Perception & Psychophysics
1983, 33 (5), 460-468

Group and individual relations between sensation magnitudes and their numerical estimates

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The study deals mainly with absolute magnitude estimation (AME) of the component loudnesses and the total loudness of pairs of heterofrequency, sequential tone bursts. Two kinds of relations are derived from the obtained group and individual data on the assumption of loudness additivity and a two-stage scaling model. They refer to numerical loudness estimates versus derived loudness magnitudes and to the loudness magnitudes versus tone sensation levels. The relations are validated by means of indirect and direct loudness matches. In an auxiliary experiment, the same subjects performed AMEs of subjective line lengths. The resulting group and individual relations between the numerical estimates and the underlying physical line lengths were found to be nearly the same as those between the numerical loudness estimates and the derived loudness magnitudes. The mutual consistency among the several sets of empirical and derived data strongly supports the assumptions of loudness additivity and the two-stage model.

Stevens (1956) advocated the method of magnitude estimation (ME) as a "direct" measurement of sensation magnitudes on the tacit assumption that the subject's numerical responses were directly proportional to the sensation magnitudes they experienced. Attneave (1962) and others (e.g., MacKay, 1963; Treisman, 1964) pointed out that such an assumption is not necessarily valid and that, without knowledge of the functional relation between assigned numbers and corresponding sensation magnitudes, the latter are indeterminate.

The indeterminacy can be removed on the assumption of additivity of sensation magnitudes (e.g., Anderson, 1974; Campbell, 1957), but such additivity has to be demonstrated experimentally. Most of the evidence gathered thus far does not appear to be conclusive and shows merely that, under some circumstances, the additivity is consistent with direct proportionality between sensation magnitudes and the numbers assigned to them (e.g., Cain, 1976; Dawson, 1971; Hellman & Zwislocki, 1963; Marks, 1978a, 1978b; Marks & Bartoshuk, 1979; Murphy, Cain, & Bartoshuk, 1977). A stronger case was made by Levelt, Riemersma, and Bunt (1972) for binaural summation of loudness and by Marks (1978b) for loudness summation in simultaneous heterofrequency tone pairs by using nonmetric conjoint measurement (Luce & Tukey, 1964).

The rationale for assuming loudness additivity was strengthened in the present study by using sequential, heterofrequency pairs of tone bursts 20 msec in duration and spaced by an interval of only 50 msec. Under such conditions, many subjects can be made to respond either to the loudness of the component bursts or to the total loudness of the burst pair (Zwislocki, Ketkar, Cannon, & Nodar, 1974; Zwislocki & Sokolich, 1974). In the first instance, the component loudnesses were practically independent of each other when the frequency difference was sufficient; in the second, the amount of loudness summation was practically the same as in simultaneous pairs of binaural, homofrequency tone bursts, which are perceptually fused (Marks, 1978a). Given a quantitatively equal summation, the sequential tone bursts have the advantage of showing that no nonlinear interaction between the loudness components takes place.

These experiments on loudness summation in sequential burst pairs were executed by matching the loudness of a third tone burst to either the loudness of the second burst in the pair or the total loudness of the pair. To facilitate the task, the sound frequency of the third burst was made identical to that of the second burst in the first instance and identical to the geometric mean of the burst-pair frequencies in the second. No numerical estimates of any kind were involved, so the summation had to take place at a perceptual, rather than cognitive, level.

In the present experiments, the subjects estimated the component loudnesses and the total loudness by the method of absolute magnitude estimation (AME) (Hellman & Zwislocki, 1963; Zwislocki & Goodman, 1980). On the assumptions of loudness additivity and a two-stage scaling model (e.g., Attneave, 1962; Curtis, Attneave, & Harrington, 1968; Curtis & Fox, 1969), the results made it possible to derive group

I thank the Syracuse Smith College Club for allowing me to perform an experiment on magnitude estimation of subjective line length at one of their informal meetings. The work was supported by NIH Grant NS-03950. My mailing address is: Institute for Sensory Research, Syracuse University, Syracuse, New York 13210.

and individual relations between loudness magnitudes and numbers assigned to them, as well as between the loudness magnitudes and the corresponding tone sensation levels (SLs) (intensity level in decibels relative to individual detection threshold). The latter relation was validated by means of indirect and direct loudness matching. In addition, AME of subjective line lengths performed by the same subjects revealed that the group and individual relations between the assigned numbers and the physical line lengths were essentially the same as between the assigned numbers and the derived loudness magnitudes. This correlation implies both that the subjective line length is directly proportional to the physical line length and that the relation between sensation magnitudes and the numbers assigned to them in AME is the same for loudness and line length.

THEORY

The method of AME produces as its direct result a numerical relation between stimulus intensities and numbers associated with sensation magnitudes the stimuli are assumed to evoke. Often the relation can be adequately described by a power function of the form

$$n = k\phi^{\alpha}, \tag{1}$$

where n is an assigned number, ϕ, the stimulus intensity, k, a dimensional constant, and α, a power exponent derived from the empirical data. For loudness, the relation holds above an SL of about 30 dB and sometimes even below. This is true for the results of this study.

According to the two-stage model (Attneave, 1962), Equation 1 consists of two nested functions, one defining the relation between the sensation magnitudes and corresponding stimulus intensities, and the other, the relation between the assigned numbers and the sensation magnitudes—

$$n = a(b\phi^{\theta})^{\beta}, \tag{2}$$

where $\theta\beta = \alpha$ and $ab^{\beta}, = k$. Explicit introduction of sensation magnitude as an intervening variable leads to

$$x = b\phi^{\theta} \tag{3}$$

and

$$n = ax^{\beta}. \tag{4}$$

Whereas Equation 1 results from direct empirical evidence, the similar Equations 3 and 4 are only products of an assumption that appears reasonable in view of the generality of power function relations found in magnitude estimation experiments. Nevertheless, Equations 3 and 4 are strongly supported in this study by the high correlations of the results they produce with two kinds of independent empirical data.

On the assumption of loudness additivity, the total loudness of a burst pair can be expressed as

$$x_{41} = x_4 + x_1, \tag{5}$$

where x_4 and x_1 are the component loudnesses belonging in this study to sequential bursts of 4- and 1-kHz tones. With the help of Equation 4, this expression can be transformed into

$$n_{41}^{1/\beta} = n_4^{1/\beta} + n_1^{1/\beta} \tag{6a}$$

or

$$n_{41}/n_1 = [1 + (n_4/n_1)^{1/\beta}]^{\beta}. \tag{6b}$$

For some computational purposes, it is convenient to normalize Equation 6b with respect to $\beta = 1$. We then have

$$n_{41}/n_{41}^{*} = [1 + (n_4/n_1)^{1/\beta}]^{\beta}/[1 + (n_4/n_1)]. \tag{6c}$$

When the numbers n_4, n_1, and n_{41} are determined experimentally, Equation 6b or Equation 6c allows calculation of β. In particular, when $n_4 = n_1$,

$$\beta = \ln(n_{41}/n_1)/\ln 2. \tag{7}$$

According to Equation 6a, the number n_{41} assigned to the total loudness is not equal to the sum of numbers n_4 and n_1, assigned to the component loudnesses, unless $\beta = 1$. In general, the individual data were inconsistent with such equality, suggesting that $\beta \neq 1$. This outcome also negates the possibility of the subjects' responding to "total loudness" by adding up numbers associated with component loudnesses.

Once the exponent β is determined, it is possible to calculate the exponent θ from Equation 2 ($\theta = \alpha/\beta$), and so to derive the relation between sensation magnitudes and stimulus intensities that produce them. The numerical value of θ can be validated by determining it independently of α and β. This can be done by increasing the intensity of one of the component bursts presented singly (x_{1_s}) so that its loudness equals the total loudness of the burst pair. On the assumption of loudness additivity, we have

$$x_{1_s} = x_4 + x_1. \tag{8}$$

Using Equation 3, we obtain

$$\phi_{1s}^{\theta_1} = \phi_4^{\theta_1} + \phi_1^{\theta_1}, \tag{9}$$

when the stimuli are selected so that their equal intensities produce equal loudnesses. By making $\phi_4 = \phi_1$, we can simplify Equation 9 to

$$\phi_{1s}^{\theta_I} = 2\phi_1^{\theta_I}, \tag{10}$$

and obtain

$$\theta_I = 10 \log 2/10 \log(\phi_{1s}/\phi_1), \tag{11a}$$

or

$$\theta_I = 10 \log 2/(10 \log \phi_{1s} - 10 \log \phi_1), \tag{11b}$$

where $10 \log \phi_{1s} - 10 \log \phi_1$ is the measured SL difference in decibels. If loudness additivity and the two-stage model hold, $\theta_I = \theta$ must be true.

METHOD

AME of Line Length

For every subject, AME of line length was performed at the beginning of the first session, preceding loudness scaling of single sound bursts. The scaling of line length appears to be relatively easy and allows the subjects to become acquainted with the concept of AME. According to previous experience (e.g., Zwislocki & Goodman, 1980), it stabilizes the results of loudness scaling and substantially decreases the regression effect (Stevens & Greenbaum, 1966).

A set of seven black lines on a light background (Life Science Associates, 1970) was projected in a near-random sequence of physical lengths on a screen situated at a 3-m distance from the subjects' eyes. They were presented only once because of an apparent strong learning effect observed informally on an earlier occasion. Neither the longest nor the shortest line appeared first, and the shortest never followed the longest, and vice versa. These restrictions were designed to decrease variability.

The subjects were instructed as follows: "You will see lines of various lengths projected on the screen in random order. Assign a number to every line so that the *subjective* magnitude of the number is equal to the *subjective* magnitude of the line length. Do not attempt to estimate the physical lengths of the lines in inches, centimeters, or other units. You can use whole numbers, fractions, or decimals, as you feel appropriate. Treat every line individually and do not worry about the numbers you assigned to the preceding lines. Respond as quickly and spontaneously as you can."

AME of Single Sound Bursts

Scaling of the loudness of single sound bursts occurred in two sessions preceding scaling of burst pairs. The bursts were 20 msec long, had 10-msec onset and decay times, and were presented monaurally with a repetition rate of 1/sec. They were cut out of either 1- and 4-kHz sinusoids or a band of random noise extending from 1.6 to 4.8 kHz. The noise was included for the purpose of subsequent loudness matching with the sinusoids.

To obtain individual SLs, the threshold of audibility for every stimulus kind was determined by the method of adjustment in every session, prior to loudness scaling. The subjects manipulated a large unmarked knob to approximate, by bracketing, that stimulus value at which they were not certain if they had heard the stimulus or not. The adjustment was made three times, and, between every two adjustments, the experimenter introduced a different amount of additional attenuation into the circuit by means of an auxiliary attenuator. Sensation level was determined by taking the difference in decibels between a given setting and the mean threshold setting. Sensation levels, rather than sound-pressure levels, were used to minimize intersubject variability (Hellman & Zwislocki, 1961) and in the expectation that equal SLs would produce equal loudnesses.

For AME, the order of presentation of the three kinds of stimuli rotated among subjects and between the two sessions, but every stimulus was scaled in a separate sequence consisting of three runs. The stimulus SLs were randomized within every run and among runs, but the first stimulus always occurred within the range between 30 and 60 dB. The restriction was introduced to minimize biases resulting from starting stimulus sequences with extreme values (e.g., Zwislocki & Goodman, 1980).

The instructions for AME of loudness were essentially the same as for line length. However, the subjects were allowed to listen to a sequence of sound bursts for as long as they wished before making a response. After every response, the sequence was interrupted and the sound intensity changed to a different SL.

To be able to select the desired combinations of SLs in burst-pair experiments, the data on AME of single bursts had to be processed beforehand. For every subject, session, stimulus kind, and SL, the row scores of the last two runs were geometrically averaged. No normalization of any kind was introduced, in order to preserve information on both the slope and position. The resulting geometric means (GM), in their turn, were geometrically averaged over the two sessions involved. For group GMs, they were geometrically averaged over the subjects. Finally, a straight line on log-log coordinates was fitted to every appropriate set of data by the least squares method, and the associated product-moment correlations were computed. The latter computations were possible for all the subjects in the SL range from 30 to 70 dB. For two subjects, the range could be extended down to 10 dB.

AME of Burst Pairs

The sequential burst pairs consisted of the 4- and 1-kHz bursts presented monaurally with an interburst time interval of 50 msec. Like the single bursts, the pairs were repeated once per second and the 1-kHz burst always followed the 4-kHz one. The sensation level of the 1-kHz burst was set randomly at one of the following decibel values: 10, 20, 30, 40, 50, 60, or 70. The sensation level of the 4-kHz burst was adjusted so as to approximate a predetermined loudness estimate ratio with the 1-kHz burst (n_4/n_1). Three ratios, 0.5, 1, and 2, were chosen. No smaller or greater ratios were used because of the expectation that response variability would preclude an unambiguous assessment of the effect of the weaker stimulus. Once a ratio was selected, it was kept constant throughout an experimental sequence consisting of three runs, as for single bursts. The sensation levels required for the three ratios were derived for every subject from his or her loudness functions for single bursts.

The scaling occurred in two sessions, which included three runs for every ratio and, in addition, three runs for single 1-kHz bursts. The latter were scaled for reference purposes. The order of ratio presentation was randomized among the subjects and between the two sessions, and the 1-kHz bursts were presented first in one session and last in the other.

The same scaling procedure was used as for single bursts, except that the subjects were instructed to respond to the *total* loudness of every burst pair rather than to the loudness of one of the bursts. A picture representing the two bursts with a circle around them was drawn on a blackboard.

The data processing was essentially the same as for single bursts and culminated in group and individual families of loudness-estimate functions represented by least squares regression lines. Every family consisted of three curves for burst pairs with the three different loudness estimate ratios and one curve for the 1-kHz bursts. The regression lines yielded directly the power exponents, α, relating numerical loudness estimates to SLs. Since the exponents varied only slightly within the curve families, they were averaged and all the curves of a family were fitted ultimately with regression lines having a common slope, α_M. These curves passed through corresponding geometric means of numerical loudness estimates at the mean SL. The ratios between the geometric means for the burst pairs and for the 1-kHz bursts determined the experimental ratios n_{41}/n_1. From these ratios, the group and individual exponents β_M were determined, using either Equation 6b or Equation 6c. Finally, the exponents θ were computed in two ways—from the relation $\theta = \alpha/\beta$ (θ_M) and from SLs corresponding to equal loudness estimates (θ_I) (indirect loudness matching) of single 1-kHz bursts and burst pairs with $n_4/n_1 = 1$ (Equation 11b).

Loudness Matching

Time limitations coupled with the tedious process of loudness adjustment allowed only two subjects to carry out the experiments to the point at which values θ_I could be computed. The subjects matched the loudness of the noise bursts to both the loudness of single 1-kHz bursts and the total loudness of the burst pairs. They made three matches for every SL. The data were evaluated in the same manner as for AME, except that SLs of noise replaced numerical estimates, and linear means of dB SL rather than geometric means were computed. The numerical values θ_I were calculated from Equation 11b, as for the indirect loudness matching, on the assumption that equal SLs of matched noise indicated equal loudnesses of the single bursts and burst pairs.

Subjects

The subject population consisted of graduate and undergraduate students and of laboratory technicians. None had had any prior experience with magnitude estimation and none were involved in psychology or a hard science. This selection was made to avoid any preconceived ideas about relations between sensation magnitudes and numbers and the abstract significance of numbers.

Five subjects concluded the AME experiments. Another five were discarded for the following diverse reasons: One assigned the same relatively high numbers to burst pairs, independent of the intensity relation between the component bursts. Two were extremely erratic in their numerical estimates of the loudness of single bursts. Finally, two were unable to respond to total loudness, but responded instead to the louder of the two bursts. It had been noted in previous experiments based on loudness matching that some subjects integrate loudness over time much more readily than others and that some cannot do it at all (unpublished data related to the articles of Zwislocki & Sokolich, 1974, and of Zwislocki et al., 1974). As a consequence, the phenomenon was not unexpected.

RESULTS

Line Length

The group data on line length are shown on the log-log plot of Figure 1 by means of filled circles indicating GMs and the solid line fitted to them by means of least squares. The bracket at 42 cm shows the double standard error of the GM, which was about the same for all lengths. The exponent of the function is $\alpha_L = .97$, not significantly different from one, and the associated correlation coefficient r = .999. To give an idea of the stability of group units in AME of line lengths, crosses in Figure 1 show GMs of estimates obtained informally at a party, using the same set of projected lines. A group of 14 persons participated in that experiment, and they were seated at distances of about 2.7 to 3.7 m from the projection screen. They had had no previous experience with psychophysical scaling. As can be seen, the informal data are fitted rather well by the same regression line. Note that both the slope and the vertical location are meaningful in absolute scaling. The present results are in an equally good agreement with those of Teghtsoonian and Beckwith (1976) and Verrillo (1981).

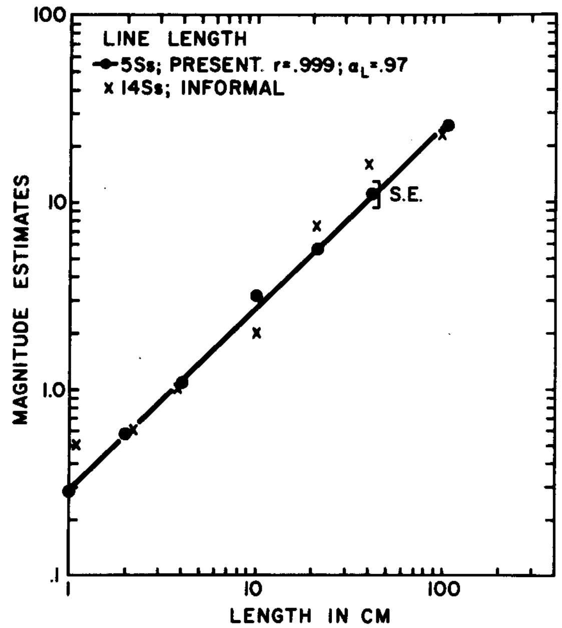

Figure 1. Group geometric means (GMs) of absolute magnitude estimation (AME) of subjective line length plotted on log coordinates as a function of physical line length. Filled circles and regression line, this study; crosses, informal scaling at a party. SE, standard error of the mean in log units; r, correlation coefficient; α_L, slope of the line.

1- and 4-kHz Tone Bursts Presented Separately

Although the main purpose of scaling the 1- and 4-kHz tone bursts was to derive the desired loudness estimate ratios between them, the experiment also allowed a comparison with preceding studies, necessary for finding out if the subject's performance conformed to typical behavior. The group GMs are shown in Figure 2 by the filled and unfilled circles for the 1- and 4-kHz bursts, respectively. Between 30 and 70 dB SL, the solid line shows the linear regression of both data sets combined. It obeys a power exponent of $\alpha = .33$ with respect to sound energy, which is in excellent agreement with the exponent of about .67, with respect to sound pressure, proposed by Stevens (1972) on the basis of the sum total of his experience with loudness scaling. At lower SLs, the solid line is fitted to the data by eye. No significant difference between the 1- and 4-kHz means is apparent, which is consistent with the large common correlation coefficient, r = .99. The double arrows show that, according to the curve, doubling of estimated loudness requires an SL increment of 9.1 dB for both frequencies.

For comparison, the crosses and asterisks show corresponding data obtained at 1 kHz on two differ-

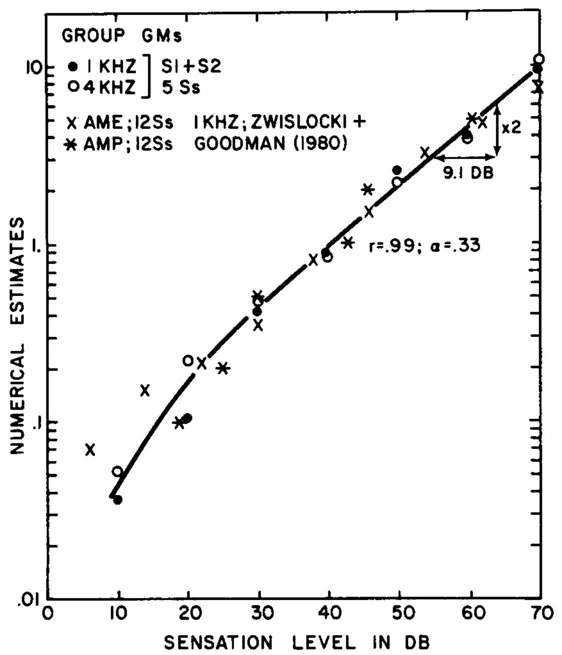

Figure 2. AME of the loudness of single 1-kHz (filled circles) and 4-kHz (unfilled circles) tone bursts, plotted as a function of SL. Group GMs of the first and second sessions combined. A regression line has been fitted to both sets of data by means of the least squares method between 30 and 70 dB SL. Below 30 dB, the line is fitted by eye. r, correlation coefficient; α, slope of the line on log coordinates. The double arrows indicate the SL increment required for doubling of the estimated loudness. The crosses and asterisks reproduce AME and AMP (absolute magnitude production) data obtained under comparable conditions in a preceding study.

ent groups of 12 subjects in a preceding study (Zwislocki & Goodman, 1980). One group (crosses) performed AME, the other, absolute magnitude production (AMP). The agreement of both sets of data with the present ones is good with respect to both slope and location above 30 dB SL. At lower SLs, the data are still consistent with each other, but a greater variability occurs. The coincidence of the results of the two studies confirms the stability of AME and AMP scales of loudness, which is found when necessary precautions are taken. Scaling line lengths by AME before AME of loudness appears to be one of them. Apparently this is not required for AMP.

It should be pointed out that loudness estimate ratios between the 1- and 4-kHz bursts required by the main experiments had to be derived from individual rather than group data. Because of the greater variability of these data, the uncertainty of the ratio determination was increased somewhat. Nevertheless, a typical example, shown in Figure 3, indicates that the variability remained within reasonable limits. The loudness estimates of the 1-kHz bursts are only about 10% greater than those of the 4-kHz bursts. Such small differences were neglected. Note that the data of J.F. in Figure 3 could be fitted by linear regression throughout the experimental range from 10 to 70 dB SL. This was not possible for all the subjects.

Burst Pairs

In connection with the scaling of burst pairs, it is necessary to mention that the reference 1-kHz function determined in the sessions in which the burst pairs were presented differs from that obtained in the preceding sessions. It is not possible to decide definitively whether the difference occurred as a result of experience with the burst pairs or followed from increased experience with AME in general. Two features of the group function obtained in the sessions including the burst pairs suggest that the latter may have been true. The average units used by the subjects are nearly equal to those used in two older studies with highly experienced subjects (Hellman & Zwislocki, 1963; Rowley & Studebaker, 1969), although the slope of the function is appreciably greater. The data obtained at the beginning of the sessions, before the presentation of burst pairs, do not differ appreciably from those obtained after their presentation, as can be seen in Figure 4, in which the GMs of these data are plotted by means of the unfilled circles and crosses, respectively. A common regression calculation yielded a correlation coefficient of r = .997. For comparison, the closed circles and solid line reproduce the 1-kHz GMs of Figure 2. As can be seen, the average unit used in the later sessions was decreased to about one half of the unit

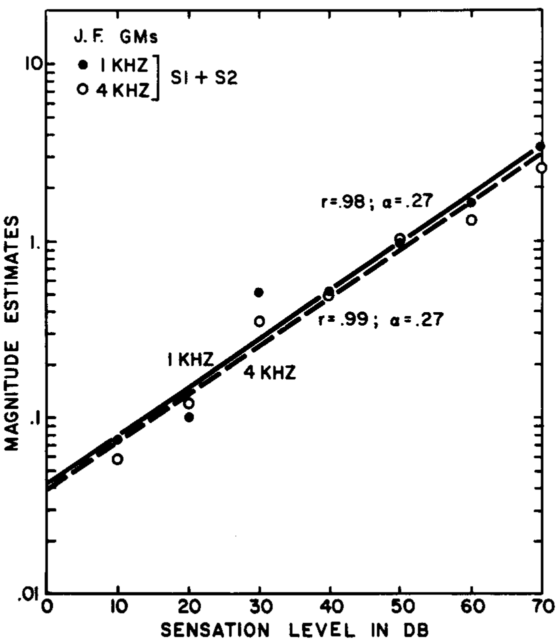

Figure 3. Similar to Figure 2, but for one subject. The solid and intermittent lines are least squares regression lines for 1 and 4 kHz, respectively.

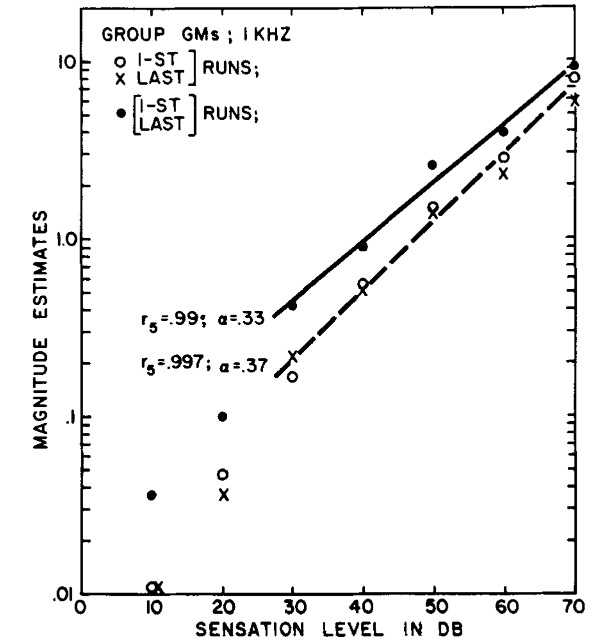

Figure 4. The filled circles and solid line reproduce the 1-kHz data and regression line of Figure 2. The unfilled circles and crosses show group GMs of AME of 1-kHz bursts presented at the beginning (circles) and end of the sessions in which burst pairs were scaled. The intermittent line is the regression line for both sets of data.

used in the earlier sessions, and the power exponent changed by about 12%. In view of these differences, the loudness functions obtained with burst pairs were not compared with 1-kHz loudness functions resulting from earlier sessions, but with data obtained within the same sessions.

The group data for the loudness of burst pairs and the reference loudness of 1-kHz bursts are shown in Figure 5 in terms of GMs and their regression lines with a common exponent α_M. The filled triangles refer to the 1-kHz bursts; the remaining symbols refer to the burst pairs with the nominal loudness estimate ratios of 0.5, 1, and 2, respectively. A perfect linear summation would have produced corresponding loudness estimate ratios between the burst pairs and the single 1-kHz bursts of 1.5, 2.0, and 3.0. The experimental ratios based on mean loudness functions of the group came out close—1.6, 2.1, and 2.7. Since the ratios between the experimental and the nominal values do not differ from 1.0 by more than 10% and their mean is almost exactly 1.0, it appears reasonable to conclude that the group data are consistent with both linear loudness summation and nearly direct proportionality between loudness magnitudes and the numbers assigned to them.

It should be pointed out that, because of the slight slope change, it was not possible to predetermine exactly, from the sessions with single tone bursts, the loudness estimate ratios, n_4/n_1, actually produced in the sessions with burst pairs. The latter could be determined only post facto. This was done, and all the computed data of Figures 6 and 7 and of Tables 1 and 2 are based on the post facto ratios. This procedure did not upset data averaging, which was based on the quotients of the experimental and post facto theoretical data—experimental (n_{41}/n_1)/theoretical (n_{41}/n_1).

As an example, individual data that deviate appreciably from mean results are shown in Figure 6. The actual loudness estimate ratios, n_4/n_1, were .65, 1.0, and 1.5. They produced loudness estimate ratios between the burst pairs and single bursts of 1.47, 1.78, and 2.23, respectively. Corresponding ratios between these experimental and the theoretical post facto values, n_{41}/n^{*}_{1} (Equation 6c), amounted to: 1.47/1.65 = .89, 1.78/2 = .89, and 2.23/2.5 = .89. Within the entire group, the latter ratios varied between .86 and 1.45, as can be seen in Table 1.

The individual exponents, β_M, were calculated from the mean individual ratios between the experimental and theoretical loudness estimates of Table 1. When these ratios are multiplied by 2, the best available estimates are obtained for the individual ratios n_{41}/n_1, when $n_4 = n_1$. To determine the power exponents, these estimates were introduced into Equation 7. The same procedure was followed in calculating the group β_M. According to Table 1,

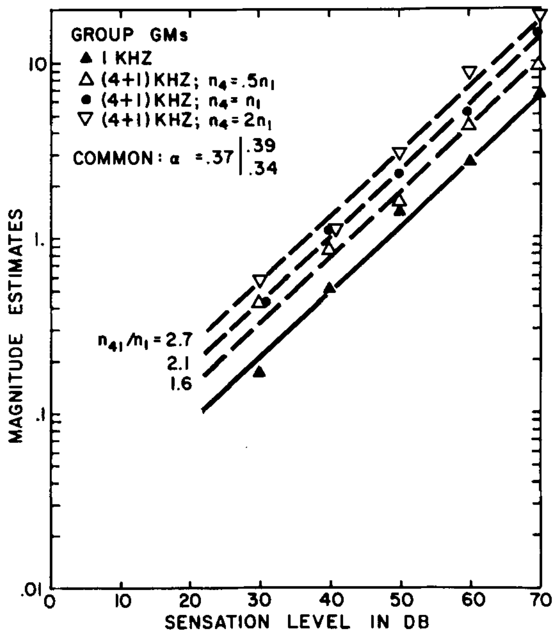

Figure 5. Group GMs for AME of burst pairs having three different loudness estimate ratios between the bursts (see text). Filled triangles reproduce the lower set of the 1-kHz data of Figure 4. Quotients n_{41}/n_1 indicate the empirical ratios between the loudness estimates of burst pairs and single 1-kHz bursts. The regression-line slopes of the individual data sets ranged from .34 to .39. The lines drawn follow the average slope of .37.

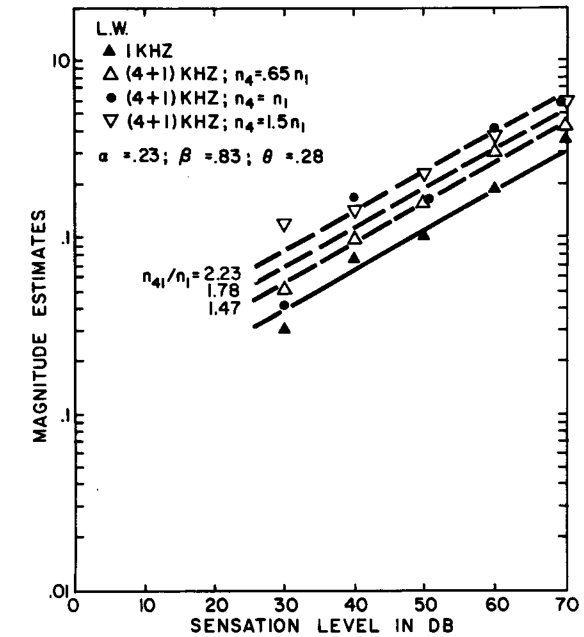

Figure 6. Same as Figure 5, but for one subject, who produced the smallest slope α. The lines follow the slope .23, which is the average slope of the regression lines of the individual data sets. The quotients n_{41}/n_1 have been calculated for the average exponent $\beta = .83$ (Equation 6c), which relates the assigned numbers to putative loudness magnitudes (text). The exponent θ relating the loudness magnitudes to SLs results from $\theta = \alpha/\beta$.

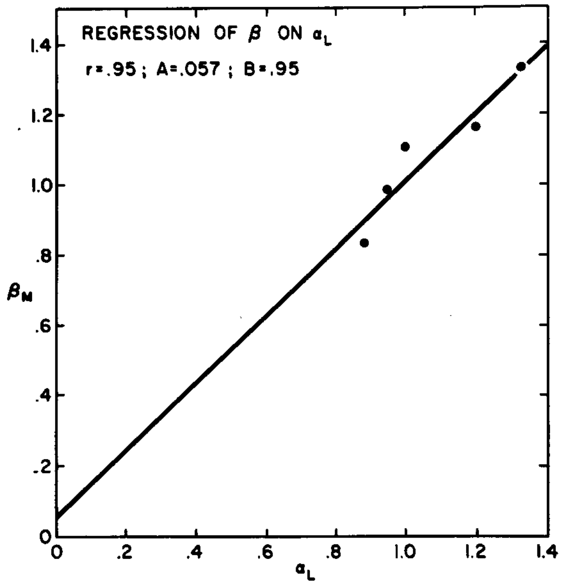

Figure 7. Correlation between the calculated individual exponents β relating the assigned numbers to associated putative loudness magnitudes and the corresponding exponents controlling the log-log slopes of subjective line-length functions.

the group $n_{41}/n_1 = 1.07 \cdot 2 = 2.14$. Inserted in Equation 7, it produces $\beta_M = 1.1$. This is in agreement with the result of very different experiments of Curtis et al. (1968), but the agreement may be coincidental. The individual values of β_M are reported in Table 2, together with their mean of 1.08. The latter is slightly lower than 1.1 because of rounding errors in the averaging procedure.

Knowledge of β makes it possible to calculate θ from the defining Equation 2, according to which $\theta = \alpha/\beta$, where α is determined by the slopes of the empirical group and individual regression lines (Figures 5 and 6). The resulting values of both α (α_M) and θ (θ_M) are listed in Table 2.

To determine θ_I values of exponent θ from indirect loudness matching, the required SL differences (Equation 11b) were obtained by measuring the horizontal distance between the regression line belonging to single bursts and that belonging to burst pairs, with $n_4 = n_1$. If the latter line is not directly available, it can be safely determined by interpolation (Figures 5 and 6). Table 2 shows that the values of θ_I so obtained are in close agreement with the values of θ_M derived from Equation 2 (perfect rank-order correlation; correlation coefficient $r = .98$).

One additional entry in Table 2 is of particular interest. It concerns the individual and group values of exponent α_L relating numbers associated with subjective line lengths to the corresponding physical line lengths. These values are in close agreement with the values of β_M (perfect rank-order correlation; $r = .95$) and suggest that the relation between sensation magnitudes and the numbers assigned to them is essentially the same for subjective line length and loudness. The agreement becomes even more evident when the regression of β_M on α_L is displayed graphically by means of an extrapolating line, and the line

Table 1
Ratios Between Experimental and Theoretical Loudness Estimates of Individual Subjects, and Their Means
[Experimental (n_{41}/n_1)/Theoretical (n_{41}/n_1)]

Subject	Nominal (n_4/n_1) 0.5	1.0	2.0	Mean
R.D	1.44	1.45	.89	1.26
M.T.	1.19	1.24	.93	1.12
J.F.	1.03	1.21	1.03	1.10
J.E.	1.23	.88	.86	.99
L.W.	.89	.89	.89	.89
Mean	1.16	1.13	.91	1.07

Table 2
Slopes of Log-Log Regression Lines

Subject	α_M	β_M	α_L	θ_M	θ_I
R.D	.56	1.33	1.33	.42	.38
M.T.	.50	1.16	1.2	.43	.43
J.F.	.25	1.10	1.0	.23	.24
J.E.	.35	.98	.95	.36	.37
L.W.	.23	.83	.88	.28	.28
Mean	.38	1.08	1.07	.34	.34

Note—α_M = loudness estimates vs. SL; β_M = loudness estimates vs. loudness; α_L = length estimates vs. line length; θ_M, θ_I = loudness vs. SL.

parameters are noted (Figure 7). They indicate a near identity between the two variables.

Note that there is a small difference (.97 vs. 1.07) between the group α_L of Figure 1 and that of Table 2. It reflects a difference in the averaging procedure coupled with rounding errors. In the first instance, all the individual raw data were pooled together for the regression calculation. In the second, individual α_Ls were calculated and then averaged. The second method was used for all the mean values of Table 2.

Loudness Matching

Although the tedious experiments on loudness matching could be completed by only two subjects, they are important in that their results corroborate the AME results on an individual basis. The data were plotted on decibel coordinates, with the 1-kHz SL along the abscissa axis and the noise SL along the ordinate axis, and were approximated by least squares regression lines ($r > .95$). Horizontal distances between the 1-kHz line and the lines for burst pairs determined the SL differences required for a constant loudness. From the distance between the 1-kHz line and the line corresponding to $n_4 = n_1$, the exponent θ_I could be calculated directly by using Equation 11b. The obtained values amounted to .43 for Subject M.T. and .38 for Subject J.E. They agree well with the corresponding values of .43 and .36 derived from AME experiments.

DISCUSSION

The results of this study on AME of total loudness and component loudnesses of pairs of heterofrequency, sequential tone bursts are consistent with complete linear loudness summation between the bursts and Attneave's two-stage model of scaling. This is true for both group and individual data.

In the group data, the derived loudness magnitudes and the numbers assigned to them are approximately linearly related. This is not necessarily true for individual subjects. Individual nonlinearities derived on the basis of loudness additivity and the two-stage model fitted power functions whose exponents were approximately symmetrically distributed about unity and deviated from it by less than 50%.

Auxiliary experiments on AME of subjective line length confirmed preceding results, showing that, on the group basis, the assigned numbers are nearly directly proportional to the physical line lengths. More importantly, they revealed that the individual nonlinear relations between the assigned numbers and the corresponding physical line lengths were almost identical to those between the assigned numbers and the derived loudness magnitudes. On the assumption of direct proportionality between the physical and subjective line lengths, this suggests that, for any given individual, the nonlinearity intervening between the sensation magnitudes and the numbers assigned to them is independent of sense modality.

The agreement between the individual nonlinearities found in AME of line lengths and those derived from AME of loudness on the assumptions of loudness additivity and the two-stage model strengthens both assumptions. The assumptions are further strengthened by the following correlations.

Once the nonlinearity between the loudness magnitudes and the numbers assigned to them is established, the relation between the former and the associated sound intensities can be derived from the two-stage model. If loudness additivity holds, the same relation must be derivable from indirect or direct loudness matching, independent of the model. The indirect matching, based on the assumption that equal numbers reflect equal loudness magnitudes, confirmed the identity (columns θ_M and θ_I of Table 2). Direct loudness matching experiments performed on two subjects led to the same result.

The assumption of loudness additivity was further tested by applying the rules of monotonicity and double cancellation of the conjoint measurement (Luce & Tukey, 1964) to the group data of Figure 5. The relative positions of the curves of Figure 5 for different n_4/n_1 ratios show that the monotonicity rule holds. To test the double cancellation rule, three pairs of points were selected on the curves so that within every pair the loudness was constant for different n_4/n_1 ratios. If we denote the points by A_1, B_1, A_2, B_2, and A_3, B_3, where $A_{1,2,3}$ stand for various n_4 values and $B_{1,2,3}$, for various n_1 values, the following equalities must hold simultaneously: $(A_1,B_1) = (A_2,B_2)$; $(A_3,B_2) = (A_1,B_3)$; and $(A_3,B_1) = (A_2,B_3)$. The following example illustrates a possible choice of pairs of n_4 and n_1 values approximating the values of Figure 6.

$$(n_4 = 0;\ n_1 = .5) = (n_4 = .25;\ n_1 = .25)$$

$$(n_4 = .5;\ n_1 = .25) = (n_4 = 0;\ n_1 = .75)$$

$$(n_4 = .5;\ n_1 = .5) = (n_4 = .25;\ n_1 = .75).$$

The finding that the power exponents relating individual loudness magnitudes to the numbers assigned to them are nearly identical to those relating the physical line lengths to the numbers assigned to the associated subjective line lengths has a practical application. Since the latter relation is obtained empirically by a simple procedure, it may be used as a correction factor to cancel the nonlinearity intervening in individual loudness scaling. More specifically, assume that α_L is the exponent relating the numerical line length estimates to the physical line lengths, and the exponent relating the numerical loudness estimates to the corresponding sound intensities is $\alpha = \beta\theta$, where β is the exponent intervening between the numerical loudness estimates and the putative loudness magnitudes, and θ, the exponent relating these magnitudes to sound intensity. Then, $\theta \cong \alpha/\alpha_L$, since $\beta \cong \alpha_L$.

The nearly linear relation between the loudness magnitudes and their numerical estimates found in this study in the group data is representative of AME experiments performed under optimum conditions. Under other conditions, an appreciable nonlinearity may intervene, as can be concluded from slope variation of magnitude estimation curves. Data from our laboratory (see Figure 5, Zwislocki & Goodman, 1980) suggest that when the subjects are inexperienced and not exposed to the estimation of line length, the group exponent tends to be relatively small. The regression effect (Stevens & Greenbaum, 1966) is synonymous with such an exponent. Of course, magnitude estimates may be quite nonlinear when based on ratios with respect to an inappropriate modulus (e.g., Hellman & Zwislocki, 1961; Stevens, 1956).

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(Manuscript received October 19, 1981;
revision accepted for publication January 18, 1983.)