

Green's Functions with Applications

DEAN G. DUFFY

CHAPMAN & HALL/CRC

Boca Raton London New York Washington, D.C.

Contents

Introduction

Definitions of the Most Commonly Used Functions

1	Some Background Material	1
1.1	Historical Development	1
1.2	The Dirac Delta Function	5
1.3	Green's Formulas	14
1.4	What Is a Green's Function?	18
2	Green's Functions for Ordinary Differential Equations	27
2.1	Initial-Value Problems	27
2.2	The Superposition Integral	33
2.3	Regular Boundary-Value Problems	35
2.4	Eigenfunction Expansion for Regular Boundary-Value Problems	44
2.5	Singular Boundary-Value Problems	51
2.6	Maxwell's Reciprocity	59

3 Green's Functions for the Wave Equation	75
3.1 One-Dimensional Wave Equation in an Unlimited Domain	79
3.2 One-Dimensional Wave Equation on the Interval $0 < x < L$	90
3.3 Axisymmetric Vibrations of a Circular Membrane	96
3.4 Two-Dimensional Wave Equation in an Unlimited Domain	97
3.5 Three-Dimensional Wave Equation in an Unlimited Domain	99
3.6 Asymmetric Vibrations of a Circular Membrane	108
3.7 Thermal Waves	113
3.8 Discrete Wavenumber Representation	115
3.9 Leaky Modes	117
3.10 Water Waves	136
4 Green's Function for the Heat Equation	177
4.1 Heat Equation over Infinite or Semi-Infinite Domains	181
4.2 Heat Equation Within a Finite Cartesian Domain	197
4.3 Heat Equation Within a Cylinder	209
4.4 Heat Equation Within a Sphere	222
4.5 Product Solution	226
4.6 Absolute and Convective Instability	231
5 Green's Function for the Helmholtz Equation	269
5.1 Free-Space Green's Functions for Helmholtz's and Poisson's Equations	273
5.2 Two-Dimensional Poisson's Equation over Rectangular and Circular Domains	291
5.3 Two-Dimensional Helmholtz Equation over Rectangular and Circular Domains	305
5.4 Poisson's and Helmholtz's Equations on a Rectangular Strip	321

5.5 Three-Dimensional Problems in a Half-Space	330
5.6 Three-Dimensional Poisson's Equation in a Cylindrical Domain	336
5.7 Poisson's Equation for a Spherical Domain	338
5.8 Improving the Convergence Rate of Green's Functions	344
Appendix A: The Fourier Transform	391
A.1 Definition and Properties of Fourier Transforms	391
A.2 Inversion of Fourier Transforms	393
A.3 Solution of Ordinary Differential Equations	396
A.3 Solution of Partial Differential Equations	398
Appendix B: The Laplace Transform	401
B.1 Definition and Elementary Properties	401
B.2 The Shifting Theorems	404
B.3 Convolution	406
B.4 Solution of Linear Ordinary Differential Equations with Constant Coefficients	407
B.5 Inversion by Contour Integration	409
B.6 Solution of Partial Differential Equations	413
Appendix C: Bessel Functions	417
C.1 Bessel Functions and Their Properties	417
C.2 Hankel Functions	421
C.3 Fourier-Bessel Series	423
Appendix D: Relationship between Solutions of Helmholtz's and Laplace's Equations in Cylindrical and Spherical Coordinates	429
Answers to Some of the Problems	433
Index	441