Growth and Distribution in an AK-model with Endogenous Impatience

Kirill Borissov[1] *and Stéphane Lambrecht*[2]

CORE DISCUSSION PAPER n° 2007/44

Abstract

This paper combines two strands of the literature on inequality and distribution issues: the classical approach, which insists on the division of society into classes characterized by different saving propensities, and the social conflict approach, which considers that inequality inflicts direct and indirect costs to economic development. An endogenous-growth model is studied. We assume that each consumer's subjective discount factor is determined endogenously and depends on economic inequality through the following two channels. On the one hand, it is positively related to the individual consumer's relative wealth. On the other hand, it is negatively affected by a simple aggregate measure of social conflict. We show that, unlike models with exogenously given discount rates, steady state equilibria in our model is indeterminate and that the set of all equilibria is a continuum which can be parameterized by a simple index of income inequality. The growth rate is ambiguously related to the inequality index. However, under some reasonable assumptions, the growth rate dependence on this index has an inverted U-shaped form.

Keywords: wealth distribution, intertemporal choice, growth, development

JEL classification numbers: D31, D90, O10

[1] Contact author: European University in St. Petersburg, 3 Gagarinskaya Str., St. Petersburg, 191187 RUSSIA (e-mail: kirill@eu.spb.ru) and St. Petersburg Institute for Economics and Mathematics (Russian Academy of Sciences), 1 Tchaikovsky Str., St. Petersburg, 191187 RUSSIA.

[2] Laboratoire EQUIPPE – Universités de Lille, Faculté des Sciences Economiques et Sociales, Université des Sciences et Technologies de Lille, Cité scientifique, bât. SH2, 59655 Villeneuve d'Ascq cedex, France, email : stephane.lambrecht@univ-lille1.fr, and CORE, Université catholique de Louvain, Belgium (voie du roman pays, 34, B-1348 Louvain-la-Neuve).

1. Introduction

Despite the reliance of mainstream growth theory on 'representative' agents and long disregard of problems of inequality and distribution, societies are patently not homogeneous, whether in incomes, wealth, or any other dimension. A concern with the importance of distribution was central to the thinking of classical economists such as David Ricardo and Karl Marx. However, up until recently, the mainstream economics profession seemed to have little to say about the impact that the distributions of income and wealth might have on efficiency of an economy and its growth rate.

Some interest in the relationship between distribution and growth restarted in the 1950s with the work of Kaldor (1956) and Pasinetti (1962) and the last decade has seen its new revival. Existing theories about the effect of income distribution on the process of development can be classified into two broad categories distinguished by their conflicting predictions. The classical approach originated by Smith (1776) and further interpreted and developed by Keynes (1920), Lewis (1954), Kaldor (1957), and Bourguignon (1981) suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas the modern approach argues in contrast that for sufficiently wealthy economies equality stimulates investment in human capital and hence may enhance economic growth.

The modern paradigm is represented by three complementary approaches. The capital markets imperfections approach, developed by Galor and Zeira (1993), Aghion and Bolton (1997) and others, has argued that, in the presence of credit markets imperfection, equality in sufficiently wealthy economies stimulates investment in human capital and in individual specific projects and hence enhances economic growth. The political economy approach initiated by Alesina and Rodrik (1994), Bertola (1993) and Persson and Tabellini (1994) has argued that equality diminishes the tendency for distortionary redistribution and hence stimulates investment and economic growth.

Another strand of the literature emphasizes the importance of social conflict as a link between inequality and efficiency. Alesina and Perotti (1996) argue that inequality can lead to less political stability, and this in turn can lead to sub-optimal investment levels. Social conflict may also have high opportunity costs caused by violence. Violence levels have recently increased sharply in both of the most unequal regions in the world (Latin America and sub-Saharan Africa), and in the one where its growth has been fastest (Eastern Europe, Russia and Central Asia). Fajnzylber et. al. (1998) documented these global trends, and find evidence to suggest that income inequality is significantly associated with violence levels, across countries. Bourguignon (2001) and others have documented the growing importance of the social and economic burden imposed on society by this rising violence, both in terms of the direct costs in lives and medical resources, and in terms of the opportunity costs of (both public and private) resources diverted from other activities towards preventing and fighting crime.

This paper proposes an endogenous-growth model combining the classical approach and the approach that considers social conflict and socio-political instability as links between inequality and development. This model is based on the following twofold assumption.

First, it is assumed that the subjective discount factor of a consumer is not given exogenously but is determined endogenously. When Uzawa (1968) noted that in an economy populated by infinitely-lived agents with different time-preference rates, the entire capital stock will eventually be owned by the most patient agent, he cast doubt on the assumption of a constant rate of time preference and assumed that a higher level of consumption increases the subjective discount rate (this assumption is needed for stability). Blanchard and Fischer (1989) argue, however, that this assumption "is difficult to defend *a priori*; indeed, we usually think it is the rich who are more likely to be patient" (p. 73). In this paper, we assume that the rich are more patient than the poor. In this respect our model agrees closely with a model developed by Schlicht (1975) and Bourguignon (1981) and does not contradict to the empirical evidence showing (see, e.g., Browning and Lusardi (1996) and Soulels (1999)) that the marginal propensity to consume is substantially higher for consumers with low wealth or low income than for consumers with high wealth or income. Our model is also akin to several dynamic macroeconomic models with heterogeneous consumers (see e.g. Becker (1980), Michel and Pestieau (1998), Smetters (1999), Mankiw (2000)), but, unlike those models, we assume that all consumers are identical in their exogenous parameters.

Secondly, it is assumed that inequality increases the impatience of economic agents. The extremely simplified reasoning behind this assumption is as follows. Income inequality increases social discontent and the probability of coups, revolutions, mass violence, etc. (see Alesina and Perotti (1996) and Perotti (1996) for empirical evidence). For simplicity, we can reduce all risks emerging from high income inequality to the threat of total collapse of the ordinary economic order with absolutely unpredictable consequences for consumers. It follows that when solving his utility maximization problem, a consumer should foresee and take into account the consequences of his saving and consumption decisions only until a possible economic collapse, after which an absolutely new economic order will be established. This is true irrespective of whether this new order will be better or worse than before the collapse. Thus, as the probability of collapse increases, the effective discount factor of each consumer decreases.

We show that unlike models with exogenously given discount rates, steady-state equilibria are indeterminate in the sense that the set of all equilibria is a continuum that can be parameterized by an index of inequality. The dependence of the growth rate on this index is not unambiguous. However, under some reasonable additional assumption, this dependence has an inverted U-shaped form.

The rest of the paper is organized as follows. In Section 2 we describe the technology of production and the firm's problem. Section 3 presents the consumers' preferences and maximization problem including the hypothesis on the endogenous formation of patience. Section 4 studies the balanced-growth competitive equilibrium maintaining the probability of collapse exogenous. This probability is endogenized in section 5 and the link between growth and inequality is analyzed. Section 6 concludes.

2 The firms

Technology is given by a production function

$$Y=F(K,AL),$$

where Y is output, K is the capital stock, L is the input of labor, and A is the state of technology. The production function $F:\Re_+^2 \to \Re_+$ is assumed to be continuous, concave, homogeneous of degree one and continuously differentiable on $\operatorname{int}\Re_+^2$. Technological progress is assumed to be Harrod-neutral and AL is interpreted as effective labor. For simplicity, the capital stock fully depreciates during one time period.

We conduct the analysis with an endogenous-growth model. This model is based on the following

***Assumption E*₁.** $A=K/\bar{k}\,L$, where $\bar{k}>0$ is exogenously given.

Borissov and Lambrecht (2005) show that the analysis can also be conducted with an exogenous-growth model, where the state of technology grows at an exogenously given rate of growth.

Assumption E_1 is borrowed from Frankel (1962) and Romer (1986). It specifies the class of so called AK-models. In what follows all markets are assumed competitive. The firms maximize their profits taking prices as given. This means that they hire the services of labor and capital up to the point where their marginal products equal their respective prices. In intensive terms, therefore the relationships between the capital intensity of effective labor, $k=K/AL$, the interest rate, r, and the wage rate earned by one unit of effective labor, w, are the following:

$$r=f'(k)-1,\ w=f(k)-f'(k)k,$$

where

$$f(k)\equiv F(k,1)$$

is the production function in intensive form. Since, by E_1, $k=\bar{k}$, we have $r=\bar{r}$ and $w=\bar{w}$, where $\bar{r}$ and $\bar{w}$ are determined by

$$\bar{r}=f(\bar{k})-1,\ \bar{w}=f(\bar{k})-f'(\bar{k})\bar{k}.$$

Time t is discrete.

3 The consumers

3.1 The objective and the constraints

There is a continuum of consumers. The population size is normalized to unity, $L=1$.

Each consumer is endowed at each time with one unit of labor. It should be noticed that, given A, the real wage rate earned by one unit of labor is $A\bar{w}$. Given the initial level of his savings at time $t=-1$, $\hat{s}_{-1}\geq 0$, at time $t=0$ the consumer solves the following maximization problem:

$$\max\{\textstyle\sum_{t=0}^{\infty}\beta^t u(c_t)\ |\ s_{-1}=\hat{s}_{-1},\ c_t+s_t=(1+\bar{r})s_{t-1}+\bar{w}A_t,$$
$$s_{t-1}\geq 0,\ t=0,1,\ldots\}, \tag{1}$$

where A_t, c_t and s_t are respectively the state of technology, consumption and savings of the consumer in period t, $\beta\in(0,1)$ is his discount factor and

$$u(c)=\log c.$$

It should be emphasized that in our models there are borrowing constraints, $s_t\geq 0$, $t=0,1,\ldots$, so that future income cannot be discounted to the present[3]. Our model may be considered as a model of sequential generations with altruism[4].

We will consider stationary solutions only. Suppose we are a given constant over time rate of technological progress, g. Let us also assume $A_0=1$ so that

$$A_t=(1+g)^t,\ t=0,1,\ldots\ .$$

[3] Nothing would be changed to our results if we relaxed this non-negativity constraint on saving and allow for some limited borrowing possibilities.

[4] We leave it for the reader to generalize them to the case of overlapping generations with altruism and/or to the case of felicity functions with constant intertemporal elasticity of substitution.

Then problem (1) can be rewritten as follows:

$$\max\{\sum_{t=0}^{\infty}\beta^t u(c_t)\ |\ s_{-1}=\hat{s}_{-1},\ c_t+s_t=(1+\bar{r})s_{t-1}+\bar{w}(1+g)^t,$$

$$s_{t-1}\geq 0,\ t=0,1,\ldots\}. \qquad (2)$$

The first-order conditions for this problem are

$$c_{t+1}\geq\beta(1+\bar{r})c_t\ (=\text{ if } s_t>0),\ t=0,1,\ldots\ . \qquad (3)$$

We focus on the solutions to this problem that grow at the same rate as the rate of technological progress, g.

Definition. *Given β and $g>-1$, a steady-growth consumer optimum is a solution* $(s^*_{t-1},c_t^*)_{t=0,1,\ldots}$ *to* (2) *at some* $\hat{s}_{-1}\geq 0$ *such that savings* s^*_{t-1} *and consumption* c_t^* *grow at the rate* g:

$$c^*_{t+1}/c_t^*=s_t^*/s^*_{t-1}=1+g,\ t=0,1,\ldots\ .$$

We now characterize steady-growth consumer optima. The following lemma follows from definitions and first-order conditions (3).

Lemma 1. *Suppose that* $g\in(-1,\bar{r})$ *and* β *are given. A steady-growth consumer optimum* $(s^*_{t-1},c_t^*)_{t=0,1,\ldots}$ *exists if and only if* $1+g\geq\beta(1+\bar{r})$. *It is characterized by*

$$s^*_{t-1}=(1+g)^t s_{-1}\ \textit{and}\ c_t^*=(1+g)^t c_0,\ t=0,1,\ldots, \qquad (4)$$

where the pair (s_{-1},c_0) *satisfies*

$$s_{-1}\geq 0\ \textit{and}\ c_0=\bar{w}+(\bar{r}-g)s_{-1}. \qquad (5)$$

If $1+g=\beta(1+\bar{r})$, *then* $s_{-1}>0$. *If* $1+g>\beta(1+\bar{r})$, *then* $s_{-1}=0$ *and hence* $c_0=\bar{w}$. □

We now turn to the examination of the key assumption of this article.

3.2 The endogenous formation of the consumer's discount factor

Our main assumption is that the formation of the effective discount factor of a consumer, β, takes place endogenously.

In models with homogenous consumers with exogenously given discount factors, the relationship between the equilibrium steady-growth rates of interest and growth, $\bar{r}$ and g, is the following:

$$1+g=\beta(1+\bar{r}). \qquad (6)$$

where g is endogenous and $\bar{r}$ considered as given.

The same is true if we consider models with heterogeneous consumers varying only in their discount factors. The only difference is that, in this case, in (6) β is not the discount factor shared by all consumers, but is the discount factor of the most patient consumers. Moreover, in steady state equilibria, all the capital belongs to the most patient consumers. This was noted by Uzawa (1968) and Becker (1980) for the case of exogenous growth and can easily be proved for the case of an AK technology.

In this paper, we explore the implications for growth and inequality of the following assumption:

Assumption E*$_2$*. *The effective discount factor of a consumer*, β, *is determined by*

$\beta=(1-p)\varphi$

where

(i) φ *is an individual-specific subjective discount factor*, *increasing in the individual relative income and*

(ii) *p is a social objective magnitude reflecting a detrimental effect of social tension and political instability on the economy*; *this magnitude is supposed to be increasing* (*or at least non-decreasing*) *in some measure of income inequality.*

Thus, φ is supposed to be an increasing function of the individual's relative income. We measure the latter by the ratio of his personal income to per capita income. By personal income we mean, as usually, the amount a person could have spent whilst maintaining the value of his wealth intact, i.e. $\bar{r}s+\bar{w}A$, where s are his personal savings. As for per capita income, it is given by $A(f(\bar{k})-\bar{k})$. Hence, the individual relative income is

$$\frac{\bar{r}s+\bar{w}A}{A(f(\bar{k})-\bar{k})}=\frac{\bar{r}s/A+\bar{w}}{f(\bar{k})-\bar{k}}$$

and, following our assumption:

$$\varphi=\varphi\left(\frac{\bar{r}s/A+\bar{w}}{f(\bar{k})-\bar{k}}\right),$$

where $\varphi:[1,\infty)\rightarrow(0,1)$ is an increasing and continuous function. The results of the paper would not change if we assumed that the subjective discount factor is an increasing function of some other measure of relative well-being, for example, of relative consumption or felicity.

Contrary to φ, the social magnitude p is common to all consumers that are assumed to be identical in their exogenous parameters. If we assume that all consumers are risk neutral, this magnitude may be interpreted as the probability of occurrence of a major episode of disruptive violence, hurting the property rights system. We shall specify later the kind of inequality measure that we choose as the argument of the p function.

The intuition behind our assumption is that the degree of impatience of an individual depends not only his personal position in society but also on the general threat on property rights of social disorders, tensions, generated by wealth inequality. This general threat equally concerns all the individuals independently of their relative position in the economy. As a consequence, the values of φ are different for consumers with different incomes, whereas p is the same for all consumers[5].

Under our assumption the individual's discount factor reads

$$\beta=(1-p)\varphi\left(\frac{\bar{r}s/A+\bar{w}}{f(\bar{k})-\bar{k}}\right),\ t=0,1,\dots \quad (7)$$

[5] Nothing would be changed in the results if, instead, we assumed that inequality inflicts losses to global output in a proportional fashion. In such a setting, through the inducted violence levels, inequality would put a wedge between potential output $F(K,AL)$and actual output $zF(K,AL)$, with z the fraction of output forgone because of violence levels.

We continue focusing on stationary solutions[6] and re-write the steady-growth consumer optimum by substituting the individual's discount factor with its definition in (7).

Definition. *Given* A_0 *and* $g>-1$, *a consistent steady-growth consumer optimum is a solution* $(s^*_{t-1}, c_t^*)_{t=0,1,...}$ *to* (2) *at some* $\hat{s}_{-1}\geq 0$ *and at*

$$\beta=(1-p)\,\varphi\left(\frac{\bar{r}\hat{s}_{-1}/A_0+\bar{w}}{f(\bar{k})-\bar{k}}\right)$$

such that savings s^*_{t-1} *and consumption* c_t^* *grow at the rate* g:

$$c^*_{t+1}/c_t^*=s_t^*/s^*_{t-1}=1+g,\ t=0,1,\ldots\ .$$

To clarify this definition, it should be noted that since the state of technology grows at the same rate g as consumption and savings,

$$A_{t+1}/A_t=1+g,\ t=0,1,\ldots,$$

we have

$$\beta=(1-p)\,\varphi\left(\frac{\bar{r}s^*_{t-1}/A_t+\bar{w}}{f(\bar{k})-\bar{k}}\right),\ t=0,1,\ldots\ .$$

The following lemma follows from Lemma 1 and our assumptions.

Lemma 2. *Suppose that* A_0, $g\in(-1,\bar{r})$ *and* p *are given. The sequence* $(s^*_{t-1}, c_t^*)_{t=0,1,...}$ *is a consistent steady-growth consumer optimum if and only if (i) it follows the path defined by the pair* (c_0, s_{-1}) *in equations* (4) *and* (5) *and (ii) the following inequality holds:*

$$1+g\geq(1-p)\,\varphi\left(\frac{\bar{r}s_{-1}/A_0+\bar{w}}{f(\bar{k})-\bar{k}}\right)(1+\bar{r}) \tag{8}$$

with a strict inequality if $s_{-1}=0$ (*in this case* $c_0=\bar{w}$) *and with equality if* $s_{-1}>0$ (*in this case* $c_0=\bar{w}+(\bar{r}-g)s_{-1}$). *We shall say that the pair* (c_0, s_{-1}) *characterizes the steady-growth consumer optimum* $(s^*_{t-1}, c_t^*)_{t=0,1,...}$.□

4 The balanced-growth competitive equilibrium with a constant p

It follows from the discussion in the previous section that on balanced-growth competitive equilibria there can be at most two types of individuals: those constrained on their savings, who spend all their wages for consumption, and those unconstrained, who contribute to capital accumulation.

---

[6] If we considered non-stationary solutions we would face the issue of time inconsistency. Indeed, for any given distribution of income, the objective function of each consumer is a well-defined concave function. However, if we tried to characterize a consumer optimum starting from an arbitrary chosen starting point, we would end up in the following issue. The discount factor accepted by a consumer at some time would not be equal to the discount factor of the same consumer accepted previously. This is due to the change in φ. The consumer's objective function changes over time. This implies that the optimal plan chosen at time t_1 need not be optimal as of time t_2. Borissov (2005) studies non-stationary solutions to this problem.

It is important to stress the consequence of this result for the analysis of competitive equilibria. One can conceive as many equilibria as the number of alternative ways to share the population in these two types.

Agents of the first type will be denoted by the subscript h and agents of the second type by the subscript l. Since the former are wealthier than the latter, they will be called the rich and the poor respectively (following Mankiw (2000), they might be called savers and spenders). We denote by σ the fraction of the rich in the population and by $1-\sigma$ the fraction of the poor.

Let us give a formal definition of a balanced-growth competitive equilibrium. As a first step, we will consider that the parameter p is given. It will appear that this equilibrium depends on the fraction σ of the rich in the population.

Definition. *Given the normalized population size*, $L=1$, *the initial capital stock* $K_0=\bar{k}$ (*and hence* $A_0=1$) *and the parameter* p, *a balanced-growth competitive equilibrium is a sequence* $\{(s^*_{l,t-1}, c^*_{l,t}), (s^*_{h,t-1}, c^*_{h,t}), K^*_t, A^*_t, Y^*_t\}_{t=0,1,2,\ldots}$ *such that*

(i) *all individual decisions*, *i.e. consumptions and savings of the poor and the rich*, $(s^*_{l,t-1}, c^*_{l,t})_{t=0,1,2,\ldots}$, $(s^*_{h,t-1}, c^*_{h,t})_{t=0,1,2,\ldots}$, *are consistent steady-growth consumer optima with some rate of growth* g^*,

(ii) *aggregate variables*, *i.e. the aggregate capital stock, the state of technology and the aggregate output*, K_t^*, A_t^*, Y_t^*, *grow at the same rate* g^* *and*

(iii) *at* (*constant over time*) *equilibrium prices* $r^*=\bar{r}$ *and* $w^*=\bar{w}$, *all markets clear and, especially*, *on the capital market*, *we have*

$$K_t^*=\sigma^* s^*_{h,t-1}+(1-\sigma^*)\, s^*_{l,t-1} \Leftrightarrow K_t^*=\sigma^* s^*_{h,t-1},\ t=0,1,2,\ldots\ . \tag{9}$$

where σ^* *is one share of the rich in the population out of an infinite number of alternative shares* σ.

Given the parameter p, the initial conditions and the equilibrium prices, a balanced-growth competitive equilibrium, as defined here above, is fully characterized by the tuple $\{(s_h^*, c_h^*), (s_l^*, c_l^*), k^*, w^*, r^*, \sigma^*, g^*\}$, where (s_h^*, c_h^*) and (s_l^*, c_l^*) are the pairs characterizing consistent steady-growth consumer optima of the rich and the poor at $A_0=1$ and $g=g^*$, k^* is the constant over time capital intensity of effective labor, i.e., $k^*=K_t^*/A_t^*L$, $t=0,1,\ldots$, σ^* is the equilibrium share of the rich in the population and g^* is the equilibrium rate of growth.

Among all these variables, the following ones are easy to determine:

$k^*=\bar{k}$, from E_1

$s_l^*=0$, $c_l^*=\bar{w}$, from Lemma 2, zero savings case.

We obtain the expression of equilibrium saving of rich individuals, s_h^*, as follows: at any time period, the stock of capital is formed by the previous period savings:

$$K^*_{t+1}=\sigma^* s^*_{h,t},\ t=-1,0,1,\ldots\ .$$

At the same time by (4) we have

$$s^*_{h,t}=(1+g^*)^{t+1} s^*_h,\ t=-1,0,1,\ldots\ .$$

Since L=1 and $A_t^*=(1+g^*)^t$, t=0,1,…, we get

$$\bar{k} = k^* = \frac{K_{t+1}^*}{A_{t+1}^* L} = \frac{\sigma^* (1+g^*)^{t+1} s_h^*}{(1+g^*)^{t+1}} = \sigma^* \, s_h^* .$$

Thus, equilibrium saving of the rich individuals writes:

$$s_h^* = \bar{k} / \sigma^*.$$

As for c_h^*, it is a simple function: of the equilibrium wage rate and the interest rate, and of the following two variables: g^* and s_h^*:

$c_h^* = \bar{w} + (\bar{r} - g^*) s_h^*$, from Lemma 2, positive savings case.

We are left now with the determination of $\sigma^*$and g^*. There remains only one equation: the consumer's first-order condition (3) with equality, which implies:

$$1 + g^* = (1-p)\varphi\left(\frac{\bar{r}\bar{k}/\sigma^* + \bar{w}}{f(\bar{k}) - \bar{k}}\right)(1+\bar{r}). \tag{10}$$

Let the relative income of a rich individual, i.e. the argument of the function φ in equation (10), be denoted by $\eta\geq 1$, a function of σ^*:

$$\eta(\sigma^*) = \frac{\bar{r}\bar{k}/\sigma^* + \bar{w}}{f(\bar{k}) - \bar{k}} .$$

Then (10) can be re-written as follows:

$$1+g^* = (1-p)\varphi(\eta(\sigma^*))(1+\bar{r}) \tag{11}$$

The following lemma says that, for a given p, the set of balanced growth competitive equilibria, is a continuum (cf. Borissov (2002)). It follows from Lemma 2.

Lemma 3. *Suppose that p is given exogenously. Then there exists an infinite number of balanced growth equilibria, i.e. there exists an infinite number of pairs* (σ^*, g^*) *which satisfy* (11). *In particular, the set of equilibrium values of the growth rate is the interval* $[g', g'')$, *where* g' *and* g'' *are given by*

$$1+g' = (1-p)\varphi(1)(1+\bar{r}),\ 1+g'' = (1-p)\varphi(\infty)(1+\bar{r}).\square$$

Note that, like in models with a representative consumer, any balanced growth equilibrium satisfies the modified golden rule:

$$1+g^* = \beta(1+\bar{r}),\ 1+\bar{r} = f'(\bar{k}).$$

Unlike most models with a representative consumer, β is not given exogenously, but is formed endogenously. Moreover, this β is not the discount factor of the representative consumer, but the effective discount factor of only one group of consumers, the rich. It depends on the relative income of this group and hence on the proportion between the number of the rich and the number of the poor. This dependence is such that the smaller the fraction of the rich in the population, the larger the equilibrium rate of growth. Indeed, after substitution of $s_h^* = \bar{k}/\sigma^*$, the equilibrium relative income of the rich becomes:

$$\eta = \frac{\alpha}{\sigma^*} + 1 - \alpha ,$$

where α is the share of capital income in national income:

$$\alpha = \frac{\bar{r}\bar{k}}{f(\bar{k}) - \bar{k}}$$

It follows that equation (11) can be re-written as follows:

$$1+g^*=(1-p)\varphi\left(\frac{\alpha}{\sigma^*}+1-\alpha\right)(1+\bar{r}).$$

Therefore, given p, the equilibrium rate of growth g^* increases as the share of the rich in the population σ^* decreases.

5. The competitive balanced-growth equilibrium with an endogenous p

To relax our previous assumption that p is given exogenously, we need to specify how to measure inequality and how does p depend on this measure. There are many ways of measuring inequality, all of which have some intuitive or mathematical appeal. Cowell (1995) contains details of at least 12 summary measures of inequality. For example, we could take the Gini coefficient as a measure of inequality. However, since we are interested only in balanced growth equilibria, our task becomes simpler since we know that, given p, at any balanced growth equilibrium the population is divided into at most two classes, the rich and the poor. In such a context we can take as an index of inequality the relative income of a rich consumer, $\frac{\bar{r}s_h^* + \bar{w}}{f(\bar{k}) - \bar{k}}$.

It should be noted that this index is equivalent to the Gini coefficient as a measure of inequality in equilibria in the sense that they represent the same inequality ordering over the set of distributions of income between the two classes. Indeed, let $\{(s_h^*, c_h^*), (s_l^*, c_l^*), k^*, w^*, r^*, \sigma^*, g^*\}$ be the vector which characterize a balanced-growth competitive equilibrium. The Lorenz curve characterizing the income distribution in this equilibrium is presented on Fig. 1. It is not difficult to check that the Gini coefficient corresponding to this Lorenz curve is $\alpha(1-\sigma^*)$. The share of capital in relative income, α, appears on the graph. We know that σ^* is inversely related with

$$\eta(\sigma^*)=\frac{\bar{r}s_h^* + \bar{w}}{f(\bar{k}) - \bar{k}}.$$

Therefore, the Gini coefficient increases as $\eta(\sigma^*)$ increases. It will be convenient for us to measure inequality by means of $\eta(\sigma^*)$.

To give a general definition of balanced growth equilibrium, let us specify our assumption on the formation of p and assume that

$$p = p\left(\frac{\bar{r}s_h^* + \bar{w}}{f(\bar{k}) - \bar{k}}\right)$$

where p:$[1,\infty)\rightarrow[0,1)$ is a continuous non-decreasing function.

The formal definition of a balanced-growth competitive equilibrium with an endogenous parameter p is similar to the one given in the previous section with the following equation defining the equilibrium value of p, p^*:

$$p^{*} = p\left(\frac{\bar{r}s_h^{*} + \bar{w}}{f(\bar{k}) - \bar{k}}\right).$$

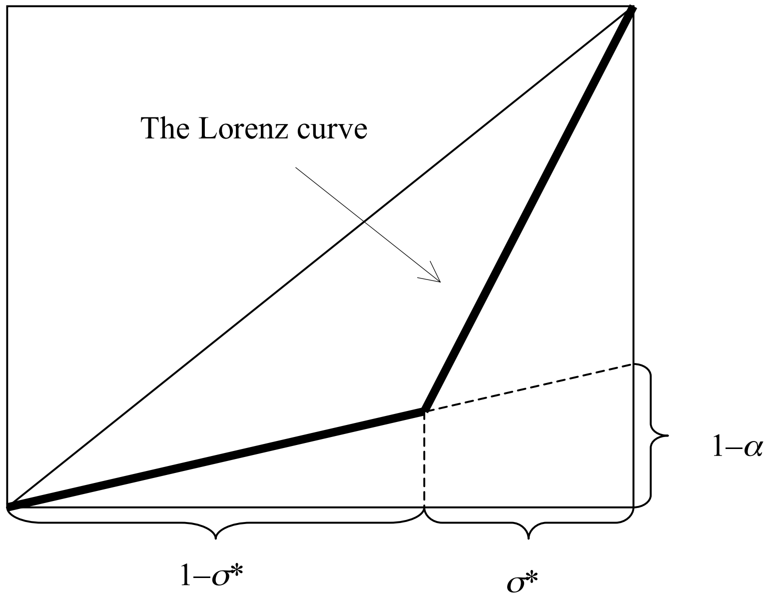

Figure 1. Lorenz curve

Similarly to the previous section discussion, a balanced-growth competitive equilibrium is fully characterized by a similarly determined tuple $\{(s_h^{*},c_h^{*}), (s_l^{*},c_l^{*}), k^{*}, w^{*}, r^{*}, \sigma^{*}, g^{*}\}$. The examination of the consumer's first-order condition with equality allow us to characterize the equilibrium values $\sigma^{*}$and g^{*} with an endogenous $p^{*}=p(\eta(\sigma^{*}))$:

$$1+g^{*}=(1-p(\eta(\sigma^{*})))\varphi(\eta(\sigma^{*}))(1+\bar{r})$$

with $\eta(\sigma^{*})\geq 1$. Define the function $\psi:[1,\infty)\to(0,1)$ by

$$\psi(\eta)=(1-p(\eta))\varphi(\eta)$$

and present the equilibrium rate of growth g^{*} as a function of η:

$$g^{*}=\psi(\eta)(1+\bar{r})-1$$

The following theorem follows from definitions and Lemma 3.

Theorem. *There exists an infinite number of balanced growth equilibria with endogenous value of* p, *i.e. there exists an infinite number of pairs* (σ^{*}, g^{*}) *satisfying*

$$g^{*}=\psi(\eta(\sigma^{*}))(1+\bar{r})-1.$$

Along this equilibrium path, variables $s_{l,t-1}^{*}$, $c_{l,t}^{*}$, $s_{h,t-1}^{*}$, $c_{h,t}^{*}$, K_t^{*}, A_t^{*} and Y_t^{*} *grow at the rate* g^{*}. □

This theorem says that in our model, like in the model proposed by Borissov (2002), the set of equilibria is a continuum. Moreover, this set can be parameterized by a simple index of inequality, η. The dependence of the growth rate on η is in general ambiguous. It is completely determined by the shape of the function $\psi(\eta)$, which in its turn depends on the shapes of $p(\eta)$ and $\varphi(\eta)$. By the moment we have assumed no more than that the functions $p:[1,\infty)\to(0,1)$ and $\varphi:[1,\infty)\to(0,1)$ are monotonically increasing and continuous. At the same time it is not unreasonable to assume in addition that as η increases, $\psi(\eta)$ first increases, peaks, and then decreases. Indeed, one might expect that $\varphi(\eta)$ is a concave function having a noticeable positive slope when η is sufficiently small and getting more and more flat as η as goes up. As for $p(\eta)$, one is inclined to think that it is zero or close to zero when η is sufficiently small, then there is an interval on which it goes up sufficiently steeply and then asymptotically $p(\eta)$ converges to some positive number (≤ 1). If on the interval where $p(\eta)$ goes up fast, $\varphi(\eta)$ is sufficiently flat, then $\psi(\eta)$ and therefore the dependence of economic development on inequality has an inverted U-shaped form.

To be more precise, suppose that the functions $\varphi(\eta)$ and $p(\eta)$ are twice continuously differentiable, that $\varphi(\eta)$ is concave ($\varphi''(\eta)<0$, $\eta>0$, see Fig. 2) and that there is an $\eta_1>0$ such that $p(\eta)$ is convex on the interval $[0,\eta_1]$ and concave on $[\eta_1,\infty)$ ($p''(\eta)>0$ for $0<\eta<\eta_1$, $p''(\eta)<0$ for $\eta>\eta_1$, see Fig. 3). Then the function $\psi(\eta)$ is concave ($\psi''(\eta)<0$) on the interval $[0,\eta_1]$.

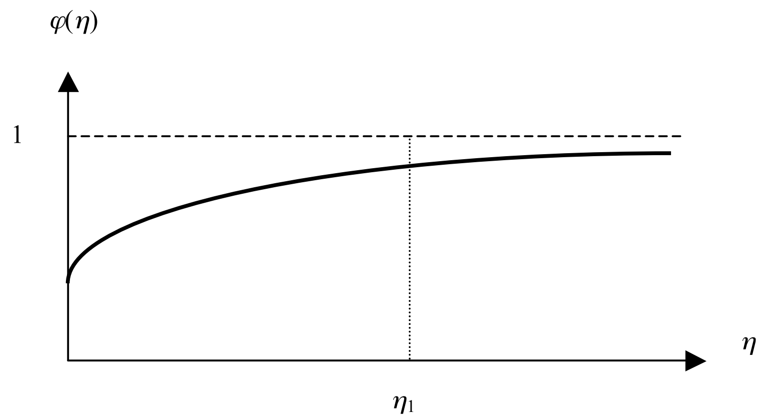

Figure 2. The shape of $\varphi(\eta)$

If, in addition, we suppose that

$$\psi'(\eta)<0,\ \eta>\eta_1, \tag{12}$$

then the function $\psi(\eta)$ reaches its maximum at some $\eta_{max}<\eta_1$ and, moreover, $\psi(\eta)$ is increasing on $(0,\eta_{max})$ and decreasing on (η_{max},∞) (see Fig. 4). It remains to note that the following conditions are sufficient for (12) to hold:

$$1-p(\eta_1)<\varphi(\eta_1),$$

$$\varphi'(\eta)<p'(\eta),\ \eta>\eta_1.$$

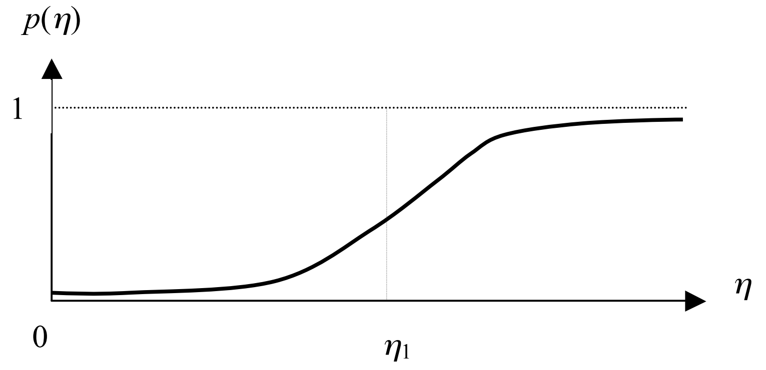

Figure 3. The shape of $p(\eta)$

Casting a glance on Fig. 2 and Fig. 3, the reader would agree that the first of these inequalities looks quite reasonable. As for the second one, though it looks somewhat peculiar, it does not seem to be extremely unreasonable.

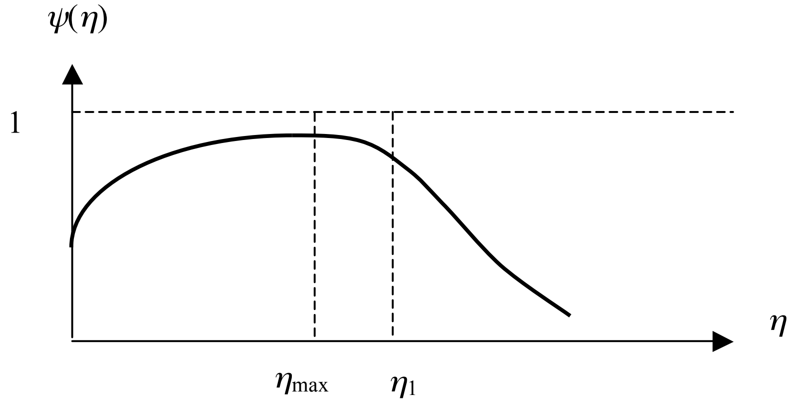

Figure 4. The shape of $\psi(\eta)$

6. Conclusion

In this paper, an endogenous-growth model with endogenous discounting has been constructed. All consumers are identical in their exogenous parameters. Our main assumption is that when maximizing his discounted utility $\sum_t \beta^t u(c_t)$, where $u(c)=\log c$, a consumer forms his effective discount factor β endogenously by computing $\beta=(1-p)\varphi$, where φ is his subjective discount factor and p is the probability of full collapse of the economy during one time period. The subjective discount factor is a monotonically increasing function of the relative well-being of the consumer and p is a monotonically increasing function of inequality. We have introduced p to reflect the empirically supported observation that inequality can lead to less political stability and hence more uncertainty the consumers bear.

We have shown that, unlike models with exogenously given discount rates, there is a continuum of steady-state equilibria. This is because the population is divided in equilibria into the rich and the poor and, what is important, the division cannot be considered as exogenous since all consumers are identical in their exogenous parameters. The set of equilibria can be parameterized by a simple index of income inequality. In general, the dependence of economic development on this index is ambiguous. However, under some reasonable assumption it has an inverted U-shaped form.

References

Aghion, P. and Bolton, P. (1997). A theory of trickle-down growth and development. *Review of Economic Studies* 64, 151-172.

Alesina, A. and Perotti, R. (1996). Income distribution, political instability, and investment. *European Economic Review* 40, 1203-1228.

Alesina, A. and Rodrik, A. (1994). Distributive politics and economic growth. *Quarterly Journal of Economics* 109, 465-490.

Becker, R. (1980). On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. *Quarterly Journal of Economics*, 95, pp. 375-82.

Benabou, R. (1996). Inequality and growth. *NBER Macroeconomics Annual*. MIT Press, Cambridge, MA.

Bertola, G. (1993). Factor shares and savings in endogenous growth. *American Economic Review¸* 83(5): 1184-1198.

Blanchard, O. J. and Fischer, S. (1989). *Lectures on Macroeconomics*. Cambridge, Mass.: MIT Press.

Borissov K. (2002). Indeterminate steady-state equilibria in a one-sector model. *Economics Letters*, 77, pp.125-130.

Borissov, K. (2005). An AK model with endogenous time preferences. European University at St. Petersburg, Department of Economics. Unpublished.

Borissov, K. and Lambrecht, S. Growth and distribution in models with endogenous impatience. European University at St. Petersburg, Department of Economics, Working Paper # 2005/03.

Bourguignon, F. (1981). Pareto superiority of unegalitarian equilibria in Stiglitz' model of wealth distribution with convex saving function. *Econometrica*, 49, pp. 1469-1475.

Bourguignon, F. (2001) Crime as a social cost of poverty and inequality: a review focusing on developing countries. In: S. Yussuf, S. Evenett and W. Wu, *Facets of globalization : international and local dimensions of development*, World Bank, Washington, DC.

Browning, M. and Lusardi, A. (1996). Household saving: micro theories and macro facts. *Journal of Economic Literature*, 34, pp 1797-1885.

Cowell, F.A. (1995). *Measuring Inequality* (2-nd edition). Harvester Wheatsheaf, Hemel Hempstead.

Galor, O., Zeira, J. (1993). Income distribution and macroeconomics. *Review of Economic Studies*, 60, 35-52.

Fajnzylber, P., D. Lederman and N. Loayza (1998) Determinants of crime rates in Latin America and the world. *A World Bank Latin America and the Caribbean Viewpoints Series Paper*, Washington, DC: World Bank.

Frankel, M. (1962). The production function in allocation and growth: a synthesis. *American Economic Review*, 52, pp. 995-1022.

Kaldor, N. (1956). Alternative theories of distribution. *Review of Economic Studies*, 23 pp. 94-100.

Kaldor, N. (1957). A model of economic growth. *Economic Journal*, 67, pp. 591-624.

Keynes, J.M. (1920). *The Economic Consequences of the Peace*. Macmillan, New York.

Kuznets, S. (1955). Economic growth and income equality. *American Economic Review* 45, pp. 1-28.

Lewis, W.A. (1954). Economic development with unlimited supply of labor. *The Manchester School*, 22, pp. 139-191.

Mankiw, N. G. (2000). The savers-spenders theory of fiscal policy. *American Economic Review*, 90, pp. 120-125.

Michel, P. and Pestieau, P. (1998). Fiscal policy in a growth model with both altruistic and non-altruistic agents. *Southern Economic Journal*, 64, pp. 682-697.

Pasinetti, L. (1962). Rate of profit and income distribution in relation to the rate of economic growth. *Review of Economic Studies*, 29, pp. 103-120

Perotti, R. (1996). Growth, income distribution, and democracy: What the data say. *Journal of Economic Growth*, 1, pp. 149-187.

Persson, T. and Tabellini, G. (1994). Is inequality harmful for growth? Theory and evidence. *American Economic Review*, 84, pp. 600-621.

Romer, P.M. (1986). Increasing Returns and Long-run Growth. *Journal of Political Economy*, 94, pp. 1002-37.

Schlicht E. (1975). A neoclassical theory of wealth distribution. *Jahrbüchern für Nationalökonomie und Statistic*, 189, pp. 78-96.

Smetters, K. (1999). Ricardian equivalence: long-run Leviathan. *Journal of Public Economics*, 73, pp. 395-421.

Smith, A. (1776) *The Wealth of Nations*. The Modern Library, New York.

Souleles, N.S. (1999). The response of household consumption to income tax refunds*, American Economic Review*, 89, pp. 947-958.

Uzawa, H. (1968). Time preference, the consumption function, and optimal asset holdings. In: Wolf J. N. (ed.), *Value, Capital and Growth; Papers in honor of Sir John Hicks*. University of Edinburgh Press, pp. 485-504.