

## **Re-Tracking Within Algebra One: A Structural Sieve With Powerful Effects for Low-Income, Minority, and Immigrant Students**

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### **Introduction**

Across the nation a key element in educational reform has been to increase the mathematical proficiency of the broad group of students making their way through K-12 schools into the workplace and higher education. The ability of the United States to identify and educate a core of highly talented students in mathematics has never been in question. The issue has been the relative importance of a much higher level of mathematical literacy for all students, and the will and capacity to create this level of mathematical competence. At stake, depending on the national policy report, is the economic competitiveness of the United States, our ability to continue to play a major international role, our viability and leadership in an increasingly technological world, our ability to thrive and exercise leadership in a world rapidly changing under the forces of globalization, our capacity to provide a more technically skilled workforce to the service sectors of our economy as well as to manufacturing and the professions.

The consensus that has emerged across the states is that the capacity to think and problem solve with the mathematical training associated with preparation for and completion of Algebra 1 can plausibly define basic mathematical literacy. This is presumed useful in three contexts. It can enhance preparation for the growing number of positions in the labor market that do not require education beyond high school but call for increasingly higher levels of mathematical and technical literacy. It can provide stronger preparation for two-year programs at community colleges that require a background in mathematics. And it can increase and strengthen the pool of students capable of mastering the higher levels of mathematics requisite for college majors in all fields, and in particular for careers in mathematics and science.

What has not emerged is a common definition for what constitutes knowledge and proficiency in Algebra 1. Indeed, each is contested ground internally within each state. The National Council for

the Teaching of Mathematics has played a major role in attempting to gain consensus through its Principles and Standards for School Mathematics that address curriculum, teaching, and assessment across K-12, but differences are deep-seated and remain fully at play. At a minimum they involve very different perceptions of the role and emphasis that should be placed on algorithmic proficiency and conceptual understanding, on abstract and applied mathematics, on open-ended materials and methods and more direct and structured approaches, on the importance of providing a comprehensive approach that includes procedural fluency, conceptual competence, strategic competence, adaptive reasoning, and productive disposition. (NCTM, 2000; Silver, 2002; Boaler, 2002)

Twenty years have gone by since the publication of *A Nation At Risk* in 1983. The commitment at the policy level to increasing the mathematical and reading literacy of all students has increased exponentially, as witnessed by the recent *No Child Left Behind* legislation with its requirements for annual testing in reading and mathematics and accountability for outcomes. The question is how are districts and schools responding at the local level and what are the outcomes for students? The prevailing paradigm has been to report changes in test scores. In this paper we take a different approach. We look at how five urban high schools with student populations that are virtually all low-income, minority, and/or immigrant have arranged for all students to take Algebra 1. And we examine the subsequent course enrollments and grades of these students in mathematics, in science, and the core set of college prep courses.

What we have found once again, at the most fundamental level, is that both preparation and these ‘arrangements’ matter. The groupings that were set up, or emerged in practice with counselor/school consent, to make it possible for more students to move forward in preparation for college, are associated with very different patterns of subsequent course enrollments and grades in mathematics, in science, and in the basic set of core college prep courses. In fact, the findings substantiate the assertions of John Meyer that where a student is in the structure of education can be more important than how a student performs within their location in the structure (Meyer, 1977). We will show in this paper that this was true not only for the full set of students, but also for the sub-set of students across all groupings who performed at the highest levels, those students that received an A or B in their initial math group course.

Our findings suggest that unless we raise achievement in mathematics by the end of 8<sup>th</sup> grade there is little that can currently be done at high schools such as the ones we studied to change the percent of students prepared for regular and competitive eligibility for college. The practical implications of the findings are numerous.

- They suggest that the major efforts underway to convince all students that they have the opportunity to prepare for college can be credible only to the extent that their schools have the capacity to prepare them effectively for Algebra 1.
- They suggest that the major efforts underway to encourage more students to be interested in, and enroll in biology, chemistry, and physics in high school and college can be successful only at the margins until there is a meaningful change in mathematics preparation.

- They suggest that programs designed to address remediation in mathematics at the high school level in order to move students forward into the college prep curriculum have to be examined carefully for effectiveness.
- They suggest that it may be wiser to place maximum effort on solving the mathematics problem at the elementary level, and then reap the rewards at the high school and college level in increased mathematical proficiency, in larger enrollments in biology, chemistry, and physics, and in higher levels of academic achievement.

### **Description of the Students in our Study**

We followed the high school experience of 3,574 students from five urban high schools with a high proportion of low-income, minority, and immigrant students. These students were 9<sup>th</sup> graders in 1997-98 and completed four year of high school in 2000-01. The students are evenly divided by gender, with 1,785 females and 1,789 males. Hispanic students are largest group, 35%, followed by White students, 24%, Asian and Black students, 16% each, and a small group of ‘Other’ students, 9%, who are either unidentified by ethnicity or are part of groups too small to separate out for analysis. There are no significant differences in the proportion of females and males across ethnic groups. Almost a quarter of our students, 23%, are English language learners. Of these, 54% are Hispanic, 34% are Asian, 6% are White and another 6% are Other.

### **Description of the High Schools**

Each of the five high schools is located in an urban area and has a high proportion of low-income, minority, and immigrant students. Each of the schools has made a commitment to increase the number of their students going to college, and to engage in systemic change to increase educational opportunity for all of their students. Each school is also an active participant in community and higher education partnerships. There is strong district support for systemic change at each school, including the creation of a school-wide college-going culture, and substantial increases in the college going rates of all sub-groups of students.

### **Description of the Algebra Groups**

The students who took Algebra 1 in 8<sup>th</sup> grade and were enrolled in geometry in 9<sup>th</sup> grade constitute our first algebra group. There are 274 of them. They make up 7.7% of the cohort of 3,574 students, and 9.1% of the 3,009 students who took Algebra 1. The criteria for assignment to 8<sup>th</sup> grade algebra 1 was specific to each of the five districts and their schools, and we could not retrieve the criteria used six years prior to our acquisition of the data.

The students who took Algebra 1 in high school in the regular one-year algebra 1 course are the second group. There are 1,442 students in this group. They make up 40.3% of the entire cohort and 47.9% of the students who took Algebra 1. They were assigned to their course based on teacher and counselor recommendation during the 8<sup>th</sup> grade articulation process. We were told that attitude, effort, grades, and test scores, are the primary criteria for assignment to all 9<sup>th</sup> grade mathematics courses and that decisions are made on an individual basis rather than with a rubric.

The students who took a two-year Algebra 1 course are the third group. There are 333 students in this third group. They make up 9.3% of the total cohort and 11.1% of the students who took Algebra 1. This two-year course was created from the premise that with slower steps and careful attention to individual learning patterns it would be possible to support students with lower mathematics grades and test scores in a mathematics program striving to make algebra available to all students. Decisions about placement in this program were made within the high school mathematics department.

It is important to establish at the outset that this two-year course was specifically designed to move less prepared students forward into the college prep curriculum as well as to meet preparation for community college courses or direct entry into the labor market. This college prep purpose is supported by the University of California which reviews the curriculum and accepts completion of the course with a grade of C or higher in each of the four semesters for fulfillment of the Algebra 1 requirement. UC certification for this 2 Year Algebra 1 course was obtained by each of the schools in our study that offered this program.

Students who enrolled in both the one-year and the two-year Algebra 1 courses form the fourth group. There are 417 students in this group. They make up 11.7% of the total cohort and 13.8% of the students taking Algebra 1. Some of them started in or completed the two-year program and then enrolled in the one-year course. Others started in the one-year course and changed into the two-year program. Some moved up and back between the two programs. They represent an ‘adaptive arrangement’ developed in practice.

Students who enrolled in only one year of the two-year program and did not complete Algebra 1, but went on to take higher mathematics courses, form the fifth group. There are 543 students in this group. They make up 15.2% of the total cohort and 18% of the students who enrolled in Algebra. They, too, represent an ad-hoc ‘adaptive arrangement’ for which there was no policy explanation.

The remaining 565 students did not enroll in Algebra 1 either in 8<sup>th</sup> grade or in high school. They make up the remaining 15.8% of the cohort of 3,574 students.

Table 1 summarizes the distribution of students across the algebra groups.

Table 1: Distribution of the Students Across the Algebra Groups				
8 <sup>th</sup> Gr Alg1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr.	1 of 2 Yr
274	1,442	333	417	543
9%	48%	11%	14%	18%

### Algebra Group as a Key Variable

It is sometimes the case in research that we can identify one factor that has a profound effect on a specific outcome. More often we identify several important factors that bear on an outcome of consequence. There are times, however, when we can identify one factor that is associated with a whole chain of events. The relationships we are about to discuss between algebra group and enrollment and grades in biology, chemistry, and physics, in higher mathematics, and in the core set of college prep courses may fall in this latter category, and to the extent that it does, the relationship

goes beyond explaining why things are as they are, to shedding light on where policy makers can best allocate resources, and where educational reformers may be able to make their most productive interventions and gains.

We are going to present our findings in ways that we hope will enable the reader to explore independently whether there is, indeed, a case to be made for algebra group as a key variable associated with an eco-system set of events. We present findings first for all students, and subsequently look at issues of gender, of race-ethnicity, and the experience of English language learners. It is important to note that in all cases the course enrollments and grades we report, are in courses, or sections of courses, approved by the University of California for college admission.

### The Relationship of Algebra Group to Enrollments in Biology, Chemistry, and Physics

	8 <sup>th</sup> Alg 1 n = 274	1 Yr Alg 1 n = 1,442	2 Yr Alg 1 n = 333	1 & 2 Yr n = 417	1 of 2 Yr n = 543
Biology	261 <b>95%</b>	1,145 <b>79%</b>	125 <b>38%</b>	217 <b>52%</b>	125 <b>23%</b>
Chemistry	238 <b>87%</b>	633 <b>44%</b>	42 <b>13%</b>	64 <b>15%</b>	10 <b>2%</b>
Physics	120 <b>44%</b>	169 <b>12%</b>	5 <b>2%</b>	6 <b>1%</b>	<b>0%</b>

\*p <.01

Enrollment in biology as well as in chemistry and physics is highly associated with algebra group. It is important to note both the strength of the relationship between algebra

group and enrollment in all three of the science courses, and the magnitude of the differences in enrollment across algebra groups.

*The strength of the relationship:* Biology enrollment is closest to the experience of all students and we focus in on it to make the relevant point. As seen in Table 2, 95% of the students from 8<sup>th</sup> grade Algebra 1 enrolled in biology, 79% from 1 Yr Algebra 1, 52% of the students who participated in both the 1 and 2 Yr Algebra 1 courses, 38% of the students from 2 Yr Algebra 1, and 23% of the students who took only one year of the two-year course and went on to take higher math. These differences are significant with  $p < .01$ . Prior observation and research would lead each of us to expect substantial differences for chemistry and physics, but this research shows the strength of the relationship for biology as well.

*The magnitude of the differences:* The findings for biology are also important in discussing the magnitude of the differences. In our findings:

- 16 percentage points separate 1 Yr Algebra 1 students from 8<sup>th</sup> grade Algebra 1 students in biology enrollment – 79% to 95%.
- 27 percentage points separate students who took both 1 and 2 year Algebra 1 courses, from 1Yr Algebra 1 students – 52% to 79%.
- 41 percentage points separate 2 Yr Algebra 1 students from 1 Yr Algebra 1 students – 38% to 79%.
- 56 percentage points separate students with 1 year of the 2 year Algebra 1 program from 1 Yr Algebra 1 students – 23% to 79%.

We do not need measures of statistical significance to appreciate the magnitude of these differences. It is also important to focus on the magnitude of the differences in chemistry enrollment, because completion of chemistry is a prerequisite for admission to both the California State University and

the University of California. Here we see that there is a 50% difference in enrollment in chemistry between 8<sup>th</sup> grade Algebra 1 students and 1 Yr Algebra 1 students, 87% to 44%. After that chemistry enrollment drops to 15%, 13%, and 2% of the respective algebra groups.

What are the factors underlying the strength and magnitude of these associations? The research literature suggests that differences in opportunities to learn and in preparation, socialized identities as ‘academic’ or ‘not academic,’ handicaps due to lack of fluency in English, differences in ability, interests, and motivation are all at play.

In our search of the literature we re-read the research of John Meyer and others on socialized identities. This research suggests that to the extent that ‘doing well in math’ forms a social lens for identifying academic ability, teachers, students, and families develop and reinforce a socialized identity of academic adequacy or inadequacy over the eight years prior to high school, that in the latter case, may be due more to lack of opportunity to learn than to lack of ability to learn. (Meyer, 1977, 1987).

We conclude that if the association between preparation in mathematics and enrollment in science is as strong as it appears to be in our findings, then the path to increasing enrollment in biology, chemistry, and physics may to an important degree be through changes in math preparation.

#### The Relationship of Algebra Group to Highest Mathematics Enrollment

In California students must complete three years of mathematics with Algebra 1 as the threshold for admission to California State University or the University of California. For all practical purposes this means completion of at least Algebra 2. What we show in Table 3 that 96% of the students from 8<sup>th</sup> grade Algebra 1 enrolled in Algebra 2, compared with 42% of the students from 1 Yr Algebra 1, and 6% or less from the other three algebra groups.

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr
	n = 274	n = 1,442	n = 333	n = 417	n = 543
Alg 2 or +	96%	42%	3%	6%	2%
Geometry	4%	27%	14%	23%	3%
Algebra 1		31%	83%	71%	95%

\* p < .01

The grade received in Algebra 2 is also very important. Before turning specifically to the issue of grades, however, we offer a brief transition analysis that reinforces the points just made about enrollment and opens up the larger discussion of grades. In this analysis we focused only on those students who received an A or B in their algebra group course. The findings on their enrollments are shown in Table 4a below, and the findings on their grades in Table 4b.

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr
Biology	97%	92%	68%	53%	
Chem	92%	62%	32%	38%	
Physics	47%	21%	11%	6%	
Algebra 2	98%	68%	16%	29%	

\*p < .01

- Two or fewer students are represented here.

		8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 Yr of 2
Biology	AB	67%	50%	46%	34%	
	C	14%	22%	15%	28%	
	DF	18%	27%	38%	39%	
	p<.01					
Chemistry	AB	75%	42%		31%	
	C	16%	31%	33%	31%	
	DF	9%	27%	67%	39%	
	p<.01					
Physics	AB	67%	47%			*50%
	C	24%	25%	*100%		
	DF	8%	27%			*50%
	p<.01					
Hi Math Alg2	AB	69%	36%		30%	
	C	21%	34%		10%	
	DF	10%	30%	100%	60%	
	p<.01					

Students with the highest grades across the algebra groups had the enrollment patterns we noted for all students. Their grades in Table 4b below show a similarly strong association between algebra program and grade received.

It is important to explain that the grades for students from 8<sup>th</sup> grade Algebra 1 are not the grades they received in algebra in 8<sup>th</sup> grade. We were not able to access those grades. They are the grades the students received in their 9<sup>th</sup> grade

math course, which was geometry. This is a very imperfect substitute, but it does put in play the math grade the student received in 9<sup>th</sup> grade, which is comparable in time to the grades of their peers.

Two recent research studies may shed some light on our findings. The first by Carbonaro and Gamoran, 2002 discusses content differences between tracks. The second by Yonezawa et.al., 2002 examines an open track system and the reluctance of students to ‘select-up’ based in part on their own acceptance of a less academic socialized identity.

We turn now to a more complete look at the relationship between algebra group and grades received by looking at the findings on grades for all students.

		8 <sup>th</sup> Alg 1 n = 274	1 Yr Alg 1 n = 1,442	2 Yr Alg 1 n = 333	1 & 2 Yr n = 417	1 Yr of 2 n = 543
AlgProgCrs	AB	45%	31%	6%	8%	
	C	27%	27%	13%	18%	
	DF	28%	42%	82%	74%	Incomplete
Biology	AB	49%	32%	13%	10%	4%
	C	21%	23%	18%	18%	6%
	DF	29%	46%	70%	73%	91%
Chemistry	AB	60%	28%	9%	8%	20%
	C	24%	32%	26%	17%	40%
	DF	15%	40%	64%	75%	40%
Physics	AB	56%	38%	20%	17%	
	C	29%	27%	40%		
	DF	16%	35%	40%	83%	
Algebra 2	AB	33%	13%	10%	12%	17%
	C	25%	30%	20%	8%	33%
	DF	42%	57%	70%	80%	50%
	*p <.01					

### Grades Received in Algebra Group Course, in Biology, Chemistry, and Physics, and Highest Mathematics Course by All Students

As expected from the findings on A and B students, there is a very strong association between location in original algebra group and grade received in each of the instances above in each of the five courses.



To illustrate how these findings may be viewed within the mathematics community, we point to a lively debate within the research literature in mathematics education about tendencies to locate differences in learning within students, within teacher preparation, between major mathematics curricula, or between broadly drawn differences in teaching methods, such as open versus structured instruction (Boaler, 2002; Sfard, 2003; Ferrini-Mundi, 2003).

A recent article by Jo Boaler from Stanford University in the Journal for Research in Mathematics Education illustrates some of the differences. In this article, Boaler reviews the research on the ability of current reform curricula to meet the needs of low-income urban youth. Noting findings that primarily locate the problem within the students themselves she reviews the counter argument. “Educators must understand the needs of different groups of students not to develop negative ideas about the students’ mathematical potential (Varenne & Mcdermott, 1999) but to become aware of the ways in which schools can serve all students. This will require a shift in focus away from what *students cannot do* ... to what *schools can do* to make the educational experience more equitable.” She then goes on to make her own specific argument that differences in “the ways in which teachers work to enact different approaches” is a critical element in effective teaching and learning and in creating more equitable outcomes in learning mathematics” Boaler 2002.

Examining our own data we found that if we were to locate the differences in enrollments and grades discussed above primarily in the students in our five high schools, we would have to conclude that 42% of the cohort of 3,574 students is incapable of succeeding in algebra and therefore in preparing effectively for the workplace or for college - the 565 students who never took Algebra 1 and the 925 who failed the course.

#### Enrollment in the Core College Prep Courses by Algebra Group

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 Yr of 2
	n = 274	n = 1,442	n = 333	n = 417	n = 543
11 - 15	68%	26%	1%	2%	0.1%
*p <.01					

We examined the number of core college prep courses completed by the students in our Algebra groups. We focused on the courses required by California State University and the University of California in English,

mathematics, science, and social science. Their common requirement is four years of English, three years of math beginning with Algebra 1, two years of science from among biology, chemistry, or physics or their equivalents (we added the equivalents into our data), together with world history and U.S. history. Students meeting the minimum threshold have 11 courses and can have a maximum of 15 courses. Our findings are displayed in Table Six below.

We see here that 68% of the students who took algebra in eighth grade met the threshold number of course enrollments, compared with 26% of their peers from 1 Yr Alg 1, 1% from the 2 Yr Algebra 1 program, 2% from both the one and two-year programs, and one tenth of a percent from students with one year of two.

Completion of the core courses with requisite grades is the key factor for college and we turn to that next.



Table 7: Completion of the Core College Prep Courses with a Grade of C or Higher

	8 <sup>th</sup> Alg 1		1 Yr Alg 1		2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr
	n = 274		n = 1,442		n = 333	n = 417	n = 543
11 - 15	76	28%	87	6%	0%	0%	0%
*p <.01							

Once grades are added into the equation, students from 2 Yr Algebra 1, those who took both the one and two year programs, or who took only one year of the two year program *disappear* from the group of students potentially eligible for CSU or UC.

Students from 8<sup>th</sup> grade algebra and the one-year algebra program are the only viable candidates, with 28% and 6% respectively potentially eligible for regular or competitive admission based on course enrollments and grades.

When we focus in more specifically on potential for competitive admission and look at the students who completed the core courses with an A or B in each course, the proportion narrows considerably as indicated in Table 8

Table 8: Completion of the Core College Prep Courses with a Grade of A or B

	8 <sup>th</sup> Alg 1		1 Yr Alg 1		2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr
	n = 274		n = 1,442		n = 333	n = 417	n = 543
11 - 15	39	14%	23	2%	0%	0%	0%
*p <.01							

below. In this case, 14% of the students from 8<sup>th</sup> grade Algebra were potentially eligible for competitively admissions and 2% of the students from the 1 Yr Algebra 1 group.

## Pathways

Table 9: Logistic Regression Models Pertaining to Completing Algebra 2

	B	S.E.	Sig.	Exp(B)
Progr6	-.968	.098	.000	.380
Grdprcrs5	-.092	.056	.103	.912
Gender(1)	-.146	.128	.253	.864
Ethnic2			.065	
Ethnic2(1)	-.026	.216	.905	.975
Ethnic2(2)	.000	.205	.999	1.000
Ethnic2(3)	.476	.206	.021	1.609
Ethnic2(4)	.079	.261	.761	1.083
School			.000	
School(1)	1.264	.232	.000	3.538
School(2)	.241	.224	.281	1.273
School(3)	1.569	.232	.000	4.800
School(4)	.701	.211	.001	2.016
Geomgra			.000	
Geomgra(1)	1.299	.195	.000	3.667
Geomgra(2)	-2.107	.247	.000	.122
Geomgra(3)	1.962	.243	.000	7.110
Geomgra(4)	.128	.199	.522	1.136
Geomgra(5)	1.996	.394	.000	7.362
Biolgra			.000	
Biolgra(1)	.213	.192	.267	1.237
Biolgra(2)	-1.255	.233	.000	.285
Biolgra(3)	.646	.203	.001	1.908
Biolgra(4)	1.571	.311	.000	4.811
Biolgra(5)	.013	.223	.955	1.013
Constant	.930	.408	.023	2.535

How do each of the elements we have been discussing come together to produce the outcomes we have observed? We used logistic regression models to explore this question. The first set of models focused on reaching algebra 2 or higher as the student's highest math course. We looked at the respective contributions of algebra group, grade in algebra group course, gender, race-ethnicity, school, and grade in geometry and biology. Algebra group and grade in geometry and biology were the most important factors. Gender and race-ethnicity were subsumed within algebra group. The odds of completing Algebra 2 or higher with a grade of A, B, or C decreased with each algebra group below 8<sup>th</sup> Alg 1, and increased with each higher grade in geometry and with A or B grades in biology.

We explained 71% of the variance in reaching Algebra 2 with these models.

The second set of models focused on potential regular eligibility to the University of California and California State University. Because number of college prep courses completed with a grade of A, B, or C defines regular eligibility within this study, we needed to use variables prior to that one to specify the path to regular eligibility. We included in our models algebra group, grade in algebra program course, gender, ethnicity, high math course, grade in high math course, grade in biology and chemistry, and number of college prep enrollments. In this case grade in algebra group course, grade in Algebra 2 or higher math course, grade in biology and chemistry, and number of college prep enrollments are the significant variables. And we explain 76% of the variance in potential regular eligibility with these models.

	B	S.E.	Sig.	Exp(B)
Progr6	-.516	.269	.056	.597
Grdpcrs5	-.244	.114	.032	.783
Ethnic2			.037	
<b>Ethnic2(1)</b>	.909	.640	.156	2.481
<b>Ethnic2(2)</b>	.401	.655	.541	1.493
<b>Ethnic2(3)</b>	-.220	.644	.733	.803
<b>Ethnic2(4)</b>	.355	.718	.622	1.426
Himacrs6	-12.607	538.597	.981	.000
Himthgr6	-2.759	.372	.000	.063
Biolgra6a	-3.283	.543	.000	.038
Chemgr6a	-2.630	.665	.000	.072
Ucrse5	1.775	.186	.000	5.902
Constant	-19.221	2.273	.000	.000

### Do Gender, Ethnicity, and Fluency in English matter?

They do matter, most specifically in differences in enrollment in algebra group, and we turn now to how they matter. First, we look at issues of gender, then issues relating to race-ethnicity. And we conclude by looking at English language learners.

Gender	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr	No Alg 1
	n = 274	n = 1,442	n = 333	n = 417	n = 543	n=565
F = 1,785	157 <b>9%</b>	762 <b>43%</b>	146 <b>8%</b>	190 <b>11%</b>	278 <b>16%</b>	252 <b>14%</b>
M=1,789	117 <b>7%</b>	680 <b>38%</b>	187 <b>10%</b>	227 <b>13%</b>	265 <b>15%</b>	313 <b>17%</b>
* p<.01						

*Is gender an important factor in student outcomes?* It is because there are important differences in access to algebra groups. More females than males were enrolled in 8<sup>th</sup> grade Algebra 1 and 1 Yr Algebra 1; and

more males in the other math groups.

When we consider only the students participating in Algebra 1 the percentages change slightly. As these are the ones we will be referring to more frequently we present them in the table below.

Three points are important about these *between-group* differences. *First*, the two percentage point difference in favor of females in 8<sup>th</sup> Algebra 1 is large in light of the outcomes noted for all students from 8<sup>th</sup> Alg 1. *Second*, when we add together participation in 8<sup>th</sup> Alg 1 and 1 Yr Alg 1 we find 60% of females

Gender	8 <sup>th</sup> Alg 1	1Yr Alg 1	2 YrAlg 1	1 & 2 Yr	1 of 2 Yr
	n = 274	n = 1,442	n = 333	n = 417	n = 543
F = 1,785	<b>10%</b>	<b>50%</b>	<b>10%</b>	<b>12%</b>	<b>18%</b>
M=1,789	<b>8%</b>	<b>46%</b>	<b>13%</b>	<b>15%</b>	<b>18%</b>
*p<.01					

and 54% of males, a statistically significant difference at  $p < .01$ . *Third*, 46% of males and 40% of females were in algebra programs that, by our metric above, did not contribute students to the eligibility pool for California State University or the University of California.

**Table 13: Number of Core College Prep Enrollments by Gender**

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 of 2 Yr
11 - 15	n = 274	n = 1,442	n = 333	n = 417	n = 543
F	69%	28%	1%	1%	0.3%
M	66%	24%	1%	3%	0%
p>.05					

A second question pertains to *within-group* differences. Once students were in their algebra group, did females and males have similar or different enrollment patterns and grades in biology, chemistry, and physics, and in higher math?

Differences were significant only for students from One Year Algebra 1, where females had higher grades in 1 Yr Algebra, higher enrollment and grades in biology and chemistry, higher grades in physics and higher enrollment in the courses from algebra2 through calculus.

We turn now to the impact of these differences on eligibility for college.

Females from 8<sup>th</sup> Alg 1 and 1 Yr Alg 1 had a modestly higher percent of enrollments in the core college prep courses, but these differences were not statistically significant. This reinforces that what is most important about gender is differences in access to the

**Table 14: Completion of College Prep Courses With a C or Higher by Gender**

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 Yr of 2
11 - 15	n = 274	n = 1,442	n = 333	n = 417	n = 543
F	28%	7%			
M	27%	5%			
	*p>.05	*p<.01			

various algebra groups, rather than differences between females and males within algebra groups. This becomes clearer when we examine completion of the core college prep courses with the grades requisite for regular and competitive admission in Tables 14 and 15. It is the differences in completion between students from 8<sup>th</sup> Alg 1 and 1 Yr Alg 1 for each gender group that are most important, not the differences between males and females within each algebra group.

It would appear from this that gender differences in initial algebra group, is the primary factor shaping different outcomes for females and males.

**Table 15: Completion of College Prep Courses With a Grade of A or B by Gender**

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 Yr of 2
11 - 15	n = 274	n = 1,442	n = 333	n = 417	n = 543
F	15%	2%			
M	14%	1%			
	*p>.05	*p<.01			

Are There Important Differences by Race-Ethnicity?

Yes, there are. Hispanic students had the smallest percent of students enrolled in 8<sup>th</sup> Alg1 and 1 Yr Alg1. African-American students had the lowest grades overall. Asian students had the highest enrollments and grades in the core college prep courses, followed by White and other students. White and Asian students had a similar percent of students potentially eligible for regular admission. But White students had the largest percent of students potentially eligible for competitive admission. *Once again, however, it is the initial differences in algebra group that are the most important, and within algebra group differences are significant only for students from one year Algebra 1 when it comes to potential eligibility for college.*

We begin by looking at the initial distribution of students across algebra groups by race-ethnicity. We identify the number of students in each racial or ethnic group in row two in Table 16. At the left, we show the total number of students in each ethnic group. At the right we show the number that enrolled in algebra 1. Percentages refer to the number of students who enrolled in algebra 1.

Table 16: Distribution of Students Across Algebra Groups by Race-Ethnicity										
	Asian		Af-Amer		Hispanic		White		Other	
	584	510	554	451	1256	1090	870	706	310	252
8 <sup>th</sup> Alg1	14%		3%		5%		15%		12%	
1 Yr Alg1	66%		62%		31%		46%		60%	
2 Yr Alg1	6%		6%		18%		9%		6%	
1 & 2 Yr	8%		10%		20%		13%		10%	
1 Yr of 2	5%		18%		26%		17%		13%	
*p<.01										

We see in Table 16 that Hispanic students had the smallest percent of their cohort enrolled in 8<sup>th</sup> Alg1 and 1 Yr Alg1, at 36%. African-American students, by comparison, had 65%. This African-American enrollment was four percentage points higher than that for White students, 65% compared with 61%. Asian students had the highest enrollment of all at 80%.

With this benchmark established, we briefly summarize subsequent course enrollment by race-ethnicity in Table 17 below. *It is important to read across each row for the relevant comparisons.* For example, focusing on biology and students from 1 Yr. Alg 1, 86% of Asian students were enrolled, compared with 83% of Hispanic students, 81% of White students, and 70% of Hispanic students.

\*Algebra 2 or higher. \*\*p<.05 for 1Yr Alg1 only.

Table 17: Enrollment in Biology, Chemistry, and Algebra 2 by Race-Ethnicity					
	Asian	Af-Amer	Hispanic	White	Other
Biology					
*8 <sup>th</sup> Alg1	96%	100%	86%	98%	100%
*1 Yr Alg1	86%	83%	70%	81%	77%
*2 Yr Alg 1	48%	59%	28%	54%	29%
*1 & 2 Yr	59%	72%	45%	54%	54%
*1 Yr of 2	46%	46%	16%	20%	15%
*p<.05					
Chemistry					
8 <sup>th</sup> Alg1	95%	83%	79%	85%	90%
*1 Yr Alg1	53%	38%	38%	48%	38%
2 Yr Alg 1	18%	18%	48%	6%	36%
*1 & 2 Yr	13%	35%	12%	17%	4%
*1 Yr of 2	12%	2%	1%	1%	3%
*p<.05					
*Algebra2					
8 <sup>th</sup> Alg1	92%	100%	89%	97%	93%
**1 Yr Alg1	54%	31%	36%	38%	34%
2 Yr Alg 1	3%		4%	3%	
1 & 2 Yr	15%	7%	4%	7%	4%
1 Yr of 2	12%	4%	1%	2%	

After students are in their algebra groups, there are additional racial-ethnic differences in enrollment in biology across all algebra groups, and more limited differences in chemistry and algebra 2. In chemistry the most important differences are among students from 1 Yr Alg 1, where Asian students have a 53% enrollment and Hispanic students 38%. In algebra 2, the 1 Yr Algebra 1 students were the only ones with additional racial-ethnic differences in subsequent enrollments. Here Asian students

had 54% enrollment and African-American students 31%. An important finding in the science data is the growing strength of African-American enrollments in biology and chemistry.

When we focus on grades received, however, we find an opposite pattern. It is African-American grades that need the most support, followed by those of Hispanic students. Differences in grades by race-ethnicity were significant in biology across all algebra groups, and in chemistry for students in 1 Yr Alg1, in 2 Yr Alg1, and in 1 Yr of 2. There were so few enrollments from the latter two groups, however, that we only show the difference in grades for 1 Yr Alg1 students by race-ethnicity. There were no significant differences in grade received by race-ethnicity in physics, or any substantive ones to note from highest math course. We show percent of A and B grades in Table 18.

Asian students had the highest percent of AB grades, followed by White students; and Hispanic students received higher grades than African-Americans.

We summarize outcome differences by race-ethnicity by looking at completion of 11 or

Table 18: Percent of AB Grades in Biology for All Students, and in Chemistry for 1 Yr Alg1 Students by Race-Ethnicity					
	Asian	Af-Am.	Hispanic	White	Other
Biology					
8 <sup>th</sup> Alg1	53%	25%	32%	62%	40%
1 Yr Alg1	43%	21%	21%	35%	33%
2 Yr Alg 1	19%	12%	9%	18%	0%
1 & 2 Yr	26%	6%	6%	12%	8%
1 Yr of 2	8%	0%	4%	8%	0%
*p<.01					
Chemistry					
1 Yr Alg1	35%	14%	26%	31%	27%
*p<.01					

more college prep courses with a grade of A, B, or C – our marker for regular eligibility to college, and completion with grades of A or B only, our marker for competitive eligibility, in Tables 19 and 20 below. There were no significant differences by race-ethnicity in the percent of students from 8<sup>th</sup> grade Alg 1 completing the college prep courses. But there were significant differences by race-ethnicity among the 1 Yr Alg1 students.

Table 19: Completion of College Prep Courses with a Grade of A, B, or C by Race-Ethnicity

	Asian	Af-Am.	Hispanic	White	Other
8 <sup>th</sup> Alg1 p>.05	25%	8%	25%	23%	35%
1 Yr Alg1 *p<.01	9%	2%	5%	4%	9%
% all stu.	9%	1%	3%	9%	5%

from 8<sup>th</sup> Alg 1 and 9% of the students from 1 Yr Alg1 completed the core set of college prep courses with a grade of C or higher in each course. These students represent 9% of the total group of Asian students who were enrolled across the algebra groups.

Table 20: Completion of College Prep Courses with a Grade of A or B by Race-Ethnicity

	Asian	Af-Am.	Hispanic	White	Other
8 <sup>th</sup> Alg 1 p>.05	14%	0	9%	20%	10%
1 Yr Alg 1 p>.01	2%	1%	1%	3%	1%
% all stu	3%	0.4%	1%	5%	2%

The findings in the ‘percent of all students’ row in Tables 19 and 20 confirm that the lower enrollments of African-American and Hispanic students in 8<sup>th</sup> Alg1 and 1 Yr Alg 1 were never overcome within the high school experience. When the 2 Yr Alg 1 program proved incapable of moving students forward into potential eligibility for college, and moving between the 1 and 2 year programs did not work, a much larger proportion of African-American and Hispanic students than Asian and White students were limited in their opportunities to prepare for college. Differences in enrollment in initial algebra group, by race-ethnicity, is the most important factor shaping racial-ethnic differences in outcomes.

### What Did It Mean To Be An English Language Learner?

We have some encouraging findings from our English language learners, who made up 23% of the cohort and were 54% Hispanic, 34% Asian, 6% White, and 5% Other. But it is important to state at the outset that we would have preferred more information about these students. The data set provided by the school districts provided a tag for ELL but did not specify length of time as an English language learner or level of English fluency. These students completed high school prior to the implementation of the statewide CELDT test to assess English language proficiency. Therefore there was no common measure of English fluency across districts, each of which used a test of their own choosing. With this in mind we present our findings as *indicative* of important strengths when English language learners are prepared in mathematics prior to 9<sup>th</sup> grade.

We turn first to our benchmark – the distribution of English language learners across the algebra groups. Our two categories are English language learners and a combined category with native English speakers and reclassified fluent students.

Table 21: Distribution of English Language Learners by Algebra Group

	8 <sup>th</sup> Alg 1	1Yr Alg 1	2 YrAlg 1	1 & 2 Yr	1 of 2 Yr	No Alg1
ELL Nat/ReCl *p<.01	3%	44%	11%	10%	19%	13%
	9%	39%	9%	12%	14%	17%



When we take away the students who did not participate in Algebra, the percentages change across the active algebra groups. That distribution is shown below.

English language learners did have a smaller percentage of enrollment in 8<sup>th</sup> grade Algebra 1. But when we look at the combined enrollment from 8<sup>th</sup> Alg 1 and 1 Yr Alg1 the differences narrow to 54% for ELL students and 58% for native and reclassified English fluent students. *Two points are crucial.* First, *over half of the ELL students are enrolled in the math programs with college eligibility outcomes.* And second there is only a four percent difference in combined enrollment in these important courses.

Table 22: Distribution of English Language Learners Across Algebra Groups

	8 <sup>th</sup> Alg 1	1 Yr Alg 1	2 Yr Alg 1	1 & 2 Yr	1 Yr of 2
ELL	3%	51%	13%	12%	21%
Nat/ReClas	11%	47%	11%	15%	7%
*p<.01					

Moreover, once ELL students are in their algebra groups, there are no significant differences in enrollment in biology between ELL and non-ELL students from any math group. And English language learners have a *higher* percentage of AB grades and a *lower* percentage of DF grades in biology than their peers.

In chemistry, ELL students had more difficulty. English language learners from 1 Yr Alg1 and 1 & 2 year had lower enrollment in chemistry than their peers, and ELL students from 8<sup>th</sup> Alg1 and 1 Yr Alg 1 had lower grades.

But the pattern for biology held in physics. Once the ELL students were in their algebra groups there were no differences in enrollment in physics. And, ELL students from 1 Yr Alg1 had *higher* grades in physics than their non-ELL peers.

Only among 8<sup>th</sup> grade Algebra 1 students were there significant differences in enrollment in high math course, with ELL students having a somewhat lower percent of enrollment in Algebra 2 or higher. But there were no significant differences in grade received in high math course between ELL and native students from each algebra group.

The inability of English language learners to access the regular English courses resulted in the completion of fewer of the core college prep courses. But the strength of their grades in the other courses, and the propensity of many to take as many advanced math and science courses as possible, produced a phenomena in potential eligibility where there were *no significant differences* for ELL students in *aggregate* completion of 11 or more courses with a grade of A, B, or C or with a grade of A or B.

Table 23: Completion of Eleven or More Core College Prep Courses With Appropriate Grades

	ELL	Native/Reclassified Student
A, B, or C in each course	5%	11%
A or B in each course	2%	4%
p. > .05		

The most important contribution we can make to English language learners beyond facilitating their rapid acquisition of academic English is to support their preparation in mathematics prior to entry in high school. English language learners are using their preparation in mathematics to gain broad



access to the college prep curriculum, and for more than a few to develop excellence in the highest course offerings in mathematics and science. Their outcomes, even more than their peers are being shaped by algebra group.

### **Grouping within Algebra One**

How prevalent are grouping arrangements within Algebra 1 as schools endeavor to extend algebra to all students, and how similar or different are those examined here to the larger set of practices? We have searched the literature in mathematics education and the larger education literature, and contacted a group of faculty at universities across the country who have contributed to policy reports or provided oversight to the NCTM standards project for answers to these questions and have found little to inform answers.

We have not located any discussion of grouping practices in the international or national reviews of research in mathematics education since the early 1990's (Bauersfeld, 1997; Kieran, 1994) or in our own perusal of articles in a sample of mathematics education journals over the past eight years. Articles on grouping in mathematics, and grouping or tracking in general, appear rather in the major national educational journals and we cite a number of these articles in our references. Most of these articles are devoted to understanding the motives for and outcomes of tracking together with discussions of the politics of grouping and tracking.

We have been unable to locate any articles that attempt to identify regional, state, or national patterns of grouping or the prevalence of particular group or tracking patterns. Rather, the assumption appears to be that grouping or tracking within mathematics has been and is ubiquitous, and that it has involved at least a basic track, an average track, and an honors track in each course. When we asked about re-tracking within Algebra 1 our respondents were puzzled because they were not aware of instances where schools were endeavoring to detrack in mathematics in order to provide an equivalent experience for all, and/or to re-track within algebra 1 to provide more equitable opportunities.

The one place within the mathematics literature where we found a reference to tracking is in a discussion of the process of providing feedback to the Discussion Draft of Principles and Standards for School Mathematics, which was released in October 1998 (Ferrini-Mundi and Martin, 2003). Approximately 700 responses were reviewed to identify a set of issues for the Writing Group to consider as general decisions were made for the final document. Tracking but not equity was on the next to final list of issues. On the final list "Issues Related to Equity", but not tracking, was forwarded to the Writing Group and the defining sub-items were 'if and how curriculum should vary for sub-populations of students,' and 'how instruction should vary for sub-populations.' The substantial volume *A Research Companion to Principles and Standards for School Mathematics* does not include an article on tracking or equity (NCTM, 2003).

As a part of our larger work we have also initiated a search for examples of schools that have been particularly successful in making algebra available to all students. To date only one school has been recommended, and this endeavor remains a work in progress.

### **Implications From Our Findings**

In the introduction to this paper we raised three questions. *First*, can algebra group be considered a key variable? *Second*, is it associated with a chain of events that are interrelated? *Third*, are the high schools able to provide value added to students who enter with low levels of preparation in math through re-tracking options within Algebra 1, or does the burden for all practical purposes lie with the K-8 schools? All three questions have major policy and opportunity implications.

*Can algebra group be considered a key variable?* The strength of the association between algebra group and each of the other variables in our study points in this direction. The magnitude of the between-group differences also point in this direction. There is a problem in introductory statistics courses that focuses on an increase in alcohol consumption in colonial American in proportion to increases in the number of ministers in the colonies. Clearly, number of ministers was a proxy variable for increases in the population. Is algebra group a proxy variable for differences in ability, in opportunity to learn, in interest and motivation to learn, in socialized identities, in ideas about what to teach and how to teach it? It certainly captures all of those dimensions. And it is precisely because it captures each of these dimensions and is so strongly associated with the most important opportunity outcomes for students that we suggest it be considered a key variable for policy and practice decisions.

*Is algebra group associated with a chain of events that are interrelated in producing a major social opportunity outcome?* To the extent that enrollment and grades received in biology, chemistry, and physics, in higher mathematics, and in the other core college prep courses differ significantly by algebra group, we may conclude that algebra group captures a set of variables associated with a chain of events that result in very different opportunities to prepare for college and for direct entry into the workforce.

*Are the high schools able to provide value added to students who enter with lower levels of preparation in math?* The options provided within algebra, to move less prepared students forward in mathematics and into eligibility for college, did not achieve anticipated outcomes. Not a single student from the two-year algebra program, from the group of students who took both one-year and two-year algebra courses, or from the group who took one year of the two-year course completed the core college prep courses we examined with grades of C or higher in each course.

Perhaps it is expecting more than it is reasonable to expect to ask high schools to overcome not only the differences in preparation, but also the socialized identities built up over eight years prior to high school about ability to learn math, and about overall academic competence measured by facility in math. The policy option is to place the required resources at the elementary school level and hold schools, teachers, students and their families, and community based organizations at this level accountable for outcomes.

Are there plausible arguments for assigning a large portion of the resources for mathematics education to the elementary school level? There are very important reasons to consider such a policy. Among them:

- Early and sustained competence in mathematics fosters strong early academic identities within the frame of reference of teachers, students, and their families.

- More is known about the cognitive processes involved in learning mathematics for elementary school students than is known about the cognitive process of older students (Siegler, 2003; Nesher and Kilpatrick, 1990).
- The NCTM Principles and Standards for School Mathematics can achieve their potential in proportion to the effective implementation of their principles and underlying dynamic *from the earliest years forward*. The metaphor is one of fruit growing naturally versus efforts to paste or tie fruit to a tree. The mistake is to view the standards as discrete stand-alone year-by-year elements. There is a generative growth process embedded in the standards that creates a path for learning with accompanying synergy. To the extent that the underlying dynamics are implemented early they can be built on sequentially to create a strong foundation for subsequent learning. From a policy viewpoint as much as for purposes of practice it is essential to understand the underlying dynamic and how it needs to be ‘grown’ from the early years forward (Sfard, 2003).
- Some of the specific ways that early implementation can make a difference can be observed by examining several of the ten fundamental needs for learning mathematics addressed in the Principles and Standards, and discussed by Sfard, 2003. Each becomes more difficult to acquire the later it is approached. The first is the need for the students to see meaning. Sfard points to the fact that in the process of making meaning, students develop misconceptions. Misconceptions that persist over time become deeply rooted and are harder to change. When they are discovered and addressed in the earlier years it is easier to use the misconceptions as “steppingstones for further development rather than as hurdles to learning”(Sfard, 2003; Easley, 1978; Nesher, 1987; Smith et.al, 1993).
- A second fundamental addressed by the Standards is the need for difficulty. Without difficulty no learning occurs, but when the level of difficulty is too great no learning occurs (Sfard, 2003). Placing additional resources at the elementary school level, and proportionately at the earliest years, makes it possible to scaffold level of difficulty before problems spin out of control.
- A third fundamental need is for social interaction in learning mathematics. The elementary school is uniquely suited to foster social interaction in learning and to bring out within young learners the social transaction rather than solo performance that Vygotsky believes is the fundamental vehicle of learning, and that recent research has found to be particularly important for females and for many African-American, Asian, and Hispanic students (Bruner, 1985). This social interaction is also the basis for meeting two other fundamental needs, for verbal-symbolic interaction, i.e., the students’ ability to ‘talk mathematics,’ and for participation in the “totality of communicative activities” practiced by the mathematics community. Early acquaintance and facility with these elements of social interaction make them natural and facilitate growing fluency. Efforts to acquire them later may be likened to learning a second language. (Sfard 2003).
- Two other needs complete the point. The first is the need for belonging, what Sfard refers to as the feeling within the learner of ‘taking part’ and ‘being a part’ of

mathematics. The second is the need for balance, the need for the learner to feel that there is opportunity for she or he to learn within the multiple ways of learning that are provided. When these feelings of belonging and balance are established early, sustained engagement with mathematics is a positive and rewarding experience. Inversely, to the extent that neither appears the learner becomes progressively disconnected and remediation becomes more and more difficult.

- Moving beyond the standards to test scores, and measures of change within the state of California, gains in the percent of students scoring at or above the 50<sup>th</sup> national percentile on the mathematics portion of the state test taken by 2<sup>nd</sup> through 11<sup>th</sup> graders between spring 1999 and spring 2002 have been largest at the early elementary school level, have grown in smaller proportions through 8<sup>th</sup> grade, and have been much more resistant to change at the high school level (California Department of Education, DataQuest).
- Economies of scale may also be possible in two ways. First, by meeting the basic mathematics needs of students as early as possible, we should be able to reduce the need for continuous and progressively less effective re-teaching and external remediation efforts at the elementary, middle or junior high, and high school levels. Second, by increasing the mathematics proficiency and academic identities that play a role in shaping enrollment and grades in biology, chemistry, and physics, and across the core college prep courses, we may be able to reduce remediation needs in these areas as well, and have a larger pool of students to draw on for careers in science and technology.

#### Policy Recommendations

1. Focus resources and programmatic initiatives for mathematics at the elementary school level, building from the earliest years forward.
2. Recognize an AIDS like problem in mathematics where lack of facility in mathematics, and negative academic identities derived from perceptions of mathematical ability, progressively affect enrollments and grades in science and the other key academic areas - and recommend allocations of resources with this in mind.
3. Support initiatives at the elementary school level, that function as a “cocktail retroviral” tightly coupling classroom, individual, family, and community level endeavors and responsibilities, with attention to dosage, a record of treatments and outcomes. Among the lessons learned in school/university partnerships is the proclivity of each of the participants above to tend to their own role and the imperative need to develop interactive endeavors and joint accountability. The effect of ‘going it alone’ is similar to separating each of the drugs; the probability of success is considerably higher when the efforts are joined in a comprehensive treatment.
4. Support the need at the high school level for a path perceived as more effective and academic than district summer school, where students with average to good grades in mathematics can deepen their understanding in the mathematics course sequence they are in, or move forward toward calculus.

5. To the extent that it is possible, provide resources to support instruction in mathematics in the home language of English language learners who are using mathematics so effectively to move forward into all parts of the college prep curriculum.
6. Provide support for a venue to disseminate the cognitive science research on how students learn mathematics including, but not limited to, a research clearinghouse, funding for workshops, a newsletter, and teacher training.

We believe that implementation of these policy recommendations can change the learning dynamic in mathematics, put in place a key component requisite to increase the pool of students interested and proficient in science, and address two of the most fundamental variables in expanding educational opportunity and equity – mathematics preparation and perceptions of academic efficacy – joined at the hip by social paradigms that link proficiency in mathematics with intelligence and academic efficacy.

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