Gross Features of Angular Momentum Distribution of Coulomb Captured Mesonic Particles

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The gross features of the angular momentum distribution of mesonic particles resulting from Coulomb capture are calculated in closed form. The calculation is based on the semiclassical model of Coulomb capture with the energy loss being due to a frictional force the mesonic particle experiences in the atomic electron gas. Atomic structure effects other than the atomic radius, and chemical bonding are neglected.

§ 1. Introduction

There are two basic questions to be answered by any theory of Coulomb capture of mesonic particles; capture ratio for capture in the individual elements (in the case of a compound, etc.) and angular momentum distribution for capture in a given element. The experimental capture ratio is more directly correlated to the yet unobserved Coulomb capture process, i.e., the transition from an unbound state to the first bound state, than the experimental X-ray intensity pattern: In all theories or models except the large mesic molecule model[1] the capture ratio is only determined by the Coulomb capture processes while there are a large number of yet unobserved Auger transitions between capture and X-ray emission. Although it is not easy to treat, theory and experiment are simplest in the case of muons. Fair agreement between a semiclassical theory and experiment has been obtained for the capture ratio. The situation is worse in the case of the X-ray intensity pattern: There are only some numerical values available as theoretical results, and one cannot speak of good agreement between these values and the experimental results.

The field is reviewed, for example, in Refs. 2) ~ 4).

It is the aim of the present paper to give a closed-form calculation of the gross features of the initial distribution immediately after Coulomb capture. Detailed effects, such as electron configurations and chemical binding will not be taken into account, but the effect of the atomic radius will be taken into account to some extent. The calculation is based on a semiclassical description of the capture process.[4]~[9] In this model the mesonic particle is treated classically while the electrons are assumed to be a Fermi gas, with corrections for the atom's finite size in condensed matter. The energy loss and the resulting angular momentum

loss are treated as being due to a frictional force arising from collisions between the mesonic particle and the electrons. In § 2 the shape of the initial distribution plot is calculated. Section 3 contains some numerical results and the discussion.

§ 2. Initial distribution

The orbital angular momentum I of an incoming mesonic particle of mass M, energy W_0 and impact parameter q at a distance from the atomic center $r > r_0$ where r_0 is the cutoff radius of the potential,[4),5),9)] is

$$I_0 = \sqrt{2MW_0'}\, q \ . \tag{1}$$

The probability that a particle enters with an impact parameter between q and $q + dq$, and an energy between W_0 and $W_0 + dW_0$, will be assumed to be proportional to $q^{9)}$ and turns out to be independent of W_0,[6),9)] respectively.

Neglecting for the moment the (small) change in angular momentum while the mesonic particle is moving through the atom during its first orbit around the center, I_0 is a constant of motion during capture. The twofold differential probability dw that an incoming mesonic particle with energy and impact parameter within the above ranges will be captured is then for $q \leqq r_0$ and $W_0 \leqq -\Delta W$ where $-\Delta W$ is the energy loss during the first orbit,

$$dw = -\frac{2q\,dq\,dW_0}{r_0{}^2\,\Delta W} \ . \tag{2}$$

With the help of Eq. (1) and the easily derived relation

$$dq = (\partial q / \partial I_0)\, dI_0 \ ,$$

Eq. (2) yields

$$dw = -\frac{I_0}{Mr_0{}^2\,\Delta W\,W_0}\, dI_0\, dW_0 \ . \tag{3}$$

Because I_0 is a constant of motion and ΔW is independent of W and I_0 in our approximation, the mesonic particle is bound with the same I_0 and an energy $W_0' = W_0 + \Delta W$. Hence the population density normalized to unity for one particle with $q < r_0$ and $W_0 < -\Delta W$ is

$$\rho(W_0', I_0)\, dI_0\, dW_0' = -\frac{I_0\, dI_0\, dW_0'}{Mr_0{}^2\,\Delta W\,(W_0' - \Delta W)} \tag{4}$$

with the normalization

$$\int_{\Delta W}^0 \int_0^{I_{0,\max}} \rho(W_0', I_0)\, dI_0\, dW_0' = 1 \ , \tag{5}$$

where

$$I_{0,\max} = \sqrt{-2M\Delta W'} \, r_0 . \tag{6}$$

The integration of Eq. (4) with $W_0' - \Delta W$ replaced by W_0 yields the angular momentum population

$$P(I_0) \, dI_0 = \int_{W_{0,\min}}^{W_{0,\max}} \rho(W_0, I_0) \, dI_0 \, dW_0$$

$$= -\frac{I_0 \, dI_0}{M r_0^2 \Delta W} \ln \frac{W_{0,\max}}{W_{0,\min}} , \tag{7}$$

where $W_{0,\max}$ and $W_{0,\min}$ are the maximum and minimum values, respectively, of W_0 consistent with the capture condition $W_0 \leqq -\Delta W$ and with Eq. (1):

$$W_{0,\max} = -\Delta W$$

and

$$W_{0,\min} = I_0^2 / (2M r_0^2).$$

With Eq. (6) we may also write Eq. (7) in the form

$$P(I_0) \, dI_0 = -\frac{2I_0 \, dI_0}{M r_0^2 \Delta W} \ln \frac{I_{0,\max}}{I_0} . \tag{8}$$

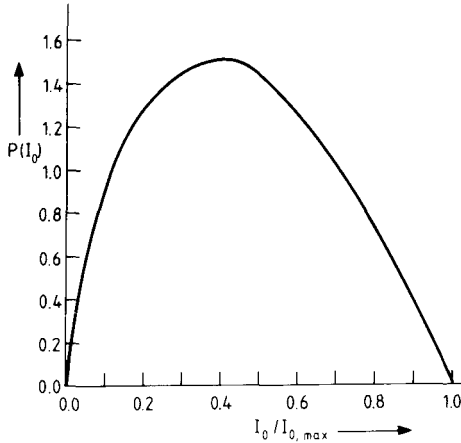Figure 1 shows the shape of $P(I_0)$.

Fig. 1. Angular momentum distribution $P(I_0)$ of muons in the first orbit after Coulomb capture vs incoming angular momentum I_0 (in terms of maximum angular momentum $I_{0,\max}$ still leading to capture).

§ 3. Numerical results and discussion

Numerical results have been computed for muons in gaseous Ar with r_0 set equal to the empirical value of 1.91 Å for the atomic radius[10] and in a hypothetical

Table I. Numerical data and results.

	$r_0[\text{Å}]$	$r_{min}[\text{Å}]$	$\varkappa$	$-\Delta W$ [eV]	U_0 [eV]	$I_{0,max}[\hbar]$
gaseous Ar	1.91	0.029	1	11.7	5.1	48.5
hypothetical Ar[a]	1.40	0.029	2.54	27.5	9.5	54.1

a) cf. text.

condensed state Ar[9] with $r_0 = 1.40$ Å and $\varkappa = 2.54$. Table I summarizes the results.

The numerical results of Table I appear at first sight surprising. Despite the smaller maximum impact parameter $q_{max} = r_0$ the denser atom is able to capture muons with higher angular momentum I_0 than the gaseous atom. The smaller r_0 value is overcompensated by the larger $-\Delta W$ value.

The results of this paper cannot be directly compared to experiment as there is no reliable quantitative prescription of how to follow the mesonic particles from the very high lying states of the Coulomb capture down to the first states whose population is observable (at principal quantum number $n = 20$[4],[11]). They disagree with most of the results from the previous semiclassical calculations[4] where nearly statistical initial distributions were found. The reason is, perhaps, the different treatment of the centrifugal barrier.[9]

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