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GREY RELATIONAL ANALYSIS METHOD FOR INTUITIONISTIC FUZZY MULTIPLE ATTRIBUTE DECISION MAKING WITH PREFERENCE INFORMATION ON ALTERNATIVES

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Abstract

The aim of this paper is to investigate intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives, in which the information on attribute weights is completely unknown and the attribute values and preference information on alternatives take the form of intuitionistic fuzzy numbers. In order to get the weight vector of the attribute, we establish an optimization model based on the basic ideal of traditional grey relational analysis (GRA) method, by which the attribute weights can be determined. Then, based on the traditional GRA method, calculation steps for solving intuitionistic fuzzy multiple attribute decision-making problems with incompletely known weight information are given. The degree of grey relation between each alternative and subjective preference is defined to determine the ranking order of all alternatives. The method can sufficiently utilize the objective information, and meet decision makers' subjective preference, can also be easily performed on computer. Furthermore, we shall extend the developed models and procedures to solve the interval-valued intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Key Words: Multiple attribute decision-making; Grey relational analysis (GRA); Intuitionistic fuzzy numbers; interval-valued intuitionistic fuzzy numbers; Incomplete weight information, Preference

1 Introduction

Atanassov [1-3] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set [4]. The intuitionistic fuzzy set has received more and more attention since its appearance [5-28]. Szmidt and Kacprzyk [5-7] considered the use of intuitionistic fuzzy sets for building soft decision-making models with imprecise information, and proposed two solution concepts about the intuitionistic fuzzy core and the consensus winner for group decision making using intuitionistic fuzzy sets. Szmidt and Kacprzyk [9] proposed a non-probabilistic type of entropy measure for intuitionistic fuzzy sets. Szmidt and Kacprzyk [10] discussed distances between intuitionistic fuzzy sets. Bustince [11] presented different theorems for building intuitionistic fuzzy relations on a set with predetermined properties. Li and Cheng [12] studied similarity measures of intuitionistic fuzzy sets and their application to pattern recognitions. Szmidt and Kacprzyk [13] proposed some solution concepts in group decision making with intuitionistic fuzzy preference relations, such as intuitionistic fuzzy core and consensus winner, etc. Szmidt and Kacprzyk[14] investigated the consensus-reaching process in group decision making based on individual intuitionistic fuzzy preference relations. Atanassov et al. [15] provided an algorithm for solving the multi-person multi-attribute decision making problems, in which the attribute weights are given as exact numerical values and the attribute values are expressed in intuitionistic fuzzy numbers. Wei [16] developed some geometric aggregation functions and applied these operators to dynamic multiple attribute decision making in intuitionistic fuzzy setting. Wei [17] utilized maximizing deviation method to solve intuitionistic fuzzy multiple attribute decision making with incomplete weight information. Wei [18] proposed some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making. Wei [19] developed some induced aggregating operators with fuzzy number intuitionistic fuzzy information and their applications to group decision making. Li [20] investigated multiple attribute decision making with intuitionistic fuzzy information and constructed several linear programming models to generate optimal weights for attribute. Lin [21] presented a new method for handling multiple attribute fuzzy decision making problems, where the characteristics of the alternatives are represented by intuitionistic fuzzy sets. The proposed method allows the degrees of satisfiability and non-satisfiability of each alternative with respect to a set of attribute to be represented by intuitionistic fuzzy sets, respectively. Furthermore, the proposed method allows the decision-maker to assign the degree of membership and the degree of non-membership of the attribute to the fuzzy concept "importance." Li[22] extended the linear programming techniques for multidimensional analysis of preference (LINMAP) to develop a new methodology for solving multiple attribute decision making problems under Atanassov's intuitionistic fuzzy (IF) environments. Xu [23] investigate the group decision making problems in which all the information provided by the decision makers is expressed as intuitionistic fuzzy decision matrices where each of the elements is characterized by intuitionistic fuzzy number, and the information about attribute weights is partially known, which may be constructed by various forms. Xu and Yager[24] developed some geometric aggregation operators, such as the intuitionistic fuzzy weighted geometric (IFWG) operator, the intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, and the intuitionistic fuzzy hybrid geometric (IFHG) operator and gave an application of the IFHG operator to multiple attribute group decision making with intuitionistic fuzzy information. Xu [25] developed some arithmetic aggregation operators, such as the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, and the intuitionistic fuzzy hybrid aggregation (IFHA) operator. Xu [26] investigated the intuitionistic fuzzy MADM with incompletely known or completely unknown weight information based on the ideal solution. However, the above approaches can't deal with the intuitionistic fuzzy MADM problems with preference information on alternatives, in which the information on attribute weights is incompletely known and the attribute values and preference information on alternatives take the form of intuitionistic fuzzy numbers.

Grey system theory [29] is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. In grey system theory, according to the degree of information, if the system information is fully known, the system is called a white system; if the information is unknown, it is called a black system. A system with information known partially is called a grey system. The grey system theory includes five major parts: grey prediction, grey relational analysis (GRA), grey decision, grey programming and grey control. GRA is part of grey system theory, which is suitable for solving problems with complicated interrelationships between multiple factors and variables. So, GRA method has been widely used to solve the uncertainty problems under the discrete data and incomplete information [30-39]. In addition, GRA method is one of the very popular methods to analyze various relationships among the discrete data sets and make decisions in multiple attribute situations. The major advantages of the GRA method are that the results are based on the original data, the calculations are simple and straightforward, and, finally, it is one of the best methods to make decisions under business environment. In compared with other approaches to MADM problems [40-47, 55-57], GRA has been proven to be useful for solving problems with complicated interrelationships in multiple attribute decision making (MADM) problems.

In the process of intuitionistic fuzzy MADM with preference information on alternatives, sometimes, the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers, and the information about attribute weights is incompletely known because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain. All of the above methods, however, will be unsuitable for dealing with such situations. Therefore, it is necessary to pay attention to this issue. The aim of this paper is to develop a new method, based on the traditional GRA method, to overcome this limitation. The rest of the article is organized as follows. In the next section, we introduce some basic concepts related to intuitionistic fuzzy sets. In Section 3 we introduce intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers. To determine the attribute weights, an optimization model based on the traditional GRA method, by which the attribute weights can be determined, is established. Then, the degree of grey relation between every alternative and subjective preference is defined to determine the ranking order of all alternatives. The method can sufficiently utilize the objective information, and meet decision makers' subjective preference, can also be easily performed on computer. In Section 4, Furthermore, we shall extend the developed models and procedures to solve the interval-valued intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives. In Section 5, an illustrative example is pointed out. In Section 6 we conclude the paper and give some remarks.

2 Preliminaries

In the following, we introduce some basic concepts related to intuitionistic fuzzy sets.

Definition 1 Let X to be a universe of discourse, then a fuzzy set is defined as:

$$A=\left\{\left\langle x,\mu_A(x)\right\rangle \middle| x\in X\right\} \tag{1}$$

which is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A [4].

Atanassov[1-2] extended the fuzzy set to the IFS, shown as follows:

Definition 2 An IFS A in X is given by

$$A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle \middle| x\in X\right\} \tag{2}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, with the condition

$$0\le \mu_A(x)+\nu_A(x)\le 1,\ \ \forall\, x\in X$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A [1, 2].

Definition 3 For each IFS A in X, if

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x),\ \ \forall\, x\in X\ . \tag{3}$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A [1, 2].

Definition 4 Let $\tilde{a}_1=(\mu_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\nu_2)$ be two intuitionistic fuzzy numbers, then the normalized Hamming distance between $\tilde{a}_1=(\mu_1,\nu_1)$ and $\tilde{a}_2=(\mu_2,\nu_2)$ is defined as follows [26]:

$$d(\tilde{a}_1,\tilde{a}_2)=\frac{1}{2}\left(|\mu_1-\mu_2|+|\nu_1-\nu_2|\right) \tag{4}$$

3 Grey relational analysis method for intuitionistic fuzzy multiple attribute decision making with preference information on alternatives

The following assumptions or notations are used to represent the intuitionistic fuzzy MADM problems with with preference information on alternatives:

(1) The alternatives are known. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives;

(2) The attributes are known. Let $G=\{G_1,G_2,\cdots,G_n\}$ be a set of attributes;

(3) The subjective preference information on alternatives is known, and let $\theta=(\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_m)$ be subjective preference value vector, $\tilde{\theta}_i=(\alpha_i,\beta_i)$ is intuitionistic fuzzy number, which is subjective preference value on alternative $A_i\ (i=1,2,\cdots,m)$.

(4)The information about attribute weights is incompletely known. Let $w=(w_1,w_2,\cdots,w_n)$ be the weight vector of attributes, where $w_j\ge 0$, $j=1,2,\cdots,n$, $\sum_{j=1}^{n}w_j=1$. H is a set of the known weight information, which can be constructed by the following forms [48-51], for $i\ne j$: **Form 1.** A weak ranking: $w_i\ge w_j$; **Form 2.** A strict ranking: $w_i-w_j\ge\alpha_i$, $\alpha_i>0$; **Form 3.** A ranking of differences: $w_i-w_j\ge w_k-w_l$, for $j\ne k\ne l$; **Form 4.** A ranking with multiples: $w_i\ge\beta_i w_j$, $0\le\beta_i\le 1$; **Form 5.** An interval form: $\alpha_i\le w_i\le\alpha_i+\varepsilon_i$,

$0 \le \alpha_i < \alpha_i + \varepsilon_i \le 1$.

Suppose that $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \left(\mu_{ij}, \nu_{ij}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix, where μ_{ij} indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij} \subset [0,1]$, $\nu_{ij} \subset [0,1]$, $\mu_{ij} + \nu_{ij} \le 1$, $i = 1,2,\cdots,m$, $j = 1,2,\cdots,n$.

In the following, we apply GRA method to solve intuitionistic fuzzy MADM with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy information. The method involves the following steps:

(Procedure Ⅰ)

Step 1. Let $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n}$ be an intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij} = \left(\mu_{ij}, \nu_{ij}\right)$, which is an attribute value, given by an expert, for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$, $w = \left(w_1, w_2, \cdots, w_n\right)$ be the weight vector of attributes, where $w_j \in [0,1]$, $j = 1,2,\cdots,n$, H is a set of the known weight information, which can be constructed by the forms 1-5. Let $\theta = \left(\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_m\right)$ be subjective preference value, $\tilde{\theta}_i = \left(\alpha_i, \beta_i\right)$ is an intuitionistic fuzzy number, which is subjective preference value on alternative $A_i \left(i = 1,2,\cdots,m\right)$.

Step 2. Calculate the grey relational coefficient of each alternative between objective preference information $\tilde{r}_{ij}$ and subjective preference information $\tilde{\theta}_i$ using the following equation:

$$\xi_{ij} = \frac{\min\limits_i \min\limits_j d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) + \rho \max\limits_i \max\limits_j d\left(\tilde{r}_{ij}, \tilde{\theta}_i\right)}{\left(\tilde{r}_{ij}, \tilde{\theta}_i\right) + \rho \max\limits_i \max\limits_j \left(\tilde{r}_{ij}, \tilde{\theta}_i\right)}$$

$$i = 1,2,\cdots,m,\ j \in 1,2,\cdots,n. \tag{5}$$

where the identification coefficient $\rho = 0.5$.

Step 3. Calculating the degree of grey relational coefficient of each alternative from subjective preference information $\tilde{\theta}_i$ using the following equation:

$$\xi_i = \sum_{j=1}^{n} \xi_{ij} w_j,\quad i = 1,2,\cdots,m. \tag{6}$$

The basic principle of the GRA method is that the chosen alternative should have the "largest degree of grey relation" from the subjective preference information. Obviously, for the weight vector given, the larger ξ_i, the better alternative A_i is. But the information about attribute weights is incompletely known. So, in order to get the ξ_i, firstly, we must calculate the weight information. So, we can establish the following multiple objective optimization models to calculate the weight information:

$$(\text{M-1}) \begin{cases} \max\ \xi_i = \sum\limits_{j=1}^{n} \xi_{ij} w_j \\ s.t.\quad w_j \in H, \sum\limits_{j=1}^{n} w_j = 1,\ w_j \ge 0 \end{cases}$$

$$i = 1,2,\cdots,m,\quad j = 1,2,\cdots,n$$

Since each alternative is non-inferior, so there exists no preference relation on the all the alternatives. Then, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

$$(\text{M-2}) \begin{cases} \max D = \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} \xi_{ij} w_j \\ s.t.\quad w_j \in H, \sum\limits_{j=1}^{n} w_j = 1, w_j \ge 0 \end{cases}$$

By solving the model (M-2), we get the optimal solution $w = \left(w_1, w_2, \cdots, w_n\right)$, which can be used as the weight vector of attributes. Then, we can get ξ_i ($i = 1,\cdots,m$) by Equations (6).

Step 4. Rank all the alternatives $A_i \left(i = 1,2,\cdots,m\right)$ and select the best one(s) in accordance with $\xi_i \left(i = 1,2,\cdots,m\right)$. If any alternative has the highest ξ_i value, then, it is the most important alternative.

4 Extension

In the following, we introduce some basic concepts and the normalized Hamming distance of interval-valued intuitionistic fuzzy numbers.

Definition 5. Let X be an universe of discourse, An IVIFS $\tilde{A}$ over X is an object having the form [53-54]:

$$\tilde{A}=\left\{\left\langle x,\tilde{\mu}_A\left(x\right),\tilde{v}_A\left(x\right)\right\rangle \middle| x\in X\right\} \tag{7}$$

where $\tilde{\mu}_A\left(x\right)\subset\left[0,1\right]$ and $\tilde{v}_A\left(x\right)\subset\left[0,1\right]$ are interval numbers, and

$$0\le\sup\left(\tilde{\mu}_A\left(x\right)\right)+\sup\left(\tilde{v}_A\left(x\right)\right)\le 1,\ \forall\ x\in X$$

For convenience, let $\tilde{\mu}_A\left(x\right)=\left[a,b\right]$, $\tilde{v}_A\left(x\right)=[c,d]$, so $\tilde{A}=\left(\left[a,b\right],\left[c,d\right]\right)$.

Definition 6. Let $\tilde{a}_1=\left(\left[a_1,b_1\right],\left[c_1,d_1\right]\right)$ and $\tilde{a}_2=\left(\left[a_2,b_2\right],\left[c_2,d_2\right]\right)$ be two interval-valued intuitionistic fuzzy values, then the normalized Hamming distance between $\tilde{a}_1=\left(\left[a_1,b_1\right],\left[c_1,d_1\right]\right)$ and $\tilde{a}_2=\left(\left[a_2,b_2\right],\left[c_2,d_2\right]\right)$ is defined as follows [26]:

$$\begin{aligned} &d\left(\tilde{a}_1,\tilde{a}_2\right)\\ &=\frac{1}{4}\left(\left|a_1-a_2\right|+\left|b_1-b_2\right|+\left|c_1-c_2\right|+\left|d_1-d_2\right|\right)\end{aligned} \tag{8}$$

In this section, we shall investigate the intuitionistic fuzzy MADM problems with with preference information on alternatives.

Let A, G and θ be presented as in section 3, and suppose that $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\left[a_{ij},b_{ij}\right],\left[c_{ij},d_{ij}\right]\right)_{m\times n}$ is the interval-valued intuitionistic fuzzy decision matrix, where $\left[a_{ij},b_{ij}\right]$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, $\left[c_{ij},d_{ij}\right]$ indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\left[a_{ij},b_{ij}\right]\subset\left[0,1\right]$, $\left[c_{ij},d_{ij}\right]\subset\left[0,1\right]$, $b_{ij}+d_{ij}\le 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

The subjective preference information on alternatives is known, and let $\theta=\left(\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_m\right)$ be subjective preference value vector, $\tilde{\theta}_i=\left(\left[\alpha_i,\beta_i\right],\left[\gamma_i,\eta_i\right]\right)$ is intuitionistic fuzzy alternative number, which is subjective preference value on $A_i\left(i=1,2,\cdots,m\right)$.

The information about attribute weights is incompletely known which is presented as in section 3.

In the following, we apply GRA method to solve interval-valued intuitionistic fuzzy MADM with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of interval-valued intuitionistic fuzzy information. The method involves the following steps:

(Procedure Ⅱ)

Step 1. Let $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\left[a_{ij},b_{ij}\right],\left[c_{ij},d_{ij}\right]\right)_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}=\left(\left[a_{ij},b_{ij}\right],\left[c_{ij},d_{ij}\right]\right)$, which is an attribute value, given by an expert, for the alternative $A_i\in A$ with respect to the attribute $G_j\in G$, $w=\left(w_1,w_2,\cdots,w_n\right)$ be the weight vector of attributes, where $w_j\in\left[0,1\right]$, $j=1,2,\cdots,n$, H is a set of the known weight information, which can be constructed by the forms 1-5. Let $\theta=\left(\tilde{\theta}_1,\tilde{\theta}_2,\cdots,\tilde{\theta}_m\right)$ be subjective preference value, $\tilde{\theta}_i=\left(\left[\alpha_i,\beta_i\right],\left[\gamma_i,\eta_i\right]\right)$ is an interval-valued intuitionistic fuzzy number, which is subjective preference value on alternative $A_i\left(i=1,2,\cdots,m\right)$.

Step 2. Calculate the grey relational coefficient of each alternative between objective preference information $\tilde{r}_{ij}$ and subjective preference information $\tilde{\theta}_i$ using the following equation:

$$\xi_{ij}=\frac{\min\limits_i\min\limits_j d\left(\tilde{r}_{ij},\tilde{\theta}_i\right)+\rho\max\limits_i\max\limits_j d\left(\tilde{r}_{ij},\tilde{\theta}_i\right)}{\left(\tilde{r}_{ij},\tilde{\theta}_i\right)+\rho\max\limits_i\max\limits_j\left(\tilde{r}_{ij},\tilde{\theta}_i\right)}$$

$$i=1,2,\cdots,m,\ j\in 1,2,\cdots,n. \tag{9}$$

where the identification coefficient $\rho=0.5$.

Step 3. See step 3 of Procedure Ⅰ.

Step 4. See step 4 of Procedure Ⅰ.

5 Illustrative Example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from [52]). There is a panel with five possible alternatives to invest the money: ① A_1 is a car company; ② A_2 is a food company; ③ A_3 is a computer company; ④ A_4 is an arms company; ⑤ A_5 is a TV company. The investment company must take a decision according to the following four attributes: ① G_1 is the risk analysis; ② G_2 is the growth analysis; ③ G_3 is the social-political impact analysis; ④G_4 is the environmental impact analysis. The five possible alternatives $A_i\left(i=1,2,\cdots,5\right)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix}(0.4,0.5) & (0.5,0.4) & (0.2,0.7) & (0.3,0.5)\\(0.6,0.4) & (0.6,0.3) & (0.6,0.3) & (0.3,0.6)\\(0.5,0.5) & (0.4,0.5) & (0.4,0.4) & (0.5,0.4)\\(0.7,0.2) & (0.5,0.4) & (0.2,0.5) & (0.1,0.7)\\(0.5,0.3) & (0.3,0.4) & (0.6,0.2) & (0.4,0.4)\end{bmatrix}$$

Decision maker's subjective preference value on alternative $A_i\left(i=1,2,3,4,5\right)$ as follows:

$$\tilde{\theta}_1=\left(0.3,0.5\right),\tilde{\theta}_2=\left(0.6,0.2\right)$$
$$\tilde{\theta}_3=\left(0.5,0.4\right),\tilde{\theta}_4=\left(0.7,0.2\right)$$
$$\tilde{\theta}_5=\left(0.4,0.3\right)$$

The information about the attribute weights is incompletely known as follows:

$$H=\{0.25\le w_1\le 0.28,0.20\le w_2\le 0.25,$$
$$0.22\le w_3\le 0.25,0.25\le w_4\le 0.30,$$
$$w_j\ge 0,\ j=1,2,3,4,\sum\nolimits_{j=1}^{4}w_j=1\}$$

Then, we utilize the procedure I developed to get the most desirable alternative(s).

Step 1. Calculate the grey relational coefficient of each alternative between objective preference information $\tilde{r}_{ij}$ and subjective preference information $\tilde{\theta}_i$.

$$\xi=\left(\xi_{ij}\right)_{5\times 4}=\begin{bmatrix}0.8462 & 0.6471 & 0.6471 & 1.0000\\0.7333 & 0.8462 & 0.8462 & 0.4400\\0.8462 & 0.7333 & 0.8462 & 1.0000\\1.0000 & 0.5789 & 0.4074 & 0.3333\\0.8462 & 0.7333 & 0.6471 & 0.8462\end{bmatrix}$$

Step 2. Utilize the model (M-2) to establish the following single-objective programming model:

$$\begin{cases}\max D(w)=4.2718w_1+3.5388w_2+3.3938w_3+3.6195w_4\\ s.t.\ w\in H\end{cases}$$

Solving this model, we get the weight vector of attributes:

$$w=\left(0.28\ \ 0.20\ \ 0.22\ \ 0.30\right)^T$$

Step 3. Calculate the degree of grey relational coefficient of each alternative from subjective preference information $\xi_i^-\left(\ i=1,\cdots,m\right)$ by Equations (6).

$$\xi_1=0.8087,\xi_2=0.6927,\xi_3=0.8697$$
$$\xi_4=0.5854,\xi_5=0.7798$$

Step 4. According to the degree of grey relational coefficient of each alternative from subjective preference information, the ranking order of the five alternatives is: $A_3\succ A_1\succ A_5\succ A_2\succ A_4$, and thus the most desirable alternative is A_3.

If the five possible alternatives $A_i\left(i=1,2,\cdots,5\right)$ are to be evaluated using the interval-valued intuitionistic fuzzy values by the decision makers under the above four attributes, and construct, the decision matrices as listed in the following matrices $\tilde{R}=\left(\tilde{r}_{ij}\right)_{5\times 4}$ as follows:

$$\tilde{R}=\begin{bmatrix}([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.1,0.2])\\([0.4,0.5],[0.3,0.5]) & ([0.5,0.8],[0.1,0.2])\\([0.2,0.3],[0.4,0.6]) & ([0.4,0.5],[0.2,0.3])\\([0.5,0.6],[0.2,0.4]) & ([0.6,0.7],[0.1,0.3])\\([0.5,0.6],[0.2,0.4]) & ([0.3,0.4],[0.4,0.6])\end{bmatrix}$$

$$\begin{matrix} ([0.4,0.5],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.2,0.5],[0.3,0.4]) & ([0.4,0.6],[0.1,0.3]) \\ ([0.3,0.4],[0.4,0.6]) & ([0.2,0.5],[0.3,0.5]) \\ ([0.4,0.5],[0.2,0.5]) & ([0.5,0.7],[0.1,0.2]) \\ ([0.2,0.4],[0.4,0.5]) & ([0.2,0.3],[0.2,0.5]) \end{matrix} \Bigg]$$

Decision maker's preference value on alternative $A_i \left(i = 1,2,3,4,5 \right)$ as follows:

$$\tilde{\theta}_1 = ([0.4,0.7],[0.1,0.3])$$
$$\tilde{\theta}_2 = ([0.6,0.7],[0.2,0.3])$$
$$\tilde{\theta}_3 = ([0.3,0.4],[0.3,0.6])$$
$$\tilde{\theta}_4 = ([0.2,0.4],[0.5,0.6])$$
$$\tilde{\theta}_5 = ([0.4,0.5],[0.3,0.4])$$

The information about the attribute weights is incompletely known as follows:

$$H = \{0.25 \le w_1 \le 0.28, 0.20 \le w_2 \le 0.25, \\ 0.22 \le w_3 \le 0.25, 0.25 \le w_4 \le 0.30, \\ w_j \ge 0,\ j = 1,2,3,4, \sum_{j=1}^{4} w_j = 1\}$$

Then, we utilize the Procedure developed to get the most desirable alternative(s).

Step 1. Calculate the grey relational coefficient of each alternative between objective preference information $\tilde{r}_{ij}$ and subjective preference information $\tilde{\theta}_i$.

$$\xi = \begin{bmatrix} 0.6667 & 0.8889 & 0.6667 & 0.6667 \\ 0.5714 & 0.7273 & 0.5333 & 0.7273 \\ 0.8000 & 0.6154 & 1.0000 & 0.8000 \\ 0.4706 & 0.3810 & 0.5714 & 0.3810 \\ 0.8000 & 0.6667 & 0.6667 & 0.6154 \end{bmatrix}$$

Step 2. Utilize the model (M-2) to establish the following single-objective programming model:

$$\begin{cases} \max D(w) = 3.3087 w_1 + 3.2792 w_2 + 3.4381 w_3 + 3.1903 w_4 \\ s.t.\ w \in H \end{cases}$$

Solving this model, we get the weight vector of attributes:

$$w = \left(0.25\ \ 0.23\ \ 0.22\ \ 0.30\right)^T$$

Step 3. Calculate the degree of grey relational coefficient of each alternative from subjective preference information $\xi_i^- \left(i = 1, \cdots, m \right)$ by Equations (6).

$$\xi_1 = 0.7178, \xi_2 = 0.6456, \xi_3 = 0.8015$$
$$\xi_4 = 0.4453, \xi_5 = 0.6846$$

Step 4. According to the degree of grey relational coefficient of each alternative from subjective preference information, the ranking order of the five alternatives is: $A_3 \succ A_1 \succ A_5 \succ A_2 \succ A_4$, and thus the most desirable alternative is also A_3.

6 Conclusions

In this paper, we have investigated intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives, in which the information about attribute weights is incompletely known, and the attribute values and preference values on alternatives take the form of intuitionistic fuzzy numbers. A modified GRA analysis method is proposed. In order to get the attribute weight, we establish the multiple objective optimization models based on the basic ideal of the traditional GRA. Then, by linear equal weighted method, the multiple objective optimization models can be transformed into a single-objective programming model. By solving the single-objective programming model, we can get the attribute weight information. Then, based on the traditional GRA method, calculation steps for solving intuitionistic fuzzy multiple attribute decision-making problems with incompletely known weight information are given. The degree of grey relational coefficient of each alternative from subjective preference information is defined to determine the ranking order of all alternatives. The method can sufficiently utilize the objective information, and meet decision makers' subjective preference, can also be easily performed on computer. Furthermore, we shall extend the developed models and procedures to solve the interval-valued intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives. Furthermore, we shall extend the developed models and procedures to solve the interval-valued intuitionistic fuzzy multiple attribute decision making problems with preference information on alternatives. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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