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# Green Supplier Selection With a Continuous Interval-Valued Linguistic TODIM Method

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**ABSTRACT** TODIM method (an acronym in Portuguese of Interactive and Multicriteria Decision Making) has attracted the attention of many scholars and achieved much success in multi-criteria decision making. Linguistic expression, as a qualitative information representation form, is much closer to human's cognition than specific numerical values. This paper combines the classical TODIM method with the continuous interval-valued linguistic term set to solve multi-criteria group decision-making problems. A distance measure of the continuous interval-valued linguistic term set is developed and then applied to develop a criteria-weighting method. In addition, the consensus level among experts are taken into account and an improved consensus reaching method is introduced to help experts reach an agreement. Subsequently, the framework of the continuous interval-valued linguistic TODIM method is introduced to show how to derive the optimal decision-making result with complicated and precise linguistic assessments in the qualitative situation. A case study concerning the green supplier selection for a food company is provided to verify the feasibility and practicality of the proposed method. Finally, the comparisons with other ranking methods are provided to demonstrate the advantages of the proposed method.

**INDEX TERMS** Multi-criteria decision making, continuous interval-valued linguistic term set, TODIM method, distance measure, group decision making, green supplier.

## I. INTRODUCTION

Multi-criteria decision-making (MCDM) problems are to select the optimal alternative from a set of finite alternatives with multiple criteria, which has achieved much success as a vital research topic of decision-making theory [1], [2]. Due to the uncertainty of MCDM problems, the experts participating in decision-making process have some difficulty in providing the certain and precise assessment information. In addition, as many criteria are shown in qualitative form, it is unrealistic for experts to give specific numerical values. People are accustomed to expressing their opinions in the form of flexible linguistic expressions which are close to human cognition [3]. Linguistic evaluations are more applicable than numerical information in the actual decision-making process [4]. To reduce the loss of decision-making information, different expression forms of linguistic information have been introduced, such as the 2-tuple fuzzy linguistic representation model [5], virtual linguistic model [6], linguistic models based on type-2 fuzzy sets presentation [7], and probabilistic linguistic model [8]. However, the single linguistic term was employed in these modes, which does not conform to the rich linguistic expressions of people.

To express rich and flexible linguistic assessments, Rodriguez *et al.* [9] proposed the concept of the hesitant fuzzy linguistic term set (HFLTS), which can elicit several linguistic terms or expressions for a linguistic variable. The linguistic assessments provided by experts, such as "*at least medium*", "*more than good*" and "*between fast and very fast*", can be transformed into the HFLTS by the text-free grammar and transformation function [9]. In decision making, the HFLTS has been applied in various fields [10]–[12]. However, the HFLTS has some limitations in expressing linguistic assessments with complex and precise expressions, and the provided linguistic terms are discrete. When decision makers or experts have a good

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understanding on decision-making problems, they may use more accurate linguistic assessments to give their opinions. For example, let $S = \{s_{-3} = \textit{very uncomfortable}, s_{-2} = \textit{uncomfortable}, s_{-1} = \textit{alittle uncomfortable}, s_0 = \textit{medium}, s_1 = \textit{a little comfortable}, s_2 = \textit{comfortable}, s_3 = \textit{very comfortable}\}$ be an LTS. When assessing the comfort of a car, the linguistic assessment provided by an expert may be *"between medium and comfortable, in which the 80% of the proportion is close to a little comfortable"*. The above linguistic assessment can be denoted by the hesitant fuzzy linguistic element (HFLE) $\{s_0, s_1\}$ or the uncertain linguistic variable $[s_0, s_1]$. Apparently, this representation cannot reflect the accurate proportion information of the linguistic assessment. To avoid the limitations of HFLTSs, Liao *et al.* [13] proposed the continuous interval-valued linguistic term set (CIVLTS) to express the linguistic assessments of experts comprehensively. Given this, the above example can be represented by a CIVLTS $[s_0, s_{0.8}]$. Compared with the HFLTS and uncertain linguistic variable, the CIVLTS not only can express more precise and accurate linguistic assessments than HFLTSs, but also can describe the collective views of a group efficiently because it considers the possible linguistic interval and the different importance degrees of experts.

Classical MCDM methods, such as TOPSIS [14] and MUTIMOORA [15], do not consider the psychological behavior of experts that they are bounded rational, and their psychological behaviors play an important role in obtaining the final decision-making problem [16]. The TODIM (an acronym in Portuguese of Interactive and Multicriteria Decision Making) method [17] is an MCDM method that was developed based on the prospect theory that can effectively describe the psychological behavior of decision makers on risk. At present, the TODIM method has been extended to different environments. For example, Yu *et al.* [18] introduced an extended TODIM approach with intuitionistic linguistic numbers. Wei *et al.* [19] extended the classical TODIM method to deal with MCDM problems in the hesitant fuzzy linguistic environment. Zhang and Xu [20] used the hesitant fuzzy TODIM method for the sustainable water management. Zhang *et al.* [21] extended the TODIM method to the probabilistic interval-valued hesitant fuzzy environment to deal with multi-criteria group decision-making problems. Considering that the CIVLTS is useful and accurate to express complex linguistic information in actual decision making, we extend the classical TODIM method to the continuous interval-valued linguistic environment to solve practical MCDM problems.

Furthermore, with the rapid development of social work, the group decision-making (GDM) that is composed of many experts is common in solving decision-making problems [22]–[28], because this can take into account the full knowledge and experience of each decision maker, so as to facilitate the acquisition of the optimal alternative and reduce the risk of decision-making. Thus, the extended TODIM method is applied to multi-criteria group decision making (MCGDM) in this study. MCGDM is a participatory process in which multiple experts assess multiple alternatives according to relevant criteria and select the best alternative [29]. When decision makers give linguistic assessments for alternatives, there are often divergences because of their different backgrounds and expertise. Due to the different background, it is tough for experts to reach an agreement. If the divergent linguistic assessments continue to be translated and integrated, the final decision-making result will be different from the ideal solution. To avoid this situation as much as possible, the consensus process among experts in the GDM problem is considered by adjustment strategy [30]. Commonly, there are two adjustment strategies to help experts achieve the satisfied consensus level: automatic optimization method and feedback adjustment method. The former one does not need to interact with experts and thus can save much time, while the latter one can communicate with experts whatever necessary. If time permits, the feedback optimization method is more appropriate than the automatic optimization method. Furthermore, for the experts who do not reach the consensus level, the consensus improvement method is used to help them reach an acceptable consensus level [31], which mainly includes identification rules and direction rules. In this paper, the consensus of a group of decision makers are considered in the extended TODIM method with the continuous interval-valued linguistic information. In this paper, the consensus process is considered into the TODIM method under the continuous interval-valued linguistic environment.

In summary, an integrated framework of the continuous interval-valued linguistic TODIM (CIVL-TODIM) method based on an improved distance measure is proposed to widen the application scope of the classical TODIM method. The contributions of this paper can be concluded as follows:

(1) We extend the classical TODIM method to the continuous interval-valued linguistic environment and propose the CIVL-TODIM method.

(2) We propose a new distance measure between continuous interval-valued linguistic elements (CIVLEs), and the weights of criteria and decision makers are determined by the new distance measure.

(3) We construct a framework of the CIVL-TODIM method to deal with the MCGDM problem and carry out the proposed method to a practical case study concerning the selection of green suppliers for a food company to illustrate the practicality and feasibility of the method.

This paper is organized as follows: Section II reviews the concepts of the CIVLTS and the TODIM method. Section III improves the distance measure of CIVLEs. The method of determining the weights of experts and criteria and the detailed procedure of the CIVL-TODIM method are introduced in this section. In Section IV, a case study is provided and some comparative analyses with other ranking methods are given. Section V points out some concluding remarks.

## II. PRELIMINARIES

In this section, the basic concepts of the CIVLTS and the classical TODIM method are reviewed.

### A. THE CONTINUOUS INTERVAL-VALUED LINGUISTIC TERM SET

There are limitations of the uncertain linguistic variable and HFLTS in terms of linguistic information expression [13]: 1) the linguistic assessments from people participating in a decision-making problem are not adequately expressed; 2) the comprehensive judgments of an expert group cannot fully reflect the whole ideas of the group. In this regard, Liao *et al.* [11] presented a novel fuzzy linguistic approach called the CIVLTS which can have ample expressions of linguistic assessments. Let $x_i \in X(i = 1, 2, \cdots, m)$ be fixed and $S = \{s_\alpha | \alpha = -\tau, \cdots, 0, \cdots \tau\}$ be a linguistic term set (LTS). The mathematical form of a CIVLTS [13] can be denoted as $\tilde{H}_S = \{< x_i, \tilde{h}_S(x_i) > | x_i \in X\}$ where $\tilde{h}_S(x_i)$ is a subset in the continuous interval-valued form of the LTS $\tilde{S} = \{s_\alpha | \alpha \in [-\tau, \tau]\}$, shown as $\tilde{h}_S(x_i) = [s_{L_i}, s_{U_i}]$, $L_i, U_i \in [-\tau, \tau]$ and $L_i \leq U_i$. $\tilde{h}_S(x_i)$ is the possible degree of the linguistic variable $x_i$ belonging to $\tilde{H}_S$, and $\tilde{h}_S(x_i)$ is called the CIVLE. $s_{L_i}$ and $s_{U_i}$ are the lower and upper bounds, respectively. For example, assume that a symmetric LTS with odd cardinality is expressed as: $S = \{s_{-3} = \textit{very bad}, s_{-2} = \text{bad}, s_{-1} = \textit{a little bad}, s_0 = \textit{medium}, s_1 = \textit{a little good}, s_2 = \textit{good}, s_3 = \textit{very good}\}$. There are two decision makers participating in the linguistic assessments concerning the dress designing level in a certain company. The linguistic assessments from one decision maker for dress designing level is given as: "*between a little good and good but closing to good with 80% proportion*", which can be denoted as the CIVLE $[s_1, s_{1.8}]$. The linguistic assessment of another decision maker is given as: *"between medium and a little good but closing to a little good with 50% proportion"*, which can be denoted as the CIVLE $[s_0, s_{0.5}]$.

*Note:* For two CIVLEs $\tilde{h}^1_S = [s_{L_1}, s_{U_1}]$ and $\tilde{h}^2_S = [s_{L_2}, s_{U_2}]$, if and only if $s_{L_1} = s_{L_2}$ and $s_{U_1} = s_{U_2}$, then $\tilde{h}^1_S = \tilde{h}^2_S$. Especially, if $(s_{L_1} + s_{U_1})/2 = (s_{L_2} + s_{U_2})/2$, but $s_{L_1} \neq s_{L_2}$ and $s_{U_1} \neq s_{U_2}$, then $s_{U_1} \neq s_{U_2}$. For example, let $\tilde{h}^1_S = [s_{0.5}, s_1]$ and $\tilde{h}^2_S = [s_0, s_{1.5}]$. We cannot deem $\tilde{h}^1_S$ and $\tilde{h}^2_S$ as the same since the interval of $\tilde{h}^2_S$ is larger than the interval of $\tilde{h}^1_S$. That is to say, $\tilde{h}^2_S$ is more uncertain than $\tilde{h}^1_S$.

Apparently, the CIVLTS can have more accurate and detailed expressions of linguistic assessments than the uncertain linguistic variable and HFLTS. The CIVLTS in mathematical form is essentially equivalent to the interval-valued virtual term set [13], while the CIVLTS can not only overcome the defense of the interval-valued LTS but also directly express the assessments of decision makers. Therefore, the linguistic expression form of the CIVLTS in decision-making problems is appropriate and accurate.

To compare the CIVLEs, Liao *et al.* [13] considered a transformation function in the situation that the semantics of linguistic terms are uniformly distributed. In this case, the specific semantic of the linguistic term $s_\alpha$ in a CIVLE can be calculated by:

$$g(s_\alpha) = (\tau + \alpha)/2\tau \tag{1}$$

Furthermore, Liao *et al.* [13] introduced the following operations of three CIVLEs $\widetilde{h}_S = [s_L, s_U]$, $\widetilde{h}^1_S = [s^1_L, s^1_U]$ and $\widetilde{h}^2_S = [s^2_L, s^2_U]$ on $S$:

1. Upper bound: $\widetilde{h}_S = s_U$;
2. Lower bound: $\widetilde{h}_S = s_L$;
3. Intersection: $\tilde{h}^1_S \cap \tilde{h}^2_S = [\max\{s_{L_1}, s_{L_2}\}, \min\{s_{U_1}, s_{U_2}\}]$; if $\max\{s_{L_1}, s_{L_2}\} > \min\{s_{U_1}, s_{U_2}\}$, then $\tilde{h}^1_S \cap \tilde{h}^2_S = \phi$;
4. Complement: $\tilde{h}^C_S = [s_{-\tau}, s_L] \cup [s_U, s_\tau]$.

### B. ABOUT THE TRADITIONAL TODIM METHOD

The classical TODIM method is to measure the partial and final dominance degree of each alternative to other alternatives based on the prospect theory [17]. According to the final dominance degree of each alternative, the ranking of alternatives can be obtained. Considering that experts are bounded rational and the behaviors of them play an important role in obtaining the final decision [19], the main advantage of the TODIM is that the psychological behavior of experts can be captured by establishing a prospect value function based on the prospect theory. The classical TODIM method is appropriate to deal with the MCDM problems in which the assessments of alternatives under criteria are represented by numerical values. Assume that there are $m$ alternatives $\{a_1, \cdots, a_i, \cdots, a_m\}$ and $n$ criteria $(c_1, \cdots, c_j, \cdots, c_n)$ with a weight vector $W = (\omega_1, \cdots, \omega_j, \cdots, \omega_n)^T$ in which $0 \leq \omega_j \leq 1$ and $\sum_j^n \omega_j = 1$. The procedure of the classical TODIM method can be shown as follows:

*Step 1:* The decision maker determines a preliminary decision matrix as:

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix}$$

where $x_{ij}$ denotes the assessment value of the alternative $a_i$ with regard to the criterion $c_j$.

*Step 2:* The numerical values are normalized by:

$$\bar{x}_{ij} = x_{ij} \Big/ \sum_{i=1}^{m} x_{ij} \tag{2}$$

In this way, we can obtain a dimensionless decision matrix as:

$$\bar{D} = \begin{bmatrix} \bar{x}_{11} & \cdots & \bar{x}_{1j} & \cdots & \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{i1} & \cdots & \bar{x}_{ij} & \cdots & \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \cdots & \bar{x}_{mj} & \cdots & \bar{x}_{mn} \end{bmatrix}$$

*Step 3:* The decision maker chooses the criterion with the greatest weight $\omega_c$ as the reference criterion. The relative weight of each criterion is calculated as:

$$\omega_{jc} = \omega_j / \omega_c, \quad \text{for } j = 1, 2, \cdots, n \tag{3}$$

*Step 4:* The partial dominance degree of the alternative $a_i$ with respect to the alternative $a_j$ under the criterion $c_j$ can be calculated by:

$$\Phi_j(a_i, a_k) = \begin{cases} \sqrt{\dfrac{(\bar{x}_{ic} - \bar{x}_{kc})\omega_{jc}}{\sum_{j=1}^{n} \omega_{jc}}}, & \bar{x}_{ic} - \bar{x}_{kc} > 0 \\ 0 & \bar{x}_{ic} - \bar{x}_{kc} = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{(\bar{x}_{ic} - \bar{x}_{kc})(\sum_{j=1}^{n} \omega_{jc})}{\omega_{jc}}}, & \bar{x}_{ic} - \bar{x}_{kc} < 0 \end{cases} \tag{4}$$

where $\theta$ is an attenuation factor of loss, denoting the risk preference of the decision maker. Then, the final dominance degree of the alternative $a_i$ with respect to the alternative $a_j$ under all criteria can be calculated by:

$$\delta(a_i, a_k) = \sum_{j=1}^{n} \Phi_j(a_i, a_k) \tag{5}$$

*Step 5:* The global dominance degree of the alternative $a_i$ is determined by:

$$\xi(a_i) = \frac{\sum_{k=1}^{m} \delta(a_i, a_k) - \min_i \left\{\sum_{k=1}^{m} \delta(a_i, a_k)\right\}}{\max_i \sum_{k=1}^{m} \delta(a_i, a_k) - \min_i \left\{\sum_{k=1}^{m} \delta(a_i, a_k)\right\}} \tag{6}$$

The ranking of alternatives can be acquired in descending order of $\xi(a_i)$ for $i = 1, 2, \cdots, m$.

## III. THE CONTINUOUS INTERVAL-VALUED LINGUISTIC TODIM METHOD

In this section, the improved distance measure is proposed to calculate the distance between CIVLEs. The consensus level of each expert is checked for the MCGDM problem. Next, the weights of criteria and experts are calculated based on the improved distance measure. Finally, the framework of the CIVL-TODIM method is introduced.

### A. THE DISTANCE MEASURE BETWEEN CIVLES

Distance measures to identify the deviation degrees between variables are vital for MCDM methods. A distance measure between CIVLEs was introduced by Liao *et al.* [13] as follows:

$$d(\tilde{h}_S^1, \tilde{h}_S^2) = \left[\frac{1}{2}\left(\frac{|L_1 - L_2|}{2\tau}\right)^2 + \frac{1}{2}\left(\frac{|U_1 - U_2|}{2\tau}\right)^2\right]^{1/2} \tag{7}$$

However, the distance measure given as Eq. (7) has some limitations in describing the distance between CIVLEs. For example, for three CIVLEs $\tilde{h}_S^1 = [s_{0.6}, s_1]$, $\tilde{h}_S^2 = [s_1, s_{1.4}]$ and $\tilde{h}_S^3 = [s_{1.4}, s_{1.8}]$, by Eq. (7), $d(\tilde{h}_S^1, \tilde{h}_S^2) = d(\tilde{h}_S^2, \tilde{h}_S^3) = 0.067$. We can see that the results do not conform to the cognition of experts in the linguistic environment since the linguistic terms with the equidistant interval may differ in their degrees. The closer the linguistic assessments approach to the endpoint value, the more difficult it will be to achieve in optimal decision. Although different CIVLEs have equal numerical distance after conversion, the actual deviations they express in their minds are diverse. We take a common example in learning to explain for easy understanding. In a certain exam that the full score is 100. There are three students A, B and C whose scores are 85, 90 and 95, respectively. The gap of A and B is 5, while the gap of B and C is also 5. Although there is no difference in numerical values, the difference between the latter is obviously greater than that between the former in mind. Because the closer the score is to the full score, the higher the requirement of students' ability to master knowledge should be. In view of this, it is inappropriate to have a quadratic square for the distance measure between CIVLTSs.

Considering the psychophysics of experts, Lootsma [32] pointed out that when making evaluations, experts usually take the expectation value or the optimal value as a reference and they are sensitive to the values which are close to the reference value. To obtain the relatively accurate comparison result, it is necessary to make a distinction between CIVLEs, especially in the case of equal numerical distance.

Let $\tilde{S} = \{s_\alpha | \alpha \in [-\tau, \tau]\}$ be an LTS where $s_\tau$ is the maximum linguistic term in $\tilde{S}$. When making evaluations, let $s_\tau$ be the optimal (or expectation) value of an object. If the criterion is the cost form, then we should translate its values into the benefit form by a negative operation that $\tilde{h}'_S = neg(\tilde{h}'_S) = [s_{-U_i}, s_{-L_i}]$. Then, we improve the distance measure between two CIVLEs $\tilde{h}_S^1 = [s_{L_1}, s_{U_1}]$ and $\tilde{h}_S^2 = [s_{L_2}, s_{U_2}]$ as follows:

$$d(\tilde{h}_S^1, \tilde{h}_S^2) = \left[\frac{1}{2}\left(\frac{\left|L_1^\beta - L_2^\beta\right|}{2\tau^\beta}\right) + \frac{1}{2}\left(\frac{\left|U_1^\beta - U_2^\beta\right|}{2\tau^\beta}\right)\right]^{1/\beta} \tag{8}$$

Here, we need to consider the value of $\beta$. If $\beta$ takes an odd number, *i.e.*, $\beta = 2n + 1$ with $n$ being natural number, two situations need to be considered. The aforementioned example is continued to be considered with Eq. (8): 1) when $n = 0$, *i.e.*, $\beta = 1$, by Eq. (8), $d(\tilde{h}_S^1, \tilde{h}_S^2) = d(\tilde{h}_S^2, \tilde{h}_S^3) = 0.167$. The calculated result is equal between two CIVLEs, which is the same as that obtained by Eq. (7), so $\beta = 1$ is not appropriate; 2) when $n \neq 0$, we can obtain $d(\tilde{h}_S^1, \tilde{h}_S^2) \neq d(\tilde{h}_S^2, \tilde{h}_S^3)$. Take $n = 1$ as an example. $\beta = 3$ can be obtained. By Eq. (8), $d(\tilde{h}_S^1, \tilde{h}_S^2) = 0.420$ and $d(\tilde{h}_S^2, \tilde{h}_S^3) = 0.622$. Similarly, when $\beta$ takes other odd numbers that are not equal to 1, we can also deduce $d(\tilde{h}_S^1, \tilde{h}_S^2) \neq d(\tilde{h}_S^2, \tilde{h}_S^3)$. The calculated results conform to the cognition of people.

If $\beta$ takes an even number, *i.e.*, $\beta = 2n + 2$ with $n$ being natural number, we can find that there are some drawbacks when the subscripts of the partial linguistic terms are negative. For example, for $\tilde{h}_S^4 = [s_{-1}, s_{-1}]$ and $\tilde{h}_S^5 = [s_1, s_1]$, we can obtain $d(\tilde{h}_S^4, \tilde{h}_S^5) = 0$. Apparently, the calculated result does not conform to the actual situation.

Therefore, $\beta = 2n+1$ with $n \neq 0$ is applicable in Eq. (8).

*Theorem 1: The improved distance measure between CIVLEs satisfies: (1)* $0 \leq d(\tilde{h}_S^1, \tilde{h}_S^2) \leq 1$*; (2)* $d(\tilde{h}_S^1, \tilde{h}_S^2) = d(\tilde{h}_S^2, \tilde{h}_S^1)$*; (3)* $\tilde{h}_S^1 = \tilde{h}_S^2$ *if only if* $d(\tilde{h}_S^1, \tilde{h}_S^2) = 0$.

*Proof:* (1) Since $L_1, L_2, U_1, U_2 \in [-\tau, \tau]$, we have $0 \leq \frac{\left|L_1^\beta - L_2^\beta\right|}{2\tau^\beta} \leq 1$ and $0 \leq \frac{\left|U_1^\beta - U_2^\beta\right|}{2\tau^\beta} \leq 1$. Then, $0 \leq \left[\frac{1}{2}\left(\frac{\left|L_1^\beta - L_2^\beta\right|}{2\tau^\beta}\right) + \frac{1}{2}\left(\frac{\left|U_1^\beta - U_2^\beta\right|}{2\tau^\beta}\right)\right]^{1/\beta} \leq 1$, *i.e.*, $0 \leq d(\tilde{h}_S^1, \tilde{h}_S^2) \leq 1$.

(2) Since $\left|L_1^\beta - L_2^\beta\right| = \left|L_2^\beta - L_1^\beta\right|$, $\left|U_1^\beta - U_2^\beta\right| = \left|U_2^\beta - U_1^\beta\right|$, we have

$$\begin{aligned} d(\tilde{h}_S^1, \tilde{h}_S^2) &= \left[\frac{1}{2}\left(\frac{\left|L_1^\beta - L_2^\beta\right|}{2\tau^\beta}\right) + \frac{1}{2}\left(\frac{\left|U_1^\beta - U_2^\beta\right|}{2\tau^\beta}\right)\right]^{1/\beta} \\ &= \left[\frac{1}{2}\left(\frac{\left|L_2^\beta - L_1^\beta\right|}{2\tau^\beta}\right) + \frac{1}{2}\left(\frac{\left|U_2^\beta - U_1^\beta\right|}{2\tau^\beta}\right)\right]^{1/\beta} \\ &= d(\tilde{h}_S^2, \tilde{h}_S^1). \end{aligned}$$

(3) $d(\tilde{h}_S^1, \tilde{h}_S^2) = 0 \Leftrightarrow \left|L_1^\beta - L_2^\beta\right| = 0, \left|U_1^\beta - U_2^\beta\right| = 0 \Leftrightarrow \tilde{h}_S^1 = \tilde{h}_S^2$.

In this paper, for the convenience of calculation, the value of $\beta$ is set as $\beta = 3$. From the above examples, we can obtain $d(\tilde{h}_S^1, \tilde{h}_S^2) = 0.420 \neq d(\tilde{h}_S^2, \tilde{h}_S^3) = 0.622$ and $d(\tilde{h}_S^4, \tilde{h}_S^5) = 0.333$. The calculated results illustrate that the distance between different CIVLEs is different and cannot exit equal situation, which also illustrates that Eq. (8) with $\beta = 3$ is more in line with the cognitive psychological behavior of people than Eq. (7).

### B. WEIGHT-DETERMINING METHODS FOR EXPERTS AND CRITERIA

In this part, the methods to determine the weights of experts and criteria are introduced.

Due to the increasing complexity of practical decision-making problems, group decision-making (GDM) have been researched by many scholars [22]–[28] to make a more appropriate decision than single person. For an MCGDM problem, suppose that there are a set of experts $E = \{e_1, \cdots, e_q, \cdots, e_Q\}$ $(Q \geq 2)$, a set of alternatives $A = \{a_1, \cdots, a_i, \cdots, a_m\}$ $(m \geq 2)$, and a set of criteria $C = \{c_1, \cdots, c_j, \cdots, c_n\} (n \geq 2)$. Here, the weight vector of criteria is denoted as $W^1 = (\omega_1, \cdots, \omega_j, \cdots, \omega_n)^T$ which satisfies $0 \leq \omega_j \leq 1$ and $\sum_j^n \omega_j = 1$, and the weight vector of experts is denoted as $W^2 = (w_1, \cdots, w_q, \cdots, w_Q)^T$ which satisfies $0 \leq w_q \leq 1$ and $\sum_q^Q w_q = 1$. In this paper, we use the subscript-symmetric LTS.

First, we consider a method to determine the criteria weights. Suppose that experts are required to evaluate the performance of criteria by linguistic expressions. Let $S = \{s_{-3} = \textit{very unimportant}, s_{-2} = \textit{unimportant}, s_{-1} = \textit{a little unimportant}, s_0 = \textit{medium}, s_1 = \textit{a little important}, s_2 = \textit{important}, s_3 = \textit{very important}\}$ be an LTS to evaluate the importance degrees of the criteria. According to the linguistic terms and the proportion information mentioned by experts, we can translate the evaluations into CIVLEs. For example, when one expert evaluates that the importance degree of a criterion is "*between a little important and important and closes to important with 50% of the proportion*", then we can convert this complex linguistic evaluation as a CIVLE $[s_{1.5}, s_2]$. Based on the CIVLEs given by the expert group concerning criterion importance, we can establish a linguistic assessment matrix as follows:

$$C = \begin{array}{c} \\ e_1 \\ \vdots \\ e_q \\ \vdots \\ e_Q \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & \cdots & c_j & \cdots & c_n \end{array} \\ \begin{bmatrix} \tilde{h}_S^{1(1)} & \cdots & \tilde{h}_S^{j(1)} & \cdots & \tilde{h}_S^{n(1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{h}_S^{1(q)} & \cdots & \tilde{h}_S^{j(q)} & \cdots & \tilde{h}_S^{n(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{h}_S^{1(Q)} & \cdots & \tilde{h}_S^{j(Q)} & \cdots & \tilde{h}_S^{n(Q)} \end{bmatrix} \end{array}$$

where $\tilde{h}_S^{j(q)}$ denotes the linguistic assessment of the expert $e_q$ concerning the criterion $c_j$.

It is remarkable that the initial weights of experts are hard to be given when linguistic assessments given by experts need to be integrated into a collective decision matrix. Here, the initial weights of experts are set as equal importance degree, and then the weights of criteria are determined according to the distance from each expert's linguistic assessment to the integrated decision matrix. Based on the matrix $C$, the linguistic assessments of experts under a certain criterion are aggregated by the weighted arithmetic aggregation operator:

$$C' = \left[\tilde{h}_S^1, \cdots, \tilde{h}_S^j, \cdots, \tilde{h}_S^n\right] \tag{9}$$

where $\tilde{h}_S^j = [s'_{L_j}, s'_{U_j}]$, with the lower and upper limits of the CIVLE being aggregated, respectively, by the following formulas:

$$s'_{L_j} = s_{\frac{1}{Q}\sum_{q=1}^{Q} L_{qj}} \tag{10}$$

$$s'_{U_j} = s_{\frac{1}{Q}\sum_{q=1}^{Q} U_{qj}} \tag{11}$$

Then, we can employ the improved distance measure to determine the weight of each criterion. The weight of the criterion $c_j$ can be calculated by:

$$\omega_j = \frac{\sum_{z=1}^{n} d_j\left(\tilde{h}_S^j, \tilde{h}_S^z\right)}{\sum_{j=1}^{n}\sum_{z=1}^{n} d_j\left(\tilde{h}_S^j, \tilde{h}_S^z\right)} \tag{12}$$

Next, it is necessary to determine the weight of each expert. Since experts have various backgrounds and expertise, their weights should be determined based on their linguistic assessments. However, in the initial stage of decision-making, the weights of experts cannot be obtained directly. Given this fact, the initial weights of experts are also regarded as the same to integrate the provided assessment information in this part. Then the weights of experts are adjusted based on the preliminary consensus levels.

According to the provided alternatives and criteria, experts can give their corresponding linguistic assessments in CIVLEs. Then, the preliminary decision matrix of each expert can be obtained, which can be shown as:

$$D(e_q) = \begin{bmatrix} \tilde{h}_S^{11(q)} & \cdots & \tilde{h}_S^{1j(q)} & \cdots & h_S^{1n(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{h}_S^{i1(q)} & \cdots & \tilde{h}_S^{ij(q)} & \cdots & \tilde{h}_S^{in(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{h}_S^{m1(q)} & \cdots & \tilde{h}_S^{mj(q)} & \cdots & \tilde{h}_S^{mn(q)} \end{bmatrix}$$

where $\tilde{h}_S^{ij(q)} = [s_{L_{ij}}^{(q)}, s_{U_{ij}}^{(q)}]$ denotes the linguistic assessment of the alternative $a_i$ with respect to the criterion $c_j$.

Since the dimensions among the criteria are the same, the normalization method to eliminate the different dimensions among criteria does not need. Next, the integrated collective decision matrix needs to be obtained by the average arithmetic aggregation operator:

$$\bar{D} = \begin{bmatrix} \bar{\tilde{h}}_S^{11} & \cdots & \bar{\tilde{h}}_S^{1j} & \cdots & \bar{\tilde{h}}_S^{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\tilde{h}}_S^{i1} & \cdots & \bar{\tilde{h}}_S^{ij} & \cdots & \bar{\tilde{h}}_S^{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{\tilde{h}}_S^{m1} & \cdots & \bar{\tilde{h}}_S^{mj} & \cdots & \bar{\tilde{h}}_S^{mn} \end{bmatrix} \tag{13}$$

where $\bar{\tilde{h}}_S^{ij} = [\bar{s}_{L_{ij}}, \bar{s}_{U_{ij}}]$, with the integrated collective CIVLE being calculated by:

$$\bar{s}_{L_{ij}} = \frac{1}{Q}\sum_{q=1}^{Q} s_{L_{ij}}^{(q)} \tag{14}$$

$$\bar{s}_{U_{ij}} = \frac{1}{Q}\sum_{q=1}^{Q} s_{U_{ij}}^{(q)} \tag{15}$$

Based on the decision matrix of each expert and the integrated collective decision matrix of all experts, the preliminary consensus level of each expert can be calculated by:

$$CL(q) = 1 - \sum_{i=1}^{m}\sum_{j=1}^{n} d(\tilde{h}_S^{ij(q)}, \bar{\tilde{h}}_S^{ij}) \tag{16}$$

Then, based on the preliminary consensus level, the weight of the expert $e_q$ can be calculated by:

$$w_q = CL(q) \Big/ \sum\nolimits_{q=1}^{Q} CL(q) \tag{17}$$

### C. CONSENSUS CHECKING AND IMPROVING AMONG EXPERTS

Multiple experts in decision making could lead to a real problem that the overall performances of alternatives may exist divergence that could be hard for experts to reach an agreement. In this regard, to obtain a robust decision-making result, it is necessary to check if all experts reach a satisfied consensus level or not.

In real decision making, the full consensus, *i.e.*, "*hard*" consensus, is unrealistic to achieve as experts have various experiential, cultural and educated backgrounds. To check whether the consensus level of each expert meets the requirement, the consensus threshold $\lambda$ should be given in advance [33], which can be determined by actual decision-making problems. The higher the requirement of the decision-making problem is, the larger the predefined consensus threshold will be. This consensus is also called "*soft*" consensus [34].

The expert whose consensus level $CL(q)$ is smaller than $\lambda$ can be identified by the moderator as follows:

$$e_q = \{e_q | CL^{(q)} < \lambda\} \tag{18}$$

To achieve the satisfactory consensus level $\lambda$ for all experts, the consensus improving method should be considered. In general, the consensus improving methods include two types: automatic optimization method and feedback optimization method [35]. The automatic optimization methods do not need to provide suggestions for the experts whose consensus levels are smaller than the consensus threshold, but automatically modify the corresponding linguistic assessments. Therefore, it can save time and thus can be used for emergency decision making. The feedback optimization methods [36], [37] need to provide suggestions for corresponding experts and let them revise their linguistic assessments. In this sense, it could make the decision-making results more realistic than the automatic optimization method.

In this paper, we employ a feedback adjustment method to enhance the consensus level. The feedback adjustment method is consisted of two aspects:

(1) Identification rules: the moderator needs to identify the experts who do not reach the predefined consensus threshold, i.e., $e_q = \{e_q | CL^{(q)} < \lambda\}$.
(2) Direction rules: the moderator needs to provide the integrated collective decision matrix for the identified expert. These experts can find out 2-5 linguistic elements to revise by comparing their own decision matrix and the integrated collective decision matrix. Then, the moderator provides some suggestions for these experts on how to enhance the consensus level. They can increase or reduce the corresponding linguistic assessment $X_{ij}$.

To ensure the freedom of experts to modify linguistic elements, experts can choose a few elements from which to modify for the obtained greatest different linguistic elements.

If the consensus levels of all experts reach the predefined consensus threshold, the feedback adjustment is not necessary.

### D. THE FRAMEWORK OF THE CONTINUOUS INTERVAL-VALUED LINGUISTIC TODIM METHOD

In the following part, the framework of the CIVL-TODIM method for MCGDM is described in detail.

*Step 1:* Collect the linguistic assessments of all experts concerning the importance degrees of the criteria. Then, by Eq. (12), the weight of each criterion can be calculated. The criterion with the greatest weight among all criteria is regarded as a reference indicator $\omega_c$, based on which, the relative weight of each criterion can be calculated by Eq. (3).

*Step 2:* Obtain the linguistic assessments of each expert concerning the alternatives under different criteria. Based on Eq. (16), the preliminary consensus level of each expert can be derived. Then, by Eq. (17), the weight of each expert can be obtained.

*Step 3:* Check the consensus level of each expert by Eq. (18). If a certain expert does not reach the acceptable consensus, the consensus improving method should be employed until all experts reach the acceptable consensus. Then, the matrix of each expert who reaches the consensus level can be obtained.

*Step 4:* Obtain the partial dominance matrix for each expert. For the expert $e_q$, the partial dominance degree of the alternative $a_i$ over the alternative $a_k$ under the criterion $c_j$ is calculated by the following formula:

$$\Phi_j^{(q)}(a_i, a_k) = \begin{cases} \sqrt{\dfrac{\omega_{jc} d(\tilde{h}_S^{ij(q)}, \tilde{h}_S^{kj(q)})}{\sum_{j=1}^{n} \omega_{jc}}}, & \Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) > 0 \\ 0, & \Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{(\sum_{j=1}^{n} \omega_{jc}) d(\tilde{h}_S^{ij(q)}, \tilde{h}_S^{kj(q)})}{\omega_{jc}}}, & \Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) < 0 \end{cases} \tag{19}$$

where $\Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) = \left((L_{ij} - L_{kj}) + (U_{ij} - U_{kj})\right)/2\tau$ illustrates the attitude of the expert towards the alternatives when comparing the alternatives in pairs under the same criterion. If $\Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) > 0$, then it represents the gain of the alternative $a_i$ over the alternative $a_j$ under the criterion $c_j$; if $\Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) = 0$, then it represents the equivalence; if $\Bbbk(\tilde{h}_S^{ij(q)}) - \Bbbk(\tilde{h}_S^{kj(q)}) < 0$, then it represents the loss. $\theta$ is the attenuation factor of the loss. In this paper, without the loss of generality, the value of $\theta$ is set as 1.

*Step 5:* Next, the dominance between alternatives for each expert can be obtained by

$$\delta^{(q)}(a_i, a_k) = \sum_{j=1}^{n} \Phi_j^{(q)}(a_i, a_k) \tag{20}$$

Then, the global dominance degree of the alternative $a_i$ for the expert $e_q$ is calculated by:

$$\phi^{(q)}(a_i) = \sum_{k=1}^{m} \delta(a_i, a_k) \tag{21}$$

*Step 6:* As the weights of experts are different, we employ the weighted arithmetic aggregation operator to obtain the global dominance degree of the alternative $a_i$ as:

$$\emptyset(a_i) = \sum_{q=1}^{Q} w_q \left(\phi^{(q)}(a_i)\right) \tag{22}$$

*Step 7:* The final normalized global dominance of the alternative $a_i$ can be calculated by

$$\xi(a_i) = \frac{\emptyset(a_i) - \min\{\emptyset(a_i)\}}{\max\{\emptyset(a_i)\} - \min\{\emptyset(a_i)\}} \tag{23}$$

The ranking of alternatives can be acquired in descending order of $\xi(a_i)$, for $i = 1, 2, \cdots, m$, and the algorithm ends.

## IV. A CASE STUDY: THE GREEN SUPPLIER SELECTION FOR A FOOD COMPANY

In this section, a case study concerning the selection of green suppliers for a food company is used to demonstrate the feasibility and efficiency of the proposed CIVL-TODIM method. Then, we compare the CIVL-TODIM method with other ranking methods to demonstrate its advantages.

### A. CASE DESCRIPTION

Today, the aim of sustainable development has become a global consensus [38], and the green development has become an important direction of economic transformation in China. Since the Third Plenary Session of the Sixteenth Central Committee of the Party, China has issued a series of policies and measures to support the energy conservation, emission reduction and green environmental protection. These policies have played a positive role in some aspects, such as promoting industrial transformation, developing green consumption, green trade and promoting the construction of ecological civilization.

From a long-term perspective, the green development is to solve the problem of harmonious coexistence between man and nature. It is a necessary condition for sustainable development and an important manifestation of people's pursuit of a better life. To better respond to the green development, the research of green supply chain management emerges with the needs of the times. It is gradually recognized by people for its social and economic benefits as well as environmental friendliness. The management of green supply chain has become a hot topic for scholars and enterprise managers [39]–[42]. The focus of the green supply chain management is to help organizations introduce green practices to reduce the negative impact on environment. In this regard, choosing the optimal green supplier has become a key strategic decision of green supply chain management, which could continuously improve the performance level of the supply chain. In addition, it also directly affects the compatibility of the supply chain and the environmental performance of the manufacturer. Therefore, the selection of green suppliers plays an indispensable role in maintaining the stable development of green supply chain management.

In response to the call of national green development policy and considering the future own development, a food company in China wants to select an optimal green supplier for raw materials as a long-term supplier. The fewer chemicals are contained in the raw materials provided by green suppliers, the less amount of chemicals that enterprises need to add to eliminate these harmful substances. The raw materials with good quality provided by green suppliers can bring three benefits: 1) the production process will bring smaller or avoid environmental pollution; 2) the safety factor of food produced can be guaranteed; 3) the green raw materials can reduce the input cost of production, and is conductive to the benign development of enterprises.

Currently, the food company has three relatively good green suppliers for raw materials. There are many factors to be considered in choosing the best supplier from them. If only one decision-maker makes the final decision, it is easy for he/she to consider poorly. After discussion with relevant departments, the food company decided to invite a group of experts $E = \{e_1, e_2, e_3\}$ in this field to anonymously assess the three green suppliers with respect to a set of qualitative criteria $C = \{c_1, c_2, c_3, c_4, c_5\}$. After reviewing relevant literature [35]–[38], the assessment criteria are selected as follows:

(1) Quality management ($c_1$): quality is the most important factor determining the competitive advantage of an enterprise. In this regard, experts can consider the supplier management activities, such as its quality planning, quality policy and quality control.

(2) Service level ($c_2$): it is necessary to satisfy the need of customers as far as possible and solve customer's problems in time. Specific contents include logistics management, marketing management, and after-sales maintenance.

(3) Green transportation ($c_3$): the goal of this criterion on transportation is characterized by energy conservation and emission reduction.

(4) Green image ($c_4$): the reputation of green supplier should be considered since good reputation is conductive for long-term cooperation.

(5) Environmental management system ($c_5$): employing an effective management technology is to seek effective balance between economy and environmental construction.

### B. SOLVING THE CASE BY THE PROPOSED CIVL-TODIM METHOD

In the following, we use the CIVL-TODIM method to solve the above case.

Let $S = \{s_{-3}, \cdots, s_0, \cdots, s_3\}$ be an LTS that experts use to evaluate alternatives under different criteria. The specific meanings of the linguistic terms are denoted as $\{s_{-3} = none$, $s_{-2} = very\ terrible$, $s_{-1} = terrible$, $s_0 = medium$, $s_1 = good$, $s_2 = very\ good$, $s_3 = perfect\}$. In addition, concerning the weights of the criteria, the specific meanings of linguistic terms are denoted as: $\{s_{-3} = very\ unimportant$, $s_{-2} = unimportant$, $s_{-1} = a\ little\ unimportant$, $s_0 = medium$, $s_1 = a\ little\ important$, $s_2 = important$, $s_3 = very\ important\}$. The linguistic assessments given by the experts are represented by CIVLEs. To save the space, the transformed linguistic expressions of each expert by the transformation function are directly shown. In addition, the consensus threshold $\lambda = 0.75$ is given by the moderator in advance.

Firstly, the experts give their corresponding linguistic assessments on the importance degrees of criteria. The provided CIVLEs of the experts concerning the criteria's importance degrees are shown as:

$$C = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} [s_{2.5}, s_3] & [s_{1.6}, s_2] & [s_2, s_{2.5}] & [s_2, s_{2.5}] & [s_2, s_3] \\ [s_2, s_2] & [s_2, s_{2.5}] & [s_{1.3}, s_2] & [s_1, s_2] & [s_{1.2}, s_2] \\ [s_2, s_{2.5}] & [s_1, s_2] & [s_{1.5}, s_2] & [s_2, s_2] & [s_2, s_{2.6}] \end{bmatrix} \end{array}$$

By Eq. (12), the weights of criteria can be obtained as: $\omega_1 = 0.25$, $\omega_2 = 0.18$, $\omega_3 = 0.17$, $\omega_4 = 0.18$ and $\omega_5 = 0.22$. The criterion $c_1$ has the greatest weight. Thus, $c_1$ should be regarded as the reference criterion $\omega_c$. By Eq. (3), the relative weights of other criteria are obtained as: $\omega_{11} = 1$,$\omega_{21} = 0.72$, $\omega_{31} = 0.68$, $\omega_{41} = 0.72$ and $\omega_{51} = 0.88$.

Then, the provided linguistic assessments of the experts concerning the alternatives under different criteria are expressed as follows:

$$D(e_1) = \begin{bmatrix} [s_{1.1}, s_2] & [s_2, s_2] & [s_{0.5}, s_1] & [s_1, s_{1.5}] & [s_{1.5}, s_2] \\ [s_1, s_{1.5}] & [s_1, s_{1.5}] & [s_{1.5}, s_2] & [s_1, s_1] & [s_{0.5}, s_1] \\ [s_1, s_{1.5}] & [s_{1.5}, s_2] & [s_1, s_{1.6}] & [s_0, s_{0.5}] & [s_1, s_2] \end{bmatrix}$$

$$D(e_2) = \begin{bmatrix} [s_{1.2}, s_2] & [s_2, s_2] & [s_{1.5}, s_2] & [s_{1.5}, s_2] & [s_2, s_{2.5}] \\ [s_0, s_1] & [s_1, s_{1.8}] & [s_{1.2}, s_2] & [s_{1.2}, s_2] & [s_{1.5}, s_2] \\ [s_1, s_{1.8}] & [s_{1.5}, s_2] & [s_1, s_2] & [s_{-0.5}, s_0] & [s_1, s_2] \end{bmatrix}$$

$$D(e_3) = \begin{bmatrix} [s_1, s_2] & [s_{1.5}, s_2] & [s_0, s_{0.8}] & [s_{0.5}, s_1] & [s_1, s_{1.8}] \\ [s_2, s_2] & [s_1, s_{1.5}] & [s_1, s_{1.2}] & [s_1, s_{1.5}] & [s_1, s_1] \\ [s_0, s_1] & [s_2, s_{2.5}] & [s_1, s_{1.5}] & [s_0, s_1] & [s_{1.5}, s_2] \end{bmatrix}$$

Then, by Eqs. (13)-(15), the collective decision-making matrix for all experts can be obtained as $\bar{D}$, as shown at the bottom of the next page.

By Eq. (16), the preliminary consensus levels of these experts can be calculated as: $CL(e_1) = 0.83$, $CL(e_2) = 0.70$ and $CL(e_3) = 0.73$. Then, by Eq. (17), the weight of each expert can be obtained as: $w_1 = 0.37$, $w_2 = 0.31$ and $w_3 = 0.32$. Firstly, according to the identification rules in feedback adjustment, the consensus level of each expert is checked by Eq. (18), the moderator can find that $e_1$ and $e_2$ should improve their linguistic assessments. Then, based on the direction rules, the moderator provides the integrated collective decision matrix for the experts. These identified experts need to find out 2-5 linguistic to revise by comparing their own decision matrix and the integrated collective decision matrix. Subsequently, the moderator can provide some suggestions to them to revise corresponding linguistic assessment, i.e., the expert $e_1$ or $e_2$ should increase their linguistic assessment for terms with lower linguistic assessment for a certain alternative under a certain criterion. Vice versa. Based on the direction rules in the feedback adjustment, $e_1$ revises the linguistic assessments $X_{31}$ (the alternative $a_3$ under the criterion $c_1$) and $X_{13}$ (the alternative $a_1$ under the criterion $c_3$). The modified linguistic decision matrices of $e_2$ and $e_3$ are represented as follows $D'(e_2)$ and $D'(e_3)$, as shown at the bottom of this page. We can find that the expert revise.

Then, by Eq. (16), the consensus levels of these experts are: $CL'(e_1) = 0.84$, $CL'(e_2) = 0.73$ and $CL'(e_3) = 0.75$. Based on the identification by Eq. (18), $e_3$ has arrived the consensus threshold, but $e_2$ needs to continue to modify the linguistic assessments according to suggestions provided by the moderator. Based on the direction rules, $e_2$ revises the linguistic assessments $X_{22}, X_{24}$ and $X_{33}$. The second modified linguistic matrix of $e_2$ is shown $D''(e_2)$, as shown at the bottom of this page.

Similarly, by Eq. (16), the consensus levels of the experts are recalculated as: $CL''(e_1) = 0.85$, $CL''(e_2) = 0.76$ and $CL''(e_3) = 0.77$. By Eq. (18), all experts have reached the consensus threshold.

Next, based on $D(e_1)$, $D''(e_2)$ and $D'(e_3)$), the partial dominance of each expert for the alternative $a_i$ over the alternative $a_j$ under a criterion can be calculated by Eq. (19). Here, we take the expert $e_1$ as an example to illustrate the calculation process. The partial dominance of alternatives under different criteria corresponding to the expert $e_1$ is shown in Table 1.

Next, the dominance matrix of alternatives under all criteria for expert $e_1$ can be obtained by Eq. (20), which is shown in Table 2.

Subsequently, the global dominance degree of each alternative for the expert $e_1$ is calculated by Eq. (21). The calculated results are: $\phi^{(1)}(a_1) = -0.858$, $\phi^{(1)}(a_2) = -7.632$ and $\phi^{(1)}(a_3) = -7.185$.

**TABLE 1. Partial dominance of alternatives under different criteria.**

| $\Phi_{c_1}^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | 0.299 | 0.299 |
| $a_2$ | -1.197 | 0 | 0 |
| $a_3$ | -1.197 | 0 | 0 |
| $\Phi_{c_2}^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | 0.293 | 0.251 |
| $a_2$ | -1.626 | 0 | -1.494 |
| $a_3$ | -1.394 | 0.269 | 0 |
| $\Phi_{c_3}^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | -1.638 | -1.399 |
| $a_2$ | 0.278 | 0 | 0.257 |
| $a_3$ | 0.046 | -1.510 | 0 |
| $\Phi_{c_4}^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | 0.225 | 0.247 |
| $a_2$ | -1.248 | 0 | 0.216 |
| $a_3$ | -1.375 | -1.199 | 0 |
| $\Phi_{c_5}^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | 0.317 | 0.248 |
| $a_2$ | -1.440 | 0 | -1.378 |
| $a_3$ | -1.128 | 0.303 | 0 |

Using the same method, the global dominance of each alternative for the experts $e_2$ and $e_3$ can be obtained respectively as follows: $\phi^{(2)}(a_1) = -1.666$, $\phi^{(2)}(a_2) = -4.409$ and $\phi^{(2)}(a_3) = -10.9$; $\phi^{(3)}(a_1) = -6.643$, $\phi^{(3)}(a_2) = -5.641$ and $\phi^{(3)}(a_3) = -3.551$.

By Eq. (22), the global dominance degree of each alternative for all experts in the MCGDM can be calculated as: $\emptyset(a_1) = -2.960$, $\emptyset\ (a_2) = -5.996$ and $\emptyset(a_3) = -7.173$. Based on Eq. (23), the final normalized global dominance of each alternative is calculated: $\xi(a_1) = 1.000$, $\xi(a_2) = 0.279$ and $\xi(a_3) = 0.000$, and thus we have $a_1 \succ a_2 \succ a_3$, where

$$\bar{D} = \begin{bmatrix} [s_{1.1}, s_2] & [s_{1.83}, s_2] & [s_{0.67}, s_{1.27}] & [s_1, s_{1.5}] & [s_{1.5}, s_{2.1}] \\ [s_1, s_{1.5}] & [s_1, s_{1.6}] & [s_{1.23}, s_{1.73}] & [s_{1.07}, s_{1.5}] & [s_1, s_{1.33}] \\ [s_{0.67}, s_{1.43}] & [s_{1.67}, s_{2.17}] & [s_1, s_{1.7}] & [s_{-0.17}, s_{0.5}] & [s_{1.17}, s_2] \end{bmatrix}$$

$$D'(e_2) = \begin{bmatrix} [s_{1.2}, s_2] & [s_2, s_2] & [s_1, s_{1.2}] & [s_{1.5}, s_2] & [s_2, s_{2.5}] \\ [s_0, s_1] & [s_1, s_{1.8}] & [s_{1.2}, s_2] & [s_{1.2}, s_2] & [s_{1.5}, s_2] \\ [s_1, s_{1.5}] & [s_{1.5}, s_2] & [s_1, s_2] & [s_{-0.5}, s_0] & [s_1, s_2] \end{bmatrix}$$

$$D'(e_3) = \begin{bmatrix} [s_1, s_2] & [s_{1.5}, s_2] & [s_0, s_{0.8}] & [s_{0.9}, s_1] & [s_1, s_{1.8}] \\ [s_2, s_2] & [s_1, s_{1.5}] & [s_1, s_{1.2}] & [s_1, s_{1.5}] & [s_{0.8}, s_1] \\ [s_0, s_1] & [s_2, s_2] & [s_1, s_{1.5}] & [s_0, s_1] & [s_1, s_2] \end{bmatrix}$$

$$D''(e_2) = \begin{bmatrix} [s_{1.2}, s_2] & [s_2, s_2] & [s_1, s_{1.2}] & [s_{1.5}, s_2] & [s_2, s_{2.5}] \\ [s_0, s_1] & [s_1, s_{1.5}] & [s_{1.2}, s_2] & [s_1, s_{1.5}] & [s_{1.5}, s_2] \\ [s_1, s_{1.5}] & [s_{1.5}, s_2] & [s_1, s_{1.3}] & [s_{-0.5}, s_0] & [s_1, s_2] \end{bmatrix}$$

**TABLE 2.** Dominance of alternatives under all criteria for expert $e_1$.

| $\delta^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | -3.376 | -3.552 |
| $a_2$ | -2.001 | 0 | -2.137 |
| $a_3$ | -1.849 | -2.399 | 0 |

"$\succ$" means "superior to". Therefore, the alternative $a_1$ is regard as the optimal green supplier for the food company.

### C. COMPARATIVE ANALYSES WITH OTHER RANKING METHODS

In this paper, we propose the CIVL-TODIM method to enlarge the application scope of the classical TODIM method. To illustrate the feasibility and applicability of the proposed method in MCGDM, we employ two common decision-making methods to make some comparative analyses for the above case study.

Based on the preliminary linguistic assessments provided by all experts under different criteria, CIVLEs in the decision-making matrix can be transformed into specific numerical values by Eq. (1). The results are shown as follows:

$$D(e_1)=\begin{bmatrix}1.55 & 2.00 & 0.75 & 1.25 & 1.75\\ 1.25 & 1.25 & 1.75 & 1.00 & 0.75\\ 1.25 & 1.75 & 1.30 & 0.25 & 1.50\end{bmatrix}$$
$$D(e_2)=\begin{bmatrix}1.60 & 2.00 & 1.75 & 1.75 & 2.25\\ 0.50 & 1.40 & 1.60 & 1.60 & 1.75\\ 1.40 & 1.75 & 1.50 & -0.25 & 1.50\end{bmatrix}$$
$$D(e_3)=\begin{bmatrix}1.50 & 1.75 & 0.40 & 0.75 & 1.40\\ 2.00 & 1.25 & 1.10 & 1.25 & 1.00\\ 0.50 & 2.25 & 1.25 & 0.50 & 1.75\end{bmatrix}$$

#### 1) SOLVING THE CASE BY THE CLASSICAL TODIM METHOD

The traditional TODIM method [17] can be used to deal with the above case and obtain the ranking of alternatives. By Eq. (2), the normalized decision-making matrix of each expert can be obtained as:

$$\bar{D}(e_1)=\begin{bmatrix}0.38 & 0.40 & 0.20 & 0.50 & 0.44\\ 0.31 & 0.25 & 0.46 & 0.40 & 0.19\\ 0.31 & 0.35 & 0.34 & 0.10 & 0.38\end{bmatrix}$$
$$\bar{D}(e_2)=\begin{bmatrix}0.46 & 0.39 & 0.36 & 0.56 & 0.41\\ 0.14 & 0.27 & 0.33 & 0.52 & 0.32\\ 0.40 & 0.34 & 0.31 & -0.08 & 0.27\end{bmatrix}$$
$$\bar{D}(e_3)=\begin{bmatrix}0.38 & 0.39 & 0.15 & 0.30 & 0.34\\ 0.50 & 0.27 & 0.40 & 0.50 & 0.24\\ 0.13 & 0.34 & 0.45 & 0.20 & 0.42\end{bmatrix}$$

The relative weights of criteria are $\omega_{11} = 1$, $\omega_{21} = 0.72$, $\omega_{31} = 0.68$, $\omega_{41} = 0.72$ and $\omega_{51} = 0.88$. Then, based on Eqs. (4)-(5), for different experts, the partial dominance degrees of alternatives under all criteria are shown in Tables 3-5.

**TABLE 3.** Dominance of alternatives under all criteria for the expert $e_1$.

| $\delta^{(1)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | -0.575 | -0.306 |
| $a_2$ | -3.057 | 0 | -1.294 |
| $a_3$ | -2.938 | -1.788 | 0 |

**TABLE 4.** Dominance of alternatives under all criteria for the expert $e_2$.

| $\delta^{(2)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | 0.732 | 0.821 |
| $a_2$ | -3.514 | 0 | -1.142 |
| $a_3$ | -4.516 | -2.259 | 0 |

**TABLE 5.** Dominance of alternatives under all criteria for the expert $e_3$.

| $\delta^{(3)}(a_i,a_k)$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | -1.911 | -1.305 |
| $a_2$ | -0.815 | 0 | -1.331 |
| $a_3$ | -1.249 | -2.035 | 0 |

For each expert, the global dominance degrees of alternatives can be calculated by Eq. (6), respectively: $\xi^{(1)}(a_1) = 1$, $\xi^{(1)}(a_2) = 0.098$ and $\xi^{(1)}(a_3) = 0$; $\xi^{(1)}(a_1) = 1$, $\xi^{(2)}(a_2) = 0.221$ and $\xi^{(2)}(a_3) = 0$; $\xi^{(3)}(a_1) = 0.059$, $\xi^{(3)}(a_2) = 1$ and $\xi^{(3)}(a_3) = 0$. Here, we employ the average arithmetic operator to obtain the final global dominance degrees of alternatives concerning all experts and the results are: $\xi(a_1) = 0.586$, $\xi(a_2) = 0.549$ and $\xi(a_3) = 0$. Then, the ranking of alternatives is: $a_1 \succ a_2 \succ a_3$. Thus, the green supplier $a_1$ should be the optimal selection for food industry.

#### 2) SOLVING THE CASE BY THE EXTENDED VIKOR METHOD

Here, the extended VIKOR method [43] in continuous interval-valued linguistic environment is used to deal with the above case. Based on the decision matrix of specific numerical values for each expert, the target-based linear normalization formula [44] can be employed:

$$y_{ij}^1 = 1 - \frac{|x_{ij} - r_j|}{\max\limits_i |x_{ij} - r_j|}$$

where $r_j$ denotes the target value on the criterion $c_j$, and $x_{ij}$ denotes the specific numerical value of the alternative $a_i$ with regard to the criterion $c_j$. If the criterion $c_j$ is in benefit type, $r_j = \min\limits_i x_{ij}$; if the criterion $c_j$ is in cost type, $r_j = \max\limits_i x_{ij}$. Here, the provided fives criteria are in benefit type. The target-based linear normalized values for three experts are illustrated in Table 6.

The compromise value of each alternative can be calculated by $CV_i^q = \gamma(GU_i - GU^-)/(GU^+ - GU^-) + (1-\gamma)(IR^+ - IR_i)/(IR^+ - IR^-)$, where the group utility of the alternative $a_i$ is represented as $GU_i = \sum_{j=1}^n \omega_j y_{ij}^1$,

**TABLE 6.** Target-based linear normalized values for three experts.

| $e_1$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|
| $a_1$ | 1 | 1 | 0 | 1 | 1 |
| $a_2$ | 0 | 0 | 1 | 0.75 | 0 |
| $a_3$ | 0 | 0.67 | 0.55 | 0 | 0.75 |
| $e_2$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
| $a_1$ | 1 | 1 | 1 | 1 | 1 |
| $a_2$ | 0 | 0 | 0.4 | 0.93 | 0.33 |
| $a_3$ | 0.82 | 0.58 | 0 | 0 | 0 |
| $e_3$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
| $a_1$ | 0.67 | 0.5 | 0 | 0.33 | 0.53 |
| $a_2$ | 1 | 0 | 0.82 | 1 | 0 |
| $a_3$ | 0 | 1 | 1 | 0 | 1 |

**TABLE 7.** Calculated results by the VIKOR method.

| | $GU_i$ | $IR_i$ | $\gamma$ | | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|
| | 0.83 | 0.17 | 0.2 | | 1 | 0 | 0.03 |
| $e_1$ | 0.31 | 0.25 | 0.5 | $CV_i^1$ | 1 | 0 | 0.07 |
| | 0.38 | 0.25 | 0.8 | | 1 | 0 | 0.11 |
| | 1 | 0 | 0.2 | | 1 | 0 | 0.1 |
| $e_2$ | 0.31 | 0.25 | 0.5 | $CV_i^2$ | 1 | 0 | 0.06 |
| | 0.31 | 0.22 | 0.8 | | 1 | 0 | 0.02 |
| | 0.43 | 0.17 | 0.2 | | 0.8 | 0.5 | 0.2 |
| $e_3$ | 0.57 | 0.22 | 0.5 | $CV_i^3$ | 0.5 | 0.69 | 0.5 |
| | 0.57 | 0.25 | 0.8 | | 0.2 | 0.88 | 0.8 |

and the individual regret value is represented as $IR_i = \max_j \left(\omega_j(1-y_{ij}^1)\right)$; $GU^+ = \max_i GU_i$, $GU^- = \min_i GU_i$, $IR^+ = \max_i IR_i$ and $IR^- = \min_i IR_i$. The parameter $\gamma$ is the relative importance on $GU_i$ and $IR_i$. The calculated results with three different values of the parameter $\gamma$ are shown in Table 7.

When $\theta = 0.2$, the compromise value $CV_i = \frac{1}{Q}\sum_{q=1}^{Q}(CV_i^q)$ for different alternatives can be obtained as: $CV_1 = 0.93$, $CV_2 = 0.17$ and $CV_3 = 0.11$. Thus, the ranking of alternatives is: $a_1 \succ a_2 \succ a_3$. When $\theta = 0.5$, we have $CV_1 = 0.83$, $CV_2 = 0.23$ and $CV_3 = 0.21$, and thus the ranking of alternatives is $a_1 \succ a_2 \succ a_3$. When $\theta = 0.8$, we have $CV_1 = 0.83$, $CV_2 = 0.23$ and $CV_3 = 0.21$, and thus the ranking of alternatives is $a_1 \succ a_3 \succ a_2$.

### 3) SOLVING THE CASE BY THE HESITANT FUZZY LINGUISTIC TODIM METHOD

The hesitant fuzzy linguistic TODIM (HFL-TODIM) method [19] is employed to solve the case and derive the overall dominance of the alternative. In the aforementioned case, there are three experts to assess alternatives. Firstly, the hesitant fuzzy linguistic decision matrix needs to be constructed. Based on the original linguistic assessments provided by three experts, we can transform them the hesitant fuzzy linguistic elements as follows:

**TABLE 8.** Overall dominance degrees of each alternatives over the others.

| $e_1$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | 0 | -0.274 | -0.407 |
| $a_2$ | -2.143 | 0 | -1.096 |
| $a_3$ | -1.516 | -1.167 | 0 |
| $e_2$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | 0.731 | 0.607 |
| $a_2$ | -3.630 | 0 | --1.160 |
| $a_3$ | -3.213 | -1.714 | 0 |
| $e_3$ | $a_1$ | $a_2$ | $a_3$ |
| $a_1$ | 0 | -1.767 | -2.036 |
| $a_2$ | -0.942 | 0 | -2.192 |
| $a_3$ | -1.108 | -1.533 | 0 |

$$D(e_1) = \begin{bmatrix} \{s_1, s_2\} & \{s_2, s_2\} & \{s_1, s_1\} & \{s_1, s_2\} & \{s_2, s_2\} \\ \{s_1, s_2\} & \{s_1, s_1\} & \{s_2, s_2\} & \{s_1, s_1\} & \{s_1, s_1\} \\ \{s_1, s_2\} & \{s_2, s_2\} & \{s_1, s_2\} & \{s_0, s_1\} & \{s_1, s_2\} \end{bmatrix}$$

$$D(e_2) = \begin{bmatrix} [s_1, s_2] & [s_2, s_2] & [s_2, s_2] & [s_2, s_2] & [s_2, s_3] \\ [s_0, s_1] & [s_1, s_2] & [s_1, s_2] & [s_1, s_2] & [s_2, s_2] \\ [s_1, s_2] & [s_2, s_2] & [s_1, s_2] & [s_{-1}, s_0] & [s_1, s_2] \end{bmatrix}$$

$$D(e_3) = \begin{bmatrix} [s_1, s_2] & [s_2, s_2] & [s_0, s_1] & [s_1, s_1] & [s_1, s_2] \\ [s_2, s_2] & [s_1, s_2] & [s_1, s_1] & [s_1, s_2] & [s_1, s_1] \\ [s_0, s_1] & [s_2, s_3] & [s_1, s_2] & [s_0, s_1] & [s_2, s_2] \end{bmatrix}$$

The relative weights of criteria are denoted as: $\omega_{11} = 1$, $\omega_{21} = 0.72$, $\omega_{31} = 0.68$, $\omega_{41} = 0.72$ and $\omega_{51} = 0.88$, and the weight of each expert can be denoted as: $w_1 = 0.37$, $w_2 = 0.31$ and $w_3 = 0.32$. Then, the overall dominance degrees of each alternative over others can be obtained by the algorithm of the proposed TODIM method with HFLTSs [19], which is shown in Table 8.

Subsequently, we can obtain the overall dominance degrees of alternatives for each expert. For expert $e_1$, $\xi(a_1) = 1$, $\xi(a_2) = 0$ and $\xi(a_3) = 0.218$. For expert $e_2$, $\xi(a_1) = 1$, $\xi(a_2) = 0.022$ and $\xi(a_3) = 0$. For expert $e_3$, $\xi(a_1) = 0$, $\xi(a_2) = 0.575$ and $\xi(a_3) = 1$. Considering the weight of each expert, the overall dominance degrees of alternatives for three experts can be obtained by the weighted arithmetic aggregation operator: $\xi(a_1) = 0.608$,$\xi(a_2) = 0.191$ and $\xi(a_3) = 0.401$. The ranking of alternatives can be denoted as: $a_1 \succ a_3 \succ a_2$. Therefore, for the food industry, the green supplier $a_1$ should be the optimal selection.

### 4) COMPARATIVE ANALYSIS

Based on the calculated results, we can find that the alternative $a_1$ is always the optimal green suppler for the food company. In the following, the proposed CIVL-TODIM method is compared with the classical TODIM method and the traditional VIKOR method in detail.

#### a: COMPARE THE CLASSICAL TODIM METHOD WITH THE CIVL-TODIM METHOD

The classical TODIM method is mainly applied to the quantitative situation to obtain the optimal alternative. However, in actual decision making, experts may prefer to use linguistic assessments to express their opinions because specific numerical values are difficult to be provided directly. Therefore, we employ the CIVLEs to express the opinions of experts because it is closer to human habitual expression than numerical values. In addition, the classical TODIM method does not consider the consensus problem of experts, and the weights of criteria are directly provided. In the MCGDM, the divergence often appears since different experts have different backgrounds and expertise. It is necessary to eliminate the divergence among experts as much as possible. In this paper, the proposed method takes the consensus checking and improving methods among experts into account, which can efficiently help experts modify their linguistic assessments. Furthermore, based on the linguistic assessments of experts, the weights of criteria and experts are calculated by the improved distance measure, which could reduce the loss of information. Thus, the proposed method is comprehensive to obtain the optimal alternative. The ranking of alternatives is relatively robust and reliable.

#### b: COMPARE THE EXTENDED VIKOR METHOD WITH THE PROPOSED METHOD

The extended VIKOR method in the continuous interval-valued linguistic environment is based on the idea of compromise optimization to obtain the ranking of alternatives. In this method, an appropriate value of the parameter $\gamma$ is hard to be determined to aggregate the group utility values and individual regret values. If the number of alternatives is large, and two relative optimal alternatives need to be selected from all alternatives, there may be multiple selections that are not conducive to decision-making. In addition, by employing the extended VIKOR method in the MCGDM, the consensus among experts is rarely taken into account. However, the CIVL-TODIM method considers the consensus levels among experts so that the optimal alternative obtained is more in line with actual decision-making problems. Therefore, the decision-making risk is relatively small, which can also help decision makers obtain an acceptable solution from all provided alternatives.

#### c: COMPARE THE HFL-TODIM METHOD WITH THE PROPOSED METHOD

Based on the ranking results of alternatives compared with the proposed method, the optimal green supplier for the food industry is still alternative $a_1$. However, based on the fact that there are usually multiple experts participated in actual decision-making problems, the divergencies among experts are easily arose. In the HFL-TODIM method, it does not consider the group decision-making problems involving multiple experts. In addition, when experts have a certain understanding of decision-making problems, the discrete linguistic terms are sometimes difficult to accurately express the ideas of experts. In this paper, the proposed CIVL-TODIM method can have relatively precise linguistic assessments for alternatives under different criteria. The consensus among experts and feedback adjustment method to improve consensus level are taken into account. Thus, the ranking result of alternatives are relatively close to the actual situation. The corresponding decision-making risk could be reduced.

## V. CONCLUSION

As is known to us, the TODIM method has been applied in many fields due to its efficient performance. Nevertheless, the traditional TODIM method is rarely applied to the continuous interval-valued linguistic environment that is close to the precise linguistic expressions of people. By using the CIVLTS, people can express individual accurate and complicated assessments. In this regard, we extended the TODIM method into the continuous interval-valued linguistic environment and developed the CIVL-TODIM method.

In addition, with the rapid development of modern society with the scientific and technological progress, great changes have taken place in the decision-making process. It is no longer a decision-maker who decides to select the optimal alternative, but multiple experts in a group to determine one. Based on the fact that multiple experts participate in the process of decision making and different experts have different backgrounds and expertise, it is difficult to avoid the conflicts and divergences among experts on the provided decision-making information. Thus, we considered the consensus of experts while employing the CIVL-TODIM method. Based on the improved distance measure of CIVLEs, the consensus level of each expert was obtained. Then, the consensus level of each expert can be checked by comparing with the predefined consensus threshold. For experts who do not reach the consensus threshold, we employed a feedback optimization method to enhance the consensus levels of experts. In addition, the weights of both experts and criteria were calculated based on the improved distance measure. In this way, the linguistic assessment information provided by experts can be transformed on the same benchmark as much as possible, which can reduce the loss of information in the process of transformation. The advantages of the proposed method were illustrated by a case study concerning a food company to select the optimal green supplier from multiple alternatives.

However, we only consider the symmetrical linguistic term when the linguistic assessments are provided. In the future, the semantics of the asymmetrical linguistic term will be taken into account. In addition, there are some research topics concerning the CIVL-TODIM method to be considered in the future. For instance, since criteria may be interactive, the combination of the CIVL-TODIM method with Choquet integral could be an interesting research issue. Besides, it is necessary to apply the CIVL-TODIM method to other fields to verify its practicability and reliability.

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