Grouping and Partner Selection in Cooperative Wireless Networks

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Abstract—Various results to date have demonstrated the advantages of one or several relay nodes assisting transmissions in a wireless network. In many practical scenarios, not all nodes in the network are simultaneously involved in every transmission; therefore, protocols are needed to form groups or subsets of nodes for the purposes of cooperation. We consider this problem in the context of regenerative nodes and non-altruistic cooperation (all nodes have data of their own to transmit). For a network-wide diversity advantage, the protocol must provide each transmitting node with enough "partners" that can decode its message with high-enough probability. Assuming that the nodes cannot communicate their control decisions (distributed scenario), and that each node chooses to help n other nodes, we point out a simple, static selection strategy that guarantees diversity $n+1$ for all transmissions. We then consider centralized control strategies and study the additional gains that arise from a central control, under various amounts of information being available to the central controller.

Index Terms—User cooperation, cooperative networks, diversity, transmit diversity

I. INTRODUCTION

USER cooperation provides diversity through a signaling scheme that allows single-antenna nodes to transmit using the antennas of several nodes. Cooperation protocols in orthogonal channels have been investigated, among others, in [1]–[7]. These studies concentrate on isolated two-node cooperation scenarios. Laneman and Wornell [8] consider regenerative cooperating protocols in orthogonal multi-user channels, where channel gains are pairwise complex Gaussian, i.i.d., and all users cooperate with every transmission. Several space-time cooperation protocols are proposed and analyzed in [8].[1]

In a practical scenario, not all nodes may wish, or be able, to be involved in every transmission, so we need protocols for assigning nodes to each cooperative transmission. This paper studies grouping and partner selection for cooperative networks. We consider the following questions: how to allocate nodes to assist other nodes, the effect of allocation policies on system performance, and how the cooperative gain scales with the number of cooperating nodes. Similar to the references above, this paper considers a model where each

node has data of its own to transmit. In other words, a *non-altruistic cooperation* without pure relays.

In general we allow non-reciprocal cooperation; e.g., Node A might give help to Node B, but not receive help from it. There are several reasons for studying non-reciprocal cooperation. First, for distributed algorithms where each node makes an individual decision on cooperation, it may not be possible to make mutual arrangements. Second, non-reciprocal cooperation allows a more flexible arrangement of partners. For example, an odd number of nodes cannot do pairwise cooperation, but it is still possible for an odd number of nodes to arrange themselves such that each node receives help from exactly one other node, and thus for the network to achieve diversity factor two. For the purposes of exposition, several of our examples (such as above) are drawn on the basis of one cooperating node per user, however, the methodology and results of the paper are completely general and apply to many-user cooperations.

We now summarize the results of this paper: In distributed scenarios, each node can help n other nodes, but must individually decide whom to help, without knowing whether other nodes can detect and cooperate with it. In this case we point out simple (non-unique) node assignment protocols that produce diversity $n + 1$ across the network. Since distributed strategies already achieve full diversity, there is no further diversity advantage to be obtained by a central controller, but nevertheless large gains are possible by central control strategies, depending on how much information is made available to the controller.

Under distributed node assignment, each node has receive-side channel state information (CSI), but no transmit-side CSI, and no feedback or handshake is available to the physical layer protocols; therefore, each node must decide whom to cooperate with, in the absence of any knowledge of who may be able to decode them and assist them in the respective time interval. As mentioned earlier, such cooperative relationships may become non-reciprocal. A simple example can shed light on the basic ideas in this setup: Consider a 5-node network, where each node has the ability to assist one other node. We will see that with any arbitrary labeling of the nodes, the arrangement $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 1$, where the arrows indicate the direction of cooperation assistance, gives network-wide diversity factor 2. Node 1, for example, will always *attempt* to decode Node 2 and help it, but may be unsuccessful in a given block. The diversity factor depends on the rate of decline of such probabilities with increasing SNR.

We generalize this idea into a protocol called *fixed priority*

[1]Cooperation is closely related to, and motivated by, earlier developments on the relay channel [9], [10].
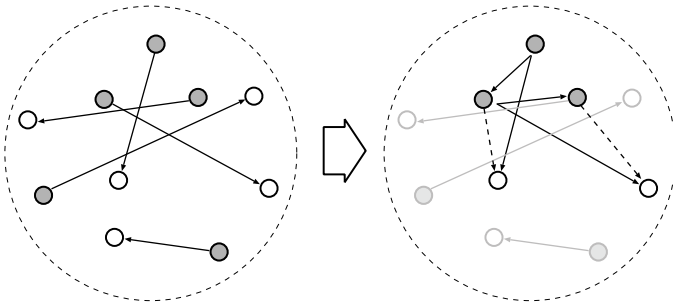
Fig. 1. Left: non-cooperative network, shaded circles are data sources and empty circles are data sinks. Right: cooperative netowrk, where two non-reciprocal cooperations are highlighted.

selection, which allows each node to assist n other nodes and provides diversity $n+1$ over the network. Since at high-SNR, the average outage probability is dominated by the worst link, average diversity of $n+1$ means all links achieve diversity of (at least) $n+1$. In this general scenario, we shall see that it is not the number of nodes that actually assist a given node for a given time block (which is a random variable) but rather the number of *decoding attempts per user* that determines the diversity factor of the network.

It is worth emphasizing that in a distributed scenario, the nodes know nothing about each other, therefore the network geometry (node positions), or channel gains, cannot be used for cooperative node selection. The relative position of other nodes, although useful, is simply unavailable to each node. Nor can the actions of the nodes be coordinated in a direct manner. The positive result arising from this paper is that, even with such heavy restrictions, it is still possible for the nodes to select partners so that full network-wide cooperative diversity is achieved. For example, if each node agrees to cooperate with (at least) two other nodes, network-wide diversity of 3 can be achieved despite unavailability of information about network geometry and lack of centralized control.

The next natural question is: how much better can we do with a centralized controller? Intuitively, a centralized controller should provide better performance. We consider a centralized node selection protocol that minimizes average node outage across the network, and study the effect of various amounts of channel state information being available to this centralized controller. The centralized controller does not provide any additional diversity compared to a well-designed distributed control, simply because distributed control already achieves the maximum possible diversity. But centralized control can provide significant gains nevertheless. We demonstrate the gains made possible by centralized control via extensive simulations.

II. SYSTEM MODEL AND CHARACTERIZATION

Our network model consists of multiple nodes, which we alternatively call *users*, randomly distributed over a circular disk according to a uniform distribution. We consider a group of M nodes that have data to transmit. Each has a destination node (outside the set of M transmitting users) at a random location within the same region. We define a given placement of the M nodes and their corresponding destinations as a *network realization* (See Fig. 1).

Each of the users is assigned an orthogonal (i.e., in time, frequency, or spreading code) multiple-access channel. The physical channel from User i to User j has instantaneous signal-to-noise ratio (SNR)

$$\gamma_{i,j} = \Gamma_{i,j} \cdot |h_{i,j}|^2, \tag{1}$$

where $|h_{i,j}|$ is the Rayleigh-distributed fading magnitude, with $E\{|h_{i,j}|^2\} = 1$. The term $\Gamma_{i,j}$ represents the average SNR of the channel over fading

$$\Gamma_{i,j} = \Gamma_T \Gamma'_{i,j} = \left(\frac{P}{N_0}\right) K S_{i,j} d_{i,j}^{-\beta}, \tag{2}$$

where $\Gamma_T = \left(\frac{P}{N_0}\right)$, P is the transmit power, and N_0 is the additive white Gaussian noise power at the receivers. For the purposes of this work, to highlight the gains from cooperation, we consider that Γ_T is constant for all users and transmissions (though in general this need not always be the case). As for the terms of $\Gamma'_{i,j}$, K is the path loss for an arbitrary reference distance, $S_{i,j}$ is a log-normal shadowing component, with $10 \log S_{i,j}$ having mean of 0dB and standard deviation σ_S (dB), $d_{i,j}$ is the distance between nodes i and j (normalized by the reference distance), and $\beta \geq 0$ is the path loss exponent. We consider quasi-static fading, such that the fading coefficients $\{h_{i,j}\}$ are constant for a given transmitted block, or code word, but are i.i.d. for different blocks.

Our studies target the regime where small scale fading between any two nodes is not completely dominated by path loss and shadowing. Practically, this means that our analysis is best applied to networks or sub-networks of up to a certain coverage area, even though technically no limitations are necessary for the diversity analysis. Also, this paper concentrates on cooperative issues and does not consider multi-hop routing. Within this model each node's communication objective is achieved, with cooperative assistance, in one hop.

We assume that cooperating nodes use regenerative transmission. In other words, the nodes that wish to cooperate must decode the intended signal. Among regenerative transmission methods, the most straightforward is arguably the decode-and-forward (DAF) [3], where the cooperating node decodes and then retransmits the decoded signal, and the receiver diversity-combines the signals from the source and relay. The cooperative transmission is integrated with characteristics of the MAC layer (e.g., FDMA, TDMA, or CDMA) such that a user is not simultaneously transmitting and receiving or relaying. References [3] and [6] provide a more detailed treatment of this issue. We consider a variation called *selection DAF* (S-DAF), in which the cooperating node will only retransmit the decoded signal if it has been correctly decoded. In an information theoretic sense, this presents no difficulty because the cooperating node can monitor its received SNR to see if it supports the intended rate. In a practical sense, S-DAF can be implemented with CRC codes.

Another closely related technique is known as coded cooperation [4]–[6]. This protocol replaces the repetition coding and diversity combining of the DAF with an incremental redundancy scheme along with code combining at the destination, resulting in better coding gain and a more flexible allocation of resources between the links involved in cooperation. We briefly explain coded cooperation below in a

scenario where two partners cooperate with each other; for a more detailed description of coded cooperation, the reader is referred to [6].

In coded cooperation, each user segments its data into coded transmit blocks, consisting of N symbols, such that the allocated rate for each block is R b/s/Hz. Users cooperate by dividing the transmission of each N-symbol block over two successive time segments, called *frames*. In the first frame each user transmits a rate R/α code word ($N_1 = \alpha N$ symbols). This itself is a valid (albeit weaker) code word which can be decoded to obtain the original information. If a user successfully decodes a selected partner's first-frame transmission, the user then calculates and transmits N_2 additional parity symbols for the partner's data in the second frame, according to some overall coding scheme, where $N_1 + N_2 = N$. This is the essence of coded cooperation: that in a coded system, each user may elect to weaken its own codewords and thus conserve power and bandwidth that it then uses to assist a partner.

Coded cooperation and S-DAF have the same diversity – their difference is only in their coding gain (because S-DAF uses repetition coding). Due to their similarities, and in the interest of brevity, in each instance we either combine the discussion of both, or present a representative example for both of them.

III. PROTOCOLS AND OUTAGE

We first characterize the outage probability of a transmission in the presence of cooperation, which will motivate the partner selection protocols. In this paper the word "partner" is meant in a specific way; namely, when we say node i is the partner of node j, we mean that node i assists, or cooperates in the transmission of node j (but not necessarily vice versa).

We characterize performance based on outage, i.e., the probabilitiy that the channel capacity $C(\gamma) = \log_2(1 + \gamma)$ cannot support the desired rate. For the case of Rayleigh fading, γ has an exponential pdf with parameter $1/\Gamma$, where Γ is the mean SNR over the fading as described in (1) and (2), resulting in the outage expression

$$P_{out} = \int_0^{2^R-1} \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right) d\gamma = 1 - \exp\left(-\frac{2^R - 1}{\Gamma}\right). \tag{3}$$

which we shall see frequently in the sequel. Now we examine outage probability and diversity of cooperative communication for a given network realization. The developments in this section, with some small variations, are similar to outage calculations in [8], [11].

Let $\mathcal{S}_i$ be the set of nodes that assist Node i. The membership of this set is a random variable that depends on the strength of the channel between Node i and other nodes, as well as the constraints set by the partner selection protocol. The overall outage probability can be expressed as follows:

$$P_{out,i} = \sum_{\mathcal{S}_i} \Pr\{\mathcal{S}_i\} \cdot P_{out,i}(\mathcal{S}_i), \tag{4}$$

where $\Pr\{\mathcal{S}_i\}$ is the probability that nodes in the set $\mathcal{S}_i$ decode and hence cooperate with Node i.[2] The remaining term,

$P_{out,i}(\mathcal{S}_i)$, is the probability of outage of Node i conditioned on $\mathcal{S}_i$. Note that the summation is over all distinct sets $\mathcal{S}_i$.[3] Clearly the diversity factor will depend on the rate of decline of each of these two terms, as SNR goes to infinity.

Each of the nodes in $\mathcal{S}_i$ will transmit the same symbols to assist Node i. Since the destination node has CSI for all channels it receives, and since the nodes use orthogonal channels, the destination can diversity-combine the symbols from from each partner (e.g., via maximal-ratio combining) such that the SNR of the partner-destination channels add at the destination. The conditional probability term, assuming mutually independent channels between all nodes, is thus calculated as:

$$P_{out,i}(\mathcal{S}_i) = \int \cdots \int_A \frac{1}{\Gamma_{i,d}} \exp\left(-\frac{\gamma_{i,d}}{\Gamma_{i,d}}\right) d\gamma_{i,d}$$
$$\times \prod_{p \in \mathcal{S}_i} \frac{n_{s,p}}{\Gamma_{p,d}} \exp\left(-\frac{n_{s,p}\gamma_{p,d}}{\Gamma_{p,d}}\right) d\gamma_{p,d} \tag{5}$$

where the subscript d denotes Node i's destination, the subscript p denotes a given partner in the set $\mathcal{S}_i$, $n_{s,p}$ denotes the total number of users (including Node i) that are assisted by Node p, and A is the set of channel conditions that do not support the required transmission rate; in other words, A is the outage event in terms of the channel gains.

For the S-DAF network, the outage event of Node i is:

$$A \equiv \left\{ 1 + \gamma_{i,d} + \sum_{p \in \mathcal{S}_i} \gamma_{p,d} < 2^{2R} \right\} \tag{6}$$

where the penalty factor of 2 in front of the rate is due to repetition. For the coded cooperation network, the transmissions by the source and relay can be viewed as parallel (conditionally) Gaussian channels, whose capacities add [12, Section 10.4]. Equivalently, they can be viewed as time sharing between sets of independent channels. We can thus write the outage event for Node i conditioned on $\mathcal{S}_i$ as

$$A \equiv \left\{ (1 + \gamma_{i,d})^\alpha \left(1 + \sum_{p \in \mathcal{S}_i} \gamma_{p,d} \right)^{1-\alpha} < 2^R \right\}, \tag{7}$$

where $0 < \alpha < 1$ describes the division of degrees of freedom between the source-destination link and the relay-destination link in coded cooperation (see [7] for details).

Equation (5) can be simplified further, but that is not necessary for our purposes and we omit the details. We wish to examine (5) in the high-SNR regime to determine the diversity. To do this, we let Γ_T in (2) go to infinity, while $\Gamma'_{i,j}$ remains as a finite constant. The common SNR parameter for all nodes Γ_T allows us to separate the effects of the transmit power of mobiles from the randomness of the network realization, and then by letting $\Gamma_T \to \infty$ (e.g., the high-SNR regime), the diversity order is obtained.

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[2]To be precise, this is the probability that nodes in $\mathcal{S}_i$, and only nodes in $\mathcal{S}_i$, cooperate with Node i.

[3]The maximum possible number of such sets is $\sum_{k=0}^{M-1} \binom{M-1}{k}$, although the constraints of the particular partner selection protocol may reduce this number in practice.

The conditional outage probability as a function of $1/\Gamma_T$, is obtained by rewriting (5) as

$$P_{out,i}(\mathcal{S}_i) = \frac{1}{\Gamma_T^{|\mathcal{S}_i|+1}} \int \cdots \int_A \frac{\prod_{p\in\mathcal{S}_i} n_{s,p}}{\Gamma'_{i,d} \cdot \prod_{p\in\mathcal{S}_i} \Gamma'_{p,d}}$$
$$\times \exp\left(-\frac{\gamma_{i,d}}{\Gamma_T \Gamma'_{i,d}} - \sum_{p\in\mathcal{S}_i} \frac{n_{s,p}\gamma_{p,d}}{\Gamma_T \Gamma'_{p,d}}\right) \cdot d\gamma_{i,d} \prod_{p\in\mathcal{S}_i} d\gamma_{p,d},$$

(8)

where $|\mathcal{S}_i|$ denotes the number of partners in the set $\mathcal{S}_i$. We can expand the exponential term in (8) using the equivalent Taylor's series representation [13, p. 299] to obtain

$$P_{out,i}(\mathcal{S}_i) = \left(\frac{1}{\Gamma_T^{|\mathcal{S}_i|+1}}\right) \times \left(\frac{\prod_{p\in\mathcal{S}_i} n_{s,p}}{\Gamma'_{i,d} \cdot \prod_{p\in\mathcal{S}_i} \Gamma'_{p,d}}\right)$$
$$\times \left(\int \cdots \int_A d\gamma_{i,d} \prod_{p\in\mathcal{S}_i} d\gamma_{p,d}\right) + O\left(\frac{1}{\Gamma_T^{|\mathcal{S}_i|+2}}\right).$$

(9)

The integral term represents the first term in the Taylor's series expansion of $\exp(\cdot)$ (namely 1), while $O\left(\frac{1}{\Gamma_T^{|\mathcal{S}_i|+2}}\right)$ denotes the order of the remaining terms.[4] The multiple integration can be simplified to a single integral; as this is not important for our results here, we omit the details.[5]

We see from (9) that, as $\Gamma_T \to \infty$, the outage probability behaves as $1/\Gamma_T^{|\mathcal{S}_i|+1}$, which demonstrates that the achieved diversity, conditioned on $\mathcal{S}_i$, is $|\mathcal{S}_i|+1$. For example, if $|\mathcal{S}_i| = 2$, Node i's transmission experiences diversity order 3 for that block.

As mentioned earlier, $|\mathcal{S}_i|$ is an integer-valued random variable, and the overall outage probability follows the Bayesian expression given in (4). The overall diversity order offered to Node i is thus equal to the smallest $|\mathcal{S}_i| + 1$ for which $\Pr\{\mathcal{S}_i\} > 0$ as $\Gamma_T \to \infty$. In general $\Pr\{\mathcal{S}_i\}$ depends on the selection protocol and may have a complicated form. Nevertheless, it is clear that in order to achieve diversity order $n + 1$, we must have $\Pr\{|\mathcal{S}_i| < n\} \to 0$ as $\Gamma_T \to \infty$, or equivalently

$$|\mathcal{S}_i| < n \quad \Rightarrow \quad \lim_{\Gamma_T\to\infty} \Pr\{\mathcal{S}_i\} = 0$$

Furthermore, in order to obtain any diversity improvement at all over non-cooperative transmission, we require that $\Pr\{|\mathcal{S}_i| = 0\} \to 0$ as $\Gamma_T \to \infty$. In other words, the probability of Node i not being assisted by any of the other nodes must go to zero in the high-SNR regime. We will use this insight in the development of partner selection protocols.

A. Distributed Partner Selection

Consider a distributed partner selection scenario, where nodes must decide individually whom to assist. Initially, we consider that each node has the capability to help only $n = 1$

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[4]Throughout this paper, $O(\cdot)$ denotes the familiar order notation; see for example [14, pp. 2–3].

[5]The simplifications of the multiple integration terms in both (5) and (9) involve rewriting the constraints, and then simplifying each integral successively in an iterative fashion. The basic technique is shown for the two-user case in [7, Appendix A].

other node in each transmission. Then, to achieve the maximum diversity factor of 2 across the network, it is required that no node remains unassisted, i.e., $\Pr\{|\mathcal{S}_i| = 0\} \to 0$ at high SNR. Since each transmitter is also a relay, and each node can assist exactly one other node, if anyone gets more than one partner, then there must be some other node in the network that is being deprived of partnerships. Thus, in this simple scenario the goal of a partner allocation protocol is to ensure an equitable distribution of cooperation.

Recall that there is no feedback or handshake available to the physical layer, therefore each node will determine for itself whom to cooperate with, and decisions are made in a distributed fashion. To demonstrate the possible pitfalls for selection algorithms under such conditions, we briefly visit two approaches that fail to achieve full diversity, and then present one that does achieve full diversity.

First, consider a system in which each node decides randomly for each transmission whom it will attempt to assist. Assuming that all nodes receive each other, the probability of Node j not selecting to assist Node i will be:

$$\Pr\{j \notin \mathcal{S}_i\} = \left(\frac{M-2}{M-1}\right)\left(\frac{M-3}{M-2}\right) \cdots \left(\frac{M-n-1}{M-n}\right)$$
$$= \frac{M-n-1}{M-1} = 1 - \frac{n}{M-1}.$$

(10)

Since each node selects independently of all other nodes, the probability that Node i is not selected by any other node is

$$\Pr\{|\mathcal{S}_i| = 0\} = \left(1 - \frac{n}{M-1}\right)^{M-1}.$$ (11)

From (11), we see that $\Pr\{|\mathcal{S}_i| = 0\} > 0$ for the random selection protocol for all $n < M - 1$; thus, the diversity order is only one, and this approach is obviously not desirable.

Now, consider another selection protocol that tries to (locally) minimize the term $P_{out,i}(\mathcal{S}_i)$, the conditional outage. To do so, each node measures its receptions and attempts to assist the node for which it has the highest received SNR; in other words, the node whose transmission it has the highest probability of decoding successfully. (Note that prioritization based on received SNR occurs prior to any decoding attempts.) However, the received SNR depends on the random spatial distribution of the nodes (path loss) as well as the random shadowing and fading components, which are independent of Γ_T. Thus, we expect the statistical behavior of receive SNR selection to be similar to that of random selection, and in fact empirically we observe (see simulation results) that $\Pr\{|\mathcal{S}_i| < n\}$ for both of them is bounded away from zero, which means full diversity cannot be achieved.

Finally, we consider a simple assignment that *does* achieve full diversity. If the total number of nodes is M, with any arbitrary labeling, the cooperation order $1 \to 2 \to \cdots \to M \to 1$ achieves full diversity. The idea is simple: the constraint set by the protocol ensures that no node will be assisted by more than one other node, so that exactly one node is available to assist a given node. Furthermore, as SNR goes to infinity, for each node i we have $\Pr\{|\mathcal{S}_i| = 1\} \to 1$, thus network-wide diversity of two will be achieved.

This basic idea can be generalized for higher degrees of cooperation; i.e., when each of the nodes can assist n other nodes. Then, from the expressions above one hopes to achieve diversity $n + 1$, but only if one can guarantee that $\Pr\{|\mathcal{S}_i| < n\} \to 0$ at high SNR. The key to this, like the last case, is to ensure that no node receives more than its fair share, thus in the asymptote everyone will be fairly served. This can be accomplished by a *fixed priority transmission* protocol, as follows:

Each node is provided with a priority list, used for cooperation purposes. The node attempts to assist n nodes, starting from the top of the list. The key to full diversity is to ensure that the lists across the network have no systematic bias. The solution is highly non-unique (more on this in the section on centralized selection), but one easy solution is to label nodes arbitrarily from 1 to M, and each node i follows a list composed via a priority vector

$$[i+1,\, i+2,\, \ldots,\, M,\, 1,\, 2,\, \ldots,\, i-1]$$

Each node will attempt to decode, and assist, the first n nodes from its priority list. For the case of $n = 1$, this clearly specializes to the example mentioned earlier. For an arbitrary n, each Node i will attempt to assist n others. More importantly, due to the construction of the lists, n other users will attempt to assist Node i. These attempts, as SNR goes to infinity, will be successful with probability one[6]. Thus full diversity on the order of $n + 1$ will be achieved.

For this protocol, each of the nodes needs to have a unique number associated with it. In any wireless network, each of the users is assigned a number (node identifier) by the MAC layer upon initialization or admission of the node to the network. Whenever a user signs off or exits the network, the ID will be released and available for reuse. The same mechanism that allocates the ID numbers may also grant a "cooperation number," or alternatively the same ID numbers may be used for cooperation.

We mention two variations of the fixed priority protocol: (a) each node will make exactly n decoding attempts, and will cooperate with those that are successful; (b) each node will make successive decoding attempts, going down the list, until n users are correctly decoded and cooperated with (or until the list is exhausted). The former will be called n *decoding attempts,* and latter will be called *full decoding attempts*. The distinction of the two diminishes at high SNR and in fact both achieve full diversity.

B. Centralized Partner Selection

It is natural that when nodes are allowed to communicate their control functions, one may achieve additional gains in network performance, compared to the case when nodes must operate in isolation. In this section, we consider a centralized controller for assigning cooperation partners, and study how its performance is affected by availability of various amounts of network channel state information. This is a suitable model

for applications such as cellular networks, where all communications are supervised by a base station, or other types of networks which are coordinated by a centralized controller or master node. In addition, since this protocol attempts to optimize partner assignment globally over all users, it provides insight into the maximum gain achievable with intelligent *a priori* partner assignment.

Earlier we saw that diversity-maximizing protocols are not unique, in fact, many different labelings of the nodes and priority listings would each result in full diversity. Naturally, some of them may perform better than others. The distinction between the distributed and centralized algorithms is that the distributed protocol would pick one of the eligible solutions randomly, while the central protocol has the opportunity to pick the *best* from among such solutions, using knowledge of path losses or channel gains.

In the following discussions, for the purposes of exposition we discuss the case where each user has one partner ($n = 1$). Extension to $n > 1$ follows a similar approach and is straight forward, therefore is omitted for the sake of brevity. We optimize the average outage probability over the M transmitting users, using a greedy algorithm as shown in Figure 2.

To implement this algorithm, we construct a matrix of pairwise path losses (we include shadowing information, when available). Then, using the outage expressions to be derived below, we find the partner assignment that maximizes the network-wide throughput (aka sum capacity of the network).

For the optimization algorithm above, actual outage values are needed, unlike the last section, where diversity calculations were sufficient. In the following, we outline the outage calculations for non-reciprocal coded cooperation. The S-DAF, which uses a repetition code, can be treated similarly and is not separately developed in the interest of brevity.

As in Section III-A, we consider outage probability and diversity for a given network realization. There are many similarities between the following developments and outage calculations in [7], [8], [11]. We apply the notational convention of Fig. 3, denoting User j as the node that assists User i, and User i as the node that assists User k. Depending on whether each partner can detect its incoming signal, we have four cases which we parameterize by $\Theta \in \{1, 2, 3, 4\}$ and express the corresponding conditional capacities and outage events for each case relative to User i as follows:

- **Case 1 ($\Theta = 1$):** In this case, Users i and j both correctly decode the rate R/α first-frame transmissions of their respective partners. In the second frame User j transmits additional parity for User i. The first and second frame transmissions of User i's coded symbols can be viewed as parallel channels, therefore:

$$\begin{aligned} C_{1,d}(\gamma_{i,d}, \gamma_{j,d}|\Theta = 1) = {} & \alpha \log_2(1 + \gamma_{i,d}) \\ & + (1-\alpha)\log_2(1 + \gamma_{j,d}) < R \end{aligned} \tag{12}$$

where again the subscript d denotes User i's destination.

- **Case 2 ($\Theta = 2$):** In this case, neither User i nor j correctly decodes its respective partner, User i therefore transmits its own additional parity, and the outage event is

$$C_{i,d}(\gamma_{i,d}|\Theta = 2) = \log_2(1 + \gamma_{i,d}) < R \tag{13}$$

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[6]In other words, as the SNR goes to infinity, the probability of channel errors goes to zero, and thus the probability of Node i not successfully decoding Node j goes to zero for all i and j.

1) Randomly assign partners such that each user attempts to assist exactly one other user, and exactly one other user attempts to assist it.

2) Compute average outage probability over all users conditioned on the available channel knowledge.

3) Assume User i is currently assisted by $\hat{i}$. Among all other candidate partners for User i, find the User $\tilde{i}$ such that exchanging the nodes that assist Users i and $\tilde{i}$ (so that User $\tilde{i}$ is assisted by $\hat{i}$ and User i is assisted by the node that was assisting User $\tilde{i}$) will minimize the average (network-wide) outage probability. Sometimes no exchange is the best exchange.

4) Repeat Step 3 for all M users.

5) If no exchanges in Steps 3, 4, stop. Else, go back to step 3

Fig. 2. Algorithm for central partner selection
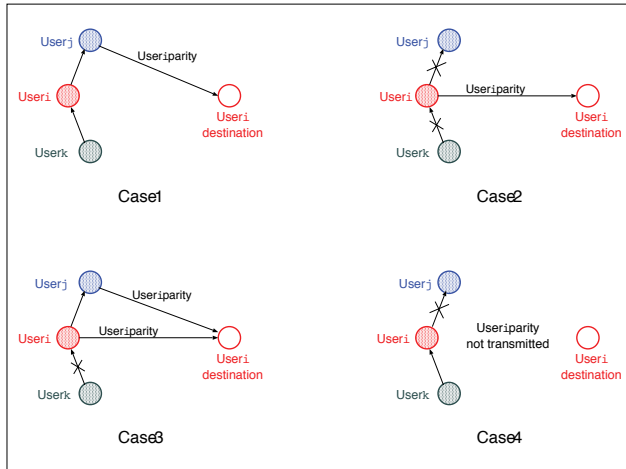
Fig. 3. Four possible cases of cooperative transmission to benefit User i. In this example, User j assists User i, while User i assists User k

- **Case 3 ($\Theta = 3$):** In this case, User j correctly decodes User i, but User i does not correctly decode User k. Therefore, in the second frame, User i's additional parity is transmitted by both User j and User i. The corresponding outage event is

$$C_{i,d}(\gamma_{i,d}, \gamma_{j,d}|\Theta = 3) = \alpha \log_2(1 + \gamma_{i,d})$$
$$+ (1 - \alpha)\log_2(1 + \gamma_{i,d} + \gamma_{j,d})$$
$$< R \quad (14)$$

- **Case 4 ($\Theta = 4$):** In this case, User j does not correctly decode User i, but User i does correctly decode User k. In the second frame, no additional parity symbols for User i are transmitted, resulting in the outage event

$$C_{i,d}(\gamma_{i,d}|\Theta = 4) = \log_2(1 + \gamma_{i,d}) < R/\alpha \quad (15)$$

Since the four cases are disjoint, and $\{\gamma_{i,j}, \gamma_{k,i}, \gamma_{i,d}, \gamma_{i,d}\}$ are mutually independent, we can write the overall outage

probability for User i as

$$P_{out,i} = \Pr\{\gamma_{i,j} > 2^{R/\alpha} - 1\} \cdot \Pr\{\gamma_{k,i} > 2^{R/\alpha} - 1\}$$
$$\cdot \Pr\{(1 + \gamma_{i,d})^\alpha (1 + \gamma_{j,d})^{1-\alpha} < 2^R\}$$
$$+ \Pr\{\gamma_{i,j} < 2^{R/\alpha} - 1\} \cdot \Pr\{\gamma_{k,i} < 2^{R/\alpha} - 1\}$$
$$\cdot \Pr\{\gamma_{i,d} < 2^R - 1\}$$
$$+ \Pr\{\gamma_{i,j} > 2^{R/\alpha} - 1\} \cdot \Pr\{\gamma_{k,i} < 2^{R/\alpha} - 1\}$$
$$\cdot \Pr\{(1 + \gamma_{i,d})^\alpha (1 + \gamma_{i,d} + \gamma_{j,d})^{1-\alpha} < 2^R\}$$
$$+ \Pr\{\gamma_{i,j} < 2^{R/\alpha} - 1\} \cdot \Pr\{\gamma_{k,i} > 2^{R/\alpha} - 1\}$$
$$\cdot \Pr\{\gamma_{i,d} < 2^{R/\alpha} - 1\}. \quad (16)$$

The form of (16) is analogous to the outage probability expression for two-user reciprocal cooperation derived in [7, (12)]. As a result, based on [7, (13)-(15) and Appendix A], we can evaluate (16) for the case of Rayleigh fading as

$$P_{out,i} = \exp\left(\frac{1 - 2^{R/\alpha}}{\Gamma_{k,i}}\right)\left[1 - \exp\left(\frac{1 - 2^{R/\alpha}}{\Gamma_{i,d}}\right)\right.$$
$$\left. - \exp\left(\frac{1 - 2^{R/\alpha}}{\Gamma_{i,j}}\right) \cdot \Psi_1(\Gamma_{i,d}, \Gamma_{j,d}, R, \alpha)\right]$$
$$+ \left[1 - \exp\left(\frac{1 - 2^{R/\alpha}}{\Gamma_{k,i}}\right)\right]\left[1 - \exp\left(\frac{1 - 2^R}{\Gamma_{i,d}}\right)\right.$$
$$\left. - \exp\left(\frac{1 - 2^{R/\alpha}}{\Gamma_{i,j}}\right) \cdot \Psi_2(\Gamma_{i,d}, \Gamma_{j,d}, R, \alpha)\right] \quad (17)$$

where

$$\Psi_1(\Gamma_{i,d}, \Gamma_{j,d}, R, \alpha) = \int_0^{2^{R/\alpha}-1} \frac{1}{\Gamma_{i,d}}\exp\left(-\frac{\gamma_{i,d}}{\Gamma_{i,d}} - \frac{a}{\Gamma_{j,d}}\right)d\gamma_{i,d}$$

$$\Psi_2(\Gamma_{i,d}, \Gamma_{j,d}, R, \alpha) = \int_0^{2^R-1} \frac{1}{\Gamma_{i,d}}\exp\left(-\frac{\gamma_{i,d}}{\Gamma_{i,d}} - \frac{b}{\Gamma_{j,d}}\right)d\gamma_{i,d}$$

$$a = \frac{2^{R/(1-\alpha)}}{(1 + \gamma_{i,d})^{\alpha/(1-\alpha)}} - 1$$

$$b = \frac{2^{R/(1-\alpha)}}{(1 + \gamma_{i,d})^{\alpha/(1-\alpha)}} - 1 - \gamma_{i,d}. \quad (18)$$

We use (17) in the Monte Carlo simulation to evaluate the performance of the centralized protocol. Note that the average SNR values $\Gamma_{i,j}$ used in (17) for Steps 2 and 3 of the centralized partner selection algorithm depend on the level of channel knowledge. If only the user locations are known, then $\Gamma_{i,j}$ does not contain the shadowing component. If all
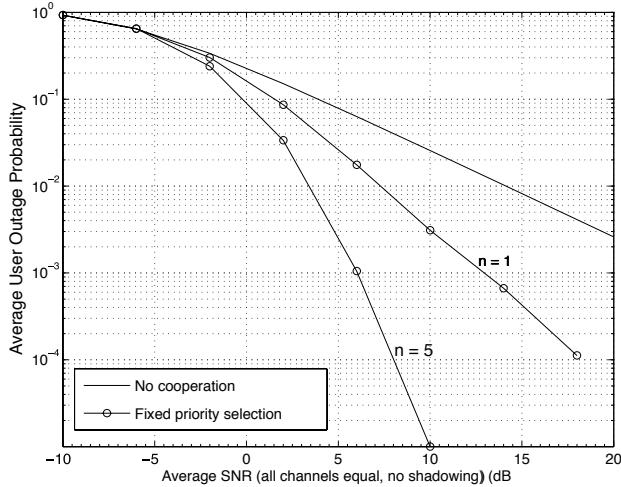
Fig. 4. Fixed priority protocol under Rayleigh fading without path loss or shadowing. Users make $n = 1, 5$ decoding attempts.

Fig. 5. Distributed protocols for $M = 10$ users, $n = 1$ decoding attempts per user, path loss $\beta = 4$, $K = -60$dB, shadowing $\sigma_S = 8$. For $M = 50$ users the curves are virtually identical.

the shadowing components are known, then $\Gamma_{i,j}$ is as defined in (2).

To determine the achieved diversity, we again let Γ_T go to infinity, and obtain outage probability as a function of $1/\Gamma_T$ by expanding the exponential terms using the equivalent Taylor's series representation. For the centralized protocol, this is analogous to the procedure in [7, Section III-B and Appendix B], and we obtain the following result:

$$P_{out,i} = \frac{1}{\Gamma_T^2} \cdot \left[\frac{(2^{R/\alpha} - 1)^2}{\Gamma'_{i,d}\Gamma'_{i,j}} + \frac{\Lambda(R,\alpha)}{\Gamma'_{i,d}\Gamma'_{j,d}} \right] + O\left(\frac{1}{\Gamma_T^3}\right), \quad (19)$$

where, for $\alpha \neq 1/2$ we have

$$\Lambda(R,\alpha) = 2^{R/(1-\alpha)} \left(\frac{1-\alpha}{1-2\alpha} \right) \left(2^{R(1-2\alpha)/\alpha(1-\alpha)} - 1 \right)$$

and for $\alpha = 1/2$ we have

$$\Lambda(R,\alpha) = R \cdot 2^{2R+1} \cdot \ln 2 - 2^{2R} + 1.$$

Equation (19) is thus analogous to [7, (20)], and we see that, as $\Gamma_T \to \infty$, the outage probability is a function of $1/\Gamma_T^2$. This shows that the centralized protocol achieves full diversity, in this case diversity order two for $n = 1$.

IV. SIMULATIONS

In this section we present simulation results to demonstrate the characteristics and behavior of various partner selection algorithms. Throughout the simulations, we use the coded cooperation protocol. Since S-DAF produces similar qualitative results, we do not present separate simulations in the interest of brevity. All simulations use $\alpha = 0.75$ and $R = 1/3$, unless noted. We emphasize that these values are only used as representative examples to demonstrate system behavior and trends.

Fig. 4 provides results without path loss and shadowing, so that the curves highlight the diversity gain achieved. All of the remaining simulation results include the effects of path loss and shadowing, to illustrate performance under all the channel impairments. For these cases, the parameters of $\Gamma'_{i,j}$ used are:
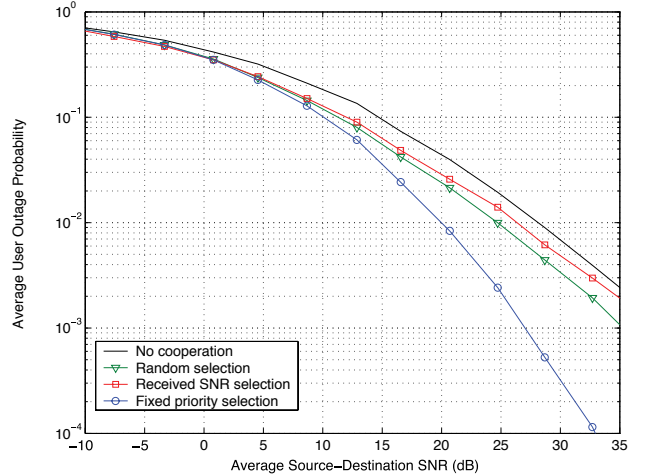
$\beta = 4$, $\sigma_S = 8$, and $K = -60$dB, which is the path loss referenced at the outer edge of the circular region (the size of the region is normalized out in the calculations, as indicated in description of the parameter $d_{i,j}$ in (2)).

Again, these values are examples only.

The simulations average the outage probabilities over the M users, and over network realizations (i.e., shadowing components and relative node locations). In other words, the simulations effectively compute the average outage probability of all users for a given network realization (by varying the fading for each transmit block with user locations and shadowing constant), then repeat this over several network realizations to get the average outage probability over network realizations. In order to be fair in the comparisons made in this work, Γ_T (i.e., transmit power and receiver noise) is the same for all nodes and all transmissions, as is the bandwidth utilized by each node. In the plots, outage probability is referenced to average source-destination SNR over all users and network realizations. This gives a dynamic range analogous to the case of a single user and channel, where the x-axis is typically the average SNR of the user-destination channel.

A. Distributed Protocol

We start by a demonstration of the fixed priority selection protocol for a system with $M = 10$ users. The results are shown in Fig. 4. It is shown that for $n = 1$ and $n = 5$, diversity order as predicted by analysis is observed. This is a basic simulation that does not include path loss and shadowing; channels are modeled simply as i.i.d. Rayleigh random variables.

We next show the distinctions between fixed priority selection, random selection, and receive SNR selection. We expect that the latter two do not achieve full diversity, and in fact this is borne out in simulations. Fig. 5 compares the outage probability of the three distributed protocols for the case of $n = 1$. The fixed priority protocol has a gain of approximately 8dB over no cooperation for outage probability 10^{-2}. In contrast, the other two protocols have a gain of only
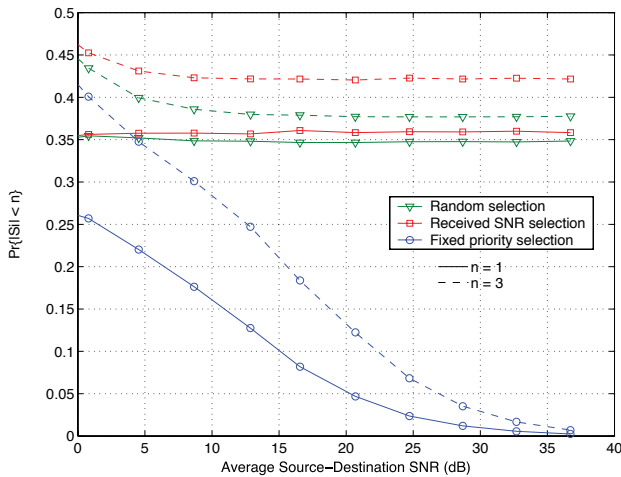
Fig. 6. $\Pr\{|\mathcal{S}_i| < n\}$ vs. average source-destination SNR for $M = 10$ users, and $n = 1, 3$, path loss $\beta = 4$, $K = -60$dB, and shadowing $\sigma_S = 8$.

2-3dB. In this, as well as succeeding simulations, we include the effects of path loss and shadowing.

We saw that full diversity requires $\Pr\{|\mathcal{S}_i| < n\} \to 0$ as $\Gamma_T \to \infty$. Fig. 6 shows that for the random selection protocol, $\Pr\{|\mathcal{S}_i| < n\}$ is bounded away from zero for all SNR. Thus, the random selection protocol does not achieve full diversity, and in fact at best achieves only diversity order 1 for $n < M - 1$.

It is interesting to note that selection based on the best received SNR actually performs worse than random selection. We observe that $\Pr\{|\mathcal{S}_i| = 0\}$ is greater for receive SNR selection than random selection for a given n. This is because the respective probabilities of Node i being selected by Nodes j and k are not uncorrelated. With $n = 1$ and $\gamma_{i,j} < \gamma_{k,j}$, Node j will likely not select Node i, and in addition it is more likely that $\gamma_{i,k} < \gamma_{j,k}$, in which case Node k will likely not select Node i either. One example of such a scenario is that Node i is isolated near the boundary of the region, while the other users are clustered relatively close together in another part of the region.

Fig. 5 also indicates that, for a given $n < M - 1$, the number of users M has little effect on the performance. In the random selection and fixed priority selection protocols, the priority list order is uncorrelated with the physical locations of the users. Thus, the partners in $\mathcal{S}_i$ are located randomly with respect to Node i and its destination, and the probability of their locations is independent of M. Consequently, the outage probability of Node i is not significantly affected by M. Results show that receive SNR selection improves slightly as M increases. With more users, it becomes less likely that, for Node i, receive SNR $\gamma_{i,j}$ is small f'or all j. However, at best, receive SNR selection approaches the performance of random selection, which is not very good. Obviously, for $n = M$, increasing M increases the achievable diversity according to (9), and thus improves performance, but at the expense of increased complexity, as each user attempts to decode all $M - 1$ other users during each transmit block.

Fig. 7 compares the outage probability of the fixed priority selection protocol for $M = 10$ and various values of n. We can characterize the complexity as the expected number

of decoding attempts per transmit block per user. Fig. 8 compares complexities for various n values when users make full decoding attempts. We note from Fig. 8(a) that random selection and fixed priority selection have identical complexity. The complexity of receive SNR selection is less and converges much faster to the minimum value n, which makes sense since a user in this protocol attempts to decode first those users for which it sees the highest received SNR. However, the computational advantage is not sufficient to recommend this algorithm, due to its inferior performance.

In Fig. 7, we see that as n increases, the diversity increases and the outage probability decreases relative to noncooperative transmission. However, cooperative gains slow down with increasing n, while the complexity increases approximately linearly, illustrating a case of diminishing returns. For example, in Fig. 7(a), $n = 9$ (the maximum n value) provides an additional 10dB gain over $n = 1$ for outage probability 10^{-2}. Setting $n = 5$ provides roughly 85% of this additional gain, with 30%-45% less complexity (depending on whether users make all possible decoding attempts, or only n attempts).

The curves in Fig. 7 correspond to full decoding attempts. As noted in the caption, the curves for n decoding attempts are virtually identical (and thus are not shown in the figure to improve readability). As an example, in Fig. 7(a), at an outage probability of 10^{-2}, the gain with n decoding attempts is within 0.5dB of the gain with full decoding attempts, for all values of n. As the SNR increases, the performance of the two cases converges, as shown in Fig. 8(a); that is, even for comprehensive decoding the average number of decoding attempts converges to n. Thus, for low rates the complexity vs. performance tradeoff favors making only n decoding attempts in each transmit block.

At higher rates (Fig. 7(b)) the outage probability at a given rate is simply determined by the average number of decoding attempts, regardless of whether a user makes all possible attempts, or n attempts only. For example, the outage probabilities for $n = 2$ with all possible decoding attempts and for $n = 3$ with n attempts only are roughly equal for rates around 2.5b/s/Hz. We see from Fig. 8(b) that this corresponds to the rates for which the average number of decoding attempts, for $n = 2$, is close to 3.

B. Centralized Protocol

Fig. 9 shows the effect of the amount of channel knowledge and number of users M on the centralized protocol. Even though additional channel knowledge gives better performance, the centralized protocol has full diversity at all levels of channel knowledge. In the low-rate regime, illustrated in Fig. 9 with $R = 1/3$, cooperation without any central channel state information[7] provides a gain of roughly 8dB at outage probability 10^{-2}. While this is certainly not insignificant, knowledge of the user locations gives an additional gain of 3dB for $M = 10$. Full channel knowledge (e.g., knowledge

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[7]The performance in this case is equivalent to a distributed protocol. When there is no central information, the protocol can only use whatever the nodes know locally. See also the simulations.
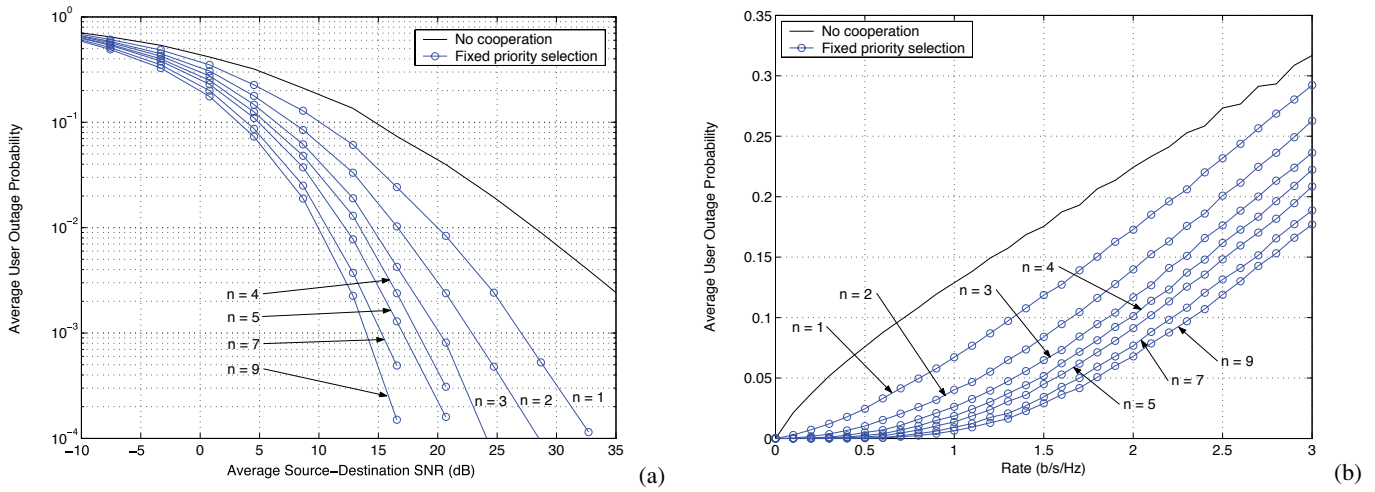
Fig. 7. Comparison of fixed priority protocol for $M = 10$ users, path loss $\beta = 4$, $K = -60$dB, shadowing $\sigma_S = 8$. (a) Outage probability vs. average source-destination SNR at $R = 1/3$. (b) Outage probability vs. rate for constant average source-destination SNR of 20dB.
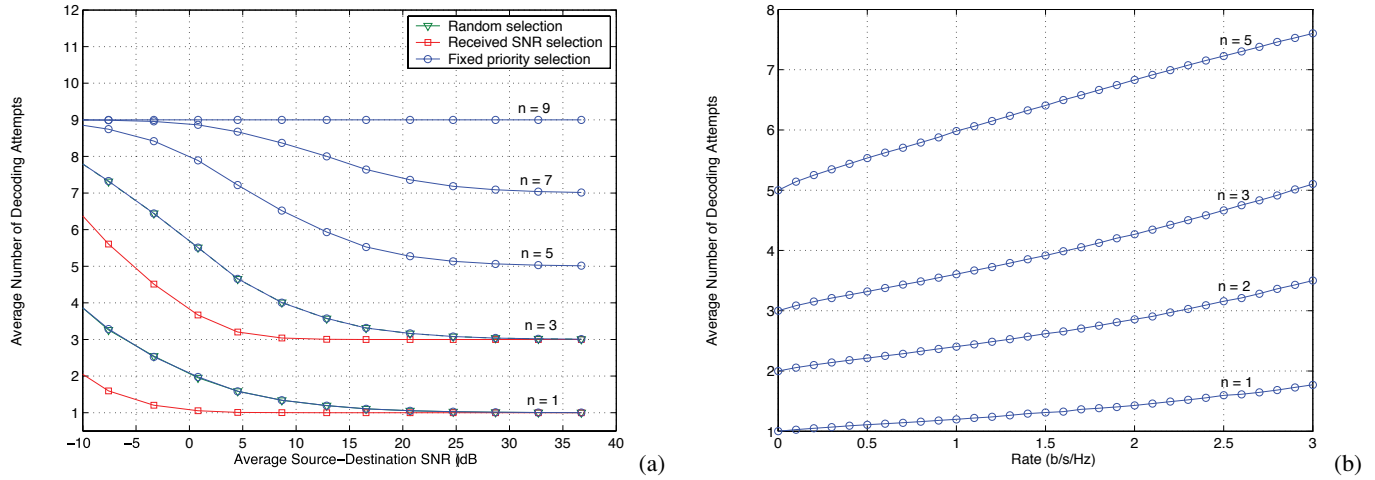

Fig. 8. (a) Complexity vs. average source-destination SNR. (b) Complexity vs. rate under average constant source-destination SNR of 20dB. Network has $M = 10$ users, path loss $\beta = 4$, $K = -60$dB, and shadowing $\sigma_S = 8$.
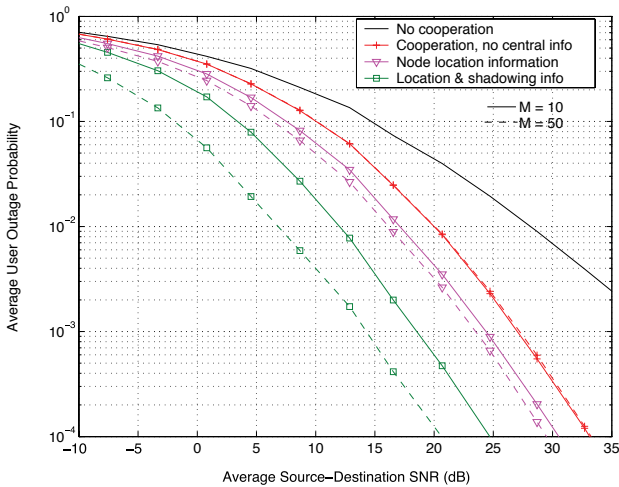

Fig. 9. Showing the effect of various amounts of information available to the centralized controller. Network has $M = 10, 50$ users, path loss $\beta = 4$, $K = -60$dB, and shadowing $\sigma_S = 8$.

of all user locations and shadowing components of each link) provides an additional gain of 8dB.

Fig. 9 shows that, when no central channel information is available, the outage probability is unaffected by the value of M. When the user locations are known, increasing M provides a small improvement, while with full channel knowledge the improvement is much more significant. In this case, increasing the pool of potential partners clearly allows for a better partner assignment, and thus better performance. Naturally when the central controller knows nothing about the quality of potential partners, increasing the size of the pool is unhelpful.

Fig. 10 shows the performance of the centralized protocol with as well as fixed priority selection for $n = 1$, and $M = 10$, with varying R. Note that the distributed fixed priority protocol has identical performance to the centralized protocol with no central channel state information. Thus, the centralized protocol with channel knowledge shows a significant gain over the distributed protocol for equal n.

V. CONCLUSIONS

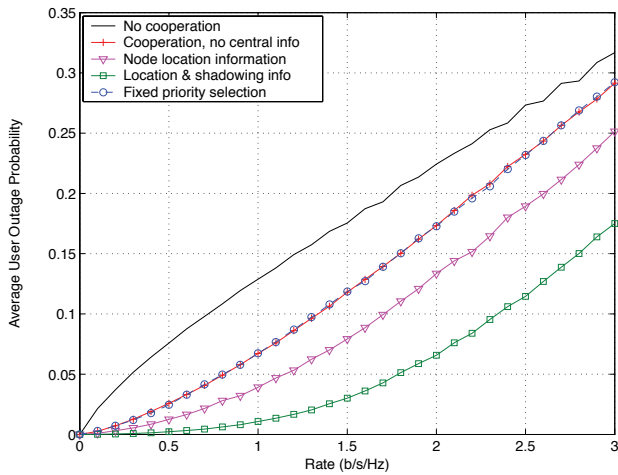In this work, we consider partner selection protocols for cooperative communication in wireless networks. In a dis-

Fig. 10. Comparison of centralized and distributed protocols. Network has $M = 10$ users, path loss $\beta = 4$, $K = -60$dB, and shadowing $\sigma_S = 8$. Source-destination SNR is fixed at 20dB to see the effect of varying rate.

tributed protocol, each user decides and acts autonomously in assisting other users. We examine the outage probability for these protocols, demonstrating that full diversity in the number of cooperating users can be achieved with a properly formulated protocol, as well as significant improvement compared to a non-cooperative system. We also study a centralized protocol that assigns partners to minimize the average outage probability over all users, based on knowledge of the channels between the users. Outage probability results show again that full diversity is achieved, as well as significant gains compared to a distributed protocol, if the centralized protocol has enough channel state information.

This work reveals basic strategies for cooperative partner assignment, however, several interesting problems remain to be solved. In particular, the present work assumes that small-scale fading is not dominated by path loss, which points to networks of up to a certain coverage area. As a result, our work does not consider multi-hop routing. For larger wireless networks, the calculation of multi-hop route jointly with cooperative partner allocation remains an interesting and challenging open problem.

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