

## Grey box modelling for river control

Björn Sohlberg and Mats Sernfält

### ABSTRACT

This paper deals with modelling and identification of a river system using physical insights about the process, experience of operating the system and information about the system dynamics shown by measured data. These components together form a linear model structure in the state space form. The inputs of the prospective model are physical variables, which are not directly measured. However, the model inputs can be found by a nonlinear transformation of measured variables. Unknown parameters of the model are estimated from measured data. The modelling work focuses on the principle of parsimony, which means the best model approach is the simplest one that fit the purpose of the application.

The goal of the model is to control the water level of the river, where the water flow is mainly determined by the demand for energy generation produced by the hydropower stations along the river. The energy requirement increases in the morning and decreases in the evening. These flow variations, caused by the energy demand, have to be compensated by controlling the power plants downstream, in such a way that the water level between the power stations is guaranteed. Simulation of the control system by using an adaptive model predictive controller shows that the water levels vary less and can be maintained at a higher level than during manual control. This means that more electric power can be produced with the same amount of water flow.

**Key words** | extended Kalman filter, grey box model, model predictive control, parameter estimation, river

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### INTRODUCTION

An important way of producing electrical energy is by using hydropower stations along rivers. Since electrical energy cannot be stored on a large scale, the energy has to be generated exactly when the order occurs. The demand for energy varies during the day; increasing during the daytime and decreasing at night. There is also a variation over a 7-day period; the demand is higher during working days compared to other days. Further, the resources of water are depending on seasonal variations. To make it possible to maintain such a varying energy production, the water is held back in reservoirs and rivers. This leads to a situation with varying water levels and flows.

There are also environmental and ecological aspects of using waterpower. Varying water levels affect the gardens of the local residents, aquatic sport and fishing

facilities, etc. Changing water flow also has an impact on erosion along the river. Furthermore, continuous operation of the turbine will increase wear of the machinery and increase maintenance cost (Meier *et al.* 1992).

To accommodate and coordinate the various interests optimally, statutory rules are indispensable. They provide a basis for cooperation aimed at doing justice to both the economic and the environmental aspects of any interference with the natural water cycle. Together with the individual concession stipulations, they provide the guidelines for operating the specific hydropower stations.

To predict and control the water flows and levels of a river automatically, it is necessary to have access to an adequate model of the system. Generally, modelling the

water flows and the levels in a river involves the experience of the specialists to determine the model structure, which is influenced by many factors, such as the water depth, water velocity, friction force, lateral inflow and boundary conditions, etc. Models constructed in this way are called white box models or simulation models.

A second way of making a model of a process is to adapt a standard parametric model to measured data obtained from the process. This will give a model that does not reflect any specific physical structure of the system, namely a black box model.

For many physical processes, there exists some, but incomplete, knowledge about the structure of the system. It implies that, between the white box models and the black box models, there is a grey zone, where so-called grey box models are appropriate (Bohlin 1991). These give a third way of making models of physical systems, whereby *a priori* knowledge concerning the process is used directly and unknown parts are estimated from measured data.

The system under investigation is a nonlinear process and is also dependent on specific weather and seasons. Therefore, the model must be able to cope with varying operating situations. This means that the model has to be adapted to the specific operation of the river and some unknown parameters have to be estimated on line.

This paper demonstrates how the grey box technique can be applied when different kinds of knowledge and information are used to formulate a model structure of a river system. It shows that knowledge about physical relations, experience from operating the system, dynamics revealed from measured data and nonlinear parameter transformation should be considered when the structure of the model is formulated.

In our case the resulting process description is a state space model. A direct way to estimate unknown parameters of a state space model is to use an Extended Kalman Filter (EKF), which estimates simultaneously both the states and the parameters. Wilson *et al.* (1998) discuss practical experience of applying EKF to industrial processes and parameter estimation is presented by Wu & Bellgardt (1998).

The river system is mainly affected by the flows at the power stations located sequentially. The flow at the

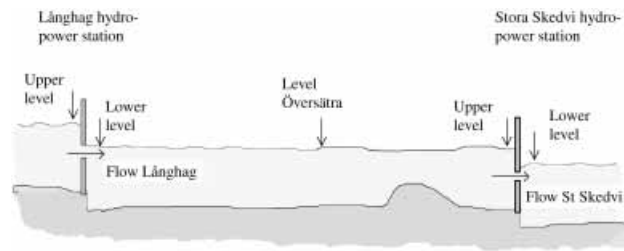


Figure 1 | Schematic outline of the process.

upstream station is estimated from energy production measurements and the flow at the downstream station is manipulated by adjusting the turbine generators to maintain the water levels at specified locations to defined limits. The system consists also of time delays. This makes the process suitable for control by a model predictive controller. An overview of model predictive control is given by Morari & Lee (1999).

## PROCESS DESCRIPTION

The system considered is part of the river Dalälven, Sweden, and consists of the section between the hydro-power plants at Långhag and Stora Skedvi. The distance between the power plants is about 12 km and an outline of the river is presented in Figure 1.

The water level is measured at five places along the route: upper and lower level at Långhag hydropower station, upper and lower level at Stora Skedvi hydro-power station and finally at a place named Översättra located between the two power plants. The water level at Översättra is restricted by law to a specific interval.

The water flow through the power stations at Långhag and at Stora Skedvi consists of two parts: one part originates from the flow through the turbine generators and one part originates from the flow passing through the floodgates. Normally, the floodgates are closed but they are opened during the spring flood. At Långhag there are two turbine generators and three floodgates and at Stora Skedvi there are two turbine generators and two floodgates.

The river flow is also influenced by inflow from small rivers along the river, rainfall, evaporation and melting snow. These flows can be regarded as slowly varying disturbances. Along the river, there is a bare hillock about two km upstream from Stora Skedvi, which to some extent influences the water flow.

The flow through the generators at Långhag power plant is regulated by energy requirements from the energy market. These flow variations at Långhag must be compensated for by controlling the energy production at Stora Skedvi power plant in such a way that the water level at Översåtra is maintained at a given level. Furthermore, the upper level at Stora Skedvi must be kept high to maintain the efficiency of the power generation and also kept constant to reduce the impact on the environment.

One of the difficulties in controlling the water levels is that there is no basin between the two power stations. Basins work as a buffer and reduce the influence of the variations in the water flows and prevent flooding. It is also a time delay system: it takes about 10–15 minutes for a variation in the flow at Långhag to influence the water level at Stora Skedvi.

Another difficulty is the backwater effect. The water level declines from the lower water level at Långhag to the upper level at Stora Skedvi and the decrease varies from 0.2 m to 0.5 m. This means that the slope for this part of the river is about 0.002–0.004%. Backwater effects will occur when the slope is less than 0.01% (Chow *et al.* 1988). Consequently, upstream propagation must be considered when the river is controlled and the water levels considered along the river cannot be controlled independently of each other; a change in the water flow will influence all levels considered in a complex way.

The flows at the hydropower stations are not measured. However, estimation of the flows can be made based on the generated electric power produced and the difference between the upper and lower levels at the power stations. The precision of the estimated flows depends on the quality of the measurements and the accuracy of the model the estimation is based on.

When the net inflow, the mean of the difference between the flows at Långhag and Stora Skedvi and the other kinds of flows affecting the river, is positive the

water levels will increase and the reverse will happen when the net inflow is negative: the water levels will decrease. To make a rough estimate of the influence of the accuracy in the estimates of the water flow, we regard the part of the river considered to be a uniform water tank, which is 12 km long and 30 m wide. Assuming the flow at Långhag and Stora Skedvi is 500 m<sup>3</sup>/s stationary; a change of 1% of the water flow at Stora Skedvi will cause the water level to deviate 2 cm per hour. This means the system is very sensitive due to bias in the estimates of the water flows.

To summarize, the system consists of two interacting hydraulic complexes situated sequentially. The hydraulic degrees of freedom contain:

- flow and level upstream of Långhag power station,
- flow and level downstream of Långhag power station,
- flow and level upstream of Stora Skedvi power station,
- flow and level downstream of Stora Skedvi power station.

This leads to eight unknown variables, although the flows are equal at each power station.

## GREY BOX MODELLING

The river system considered is a distributed parameter process, because the flow rate, the velocity and the depth vary in space throughout the watershed. Based on physical laws, this type of process can be described by using partial differential equations from the Saint-Venant equations (Chow *et al.* 1988). This model is not well suited, because the control problem will be ill conditioned and sensitive due to bias in the estimates of the flows.

The situation focuses on the principle of parsimony, which was first formulated by the medieval philosopher Ockham. The rule says the best model approach is the simplest one that fits the purpose of the application. This means that a description of the system has to model the main behaviour of the process, which is important to the control design, and operation of the process.

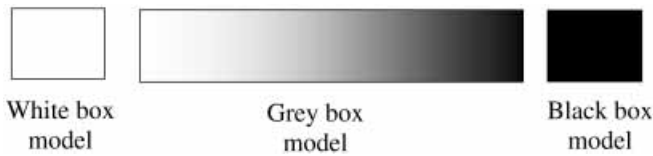


Figure 2 | Grey scale for grey box modelling.

To solve the problem we apply the grey box method, which is based on *a priori* knowledge about the process and measured data. The amount of knowledge varies from one process to another. The grey zone in Figure 2 represents the area between the two extremes of the white box and black box models, where the grey box model can be found. In this application, the model may be located somewhere in the middle of the grey zone.

The grey box modelling technique has been focused in different ways. One way emanates from the black box model frame. *A priori* knowledge is incorporated as constraints on model parameters or variables (Bohlin & Graebe 1994). Further, Lindskog & Ljung (2000) incorporate constraints on the model gain where the static gain is restricted to being a nonlinear function.

Another way is to start with a model originating from mathematical relations, which describe the behaviour of the system. This means the starting point is a specific model structure based on physical relations. Basing our conclusions on hypotheses concerning the process, we can formulate the relations between the variables for the process. Typically, the equations consist of three types: balance equations, algebraic equations and logical expressions.

The balance equations relate dependent variables to the independent variables and are derived from the principle of conservation, which is formulated as (Stephanopoulos 1984)

$$\frac{\text{Accumulation of } S}{\text{Time period}} = \frac{\text{Total input of } S}{\text{Time period}} - \frac{\text{Total output of } S}{\text{Time period}} + \frac{\text{Total generation of } S}{\text{Time period}} - \frac{\text{Total consumption of } S}{\text{Time period}} \quad (1)$$

Equation (1) means that the accumulation of a quantity,  $S$ , per time period within the process is equal to the net

sum of total input to the process plus the net sum of  $S$  generated inside the process. For example, the quantity  $S$  may denote energy, mass or momentum.

Recently, several successful attempts have been made in the area of grey box modelling based on mathematical relations (Sohlberg 1998; Bechmann *et al.* 2000).

Another possible way, which deals with the nonlinearities of a process, is to make a nonlinear transformation of the measured data into new variables. In this case, *a priori* knowledge of the process is used to generate new input output variables, which are used to estimate unknown parameters in a black box model. The resulting model will typically be linear in the new variables and the method is named semi-physical modelling (Lindskog & Ljung 1994; Sjöberg *et al.* 1995).

In this paper we use a combination of the grey box methods discussed earlier. Physical insight is used to form a linear *model structure* with both known and unknown parameters. The inputs of the prospective model are physical variables, which are not directly measured. However, the model inputs can be found by a nonlinear *transformation of the measured input* variables. The process operators, who have experience of operating the process over several years, have an intuitive understanding of the relation between different process variables. This knowledge is considered when formulating the *expanded model structure*, where the model structure is extended with supplementary relations about important variables. Furthermore, there is information from measured data to be considered, which reflects the behaviour of the system. This is a kind of pattern recognition information, which is also incorporated into the model structure. The expanded model consists of several unknown parameters, which need to be estimated. This is done by *parameter estimation* using established black box identification procedures.

## APPLICATION OF GREY BOX MODELLING

### Model structure

Consider a demarcated volume of the river, with water depth  $y$  and cross section area  $A$ . Let the inflow to the

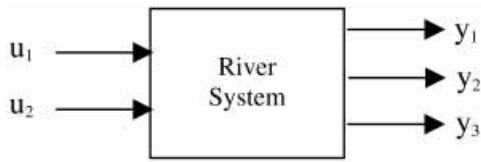


Figure 3 | Schematic diagram of the system.

volume be  $q_{in}$  and outflow  $q_{out}$ . From the principle of conservation, a reasonable basic lumped parameter model is given as

$$A \frac{dy}{dt} = q_{in} - q_{out} \quad (2)$$

Using the Euler approximation (Åström & Wittenmark 1997), Equation (2) is converted into a discrete form, where  $\Delta t$  is the sample interval:

$$y(k) = y(k-1) + \frac{\Delta t}{A} [q_{in}(k-1) - q_{out}(k-1)]. \quad (3)$$

Equation (3) means that the river is considered as a system with water flows at the two power stations as inputs and the water levels at the power stations together with the water level at Översätra as outputs. To be more specific, let the input  $u_1$  be the sum of flows caused by the generators at Långhag power station and the input  $u_2$  be the sum of the flows from the power generation at Stora Skedvi. The outputs  $y_1$ ,  $y_2$  and  $y_3$  are the lower water level at Långhag, the water level at Översätra and the upper water level at Stora Skedvi, respectively. Consequently, the process is characterized by two inputs and three outputs shown by Figure 3.

From Equation (3), a model structure is formed as

$$y_1(k) = y_1(k-1) + p_{11}u_1(k-n_{11}) - p_{12}u_2(k-n_{12}) \quad (4)$$

$$y_2(k) = y_2(k-1) + p_{21}u_1(k-n_{21}) - p_{22}u_2(k-n_{22}) \quad (5)$$

$$y_3(k) = y_3(k-1) + p_{31}u_1(k-n_{31}) - p_{32}u_2(k-n_{32}) \quad (6)$$

where  $p_{ij}$  are unknown parameters and  $n_{ij}$  are unknown time delays,  $i = 1, 2, 3$  and  $j = 1, 2$ .

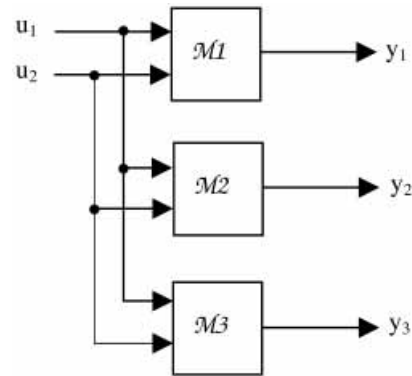


Figure 4 | Schematic model structure.

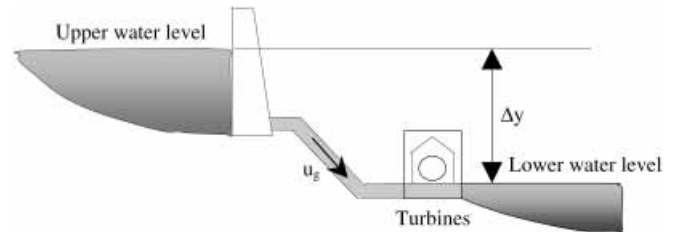


Figure 5 | Hydropower generation.

Principally, Equations (4)–(6) give a linear model describing the behaviour of the water levels at three different places along the river. The model consists of three sub-models M1, M2 and M3, where each sub-model is formulated with two inputs and one output. A schematic structure of the model is shown in Figure 4.

### Transformation of the measured inputs

Water flowing through the turbine generates electricity by changing the potential energy into electrical energy. The power generated is proportional to the product of the flow rate through the turbine and the difference between the water levels above and below the power station, see Figure 5. The difference in water levels is named the net head and is represented by  $\Delta y$ . The net head is a function of the flow through the turbine, but over a short time horizon  $\Delta y$  does not vary much. The loss of power when water energy is transformed into electricity is modelled by the efficiency of the turbine and is denoted by  $\eta$ . The

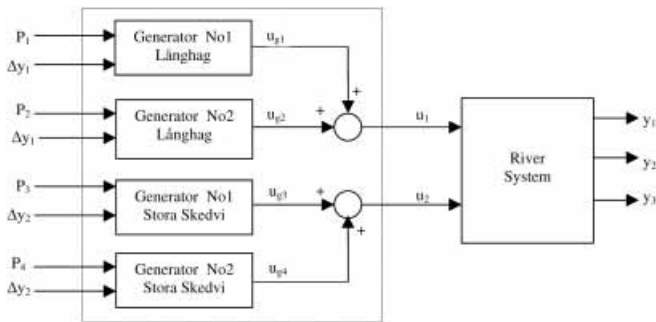


Figure 6 | Model structure.

efficiency is a nonlinear function of the flow  $u_g$  and the net head  $\Delta y$ . Generally; the power generated by a turbine is modelled as (Philpott *et al.* 2000)

$$P(u_g, \Delta y) = u_g \Delta y \eta(u_g, \Delta y). \quad (7)$$

Referring to Equation (7), it is possible to measure the generated power and the net head. The efficiency is not known directly, but from the flow–power curves produced from tests with the turbines, it is possible to form a relation between the flow and the measured electricity power and the net head.

By parameter estimation and validation, the relation estimating the flow through the turbines are approximated with Equation (8). This equation is valid for both power plants, but the estimates of the parameters  $a_1$ ,  $a_2$  and  $a_3$  differ:

$$u_g = a_1 \left( \frac{P}{\Delta y} \right) + a_2 \left( \frac{P}{\Delta y} \right)^2 + a_3 \left( \frac{P}{\Delta y} \right)^3. \quad (8)$$

Since Equation (8) is a nonlinear function of the power and the net head, the computation of water flow has to be made for each turbine separately. A layout of the transformation, from the measured variables net-head and generated power to flows, is shown in Figure 6. The variables  $\Delta y_1$  and  $\Delta y_2$  represent the net head at Långhag and Stora Skedvi, the generated electric power is represented by  $P_1$ – $P_4$  and the flow through the turbines by  $u_{g1}$ – $u_{g4}$ .

A typical data sequence achieved from the river system is shown in Figures 7 and 8, where the flows are estimated from Equation (8). The sample interval is 5 minutes and the duration of the measurement is 160 hours or about one week. The sample interval is a compromise between a desired short time due to the Euler approximation and a desired longer time due to rise times and time delays of the system.

The flow through the turbines at Långhag is increased in the morning due to increased demand for energy and consequently the flow at Stora Skedvi is increased to maintain the water levels at the desired height. In the evenings the flows are decreased again. Due to the variations in the flows the water levels will also vary. Since the variations in both the water levels and water flows are significant, the variables can be considered as informative and may be used for parameter estimation.

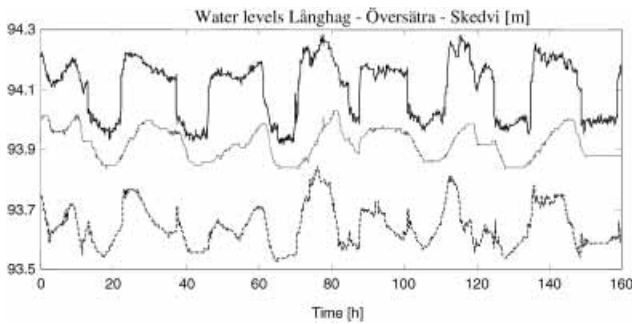
### Expanded model structure

The estimation and validation procedure is not very successful, since the sub-models given by Equations (4) and (5) are not able to describe the variation in water. This means that additional sources of information need to be considered when forming the model structure.

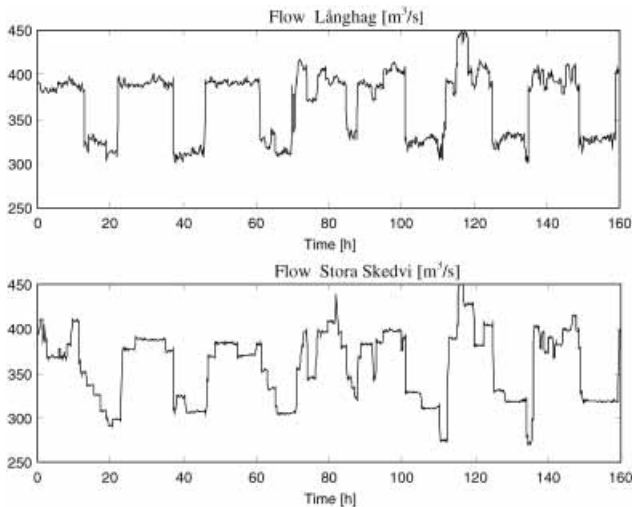
Figure 7 shows that the shapes of the three water levels are somewhat different. The level at Långhag responds very fast to the change in the turbine flows. The water level is measured close to Långhag power station, so the fast variation is attributable mainly a result of the changes in the flows through the turbines at this station. It is possible to generate a principal curve, which describes the dynamics of the water level, by means of simulation based on a transfer function  $H(q^{-1})$ , given by Equation (9), see Figure 9. This curve is achieved by letting the input signal be a square wave, which imitates the flow variations at Långhag:

$$H(q^{-1}) = \frac{p_{11} - p_{12}q^{-1}}{1 - q^{-1}} \quad (9)$$

where  $q^{-1}$  is the shift operator.



**Figure 7** | Water levels at Långhag, Översätra and Stora Skedvi. Solid: Långhag, dotted: Översätra and dashed: Skedvi.

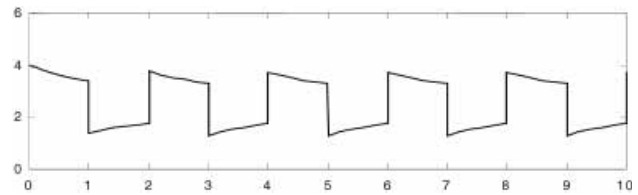


**Figure 8** | Estimated water flow through the turbines at Långhag and Stora Skedvi.

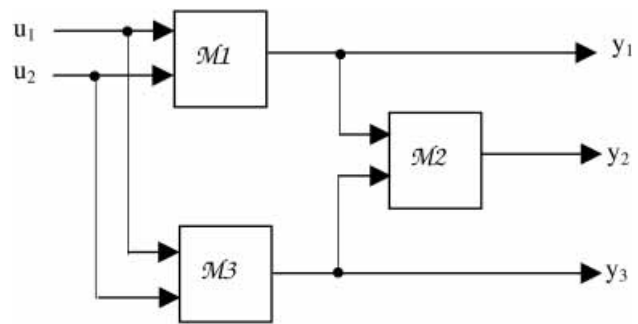
This implies that the model given by Equation (4) should be expanded into the form given by Equation (10):

$$\begin{aligned} y_1(k) = & y_1(k-1) \\ & + p_{11}u_1(k-n_{11}) - p_{12}u_1(k-n_{11}-1) \\ & - p_{13}u_2(k-n_{12}) \end{aligned} \quad (10)$$

The sub-model M2 given by Equation (5) is not suitable for use as a model structure, since estimation of the unknown parameters gives large variations in the estimates depending on the actual measured data sequence used. From empirical knowledge, gained during discussion with the process operators, the water level at



**Figure 9** | Principal appearance of the water level at Långhag.



**Figure 10** | Expanded schematic structure of the model.

Översätra depends very much on the variations of the water levels at Långhag and Stora Skedvi. This justifies changing the model structure to a structure given by the following equation:

$$y_2(k) = p_{21}y_2(k-1) + p_{22}y_1(k-n_{21}) + p_{23}y_3(k-n_{22}) \quad (11)$$

Equation (11) can be interpreted as that the flows  $u_1(k)$  and  $u_2(k)$  affect the water level at Översätra via the levels at Långhag and Stora Skedvi.

The water level changes fast at Stora Skedvi but not so fast as at Långhag. The model structure may still be given as Equation (6):

$$y_3(k) = y_3(k-1) + p_{31}u_1(k-n_{31}) - p_{32}u_2(k-n_{32}). \quad (12)$$

From the discussion in this section, the model structure given by Figure 4 should be changed to the structure given by Figure 10. This means that the inputs of the sub-model M<sub>2</sub> are exchanged to be the water levels at Långhag and Stora Skedvi.

### Parameter estimation

Since the system is a multi-input–multi-output process, a state space form is often preferred:

$$x(k+1) = F(\theta)x(k) + G(\theta)u(k) + v(k)$$

$$y(k) = Cx(k) + w(k) \quad (13)$$

where  $x(k)$  is the state vector,  $y(k)$  is the output vector,  $u(k)$  is the control vector and  $\theta$  is a vector of unknown parameters. The matrices  $F(\theta)$  and  $G(\theta)$  consist of the unknown parameters. The matrix  $C$  consists entirely of known parameters. The disturbance vectors  $v(k)$  and  $w(k)$  are considered as white noise with covariance matrices  $R_v$  and  $R_w$ .

The unknown parameters are estimated by using the Matlab® System Identification Toolbox (Ljung 2000). The estimation routine minimizes the quadratic prediction error criterion,  $V_N(\hat{\theta})$ , with an iterative Gauss–Newton algorithm. The prediction error is formulated by Equation (14) and the criterion by Equation (15) (Ljung 1999), where  $\hat{y}(k | \theta)$  is the predicted output:

$$\varepsilon(k, \theta) = y(k) - \hat{y}(k | \theta) \quad (14)$$

$$V_N(\hat{\theta}) = \det \left[ \frac{1}{N} \sum_{k=1}^N \varepsilon(k, \theta) \varepsilon^T(k, \theta) \right]. \quad (15)$$

The flow at Långhag,  $u_1(k)$ , is reduced by the mean value,  $u_m$ , of the difference between the two flows  $u_1(k)$  and  $u_2(k)$ . This parameter includes evaporation, rainfall, inlet from small flows along the river and error resulting from estimation of the flows  $u_1(k)$  and  $u_2(k)$ . During the parameter estimation the mean value between the flows is eliminated:

$$\bar{u}_1(k) = u_1(k) - u_m. \quad (16)$$

In the state space model, given by Equations (17) and (18), the water levels at Långhag, Översätra and Stora Skedvi are modelled by the state variables  $x_4(k)$ ,  $x_5(k)$  and  $x_7(k)$ . The flows are delayed via the states  $x_1(k)$ ,  $x_2(k)$ ,  $x_3(k)$  and  $x_6(k)$ . The number of delayed samples is selected as the number minimizing the loss function given by Equation (15).

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \\ x_7(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ p_{12} & 0 & p_{13} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{22} & p_{21} & 0 & p_{23} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{31} & 1 \end{bmatrix} \times \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ p_{11} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & p_{32} \end{bmatrix} \begin{bmatrix} \bar{u}_1(k) \\ u_2(k) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ v_2(k) \\ v_3(k) \\ v_4(k) \\ v_5(k) \\ v_6(k) \\ v_7(k) \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \\ w_3(k) \end{bmatrix} \quad (18)$$

In Equation (17), all eight unknown parameters are estimated based on the measurement sequence presented in Figures 7 and 8. Simulation of the model is presented in Figure 11 together with the measured water levels.

### Comments on the water level at Långhag

The water level varies very fast when the flows change. It is seen that the model is able to predict these variations. The model predicts even the slow variations between the fast changes.

### Comments on the water level at Översätra

The model is able to predict the variations in the water level. Note: the transducer of the water level indicates a constant level several times when the water level reaches



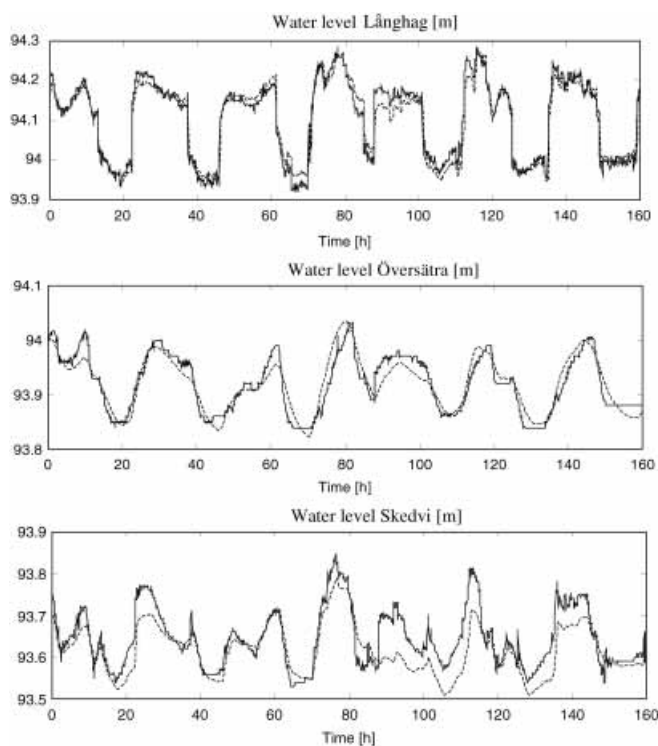


Figure 11 | Measured and simulated water levels. Solid: measured, dashed: simulated.

its lowest values. It is possible that some problem with the transducer arises when the level has been decreased and increases again.

#### Comments on the water levels at Stora Skedvi

The model describes the dynamics of the water level satisfactorily. However, the model is not able to predict the very fast increase in the water level after approximately 90 h.

#### Model analysis

To analyse the influence of weather conditions, seasonal variations and other varying conditions, the parameter estimation is made based on eleven different measurement sequences spread over a year. Each measurement sequence is approximately seven days. From the estimation based on the different sequences, it is shown that

Table 1 | Parameter estimates of the resulting model

	Parameter	Mean value	Standard deviation
Flow difference	$u_m$	8.2	3.2
Långhag M1	$p_{11} 10^{-3}$	$1.422 \pm 0.019$	0.295
	$p_{12} 10^{-3}$	$-1.259 \pm 0.013$	0.297
	$p_{13} 10^{-3}$	$-7.165 \pm 2.06$	3.304
Översätra M2	$p_{21}$	$0.981 \pm 0.0004$	0.011
	$p_{22} 10^{-3}$	$7.849 \pm 0.375$	3.807
	$p_{23} 10^{-3}$	$12.496 \pm 0.555$	8.916
Stora Skedvi M3	$p_{31} 10^{-3}$	$1.049 \pm 0.017$	0.478
	$p_{32} 10^{-3}$	$-1.050 \pm 0.017$	0.479

the parameter values differ, depending on which measurement sequence is used to estimate the parameters. The result of the estimation is presented in Table 1, showing the mean together with the 95% confidence interval and standard deviation of each parameter listed.

#### Comments on the flow difference

The mean value,  $u_m$ , varies and depends on the measurement sequence used. Consequently, it is necessary to adapt the model for the influence from this parameter; otherwise the water level will not be stable.

#### Comments on estimation of sub-model M1

The estimation gives that the sign of the parameters  $p_{11}$  and  $p_{12}$  is reversed for two of the measurement sequences. However, simulation based on the mean value of the parameters listed gives a satisfactory result for all measurement sequences, provided that we compensate for the mean value of flow difference.

### Comments on estimation of sub-model M2

The variance of parameter  $p_{21}$  is small compared to the estimate, which indicates that the estimate of the parameter is accurate. On the other hand, the model is very sensitive to this parameter. The parameters  $p_{22}$  and  $p_{23}$  vary approximately the same amount from one measurement sequence to another. Estimation and simulation show that it is possible to merge the parameters  $p_{22}$  and  $p_{23}$  into one parameter and estimate this parameter together with parameter  $p_{21}$  for each measurement sequence. This implies that two parameters need to be adapted to different operating situations.

### Comments on estimation of sub-model M3

The estimates of the parameters are almost equal, but depend on the measurement sequence is used. This means that the parameters  $p_{31}$  and  $p_{32}$  can be merged to one common parameter.

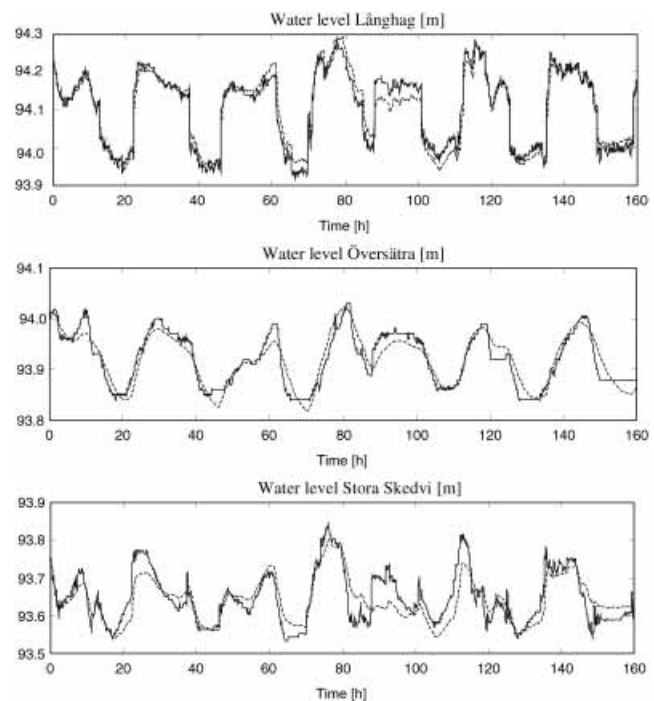
From the discussion above, we have a reduced model consisting of four parameters, which need to be estimated from measured data. The other parameters are regarded as constants and assigned the mean values shown in Table 1. The results from simulation of the reduced model are presented in Figure 12 and the corresponding parameter estimates are given in Table 2.

### Comments on the reduced model

The simulation with the reduced model gives approximately the same results as the previous model presented in Figure 11. From Table 2, we can see that parameter  $p_{21}$  is about the same as the mean value given in Table 1. The common parameter for  $p_{22}$  and  $p_{23}$  gives an estimate, which lies between the estimates given in Table 1. The common parameter for  $p_{31}$  and  $p_{32}$  gives an estimate higher than the mean value presented in Table 1.

## REAL-TIME ESTIMATION

From the discussion in the previous sections, we conclude that four parameters need to be estimated recursively. The



**Figure 12** | Measured and simulated water levels based on the reduced model. Solid: measured, dashed: simulated.

**Table 2** | Parameter estimates of the reduced model

	Parameter	Set value	Estimated value
Flow difference	$u_m$		8.58
Långhag	$p_{11} 10^{-3}$	1.42	
	$p_{12} 10^{-3}$	-1.26	
	$p_{13} 10^{-5}$	-7.20	
Översätra	$p_{21}$		$0.9815 \pm 0.0002$
	$p_{22} = p_{23} 10^{-3}$		$9.252 \pm 0.110$
Stora Skedvi	$p_{31} = -p_{32} 10^{-4}$		$1.203 \pm 0.018$

other parameters are set to constant values and equal the mean values given by Table 1. Recursive identification methods are necessary to solve the real-time estimation

problem and the subject is treated in detail by Anderson & Moore (1979) and Ljung & Söderström (1983).

From Equation (13), the process is described by a model with time varying parameters:

$$x(k+1) = F[\theta(k)]x(k) + G[\theta(k)]u(k) + v(k)$$

$$y(k) = Cx(k) + w(k) \quad (19)$$

where  $v(k)$  and  $w(k)$  represent white noise with covariance matrices  $R_v$  and  $R_w$ . The time varying vector  $\theta(k)$  given by Equation (20) consists of the unknown parameters to be estimated:

$$\theta(k) = [u_m(k) \ p_{21}(k) \ p_{22}(k) \ p_{31}(k)]^T. \quad (20)$$

An attractive approach to the parameter estimation problem is to estimate simultaneously both the process states and the unknown parameters by an augmented model in which the unknown parameters are regarded as additional states and then apply the Extended Kalman Filtering (EKF) algorithms to the augmented model; see Appendix A. Theoretical and practical aspects of Kalman filters are presented by Grewal & Andrews (1993).

Generally,  $\theta(k)$  is modelled by the following difference equation:

$$\theta(k+1) = \theta(k) + v_\theta(k). \quad (21)$$

The discrete white noise is represented by  $v_\theta(k)$ , with zero mean and covariance matrix  $R_\theta$ .

At start-up of the Extended Kalman Filter, the state vector is initialized with probable values. The initial value of the covariance matrix of the estimation error is chosen by repeated simulation so  $\theta(k)$  will have an acceptable settling time without unnecessary overshoots, although the choice is not critical. The covariance matrices of  $R_v$  and  $R_w$  are set to the same values as were used during off-line parameter estimation, while  $R_\theta$  is chosen by means of repeated simulations until acceptable results have been achieved for the parameter vector  $\theta(k)$ . Note that the elements of  $R_\theta$  are chosen at smaller values than the values of  $R_v$ , which means that the

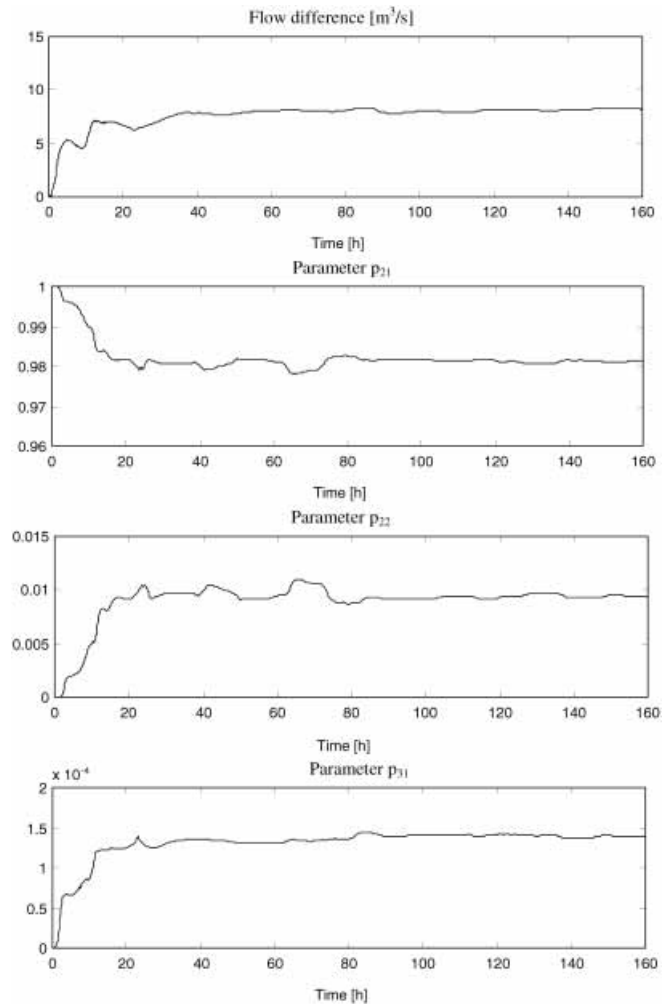


Figure 13 | Real-time estimation based on measured data.

process disturbances will be modelled mainly by  $v(k)$  and less by  $v_\theta(k)$ . Real-time estimation of the unknown parameter vector is shown in Figure 13 in which the estimation is based on the same sequence of data as used for the off-line estimation.

### Comments on the real-time estimation

The estimates of the flow difference converge towards approximately  $8 \text{ m}^3/\text{s}$ , which is about the same result as achieved from offline estimation. The other parameters converge also towards approximately the same values as presented in Table 2. The other measurement sequences

collected during a year show about the same performance but converge to other values. This means the Extended Kalman Filter is applicable in this case.

## CONTROL OF THE SYSTEM

The purpose of controlling the system is to keep important process variables within specified levels so that the energy production is efficient. In our case, it means that the water levels at the specific locations are limited to certain intervals restricted by law and practice. To avoid too much impact on the environment, there are also constraints on changes of the water flows. The control strategy can be formulated as follows: keep the net level at the power station Stora Skedvi as high as possible without violating the restriction of the water levels and without changing the water flow too fast.

### Controller design

To formulate the optimal control problem for a dynamic system, the following are needed (Kirk 1970): a mathematical model of the process to be controlled, concretization of the physical limits of the system and a specification of a performance measure.

The Model Predictive Controller, MPC (Camacho & Bordons 1995), is closely related to the optimal control problem. The MPC system is not a specific control strategy but more a range of control methods developed around certain ideas. The main differences between the optimal controller and the general MPC are that, for the MPC, two predictive horizons can be used, one for the control variable and other for the process output.

The model of the river is given in a state space form and is characterized by two input variables, of which one can be manipulated while one is only measurable. There are also constraints on both the manipulated and controlled variables and considerable time delays, which motivates application of a model predictive controller.

The computation of the control signal is performed by minimization of a loss function; see Appendix B. The

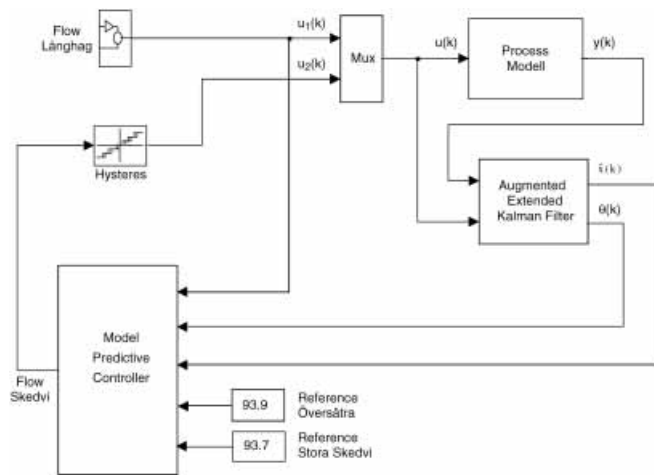


Figure 14 | Model predictive control scheme.

Table 3 | Statistical properties

	Översåtra		Stora Skedvi	
	Measured	Simulated control	Measured	Simulated control
Mean value (m)	93.92	93.96	93.64	93.70
Standard deviation (m)	0.05	0.02	0.07	0.03

procedure is repeated at every new sample index with a moving horizon and only the control variable from sample  $k$  is used as a control action.

### Simulation of the control system

For simulation of the control system, we use the same sequence of the flow at Långhag as used for the previous parameter estimation. This sequence is considered as a measurable disturbance since this input variable is a result of the energy demand and not possible to use as a manipulated variable to control the water levels. The system is controlled by merely manipulating the flow at Stora Skedvi.

All simulation is done by using the Matlab/Simulink® environment. The implementation of the process model is based on Equations (17) and (18). The process and

measurement noise are generated as white noise and the difference between the flows at Långhag and Stora Skedvi is set to the same value as achieved from the measurement sequence,  $8.5 \text{ m}^3/\text{s}$ .

Implementation of the Extended Kalman Filter is based on the equations given in Appendix A, which estimates both the model states and the unknown parameters of the reduced model. In the Simulink scheme the process model and the EKF are implemented, by using an S-function from the Simulink blocks, see Figure 14. The MPC block is based on a routine from the Model Predictive Toolbox (Morar and Ricker 1994) and is an MPC-type controller for constrained problems. The MPC routine is reconstructed to compute the control variable based on the reduced model and the equations given in Appendix B at every sample index. A hysteresis block prevents the control equipment from wearing out. From experience of operating the river system, the hysteresis level is set to  $10 \text{ m}^3/\text{s}$ .

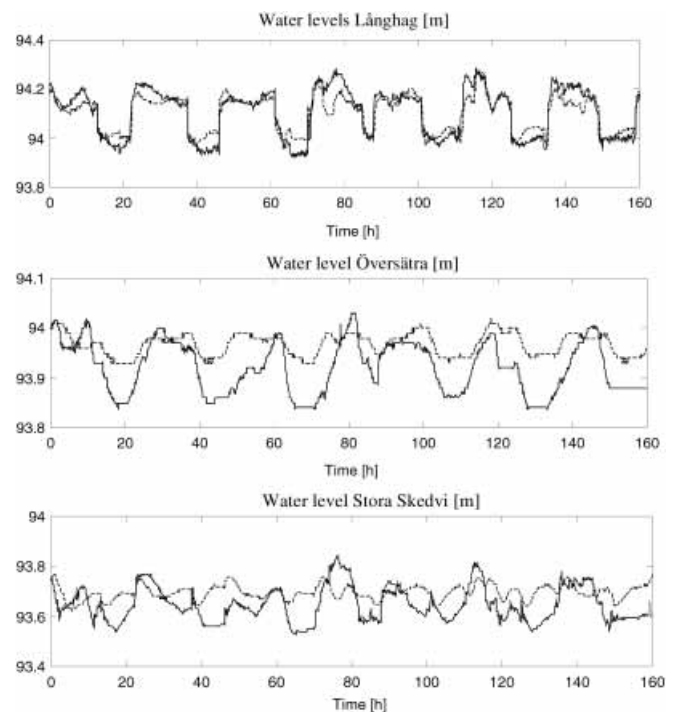
The Model Predictive Controller is tuned by repeated simulation and the control horizon is set to  $m=5$  and  $p=40$  samples, see Appendix B and Figure 19. The reference levels at Översätra and Stora Skedvi are set to 93.9 m and 93.7 m above sea level. These levels are also restricted to remain within the intervals 93.75–94.00 m and 93.50–94.00 m. The water level at Långhag has no restrictions. The results of the simulation of the automatic control system are presented in Figures 15–18.

### Comments on the water levels

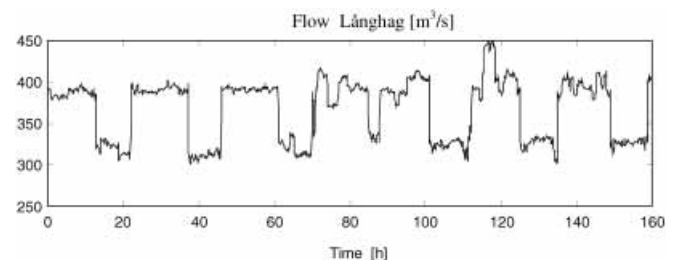
It is seen from Figure 15 that the water levels at Översätra and Långhag vary less under simulated automatic control than during manual control. The mean and standard deviation for the water levels are presented in Table 3. Further, the water level at Stora Skedvi can be kept at a higher level than during manual control. This gives that about 5% more electric power can be achieved from this hydropower station.

### Comments on the controlled water flow

Approximately the same amount of water is used during manual operating and simulated automatic control of the



**Figure 15** | Measured and under simulated automatic control of water levels. Solid: measured, dashed: simulated.

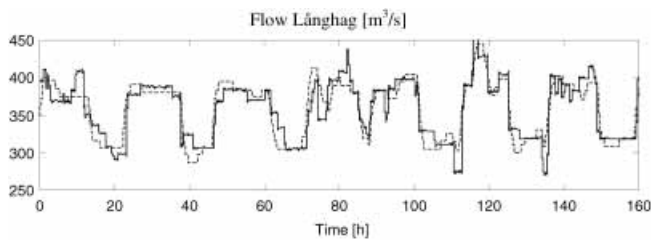


**Figure 16** | Estimated water flow through the turbines at Långhag.

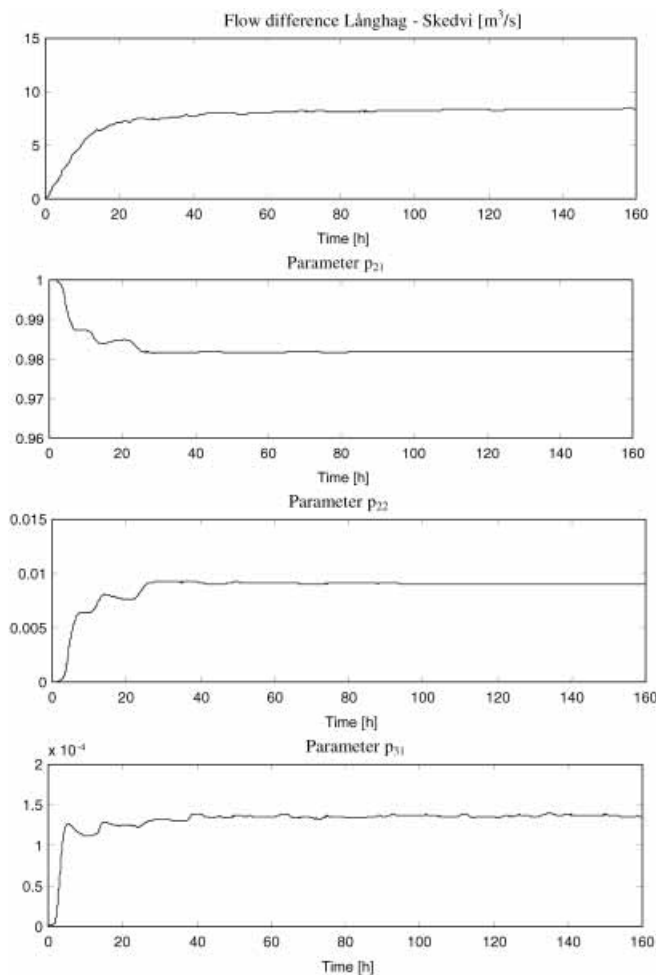
system. The results from the simulation shows also that it is possible to change the control signal in smaller steps and, by that, avoid too fast changes in the water flow, which reduces the impact on the environment, see Figure 17.

### Comments on the estimated parameters

The estimate of the flow difference converges towards the same value given in the process model, see Figure 18. It is



**Figure 17** | Estimated and under simulated automatic control of flow at Skedvi. Solid: measured, dashed: simulated.



**Figure 18** | Real-time estimation under simulated control.

also seen that the dynamic of the estimates has about the same appearance and converges towards about the same values as achieved when real data are used, see Figure 13 and Table 2.

## CONCLUSIONS

This application shows that the grey box method can be a successful way to construct process models by using different kinds of information. Knowledge gained from various areas is collected to construct a useful model structure. Physical insight is used to form a linear model structure with both known and unknown parameters. Experience from manual control of the river system indicates that there is a relation between the water levels at the power stations and a specific location along the river. The inputs of the prospective model are physical variables, which are not directly measured. However, the model inputs can be found by a nonlinear transformation of the measured input variables. Furthermore, the appearance of estimated flows and the measured water levels give additional insight into the process behaviour.

The resulting model is given in the state space form, which consists of unknown parameters, which need to be estimated on line. The model consists also of a parameter describing the flow difference between the flows at the two hydropower stations including evaporation, rainfall, inlet from small flows along the river and error resulting from computation of the flows. The state space model is augmented to include the unknown parameters as additional states and an Extended Kalman Filter is used to estimate both the states and the unknown parameters.

The river system is characterised by constraints on the manipulated and controlled variables, time delays and measurable disturbances. This makes the system suitable for manipulation by a Model Predictive Controller. Simulation of the control system based on the model predictive control system shows that the water levels vary less than during manual control. Further, the level at Stora Skedvi can be maintained at a higher level than during manual control, which means that about 5% more electric power can be produced with the same amount of water flow.

## ACKNOWLEDGEMENTS

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## APPENDIX A. EXTENDED KALMAN FILTER

Establish the augmented state vector and referring to the variables introduced earlier:

$$\xi(k) = \begin{bmatrix} x(k) \\ \theta(k) \end{bmatrix}. \quad (22)$$

The state equations of the augmented system are described by the following system:

$$\xi(k+1) = F'[\xi(k), u(k)] + v_{\xi}(k)$$

$$y(k) = C'\xi(k) + w(k) \quad (23)$$

where the process noise of the augmented state vector is  $v_{\xi}(k)$  with covariance matrix  $R_{\xi}$ :

$$v_{\xi}(k) = \begin{bmatrix} v(k) \\ v_{\xi}(k) \end{bmatrix} \quad R_{\xi} = \begin{bmatrix} R_1 & 0 \\ 0 & R_{\theta} \end{bmatrix}.$$

Application of the EKF on the augmented system, which estimates both the process states and the unknown parameters according to the prediction–estimation algorithm is given by Equation (24)–(30) (Grewal & Andrews 1993). The linearizing of the model around a nominal state trajectory gives the transition and the measurement matrices as

$$\Gamma(k) = \left. \frac{\partial C'}{\partial \xi} \right|_{\xi=\bar{\xi}(k), u=\bar{u}(k)} \quad \Phi(k) = \left. \frac{\partial F'}{\partial \xi} \right|_{\xi=\bar{\xi}(k), u=u(k)} \quad (24)$$

### Prediction

$$\bar{\xi}(k+1 | k) = F'[\hat{\xi}(k | k), u(k)] \quad (25)$$

$$\bar{y}(k) = C' \bar{\xi}(k) \quad (26)$$

$$P(k | k-1) = \Phi(k-1)P(k-1 | k-1)\Phi(k-1)^T + R_{\xi} \quad (27)$$

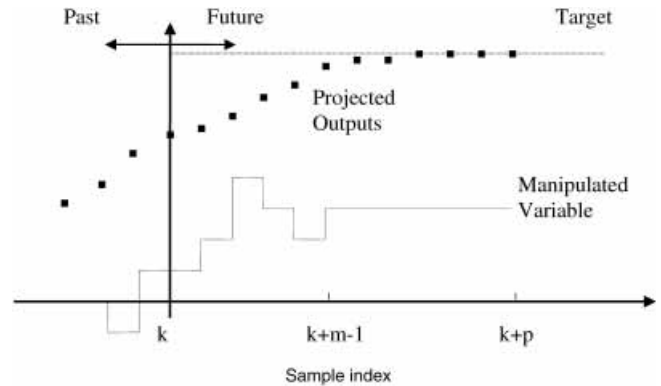


Figure 19 | Model predictive control.

### Estimation

$$K(k) = P(k | k-1)\Gamma(k)^T[\Gamma(k)P(k | k-1)\Gamma(k)^T + R_2]^{-1} \quad (28)$$

$$\hat{\xi}(k | k) = \bar{\xi}(k | k-1) + K(k)[y(k) - \bar{y}(k)] \quad (29)$$

$$P(k | k) = [I - K(k)\Gamma(k)]P(k | k-1)[I - K(k)\Gamma(k)]^T + K(k)R_2K(k)^T \quad (30)$$

where the indices  $k$  and  $k-1$  denote values pertaining to the current and previous sample, respectively.

## APPENDIX B. MODEL PREDICTIVE CONTROLLER

Referring to Figure 19, for a set of control moves  $\Delta u(k), \Delta u(k+1), \dots, \Delta u(k+m-1)$ , the future behaviour of the process outputs  $y(k+1 | k), y(k+2 | k), \dots, y(k+p | k)$  can be predicted over a horizon  $p$ . The  $m$  moves of the manipulated variable ( $m \leq p$ ) are computed to minimize a quadratic function:

$$J = \min_{\Delta u(k), \dots, \Delta u(k+m-1)} \sum_{j=1}^p \left\| \Gamma_j^y [y(k+j | k) - r(k+j)] \right\|^2 + \sum_{j=1}^m \left\| \Gamma_j^u [\Delta u(k+j-1)] \right\|^2 \quad (31)$$

where  $\Gamma_j^y$  and  $\Gamma_j^u$  are weighting matrices to penalise particular components of  $y$  and  $u$  at certain future intervals.

The references are denoted by  $r(k+j)$ . Only the first move,  $\Delta u(k)$ , is implemented, because the quadratic function is minimized at every new sample instance.

Computation of the control signal is based on constraints on the manipulated variables and outputs:

$$u_{\min}(j) \leq u(k+j) \leq u_{\max}(j)$$

$$|\Delta u(k+j)| \leq \Delta u_{\max}(j) \quad (32)$$

$$y_{\min}(j) \leq y(k+j) \leq y_{\max}(j)$$

where Equation (32) limits the values of the control signal and the outputs.

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