_0^\infty \frac{p(t)}{p(0)}c^*(t)dt = w(0)$.

Case (ii): $u(c) = \frac{c^\varrho}{\varrho}$, $\varrho < 1$, $\varrho \neq 0$. Since, by Lemma 1(i), $\lambda(t)\varrho c^*(t)^{\varrho-1} = p(t)$ for all t,

$$\left(\tilde{r}_\infty(0)\int_0^\infty \tfrac{p(t)}{p(0)} c^*(t)dt\right)^\varrho = \frac{\left(\int_0^\infty p(t)c^*(t)dt\right)^\varrho}{\left(\int_0^\infty \lambda(t)dt\right)^\varrho \cdot \left(\frac{\int_0^\infty \lambda(t)(p(t)/\lambda(t))^{\varrho/(\varrho-1)}dt}{\int_0^\infty \lambda(t)dt}\right)^{(\varrho-1)}}$$

$$= \frac{\varrho^\varrho \cdot \left(\int_0^\infty \lambda(t)c^*(t)^\varrho dt\right)^\varrho}{\left(\int_0^\infty \lambda(t)dt\right)^\varrho \cdot \varrho^\varrho \cdot \left(\frac{\int_0^\infty \lambda(t)c^*(t)^\varrho dt}{\int_0^\infty \lambda(t)dt}\right)^{(\varrho-1)}} = \frac{\int_0^\infty \lambda(t)c^*(t)^\varrho dt}{\int_0^\infty \lambda(t)dt} = w(0)^\varrho.$$

Hence, $\tilde{r}_\infty(0)\int_0^\infty \frac{p(t)}{p(0)} c^*(t)dt = w(0)$.

In both cases it follows from the definition of $h(0)$ that $\tilde{r}_\infty(0)\int_0^\infty \frac{p(t)}{p(0)} c^*(t)dt = \frac{\tilde{r}_\infty(0)}{r_\infty(0)} h(0)$. The second part of the proposition follows since $\int_0^\infty \frac{p(t)}{p(0)} c^*(t)dt = \frac{1}{r_0(0)}\left(g(0) + \dot{\mathbf{Q}}(0)\mathbf{k}^*(0)\right)$ under the assumption of constant returns to scale (by Proposition 4 and the definition of $h(0)$). □

Proof of Proposition 9 is provided in the main text.

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