Growth and Slowdown of Nations: What Role for the Elasticity of Substitution?

Debdulal Mallick*
Deakin University

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Abstract

Although the importance of the elasticity of substitution between capital and labor (σ) has long been recognized in several branches of economics, it has received too little attention in the growth literature. This paper aims to partly rectify this omission by exploring the growth potentials with σ as a yardstick and studying how different values of σ impact upon the balanced growth paths in theoretical model. When σ is high, the incremental capital is easily substituted for labor, resulting in a nearly equiproportionate increase in both factors. Under constant returns to scale, diminishing returns sets-in very slowly, and the marginal and average products of capital can remain sufficiently large so that output can grow indefinitely.

The theoretical model is built upon the work of de La Grandville and Solow (2004) who show that perpetual growth is possible in the Solow (1956) model even without technological progress, if value of σ exceeds a critical value that is greater than unity (σ_H^c). I extend the model to show that output level, capital stock and consumption follow perpetual decline if σ is less than another critical value (σ_L^c) that lies between zero and unity. The critical values depend on saving, population growth and depreciation rates, and the initial share of capital in total output; hence each country has at most one critical value. I show that the above results also carry into in a model of endogenous saving, and analytically prove that the balanced growth path exists only if σ lies between two critical values- σ_L^c and σ_H^c. I calibrate the critical value of σ from the data for each country. These values are then compared to $\hat{\sigma}$'s estimated from country time series data. A number of countries, mainly from Africa, have $\hat{\sigma} < \sigma_L^c$. Average per capita output growth in these countries is either negative or very low. Although many countries have σ_H^c indicating bright growth potential, none of them has $\hat{\sigma}$ sufficiently large (i.e., $\hat{\sigma} > \sigma_H^c$).

JEL Nos.: O40, O41

Correspondence:
Debdulal Mallick
School of Accounting, Economics and Finance
Deakin University
221 Burwood Highway, Vic 3125
Australia
Email: dmallic@gmail.com

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1 Introduction

Although the importance of the elasticity of substitution between capital and labor (σ) has long been recognized in several branches of economics, it has received too little attention in the growth literature. This paper aims to partly rectify this omission by exploring the growth potentials with σ as a yardstick and studying how different values of σ impact upon the balanced growth paths in growth models. To better understand the role of σ, we abstract from technological progress.

It is generally presumed that in the exogenous growth models[1], no long-run growth of per capita output is possible without technological progress. Due to diminishing factor returns, the capital-labor ratio and per capita output settle down to some steady state level, and total output grows precisely at the same rate of population growth. In these models, the saving rate affects only the level of long-run output, but not the growth rate. However, Solow, in his seminal 1956 article, raised the issue that per capita output can grow indefinitely, even in the absence of technological progress, if the marginal product of capital is bounded below by a sufficiently high positive number when capital-labor ratio approaches infinity.[2] The condition for sufficiently high marginal and average products of capital is that the σ elasticity must be large enough. The higher is σ, the greater the similarity between capital and labor, and thus an increase in capital with labor held fixed does not substantially change the capital-labor ratio, which in turn resists the pull of diminishing returns to capital (Brown, 1968; p. 50). We begin Section 2 with a discussion of the relationship between σ and growth rate of output per capita.

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[1] By exogenous growth model, we mean the model in which technology is exogenously determined. Both Solow (1956) and Koopmans (1965) models fall in this definition.

[2] A similar possibility has been raised by Pitchford 1(960), Barro and Sala-i-Martin (1995), and Srinivasan (1995).

Section 3 discusses the possibility of perpetual growth and slowdown. De La Grandville and Solow (2004) have demonstrated that for a country to grow indefinitely without technological progress, σ must exceed a critical value (σ_H^c) that is greater than 1. This critical value depends on saving, population growth, and depreciation rates, and initial capital share of output. However, such a high critical value does not exist for many countries. In this section, we demonstrate another possibility that perpetual decline is also possible if the marginal product of capital is bounded above by a sufficiently low number as capital-labor ratio approaches zero. The condition for sufficiently low marginal and average products of capital is that σ must be less than another critical value (σ_L^c). This critical value lies between 0 and 1 (σ_L^c), and is that value of σ below which output level, capital stock and consumption would decline and approach zero asymptotically. The σ_L^c is determined by the same parameters that determine σ_H^c. Since countries differ in these structural features, each country will have at most one critical value (σ^c). We interpret σ^c as the growth potential of a country, and actual σ, which characterizes production, as the capability to realize that potential. We also encounter a third possibility in which σ^c becomes negative. Since actual σ must always be non-negative, such a critical value implies that a country does not possess potential for perpetual growth or risk of perpetual slowdown.

Depending on the relative magnitudes of σ and σ^c**,** a steady state in the conventional sense may or may not exist where capital–labor ratio settles down to some constant. To replicate such a steady state, σ must lie between two critical values-σ_H^c and σ_L^c. If σ falls outside this plateau, then an economy can either grow or shrink indefinitely. We demonstrate that the above results also carry into a model of endogenous saving rate. Although steady state behavior is similar in both models with exogenous and endogenous saving rate, the optimization framework allows us to rigorously prove that the balanced growth path is locally saddle-path stable only if $\sigma_L^c < \sigma < \sigma_H^c$. On the other hand, no balanced growth path exists when $\sigma > \sigma_H^c$ or $\sigma < \sigma_L^c$.

In section 4, we calibrate the critical values, σ^c at the country level to get a sense about the growth potential of the countries. In section 5, we estimate σ from country time series. Section 6 compares these estimated values with σ^c to investigate whether the countries are able to realize their growth potentials. Our comparison shows that few countries from Africa have $\sigma < \sigma_L^c$. Average per capita output growth in these countries is either negative or very low. Although many countries have σ_H^c indicating bright growth potential, none of them has $\sigma > \sigma_H^c$ necessary to realize the potential.

Finally, section 7 concludes.

2 The role of σ in economic growth

The importance of σ in economic growth can be understood by investigating the properties of the CES production function. The CES production function in its normalized form is given by.[3]

$$Y_t/Y_0 = \left[a_0 \left(K_t/K_0\right)^{\frac{\sigma-1}{\sigma}} + \left(1-a_0\right)\left(L_t/L_0\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{--- (1)}$$

where, Y_t is real output, K_t is real capital stock, L_t is labor input, and σ is the elasticity of substitution between capital and labor. Y_0, K_0, L_0 and a_0 are benchmark values. With normalization, a_0 now represents the partial elasticity of output with respect to capital or initial capital share of output (Rutherford, 2003) and is given by $\left(\partial Y_0/\partial K_0\right)\left(K_0/Y_0\right) = p_{K_0}K_0 \big/ \left(p_{K_0}K_0 + p_{L_0}L_0\right)$. We assume constant returns to scale, and no

[3] The CES production function approaches the Cobb-Douglas as σ approaches 1.

technological progress. For simplicity and without loss of generality, we set the benchmark values of Y_0, K_0 and L_0 to 1, so that the production function is written as

$$Y_t = \left[a_0 K_t^{\frac{\sigma-1}{\sigma}} + \left(1-a_0\right) L_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{--- (2)}$$

To establish the relationship between σ and growth rate, countries are distinguished only by their values of σ, so common benchmark points for variables and marginal rate of substitution are required. Without normalization, a change in σ in the CES function not only alters the curvature of the isoquant but also shifts the whole isoquant map so that comparison of growth paths at different values of σ becomes difficult. Moreover, the unusual situation, that shares of capital and labor in total output approach one-half in the special case of Harrod-Domar in which σ =0, is avoided with normalization to the CES production function (Klump and de La Grandville, 2000, p. 287; Klump and Preissler, 2000, p. 46).

When σ>1, the CES production function in equation 2 does not possess any limit, i.e., $\lim_{K\to\infty} Y\Big|_{\sigma>1} = \lim_{L\to\infty} Y\Big|_{\sigma>1} = \infty$ but it does when σ<1. In other words, output can grow indefinitely if either capital or labor is also allowed to grow indefinitely. When the value of σ is high, both capital and labor become similar, and thus an increase in one input with another input held fixed does not substantially change the input ratio, which in turn resists the pull of diminishing factor returns. Brown (1968, p. 50) has provided the following rationale.

> When σ>1, the factors of production resemble each other from a technological point of view, so that if one increases indefinitely, the other being held constant, the technology permits the expanding factor to be substituted relatively easily for the constant factor. Hence, both factors seem to be increasing indefinitely, and the product to which they contribute increases indefinitely. If σ<1, the technology views the factors as being relatively dissimilar so that it is difficult to substitute the expanding factor for the constant factor. Even though one factor increases indefinitely, the growth of the product is restrained by the technologically scarce-constant factor.

Figure 1 shows the relation between σ and output growth.[4] The isoquant is L-shaped for σ equals zero. It becomes a straight line when σ approaches infinity. Finally, it is regular convex-shaped for the Cobb-Douglas case of σ equals 1. Despite very different values of σ ranging from 0 to infinity, all the isoquants for the baseline values of the variables go through the common point A. Comparison of the isoquants shows that when value of σ is higher, the same amount of output can be produced with less amount of inputs; in other words, larger output can be produced with the same amount of inputs.

3 Perpetual growth and slowdown in the Solow model

3.1 Solow-CES model with exogenous saving rate

In this section, we first draw on de La Grandville and Solow (2004) to show the case in which perpetual growth is possible even without technological progress. We then demonstrate another possibility of slowing down of an economy without technological progress. First, we rewrite equation 2 in per capita terms as

$$y = f_\sigma(k) = \left[a_0 k^{\frac{\sigma-1}{\sigma}} + (1-a_0)\right]^{\frac{\sigma}{\sigma-1}} \qquad \text{--- (3)}$$

where, y is per capita output, and k is the capital-labor ratio. For notational convenience, we omit the time subscripts.

The marginal and average products of capital are given by

$$f'_\sigma(k) = a_0\left[a_0 + (1-a_0)k^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{\sigma-1}} \quad \text{and} \quad f_\sigma(k)/k = \left[a_0 + (1-a_0)k^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

[4] Figure 1.1 is drawn from Miyagiwa and Papageorgiou (2003, p. 157). However, they demonstrate that a monotonic relationship between σ and growth may not exist in the Diamond overlapping-generations model. They showed that, if capital and labor are relatively substitutable, an economy with a higher σ may exhibit lower per capital income growth in transition and in the steady state. They conclude that the role of σ for the economic growth depends on choice of particular model (Solow vs. Diamond).

and these two are related by $f(k)/k = \left[f'(k)/a_0\right]^{\sigma}$. If σ>1, both marginal and average products of capital approach a positive constant when capital-labor ratio approaches infinity, thus violating one of the Inada conditions.

$$\lim_{k\to\infty}\left[f_\sigma'(k)\right] = \lim_{k\to\infty}\left[f_\sigma(k)/k\right] = a_0^{\frac{\sigma}{\sigma-1}} > 0 \qquad \text{--- (4)}$$

Both capital and labor now become similar and therefore, capital-labor ratio does not substantially change even if capital is increased with relatively fixed labor, therefore diminishing returns to capital sets in very slowly.

On the other hand, if σ<1, the marginal and average products of capital approach the same positive constant when capital-labor ratio approaches zero, thus violating another Inada condition.

$$\lim_{k\to 0}\left[f_\sigma'(k)\right] = \lim_{k\to 0}\left[f_\sigma(k)/k\right] = a_0^{\frac{\sigma}{\sigma-1}} > 0 \qquad \text{--- (5)}$$

Capital and labor are very dissimilar inputs because of low substitutability. Initial average and marginal products of capital are very low and also decline very rapidly as capital-labor ratio increases.

The constant $a_0^{\frac{\sigma}{\sigma-1}}$, which is independent of the size of the economy (*Y*, *K* and *L*), will play an important role in determining the asymptotic growth rate of per capita output. De La Grandville and Solow (2004) have studied the properties of $a_0^{\frac{\sigma}{\sigma-1}}$. For σ > 1, $a_0^{\frac{\sigma}{\sigma-1}}$ starts at 0 and is first strictly convex in σ up to an inflexion point at $\sigma = 1 - \frac{1}{2}\log a_0$. It then becomes concave asymptotically approaching a_0. For $0 \le \sigma < 1$, $a_0^{\frac{\sigma}{\sigma-1}}$ starts at 1 and is strictly convex approaching infinity as σ approaches 1. $a_0^{\frac{\sigma}{\sigma-1}}$ is always increasing in σ, except at the point of discontinuity at σ = 1. Figures 2 and 3 show this behavior.

The equation describing the dynamics of the Solow growth model is given by[5]

$$g_k = \dot{k}/k = sf_\sigma(k)/k - (n+\delta) \qquad \text{--- (6)}$$

where, g_k is the growth rate of capital-labor ratio, and s, n and δ are the constant saving, population growth and depreciation rates respectively.

Evolution of per capita output is derived from equation 6. Growth rates of per capita output and capital-labor ratio are related by $\dot{y}/y = \alpha_\sigma \, \dot{k}/k$, where $\alpha_\sigma = kf'_\sigma(k)/f_\sigma(k)$ is the capital share of output.[6] When $\sigma > 1$ and $k \to \infty$, capital share of output α_σ approaches unity[7], and $f_\sigma(k)/k$ approaches $a_0^{\frac{\sigma}{\sigma-1}}$. The evolution of per capita output is therefore given by

$$g_y = \dot{y}/y = sa_0^{\frac{\sigma}{\sigma-1}} - (n+\delta) \qquad \text{--- (7)}$$

If saving rate is high enough so that $sa_0^{\frac{\sigma}{\sigma-1}} > (n+\delta)$, per capita output can grow indefinitely without technological progress.

On the other hand, capital share also approaches unity, and $f_\sigma(k)/k$ approaches $a_0^{\frac{\sigma}{\sigma-1}}$ if $\sigma < 1$ and $k \to 0$. The evolution of per capita output is also governed by equation 7. If an economy starts with very low saving rate and/or high population growth rate, so that $sa_0^{\frac{\sigma}{\sigma-1}} < (n+\delta)$, growth rate becomes negative and the economy continues to slow down with per capita output approaching zero asymptotically. It may seem counter intuitive that capital

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[5] For derivation, see Barro and Sala-i-Martin (1995, chapter-1, p. 18).

[6] This is different from a_0, which is the initial capital share of output. To show that, we first take log at both side of the production function $y = f_\sigma(k)$, and then take derivative with respect to time to obtain $\dot{y}/y = f'_\sigma(k)\dot{k}/f_\sigma(k) = \alpha_\sigma \dot{k}/k$, where α_σ is the capital share of output, because in a competitive equilibrium rental income of each unit of capital is equal to its marginal product.

[7] This can be shown by taking limits of the expression for α_σ.

$$\lim_{k\to\infty}\alpha_\sigma\Big|_{\sigma>1} = \lim_{k\to 0}\alpha_\sigma\Big|_{\sigma<1} = \lim_{k\to\infty} a_0\left[a_0 + (1-a_0)k^{\frac{1-\sigma}{\sigma}}\right]^{-1}\Bigg|_{\sigma>1} = \lim_{k\to 0} a_0\left[a_0 + (1-a_0)k^{\frac{1-\sigma}{\sigma}}\right]^{-1}\Bigg|_{\sigma<1} = 1.$$

share approaches unity in this case. When capital-labor ratio is continuously falling, labor must be increasingly substituted for capital in order to maintain full employment of both factors. With poor substitutability between labor and capital, more and more labor can be employed only at the expense of lowering marginal product of labor. In this case, marginal product of labor falls more rapidly than per capita output (i.e., $F'(L)$ falls more rapidly than L/Y rises). Therefore, the labor's share of output ($1-\alpha_\sigma = F'(L).(L/Y)$) approaches zero (Pitchford, 2004).

3.2 Critical value of σ

Why does σ need to exceed a critical value to generate perpetual growth, when it is already established that output is unbounded above with $\sigma > 1$? The reason is that capital accumulation needed to ensure full employment of labor may be constrained by higher population growth and depreciation rates. To overcome the constraints, σ must be large enough to exceed a critical value to make possible faster capital accumulation.

To solve for the critical value, we set equation 7 to zero and then solve for σ.

$$\sigma^c = g(a_0, s, n, \delta) = \frac{\log[s/(n+\delta)]}{\log[a_0 s/(n+\delta)]} = \frac{1}{1+\log a_0 / \log[s/(n+\delta)]} \quad \text{--- (8)}$$

The critical value, σ^c can be greater than 1 (σ_H^c) or less than 1 (σ_L^c) depending on initial capital share, saving, population growth and depreciation rates.

σ^c >1 (σ_H^c):

The σ_H^c is that value of σ above which the asymptotic growth rate of per capita output is positive. In other words, if actual σ exceeds σ_H^c ($\sigma > \sigma_H^c > 1$), then perpetual growth is possible without technological progress. In this case, the asymptotic growth rate depends on saving rate. This is similar to "warranted rate of growth" in the Domar (1946) model, but the difference is that labor now becomes a redundant factor.

Proposition-1: For $\sigma^c > 1$, the saving rate must be sufficiently large so that $a_0 s > (n+\delta)$.

Proof: In equation 8, the condition $\sigma^c > 1$ implies that $-1 < \log a_0 / \log[s/(n+\delta)] < 0$. Since, $\log a_0 < 0$ because $1 > a_0 > 0$, $\log[s/(n+\delta)]$ must be positive to satisfy the last inequality, which in turn implies that $s > (n+\delta)$. Again, since $\log[s/(n+\delta)] > 0$, for the first inequality to hold it must be that $a_0 s > (n+\delta)$.

Capital accumulation per worker is expedited by higher saving, and retarded by higher population growth and depreciation. In this case, total capital accumulation is so high that only a fraction (given by capital share) of it is more than necessary to raise the capital-labor ratio that is diminished at the rate (n + δ). Now, if the substitutability between capital and labor is large so that marginal product is bounded below, output will grow indefinitely.

$\sigma^c < 1$ (σ_L^c):

On the other hand, σ_L^c is that value of σ below which the asymptotic growth rate of per capita output is negative. In other words, if actual σ is less than σ_L^c ($\sigma < \sigma_L^c < 1$), then output continues to slow down in the absence of technological progress.

Proposition-2: For $0 < \sigma^c < 1$, the saving rate must be sufficiently low and/or population growth rate high so that $s < (n+\delta)$.

Proof: The condition $\sigma^c < 1$ implies $\log a_0 / \log[s/(n+\delta)] > 0$, which in turn implies that $s < (n+\delta)$ because $\log a_0 < 0$.

Saving rate is so low that a country cannot even accumulate capital at a rate necessary to prevent total capital stock from diminishing that occurs at the rate (n + δ). Under this circumstance, labor must be increasingly substituted for capital to ensure full employment of both factors. But with low σ, marginal product of capital falls more rapidly than per capita output falls. Therefore, the economy will suffer perpetual slowdown.

Negative σ^c *:*

Since actual σ must be non-negative by definition, only a non-negative value of σ^c can explain a country's growth potential; a negative value implies that a country does not possess potential to grow indefinitely or risk of perpetual slowdown. Value of σ^c becomes negative when $s > (n+\delta)$ but $a_0 s < (n+\delta)$. This implies that a country's rate of capital accumulation is higher than the rate necessary to maintain per worker capital stock constant, but not large enough to ensure perpetual growth. For $\sigma^c > 0$, the saving rate has to be too high or too low. For the intermediate range of saving rate $s \in \left((n+\delta),(n+\delta)/a_0\right)$, σ^c becomes negative. The reason is that σ^c has been calculated under two extreme circumstances in which either $\sigma > 1$ and $k \to \infty$, or $\sigma < 1$ and $k \to 0$, and only under these circumstances $f_\sigma(k)/k$ approaches $a_0^{\frac{\sigma}{\sigma-1}}$. If $k \to k^* \neq (0 \; or \; \infty)$ (where, k^* is the steady state value of k), the limit of $f_\sigma(k)/k$ also depends on k^* and an analytical solution for σ^c does not exist.

3.3 Behavior of σ^c

The critical value σ^c reflects the growth potential of a country. The lower the value of σ^c, the easier for a country to realize its growth potential because given σ, a lower value of σ^c minimizes $(\sigma^c - \sigma)/\sigma$. To understand why growth potentials vary across countries, it is imperative to study the response of σ^c with respect to the parameters that determine it.

The response of σ^c to a change in initial capital share of output is conditional on the value of *s*, *n* and *δ*.

$$\frac{\partial\sigma^c}{\partial a_0} = \frac{-1}{a_0[1+\log a_0/\log\{s/(n+\delta)\}]^2\log\{s/(n+\delta)\}}$$

This is negative if $s > (n+\delta)$, and positive if $s < (n+\delta)$. The reason is that an increase in the capital share of output increases marginal product of capital relative to labor thus augmenting capital. With capital augmenting technological change in place, an increase in capital accumulation implied by $s > (n+\delta)$ indicates an economy's better growth potential that is reflected in its lower σ^c. Figure 4 shows the behavior of σ^c when $s > (n+\delta)$. Suppose, an economy saves and invests 25% of its GDP, population grows at 1% and capital stock depreciates at 4%, then σ^c decreases from 3.97 to 2.32 and 1.75 when capital share increases from 0.3 to 0.4 and 0.5 respectively.

The response of σ^c to a change in saving rate is not conditional on other parameters; σ^c is monotonically decreasing in saving rate.

$$\frac{\partial\sigma^c}{\partial s} = \frac{\log a_0}{s[\log\{a_0 s/(n+\delta)\}]^2} < 0$$

This is understandable. It is evident from equation 7 that steady state growth rate of per capita output is increasing with higher saving rate. Therefore, higher saving rate lowers the distance between σ^c and σ. Figure 5 shows the behavior of σ^c with respect to saving rate. For the values of population growth and depreciation rates reported earlier, and capital share of 0.4, σ^c decreases 2.95 to 2.04 and 1.89, if a country is able to increase its saving rate from 20% to 30% and 35% of GDP respectively.

Higher population growth and depreciation rates make worse the growth potential by raising the value of σ^c.

$$\frac{\partial\sigma^c}{\partial n} = \frac{\partial\sigma^c}{\partial\delta} = -\frac{\log a_0}{(n+\delta)[\log\{a_0 s/(n+\delta)\}]^2} > 0$$

When population grows or capital depreciated at a high rate, larger saving and investment is required to maintain capita stock per worker, and therefore σ^c increases.

3.4 Solow-CES model with endogenous saving rate

The previous model with exogenous saving rate is analogous to the situation in which a central planner decides how much to save and invest. In a decentralized economy, saving and investment decisions are made by optimizing consumers and firms that interact in the competitive markets. Although the steady state behavior of the model does not change qualitatively with endogenous saving rate, the model allows a rigorous proof of the existence and stability of balanced growth path for different values of σ.

A representative household maximizes utility U given by

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} e^{(n-\rho)t} dt$$

s.t. $\dot{k} = w + rk - c - nk$

where, c is per capita consumption, w is real wage, ρ is subjective discount rate, and $1/\theta = -u'(c)/[u''(c)/c]$ is intertemporal elasticity of substitution between consumption at two points in time. The flow budget constraint indicates that capital-labor ratio[8] rises with per capita income $w + rk$, and falls with per capita consumption and population growth rate $c + nk$.

A representative firm maximizes the flow of net profits

$\Pi = L[f_\sigma(k) - (r+\delta)k - w]$, where $f'_\sigma(k) = (r+\delta)$ is the rental rate to capital.

The transversality condition is given by $\lim_{t\to\infty} k(t)\exp\left\{-\int_0^\infty [r(v) - n]dv\right\} = 0$.

The dynamics of the model is given by the following system of two equations.

$$\dot{k}/k = f_\sigma(k)/k - c/k - (n+\delta) \qquad \text{--- (9)}$$

[8] In fact, it is per capita asset. These two are equal because capital is the only asset that households can accumulate.

$$\dot{c}/c = (1/\theta)\left[f'_\sigma(k) - (\rho+\delta)\right] \qquad \text{--- (10)}$$

Revisiting the critical value of σ:

To derive the critical value of σ, we rewrite equations 9 and 10 under the conditions that $\sigma > 1,\ k \to \infty$ and $\sigma < 1,\ k \to 0$, and set $\dot{k}/k = \dot{c}/c = 0$. In both cases, average and marginal products of capital approach $a^{\frac{\sigma}{\sigma-1}}$.

$$sa^{\frac{\sigma}{\sigma-1}} - (n+\delta) = 0 \qquad \text{--- (11)}$$

$$a^{\frac{\sigma}{\sigma-1}} - (\rho+\delta) = 0 \qquad \text{--- (12)}$$

In equation 11, we have used the definition of saving rate, $s = 1 - c/f_\sigma(k)$. Equation 11 is the same as equation 7. A critical value of σ can be derived from either equation 11 or 12. However, we show that either equation gives the same σ^c. We have already derived σ^c in equation 8 by solving equation 7 or equation 11. Now, we solve for equation 12 to derive another expression for σ^c, which is given by

$$\sigma^c = \frac{\log\left[1/(\rho+\delta)\right]}{\log\left[a_0/(\rho+\delta)\right]} = \frac{1}{1+\log a_0/\log\left[1/(\rho+\delta)\right]} \qquad \text{--- (13)}$$

This value of σ^c can be shown to be the same as that in equation 8. In order to show that, we solve for equations 9 and 10 by setting $\dot{k}/k = \dot{c}/c = 0$ to derive an expression for the steady state saving rate, $s = \alpha_\sigma(n+\delta)/(\rho+\delta)$. The transversality condition requires $\rho - n > 0$, so that $s < \alpha_\sigma$. In both cases, when $\sigma > 1,\ k \to \infty$ and $\sigma < 1,\ k \to 0$, the capital share

α_σ approaches 1, and the steady state saving rate becomes $s = (n+\delta)/(\rho+\delta)$. Substituting this expression for *s* into equation 8, one can see that both equations 8 and 13 are exactly the same.[9]

Asymptotic and balanced growth path:

Much of growth theory is about the structural characteristics of the steady states and their asymptotic stability i.e., whether equilibrium paths from arbitrary initial conditions tend to a steady state (Solow, 1999; p. 639-40). There are some reasons for that. Growth theory has been developed and still considered as a theory that would be able to explain long run growth of advanced industrialist countries. It has proven useful in explaining some of the Kaldor's (1961) "stylized facts" that are usually regarded as the characteristics of the steady state. In the following, we examine what values of σ are consistent with the existence of a steady state.

Our definitions of asymptotic path (AP) and balanced growth path (BGP) are similar to Acemoglu (2003, p. 11). We define an AP as an equilibrium path that an economy tends to as $t \to \infty$ and does not include limit cycles.[10] In the AP, output, capital stock and consumption can grow or decline more than exponentially or at a constant rate. A BGP is a special case of AP where output, capital stock and consumption grow at the same finite constant rate including zero.

[9] Solving equations 1.11 and 1.12 jointly also gives the same value of σ^c. To show that, we combine the equations to obtain $a^{\frac{\sigma}{\sigma-1}} = (\rho-n)/(1-s)$. Solving this equation for σ, a critical value is derives as $\sigma^c = \log\left[(1-s)/(\rho-n)\right] \Big/ \left\{\log\left[(1-s)/(\rho-n)\right] + \log a_0\right\}$. Now, substituting the value of steady state saving rate $s = (n+\delta)/(\rho+\delta)$ into this expression, we obtain the same formula as in equation 1.13.

[10] A limit cycle is an isolated closed integral curve to which all nearby paths approach from both sides in a spiral fashion (Gandolfo, 1997; p. 355).

Proposition-3: If $\sigma < \sigma_H^c$ or $\sigma > \sigma_L^c$, the BGP is defined by a singular point in the form of a saddle-path, which is locally stable. But if $\sigma > \sigma_H^c$ or $\sigma < \sigma_L^c$, no singular point at the origin exists.

Proof: See Appendix A.1.

We show in Appendix A.1 that when $\sigma < \sigma_H^c$ or $\sigma > \sigma_L^c$, the linearized system of two differential equations 9 and 10 has one positive and one negative eigenvalues, and is thus locally saddle-path stable. If $\sigma < \sigma_H^c$ or $\sigma > \sigma_L^c$, then $k \to k^*$ in the steady state and per capita output, consumption and capital stock do not grow without technological progress. A BGP that replicates the conventional steady state exists. Figure 6 also depicts this.

The reason for the nonexistence of singular point[11] when $\sigma > \sigma_H^c$ or $\sigma < \sigma_L^c$ is that the determinant of the characteristic matrix of the linearized system becomes zero. The linearized system of two equations reduces to $\dot{c} = b\dot{k}$, where b is a constant. In this case, the integral curves are straight lines, which no longer possess a singularity at the origin (Gandolfo, 1997, p. 359). There is no steady state equilibrium. If $\sigma > \sigma_H^c$, total output, consumption and capital stock grow more than exponentially. Per capita output and capital stock grow at the same rate (because capital share approaches 1) but growth rate of per capita consumption is lower than per capita output or capital stock. Steady state in the conventional sense does not exist because capital-labor ratio, per capita output and consumption increase at varying rates. On the other hand, if $\sigma < \sigma_L^c$, per capita output and capital stock decrease at the same rate that is higher than the rate of decline of per capita consumption. Steady state does not also exist because of differential growth rates. This is similar to the second case of Proposition 2 in Acemoglu (2003) where consumption grows faster than exponentially and technological progress is purely capital augmenting.

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[11] Any point in which two functions $\dot{c}/c$ and $\dot{k}/k$ will be simultaneously zero is called a singular point. The elementary singular points are *node*, *saddle point*, *focus* and *center* (Gandolfo, 1997; p. 349-50).

The behavior of output, capital stock and consumption can be better understood by studying the behavior of saving rate in the steady state.[12] Solving equations 9 and 10 at the steady sate, and using the relationship between average and marginal products of capital that $f(k)/k = \left[f'(k)/a_0\right]^{\sigma}$, we derive an expression for the steady sate saving rate that depends on the value of σ.

$$s = (n+\delta)\left(a_0 /(\rho+\delta)\right)^{\sigma}.$$

The response of the saving rate with respect to σ can be derived as

$$\frac{\partial s}{\partial \sigma} = (n+\delta)\left(a_0 /(\rho+\delta)\right)^{\sigma} \log \sigma .$$

For $\sigma > 1$, the steady state value of saving rate increases with the value of σ implying that per capita output and capital stock increases at a higher rate than consumption. On the other hand, for $\sigma < 1$, steady state saving rate decreases implying that per capita output and capital stock declines at a higher rate than consumption.

4 Calibration of σ^c

In the previous section, we have explored the role of σ in economic growth. We have shown that a country's asymptotic growth path depends on two parameters—σ^c that depends on structural parameters such as initial capital share, saving, population growth and depreciation rates, and actual σ that characterizes production. In the following two sections, we calibrate σ^c from data and compare σ^c with σ estimated from country time series.

[12] Smetters (2003, p. 700-701) has studied the behavior of saving rate during the transitional dynamics in a Cass-Koopmans model with CES production function. He showed that for $0 < \sigma < 1$, saving rate decreases along the transitional path after the capital-labor ratio reaches a critical value. On the other hand, for $\sigma > 1$, saving rate increases along the transition path after the critical value reaches a critical value.

4.1 Data

We collect all but capital share data from Penn World Table (PWT) 6.1 for the period 1950-2000. For some countries data are not available for the entire period. We retain 114 countries (Appendix A.2) for which at least 30 consecutive years of data are available. We divide the countries into 15 regions following the World Bank classification (Appendix A.2). It is important to note that two countries having data for the same length may have different beginning and ending years, especially if they are from different regions. But the beginning and ending years are usually the same for countries in the same region. Therefore, descriptive statistics may not be strictly comparable across regions.

Data on per capita real GDP at constant price (RGDPL), real GDP per worker at constant price (RGDPWOR), investment share of RGDPL (KI), and population (POP) are obtained from PWT 6.1. We calculate the labor force as (RGDPL*POP/RGDPWOR). We construct capital stock series from investment data using the perpetual inventory method (Appendix A.3)

Capital share of output is taken from Bernanke and Gurkaynak (2001) for the year 1996. This share is computed as one minus the labor share in GDP. The labor share is employee compensation in the corporate sector from National Accounts after making a number of adjustments that include the labor income of the self-employed and non-corporate employees.[13]

4.2 Descriptive Statistics

Saving/Investment rate:

The mean investment share of GDP for 114 countries is 15.6% with a standard deviation 7.86. It is less than 10% of GDP for 30 countries of which 24 countries are from Africa. Other countries, which invested less that 10% of GDP are El Salvador, Guatemala, Paraguay, Haiti, Bangladesh and Sri Lanka. Average investment rate in Uganda is less than 2% of GDP—the lowest in the sample. (Appendix A.4). Thirty-five countries invested more than 20% of GDP with

[13] For a detail discussion of the data set, see Bernanke and Gurkaynak (2001), Caselli and Feyrer (2006).

Singapore being on the top of the list investing 41.2%. Most countries in this list are from Europe and South East Asia. Three African countries with investment more than 20% of GDP are Republic of Congo, Tanzania and Zimbabwe.

We partition the sample period for each country into two equal intervals to see how saving rate and other variables have changed over time (Appendix A.4). The countries are heterogeneous so that the partition has not made based on any particular economic or political event. Average investment rate varied considerably in the two intervals for some countries predominantly from Africa. Most notable is Zimbabwe for which average investment rate was more than 50% of GDP in the first interval, while it declined to less than 14% in the second interval. Some other countries that experienced large decline in average investment share are Republic of Congo, Zambia, Tanzania, Namibia, Ghana, Chad, Romania, Peru, Guyana and Jamaica. On the other hand, some countries that are successful in raising their investment share include Nigeria, Lesotho, Nepal, Indonesia, Jordan, Turkey, China, Taiwan, Ireland, Malaysia and South Korea.

GDP growth:

Average annual per capita real GDP growth for the sample period was negative for 9 countries (Central African Republic, Democratic Republic of Congo, Niger, Angola, Madagascar, Mozambique, Comoros, Sierra Leone and Senegal). Twelve countries grew at less than half a percent a year, and 22 countries at less than 1% a year. All these countries were from Africa except Bolivia, Venezuela, Honduras, Nicaragua, Papua New Guinea and El Salvador. Average annual per capita real GDP growth rate is higher than 3% for 33 countries, and more than 4% for 12 countries. These countries are mostly from South East Asia with Singapore experiencing the highest annual per capita growth at 7.25%, followed by Taiwan (6.26%).

From regional perspectives, growth performance was poor in the West, Central and East Africa (Appendix A.4). For example, average annual per capital growth rate of real GDP was only 0.52% and 0.87% in the Central and West African region respectively. Growth rate indeed declined in the second interval in all African regions. It was negative in the Central African region (-1.36%), while in the first interval the region grew at a modest rate of 2.6%. Growth was most impressive in the East, and South East Asia, and Eastern Europe (5.2%, 4.0% and 4.5% respectively).

Population Growth:

The African region has very high population growth. Average population growth rate over the sample period is the highest in the North Africa and Middle East (2.88%) followed by Central Africa (2.63%), East and West Africa (2.6%). The South East and South West Asia also have a higher population growth rate slightly below 2.5%. Population growth rate is low in both Eastern and Western Europe—0.69% and 0.61% respectively. However, the population growth rate has declined in all regions except in African countries where the growth rate was higher in the second than the first interval (Appendix A.6).

Capital share of income:

In the Bernanke and Gurkaynak (2001) sample, the mean value of the capita share of output is 0.35. It is large for the developing countries, and low for the developed countries. For example, among the 16 countries that have a value of capital share larger than 0.4, only Singapore is a developed country. Twenty countries have capital share less than 0.3 of which only six are developing countries. In the sample, capital share data are available for 53 countries. We replace the missing values by the average value of the cluster where a country belongs to. Countries are clustered into four groups according to real per capita GDP measured using purchasing power parity—per capita real GDP less than $5,000, from $5,000 to less than $10,000, from $10,000 to less than $20,000, and $20,000 or above. This classification has been made based on the observation that low-income countries have relatively larger capital share.

Rate of Depreciation:

Choice of the depreciation is important not only for calibration of σ^c, but also for construction of the capital stock series. The OECD, in its estimates of the capital stock for several industrial countries, estimated the depreciation rate to be 4.1% in France, 1.7% in Germany, 2.6% in Great Britain, 4.9% in Japan, and 2.8% in the USA (OECD, 1991). Estimates of the depreciation rate for the developing countries are not available. Therefore, following the growth accounting literature we use a common depreciation rate of 4% for all countries (Mankiw, Romer and Weil, 1992; Nehru and Dhareswar, 1993).

4.3 Calibrated value of σ^c

It is clear from the description in the subsection 4.2 that African countries had low investment and higher population growth over the last several decades. The region also had lower per capita output growth, negative in many instances. Investment and per capita output growth rate was higher in the East and South East Asia, and Eastern Europe. Since σ^c is increasing in the population growth rate and decreasing in the saving/investment rate, it is, therefore, expected that countries mainly from the Africa will have $\sigma^c < 1$.

Our calibration uses the averages of investment share of GDP and population growth rate for the second interval. The reason is that many developing countries from Asia, Africa and Latin America were freed from their colonial masters immediately after the World War II that continued till 1960's, and these countries needed time for stabilization of their economies.

Appendixes A.7.1-A.7.3 provide a list of σ^c's for the depreciation rate of 0.04. There are 15 countries that have critical values σ_L^c, all of them except Haiti are from Africa (Appendix A.7.1). Five countries have a critical value larger than 0.4. These are Madagascar (0.45), Mozambique (0.46), Rwanda (0.41), Sierra Leone (0.42), and Uganda (0.55). Countries with very low value of the critical value (less than 0.1) are Benin, Mauritania, Niger, and Nigeria.

There are 49 countries with critical values σ_H^c. Singapore has the lowest value of σ_H^c of 1.67 among these countries. Other countries that have relatively low σ_H^c are Hong Kong (2.38), Japan (2.82), Norway (1.96), Thailand (2.80), and Zimbabwe (2.84). On the other hand, United Kingdom has the largest critical value of 252 (Appendix A.7.2).

The remaining 50 countries have negative σ^c (Appendix A.7.3). However, these results are based on the benchmark value of 4% depreciation rate. More countries will have critical value σ_L^c, and fewer countries will have σ_H^c for a choice of larger depreciation rate. Many of the countries with a negative critical value will also move out of this category for a different choice of depreciation rate.

5 Estimated values of σ ($\hat{\sigma}$)

In the previous section, we have calibrated σ^c. We now estimate the actual σ ($\hat{\sigma}$) from country time series data to compare those to σ^c.

The most popular and frequently used equations to estimate σ in the literature are the three first-order conditions of the CES production function for the capital-output, labor-output and capital-labor ratios. These equations are linear in parameters and therefore, convenient for estimation. The first of these three equations relates capital-output ratio with the Jorgensonian user cost of capital, which combines interest, depreciation, and tax rates and the relative price of investment goods. Under constant returns to scale, the estimated coefficient of the user cost is the aggregate σ. The user cost variable cannot be constructed as data on the tax rates are not available at the cross-country level. The simplest way to overcome the problem could be to treat the tax rates invariant over time so that only the constant term in the equation would be affected. But this would undoubtedly be a flawed assumption as taxes on capital goods have decreased in many countries over last couple of decades. In addition, Chirinko and Mallick (2007, p. 3) have raised concerns about the estimation of σ from the capital-output equation using aggregate data. They show that if capital-output ratio and user cost of capital are I(1) and cointegrated, and factor shares are constant in the long the run, then capital-output equation will always give a value of σ equal 1 independent of the production technology. The second equation equates labor-output ratio with real wage. Data for the latter variable are also not available at the cross-country level. The third equation that equates capital-labor ratio with the ratio of two input prices can also not be estimated because of the reason mentioned above.

Another possibility could be estimation of the second-order Taylor approximation to the CES production function around σ =1, first introduced by Kmenta (1967, p. 180) and estimated by, among others, Zarembka (1970) and Duffy and Papageorgiou (2000). This equation is also linear in parameters, and requires data on output-labor and capital-labor ratios.[14] However, Thursby and Lovell (1978) showed that σ is estimated from the Kmenta approximation of the CES function with large bias and mean square error. The direction of bias can be upward or downward and does not get smaller with larger sample size. When σ departs from 1, the bias in all parameter estimates increases. Since the Kmenta approximation is a truncated series of second order, the remainder term becomes an omitted variable in the regression. Moreover, the Taylor series itself converges to the underlying CES function only on a region of convergence and the Kmenta approximation is a divergent Taylor series outside that region.

Given the limitations mentioned above, we are led to estimate the following normalized CES production function using non-linear least squares (NLS) to obtain σ for each country.

$$Y_t/Y_0 = A_t\left[\alpha\left(K_t/K_0\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\alpha\right)\left(L_t/L_0\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \text{--- (14)}$$

[14] The second-order Taylor approximation to equation-2 is $\log y_t = c + \alpha \log k_t + \beta\left\{\log k_t\right\}^2 + e_t$, where y_t is that output-labor ratio, k_t is the capital-labor ratio, and $\beta = \alpha(1-\alpha)\left(\sigma-1\right)/2\sigma$. The value of σ is recovered as $\sigma = \left[\alpha(1-\alpha)/\left(\alpha(1-\alpha)-2\beta\right)\right]$. For detail, please see, Mallick (2006, p. 9-10).

To calculate the normalized value of each variable, we divide each series by its initial value. In equation 14, a Hicks neutral technology term appears and we assume its exponential growth, $A_t = A_0 \exp(\lambda t)$, where A_0 is the initial level of technology and λ is its constant growth rate. By taking logarithm to both side of equation 14, we obtain

$$\log \overline{Y}_t = \log(A_0) + \lambda t + \rho \log\left[\alpha \overline{K}_t^{\frac{1}{\rho}} + (1-\alpha)\overline{L}_t^{\frac{1}{\rho}}\right] \qquad \text{--- (15)}$$

where, $\rho = \sigma/(1-\sigma)$, $\overline{Y}_t = Y_t/Y_0$, $\overline{K}_t = K_t/K_0$ and $\overline{L}_t = L_t/L_0$. We estimate ρ by NLS and then recover value of σ, and calculate its standard error by "delta method".

The estimated values of σ ($\hat{\sigma}$) are presented in Appendixes A.7.1-A.7.3. We report the $\hat{\sigma}$ s only if these are statistically significant at least at 10% level. The value of $\hat{\sigma}$ is less than 0.1 for three countries all of which are from Sub-Saharan Africa. These countries are Central African Republic, Ethiopia and Mauritania with value of $\hat{\sigma}$ of 0.9, 0.8 and 0.9 respectively. The value of $\hat{\sigma}$ is the largest for Hong Kong of 2.18, and it is the only country that has $\hat{\sigma}$ larger than 2. Eight countries have a value of $\hat{\sigma}$ greater than 1, among which five are from East Asia.

6 Comparison of $\hat{\sigma}$ with σ^c

In the previous two sections, we have calibrated σ^c, a measure of growth potential and have estimated $\hat{\sigma}$, the ability to realize that potential. In this section, we compare these two values to understand whether countries are capable of realizing their potentials or escaping growth tragedy.

Appendix A.7.1 show that there are only two countries that have $\hat{\sigma}$ less than the critical value σ_L^c. The $\hat{\sigma}$ for Central African Republic is 0.09, which is lower than its σ_L^c of 0.22. The value of $\hat{\sigma}$ for Ethiopia is 0.08, and its σ_L^c is 0.32. Mauritania has $\hat{\sigma}$ of 0.9, which is marginally larger than its σ_L^c (0.7) but it still falls within 95% confidence interval of $\hat{\sigma}$. Four countries have a very large value of σ_L^c above 0.4 (Mozambique, Rwanda, Sierra Leone and Uganda), but their values of $\hat{\sigma}$ are estimated with large standard errors so that we do not compare those (although we have found $\hat{\sigma} < \sigma_L^c$). However, all other countries with σ_L^c experienced very low or negative growth rate of per capita GDP, even if $\hat{\sigma} > \sigma_L^c$.

On the other hand, there is no country that has $\hat{\sigma}$ larger than the critical value σ_H^c. Only Hong Kong has $\hat{\sigma}$ (2.18) close its σ_H^c (2.38), which falls within 95% confidence interval of $\hat{\sigma}$. All other countries with relatively low value of σ_H^c have $\hat{\sigma}$ less than 1, and the 95% confidence intervals fall outside σ_H^c.

7 Discussions and Conclusion

In this paper, we have discussed the role of σ in economic growth, especially the possibility of perpetual growth and decline. De La Granville and Solow (2004) derived the condition for perpetual growth that σ exceeds a critical value that is greater than 1. We have derived another condition under which perpetual decline is possible; actual σ must fall below another critical value that is less than 1. We have shown that the above results also carry into a model of endogenous saving. We have provided an analytical proof that steady state equilibrium exists only if σ lies between the two critical values.

Our calibration shows that many countries have σ_H^c s indicating their growth potential, but that their $\hat{\sigma}$ s are not large enough to realize this potential. We have identified several countries predominantly from Africa that have σ_L^c s. Average per capita growth of GDP in these countries is negative or very low. A small number of countries also have their actual σs less than σ_L^c s. There is a burgeoning literature devoted to explaining the African growth tragedy. The debate has mainly concentrated on the relative importance of low investment or low total factor productivity growth. In fact, growth literature emphasizes many factors including the above two as important determinants of economic growth, but it has so far ignored σ as one of the possible candidates. This paper shows that this is a costly omission.

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Figure 1: σ and growth of output.

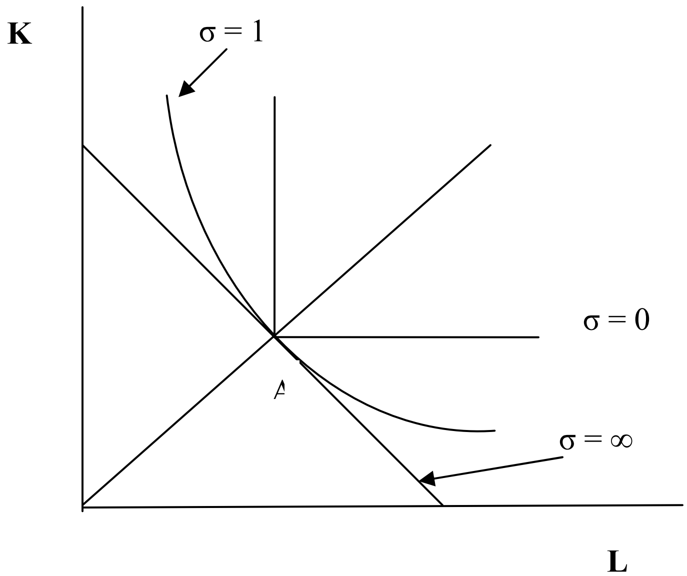


Figure 2: Behavior of $a_0^{\frac{\sigma}{\sigma-1}}$ for different values of σ >1.

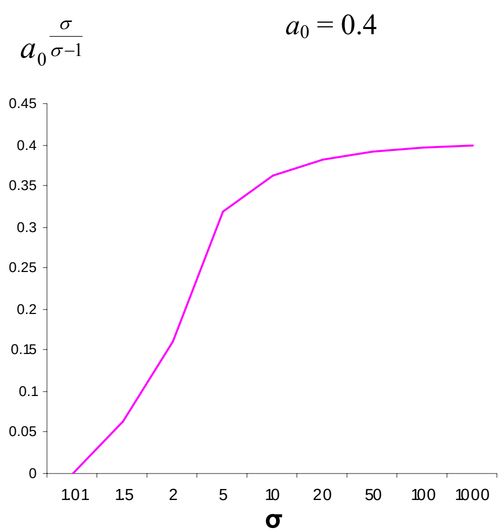


Figure 3: Behavior of $a_0^{\frac{\sigma}{\sigma-1}}$ for different values of σ <1.

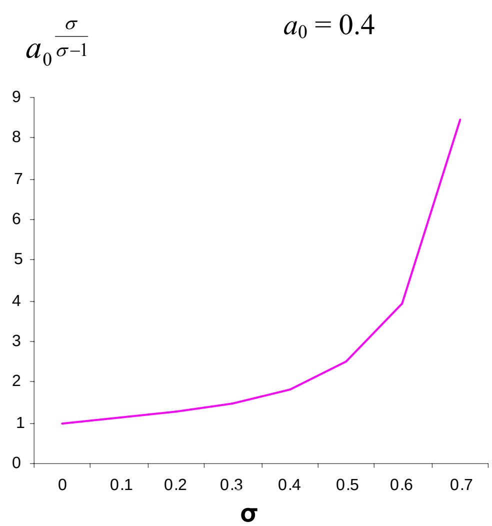


Figure 4: σ_H^c **for different capital share of income and** $s > (n+\delta)$.

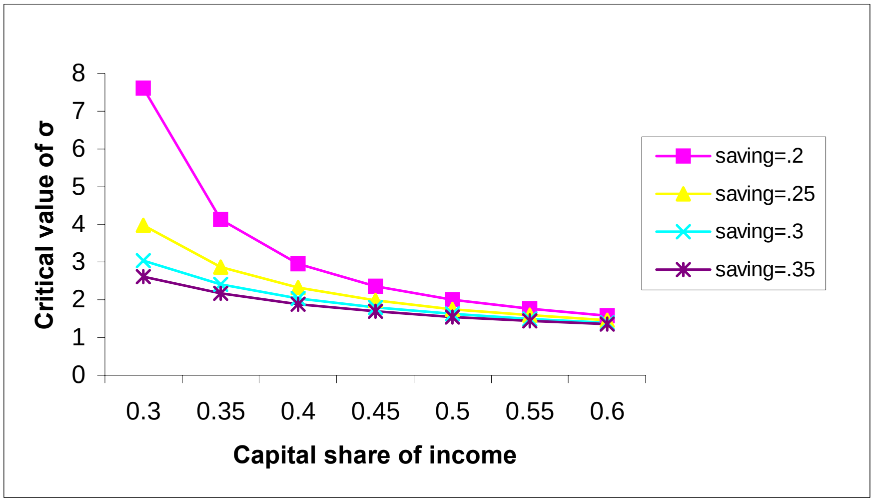


Figure 5: σ_H^c **for varying saving rate**

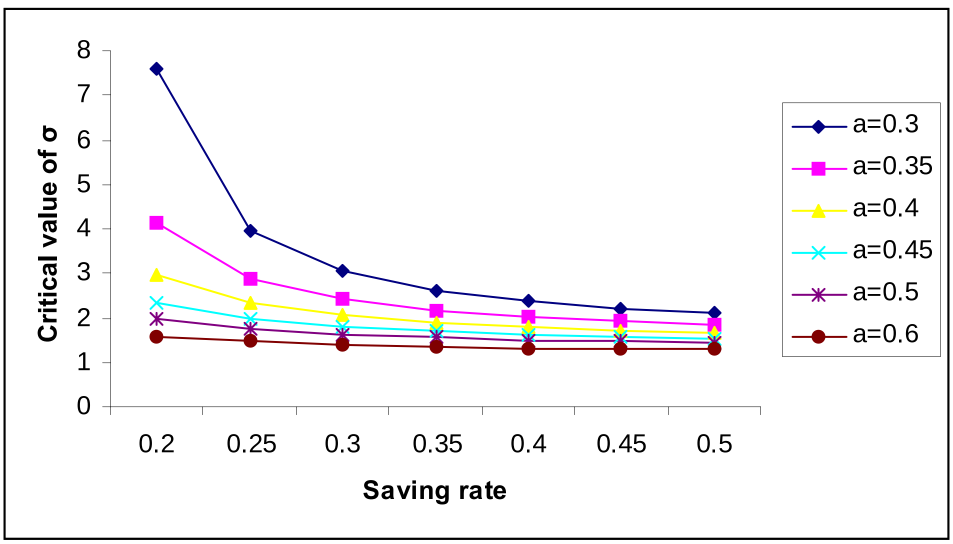
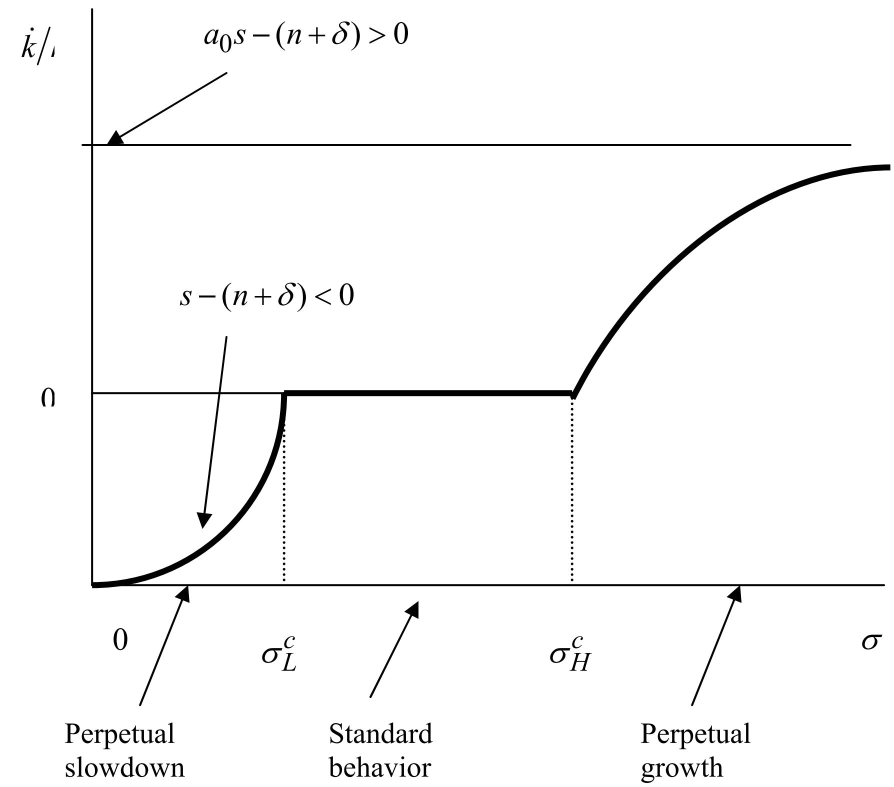


Figure 6: Value of σ and steady state growth.



Appendix

A.1: Proof of Proposition-3

Equations 9 and 10 are given by

$$\dot{k}/k = f_\sigma(k)/k - c/k - (n+\delta) \qquad \text{--- (1.9)}$$

$$\dot{c}/c = (1/\theta)\left[f'_\sigma(k) - (\rho+\delta)\right] \qquad \text{--- (1.10)}$$

In the steady state these equations become

$$f_\sigma(k^*)/k^* - c^*/k^* = (n+\delta)$$

$$f'_\sigma(k^*) = (\rho+\delta)$$

Combining these two conditions, we obtain $c^*/k^* = (\rho+\delta)/\alpha - (n+\delta)$.

Define, $\hat{x} = \log x - \log x^*$, where x^* is the steady state value of x. Log-linearization of equations 9 and 10 around the steady state gives

$$\dot{k}/k \approx \left[(\alpha-1)f_\sigma(k^*)/k^* + c^*/k^*\right]\hat{k} - (c^*/k^*)\hat{c} = (\rho-n)\hat{k} + \left[(n+\delta) - (\rho+\delta)/\alpha\right]\hat{c}$$

$$\dot{c}/c \approx \frac{1}{\theta}\left[f''_\sigma(k^*)k^*\right]\hat{k} = -\frac{(1-a)}{\theta\sigma m}\alpha^{\frac{\sigma-1}{\sigma}}(\rho+\delta)^{\frac{1}{\sigma}}\hat{k}$$

where, $m = k^{*\left(\frac{\sigma-1}{\sigma}\right)}$. If either σ>1 and $k^* \to \infty$, or if σ<1 and $k^* \to 0$, then $m \to \infty$.

The characteristic matrix of the system of equation is

$$\begin{bmatrix} \dot{k}/k \\ \dot{c}/c \end{bmatrix} = \begin{bmatrix} \rho - n & (n+\delta) - (\rho+\delta)/\alpha \\ -\frac{(1-a)}{\theta\sigma m}\alpha^{\frac{\sigma-1}{\sigma}}(\rho+\delta)^{\frac{1}{\sigma}} & 0 \end{bmatrix} \begin{bmatrix} \hat{k} \\ \hat{c} \end{bmatrix}$$

To compute the eigenvalues (λ), we write the determinant matrix

$$\det\begin{bmatrix} (\rho-n) - \lambda & (n+\delta) - (\rho+\delta)/\alpha \\ -\frac{(1-a)}{\theta\sigma m}\alpha^{\frac{\sigma-1}{\sigma}}(\rho+\delta)^{\frac{1}{\sigma}} & -\lambda \end{bmatrix}.$$

The quadratic equation in λ is given by

$$\lambda^2 - \lambda(\rho - n) - q = 0$$

where, $q = \left[(\rho+\delta)/\alpha - (n+\delta)\right]\dfrac{(1-a)}{\theta\sigma m}\alpha^{\frac{\sigma-1}{\sigma}}(\rho+\delta)^{\frac{1}{\sigma}}$. Now, $q \geq 0$, because $0 < \alpha < 1$, and from the transversality condition, $\rho > n$.

The quadratic equation has two solutions

$$2\lambda = (\rho - n) \pm \left[(\rho - n)^2 + 4q\right]^{1/2}.$$

If $q > 0$, then the two roots have opposite sign—one positive and another negative. This implies saddle-path stability. Therefore, if $k \to k^*$ when $\sigma_L^c < \sigma < \sigma_H^c$, the balanced growth path is locally saddle-path stable.

On the other hand, if $q \to 0$, the determinant of the characteristic matrix is zero, the linearized system reduces to $\dot{c} = b\dot{k}$, where b is a constant. In this case, the integral curves are straight lines, which no longer possess a singularity at the origin (Gandolfo, 1997, p. 359). Now, $q \to 0$, when $m \to \infty$ (i.e., σ>1 and $k^* \to \infty$ or σ<1 and $k^* \to 0$). The first situation occurs when $\sigma > \sigma_H^c$, and the second when $\sigma < \sigma_L^c$. No steady state equilibrium exists in either case.

A.2: Calculation of capital stock

We use the perpetual inventory method to construct capital stock series. Suppose, I_t is the gross investment at time period t, and δ is the constant rate of depreciation, then the capital stock at t, K_t is given by

$$K_t = I_t + (1-\delta)K_{t-1} \qquad \text{--- (A.2.1)}$$

Initial capital stock, K_0, is constructed using the following method. We first rearrange equation A.2.1 to get an expression for investment.

$$I_t = \left[\frac{K_t}{K_{t-1}} - (1-\delta)\right]K_{t-1} = (g+\delta)K_{t-1} \qquad \text{--- (A.2.2)}$$

where, g is the constant growth rate of capital stock.

Substituting equation A.2.2 into equation A.2.1, we obtain $K_t = (1+g)K_{t-1}$. Working backward recursively we can express capital stock in period t-1 in terms of initial capital stock as K_0, $K_{t-1} = (1+g)^{t-1}K_0$. Next, we substitute this equation into the investment equation A.2.2 to express investment in period t in terms of initial capital stock, K_0 as

$I_t = \frac{g+\delta}{1+g}(1+g)^t K_0$. Finally, take logarithms to both sides to obtain

$$\ln I_t = \ln\left(\frac{g+\delta}{1+g}K_0\right) + t\ln(1+g) = \alpha_1 + t\beta \qquad \text{--- (A.2.3)}$$

where, $\alpha_1 = \ln\left(\frac{g+\delta}{1+g}K_0\right)$, and $\beta = \ln(1+g) \approx g$. We estimate equation A.2.3 to obtain $\hat{\alpha}_1$ and $\hat{\beta}$, and given the depreciation rate we can recover K_0 as

$$K_0 = \exp(\hat{\alpha}_1)\frac{1+\hat{\beta}}{\hat{\beta}+\delta}.$$

Advantage of this method is that it uses all available information to estimate the initial capital stock.

The choice of the depreciation rate is no less important than the initial capital stock. Even if the initial capital stock is measured erroneously, the errors in the subsequent stocks are dampened over time by the depreciation rate. On the contrary, if the choice of the depreciation rate is higher (lower) than the actual, not only the initial capital stock estimate would be lower (higher), but also the capital stocks in the subsequent years would also be lower (higher) by greater amounts, because the errors are compounded in the subsequent stocks (Nehru and Dhareswar, 1993). Data on depreciation rate is not available for most of the countries. This has led the cross-country growth accounting studies to use a common depreciation rate for all countries. Following the growth accounting literature (Mankiw, Romer and Weil, 1992; Nehru and Dhareswar, 1993; Easterly and Levine, 2001) we use a common 4% depreciation rate for all countries.

A.3: List of countries by region

Region	Countries
1. Africa, West	Benin, Burkina Faso, Cameroon, Cape Verde, Cote d'Ivoire, Equatorial Guinea, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Mali, Mauritania, Niger, Nigeria, Senegal, Sierra Leone, Togo
2. Africa, Central	Burundi, Central African Republic, Chad, Democratic Republic of Congo, Republic of Congo, Malawi, Rwanda, Zambia
3. Africa, East	Comoros, Ethiopia, Kenya, Madagascar, Mauritius, Seychelles, Tanzania, Uganda
4. Africa, South	Angola, Botswana, Lesotho, Mozambique, Namibia, South Africa, Zimbabwe
5. North Africa and Middle East	Algeria, Egypt, Israel, Jordan, Morocco, Syria, Tunisia
6. America, North	Canada, Costa Rica, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, USA
7. America, South	Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Uruguay, Venezuela
8. Caribbean	Barbados, Dominican Republic, Grenada, Haiti, Jamaica, Puerto Rico, Trinidad &Tobago
9. Asia, Central	Turkey
10. Asia, East	China, Hong Kong, Japan, South Korea, Taiwan
11. Asia, South East	Indonesia, Malaysia, Philippines, Singapore, Thailand
12. Asia, Southwest	Bangladesh, India, Iran, Nepal, Pakistan, Sri Lanka
13. Europe, Eastern	Cyprus, Romania
14. Europe, Western	Austria, Belgium, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom,
15. Oceania	Australia, Fiji, New Zealand, Papua New Guinea

A.4: Investment (% of GDP) and growth rate of per capita GDP (%) by country

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	**Entire period**	
Angola	1960-1977	7.68 (1.84)	7.39 (2.21)	-1.02 (10.05)
Angola	1978-1996	7.11 (2.53)		
Argentina	1950-1974	17.26 (2.49)	17.02 (2.63)	1.19(5.75)
Argentina	1975-2000	16.80 (2.79)		
Australia	1950-1974	26.02 (2.56)	24.61 (2.58)	2.12 (2.81)
Australia	1975-2000	23.26 (1.77)		
Austria	1950-1974	24.44 (3.75)	24.96 (2.80)	3.49 (2.68)
Austria	1975-2000	25.46 (1.31)		
Burundi	1960-1979	3.02 (1.043)	5.01 (3.11)	0.48 (9.76)
Burundi	1980-2000	6.90 (3.26)		
Belgium	1950-1974	25.07 (2.12)	23.75 (2.41)	2.79 (2.06)
Belgium	1975-2000	22.48 (1.97)		
Benin	1959-1979	4.76 (1.80)	6.38 (2.85)	0.45 (3.73)
Benin	1980-2000	8.00 (2.82)		
Burkina Faso	1959-1979	6.40 (3.03)	8.37 (3.42)	0.54 (4.29)
Burkina Faso	1980-2000	10.33 (2.58)		
Bangladesh	1959-1979	9.35 (2.90)	9.88 (2.36)	1.24 (4.30)
Bangladesh	1980-2000	10.41 (1.54)		
Bolivia	1950-1974	11.55 (2.10)	10.27 (2.73)	.07 (4.08)
Bolivia	1975-2000	9.04 (2.73)		
Brazil	1950-1974	22.33 (2.95)	21.03 (3.97)	3.04 (3.66)
Brazil	1975-2000	19.78 (4.45)		
Barbados	1960-1979	22.62 (2.75)	16.86 (8.49)	4.28 (6.15)
Barbados	1980-2000	11.38 (8.50)		
Botswana	1960-1979	15.17 (9.42)	16.06 (7.02)	5.56 (6.73)
Botswana	1980-1999	16.95 (3.27)		
Central African Republic	1960-1978	4.73 (1.03)	4.64 (1.22)	-1.82 (6.40)
Central African Republic	1979-1998	4.56 (1.40)		
Canada	1950-1974	19.23 (1.34)	21.39 (2.84)	2.25 (2.69)
Canada	1975-2000	23.47 (2.30)		
Switzerland	1950-1974	28.38 (4.17)	26.99 (3.49)	1.89 (3.19)
Switzerland	1975-2000	25.66 (1.98)		
Chile	1951-1975	17.35 (4.92)	16.68 (5.19)	2.41 (5.27)
Chile	1976-2000	16.01 (5.46)		
China	1952-1975	10.81 (3.38)	14.82 (5.00)	4.01 (4.69)
China	1976-2000	18.67 (2.75)		
Cote d'Ivoire	1960-1979	10.55 (2.30)	8.08 (3.42)	.51 (5.07)
Cote d'Ivoire	1980-2000	5.74 (2.55)		
Cameroon	1960-1979	5.68 (2.01)	6.84 (2.73)	0.66 (6.26)
Cameroon	1980-2000	7.95 (2.91)		
Congo, Republic of	1960-1979	33.49 (20.12)	22.97 (19.17)	4.15 (12.47)
Congo, Republic of	1980-2000	12.95 (11.59)		

A.4 continued

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	Entire period	
Colombia	1950-1974	11.88 (1.26)	11.75 (1.61)	1.83 (2.02)
Colombia	1975-2000	11.63 (1.91)		
Comoros	1960-1979	6.70 (1.76)	7.24 (2.03)	-0.29 (6.54)
Comoros	1980-2000	7.75 (2.18)		
Cape Verde	1960-1979	15.04 (4.59)	16.35 (4.17)	3.89 (8.74)
Cape Verde	1980-2000	17.60 (3.38)		
Costa Rica	1950-1974	11.71 (1.35)	13.58 (2.71)	1.82 (3.98)
Costa Rica	1975-2000	15.37 (2.46)		
Cyprus	1950-1972	30.54 (4.88)	27.36 (5.52)	4.49 (7.59)
Cyprus	1973-1996	24.30 (4.29)		
Denmark	1950-1974	23.15 (4.28)	22.54 (3.54)	2.37 (2.84)
Denmark	1975-2000	21.95 (2.58)		
Dominican Republic	1951-1975	10.57 (3.25)	12.11 (3.05)	3.03 (4.37)
Dominican Republic	1976-2000	13.66 (1.87)		
Algeria	1960-1979	19.25 (7.68)	17.88 (6.62)	1.80 (8.00)
Algeria	1980-2000	16.58 (5.29)		
Ecuador	1951-1975	23.62 (2.39)	20.70 (4.57)	1.64 (4.38)
Ecuador	1976-2000	17.79 (4.38)		
Egypt	1950-1974	4.58 (1.12)	6.40 (2.83)	2.35 (4.09)
Egypt	1975-2000	8.15 (2.88)		
Spain	1950-1974	22.98 (4.12)	23.35 (3.14)	3.83 (4.26)
Spain	1975-2000	23.70 (1.78)		
Ethiopia	1950-1974	3.96 (1.52)	4.03 (1.27)	0.73 (5.28)
Ethiopia	1975-2000	4.09 (0.99)		
Finland	1950-1974	27.38 (2.98)	26.17 (3.68)	3.20 (3.59)
Finland	1975-2000	25.02 (3.97)		
Fiji	1960-1979	18.46 (2.36)	15.55 (4.86)	1.91 (5.11)
Fiji	1980-1999	12.64 (5.01)		
France	1950-1974	23.28 (3.86)	23.61(2.93)	2.86 (1.99)
France	1975-2000	23.92 (1.62)		
Gabon	1960-1979	15.61 (9.90)	13.52 (8.16)	3.13 (10.57)
Gabon	1980-2000	11.53 (5.60)		
United Kingdom	1950-1974	17.04 (2.95)	17.45 (2.45)	2.17 (1.98)
United Kingdom	1975-2000	17.84 (1.81)		
Ghana	1955-1977	15.78 (6.22)	11.01 (6.53)	1.24 (8.06)
Ghana	1978-2000	6.24 (0.94)		
Guinea	1959-1979	12.94 (2.00)	11.52 (2.41)	0.11 (3.73)
Guinea	1980-2000	10.11 (1.93)		
Gambia, The	1960-1979	2.67 (1.79)	5.36 (3.08)	0.81 (6.54)
Gambia, The	1980-2000	7.93 (1.31)		
Guinea-Bissau	1960-1979	22.41 (10.86)	20.53 (10.40)	2.34 (15.33)
Guinea-Bissau	1980-2000	18.74 (9.87)		
Equatorial Guinea	1960-1979	2.83 (0.74)	10.50 (17.09)	1.96 (20.75)
Equatorial Guinea	1980-2000	17.80 (21.65)		

A.4 continued

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	**Entire period**	
Greece	1951-1975	26.56 (7.92)	24.36 (6.36)	3.42 (3.98)
Greece	1976-2000	22.15 (3.12)		
Guatemala	1950-1974	8.30 (1.82)	8.14 (1.85)	1.23 (2.32)
Guatemala	1975-2000	8.00 (1.90)		
Guyana	1950-1974	25.76 (8.01)	20.46 (9.28)	0.96 (8.58)
Guyana	1975-1999	15.15 (7.28)		
Hong Kong	1960-1979	26.38 (4.67)	25.83 (3.92)	5.70 (5.10)
Hong Kong	1980-2000	25.31 (3.07)		
Honduras	1950-1974	10.29 (2.04)	11.67 (3.48)	0.37 (4.49)
Honduras	1975-2000	12.98 (4.07)		
Haiti	1960-1979	3.38 (1.91)	4.42 (2.28)	2.82 (10.12)
Haiti	1980-1998	5.46 (2.19)		
Indonesia	1960-1979	7.34 (2.98)	12.21 (5.66)	3.46 (3.94)
Indonesia	1980-2000	16.84 (3.08)		
India	1950-1974	9.50 (2.01)	10.73 (1.97)	2.62 (3.16)
India	1975-2000	11.91 (0.97)		
Ireland	1950-1974	13.46 (3.31)	16.47 (4.07)	3.73 (3.05)
Ireland	1975-2000	19.36 (2.21)		
Iran	1955-1977	15.93 (5.35)	17.89 (5.67)	3.09 (7.97)
Iran	1978-2000	19.84 (5.39)		
Iceland	1950-1974	28.64 (4.08)	26.65 (4.41)	2.91 (4.42)
Iceland	1975-2000	24.73 (3.88)		
Israel	1950-1974	34.06 (5.36)	29.80 (6.33)	3.20 (4.96)
Israel	1975-2000	25.70 (4.12)		
Italy	1950-1974	28.10 (2.86)	25.18 (3.71)	3.43 (2.45)
Italy	1975-2000	22.38 (1.72)		
Jamaica	1953-1976	25.91 (3.98)	20.51 (6.81)	1.86 (4.96)
Jamaica	1977-2000	15.12 (4.26)		
Jordan	1954-1976	8.84 (2.55)	12.29 (4.78)	2.34 (8.90)
Jordan	1977-2000	15.59 (4.03)		
Japan	1950-1974	24.87 (7.64)	28.31 (6.40)	4.82 (3.61)
Japan	1975-2000	31.62 (1.64)		
Kenya	1950-1974	17.48 (5.55)	13.30 (6.00)	1.40 (5.86)
Kenya	1975-2000	9.27 (2.84)		
Korea, Republic of	1953-1976	15.98 (6.12)	24.86 (10.54)	5.40 (4.24)
Korea, Republic of	1977-2000	33.75 (4.98)		
Sri Lanka	1950-1974	5.47 (0.91)	9.23 (4.18)	2.02 (2.76)
Sri Lanka	1975-2000	12.85 (2.52)		
Lesotho	1960-1979	5.20 (3.51)	14.93 (12.98)	2.22 (6.79)
Lesotho	1980-2000	24.20 (11.82)		
Luxembourg	1950-1974	27.69 (3.42)	24.64 (4.53)	3.05 (3.70)
Luxembourg	1975-2000	21.71 (3.41)		
Morocco	1950-1974	13.53 (5.24)	13.72 (4.37)	2.29 (5.15)
Morocco	1975-2000	13.91 (3.42)		

A.4 continued

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	**Entire period**	
Madagascar	1960-1979	2.95(0.49)	2.85 (0.56)	-0.93 (2.75)
Madagascar	1980-2000	2.75 (0.61)		
Mexico	1950-1974	17.54 (1.85)	17.92 (2.80)	2.22 (3.27)
Mexico	1975-2000	18.29 (3.48)		
Mali	1960-1979	6.78 (1.60)	7.32 (1.57)	0.14 (6.21)
Mali	1980-2000	7.83 (1.39)		
Mozambique	1960-1979	1.86 (0.47)	2.48 (1.04)	-0.70 (8.16)
Mozambique	1980-2000	3.07 (1.11)		
Mauritania	1960-1979	3.42 (2.20)	5.95 (3.46)	1.13 (12.48)
Mauritania	1980-1999	8.49 (2.48)		
Mauritius	1950-1974	10.81 (3.90)	11.80 (3.25)	2.26 (7.61)
Mauritius	1975-2000	12.75 (2.15)		
Malawi	1954-1976	14.11 (7.59)	12.52 (7.14)	1.61 (7.49)
Malawi	1977-2000	11.00 (6.48)		
Malaysia	1955-1977	13.85 (3.39)	18.92 (6.76)	3.66 (2.97)
Malaysia	1978-2000	23.99 (5.31)		
Namibia	1960-1979	27.86 (8.13)	19.00 (10.94)	1.04 (6.73)
Namibia	1980-1999	10.15 (3.80)		
Niger	1960-1979	8.00 (2.85)	6.99 (3.54)	-1.33 (5.99)
Niger	1980-2000	6.03 (3.92)		
Nigeria	1950-1974	3.71 (1.57)	6.57 (4.71)	0.27 (8.69)
Nigeria	1975-2000	9.32 (5.10)		
Nicaragua	1950-1974	9.21 (2.52)	10.50 (3.53)	0.44 (5.25)
Nicaragua	1975-2000	11.80 (3.94)		
Netherlands	1950-1974	25.01 (3.42)	23.75 (2.84)	2.55 (2.55)
Netherlands	1975-2000	22.53 (1.34)		
Norway	1950-1974	33.08 (2.75)	31.90 (3.94)	2.87 (1.74)
Norway	1975-2000	30.76 (4.59)		
Nepal	1960-1979	6.66 (3.78)	11.16 (5.30)	1.59 (3.35)
Nepal	1980-2000	15.45 (1.71)		
New Zealand	1950-1974	21.96 (2.43)	21.34 (2.42)	1.43 (3.93)
New Zealand	1975-2000	20.75 (2.30)		
Pakistan	1950-1974	11.44 (6.26)	11.46 (4.39)	2.28 (4.14)
Pakistan	1975-2000	11.48 (0.92)		
Panama	1950-1974	19.12 (5.90)	19.09 (6.49)	2.3 (4.53)
Panama	1975-2000	19.05 (7.12)		
Peru	1950-1974	30.56 (10.64)	23.90 (10.19)	1.45 (5.55)
Peru	1975-2000	17.51 (3.49)		
Philippines	1950-1974	12.47 (1.20)	14.06 (2.57)	1.94 (3.30)
Philippines	1975-2000	15.59 (2.62)		
Papua New Guinea	1960-1979	12.42 (7.04)	11.80 (5.21)	0.92 (6.72)
Papua New Guinea	1980-1999	11.18 (2.33)		
Puerto Rico	1950-1968	23.48 (4.42)	21.39 (6.78)	3.59 (3.71)
Puerto Rico	1969-1998	19.40 (8.06)		

A.4 continued

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	Entire period	
Portugal	1950-1974	17.82 (3.51)	19.53 (4.06)	4.05 (3.35)
Portugal	1975-2000	21.18 (3.93)		
Paraguay	1951-1975	6.57 (1.87)	9.73 (3.82)	1.37 (3.93)
Paraguay	1976-2000	12.89 (2.34)		
Romania	1960-1979	34.97 (6.07)	28.25 (12.53)	4.41 (12.71)
Romania	1980-2000	21.85 (13.81)		
Rwanda	1960-1979	2.33 (0.85)	3.36 (1.39)	0.47 (10.37)
Rwanda	1980-2000	4.34 (1.06)		
Senegal	1960-1979	7.69 (1.69)	7.08 (1.42)	-0.19 (4.95)
Senegal	1980-2000	6.50 (0.77)		
Singapore	1960-1977	38.00 (11.02)	41.20 (8.80)	7.25 (8.99)
Singapore	1978-1996	44.23 (4.52)		
Sierra Leone	1961-1978	1.90 (0.39)	2.78 (1.41)	-0.22 (6.72)
Sierra Leone	1979-1998	3.62 (1.52)		
El Salvador	1950-1974	5.79 (1.21)	6.59 (1.73)	0.96 (3.50)
El Salvador	1975-2000	7.36 (1.83)		
Sweden	1951-1975	23.56 (2.12)	22.05 (2.59)	2.32 (2.09)
Sweden	1976-2000	20.54 (2.10)		
Seychelles	1960-1979	10.31 (5.39)	12.62 (5.39)	3.29 (6.92)
Seychelles	1980-2000	14.81 (4.48)		
Syria	1960-1979	13.27 (4.50)	12.44 (4.14)	3.39 (11.68)
Syria	1980-2000	11.64 (3.70)		
Chad	1960-1980	13.66 (2.20)	9.89 (4.27)	0.54 (14.19)
Chad	1981-2000	6.12 (1.64)		
Togo	1960-1979	6.66 (3.64)	7.07 (3.43)	0.29 (8.53)
Togo	1980-2000	7.47 (3.27)		
Thailand	1950-1974	21.58 (7.68)	26.39 (8.64)	3.90 (4.90)
Thailand	1975-2000	31.01 (6.89)		
Trinidad &Tobago	1950-1974	9.16 (2.12)	10.03 (3.09)	3.41 (7.08)
Trinidad &Tobago	1975-2000	10.87 (3.64)		
Tunisia	1961-1980	22.25 (4.27)	18.25 (5.38)	3.26 (3.75)
Tunisia	1981-2000	14.24 (2.72)		
Turkey	1950-1974	10.50 (3.37)	13.59 (5.27)	2.81 (5.44)
Turkey	1975-2000	16.57 (5.08)		
Taiwan	1951-1974	11.51 (4.35)	15.46 (5.27)	6.26 (2.82)
Taiwan	1975-1998	19.41 (2.29)		
Tanzania	1960-1979	30.40 (6.44)	24.51 (11.03)	0.98 (8.65)
Tanzania	1980-2000	18.89 (11.67)		
Uganda	1950-1974	1.29 (0.27)	1.89 (0.96)	1.37 (7.65)
Uganda	1975-2000	2.48 (1.02)		
Uruguay	1950-1974	12.16 (2.81)	12.31 (3.09)	1.36 (5.04)
Uruguay	1975-2000	12.45 (3.40)		
USA	1950-1974	15.61 (1.47)	17.93 (2.96)	2.35 (2.50)
USA	1975-2000	20.16 (2.23)		

A.4 continued

Country	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)
		Sub-period	**Entire period**	
Venezuela	1950-1974	19.29 (5.19)	17.94 (5.70)	0.32 (4.22)
Venezuela	1975-2000	16.64 (5.95)		
South Africa	1950-1974	14.59 (2.90)	12.62 (3.99)	1.25 (2.16)
South Africa	1975-2000	10.72 (4.02)		
Congo, Dem. Rep.	1950-1973	4.82 (1.72)	5.15 (2.15)	-1.57 (7.76)
Congo, Dem. Rep.	1974-1997	5.48 (2.51)		
Zambia	1955-1977	31.56 (11.23)	20.74 (13.66)	0.31 (6.36)
Zambia	1978-2000	9.91 (3.30)		
Zimbabwe	1954-1976	50.95 (21.15)	32.02 (23.88)	2.47 (7.60)
Zimbabwe	1977-2000	13.87 (3.24)		
All Countries			**15.6 (7.86)**	**2.06 (1.65)**

(Figures in the parentheses are standard errors)

A.5: Investment (% of GDP) and growth rate of per capita GDP (%) by region

Region	Sub-period (interval)	Investment (% of GDP)		Per capita real GDP growth (%)	
		Sub-period	Entire period	Sub-period	Entire period
1. Africa, West	1	8.49 (5.72)	9.01 (4.30)	1.41 (2.05)	0.87 (1.27)
	2	9.51 (4.34)		0.38 (2.08)	
2. Africa, Central	1	13.47 (12.61)	10.54 (7.64)	2.56 (2.22)	0.52 (1.86)
	2	7.66 (3.22)		-1.36 (2.19)	
3. Africa, East	1	10.49 (9.59)	9.78 (7.48)	1.58 (1.91)	1.10 (1.34)
	2	9.10 (6.01)		0.67 (1.96)	
4. Africa, South	1	17.62 (16.99)	14.93 (9.37)	2.93 (2.89)	1.55 (2.21)
	2	12.30 (6.89)		0.27 (1.97)	
5. North Africa and Middle East	1	16.54 (9.74)	15.82 (7.33)	3.50 (1.23)	2.66 (0.61)
	2	15.12 (5.43)		1.90 (0.92)	
6. America, North	1	12.98 (5.02)	14.09 (5.22)	2.37 (0.89)	1.55 (0.82)
	2	15.16 (5.56)		0.78 (1.28)	
7. America, South	1	18.03 (7.17)	16.53 (4.86)	1.90 (1.40)	1.42 (0.85)
	2	15.06 (3.19)		0.98 (1.27)	
8. Caribbean	1	15.85 (9.31)	14.22 (6.57)	3.77 (1.97)	3.16 (0.82)
	2	12.65 (4.67)		2.58 (1.66)	
9. Asia, Central	1	10.50 (0.00)	13.59 (0.00)	3.59 (0.00)	2.81 (0.00)
	2	16.57 (0.00)		2.08 (0.00)	
10. Asia, East	1	17.91 (7.34)	21.86 (6.27)	5.50 (2.24)	5.24 (0.86)
	2	25.75 (6.87)		5.00 (1.64)	
11. Asia, South East	1	18.65 (11.96)	22.56 (11.78)	4.33(2.64)	4.04 (1.95)
	2	26.33 (11.76)		3.79 (1.79)	
12. Asia, South West	1	9.73 (3.72)	11.73 (3.13)	1.82 (1.84)	2.14 (0.67)
	2	13.66 (3.48)		2.43 (0.86)	
13. Europe, Eastern	1	32.76 (3.13)	27.8 (0.63)	5.73 (1.43)	4.45 (0.05)
	2	23.08 (1.73)		3.29 (1.40)	
14. Europe, Western	1	24.45 (4.81)	23.73 (3.63)	3.84 (1.13)	2.997 (0.61)
	2	23.02 (2.91)		2.21 (0.88)	
15. Oceania	1	19.71 (5.76)	18.32 (5.74)	2.49 (0.38)	1.60 (0.54)
	2	16.96 (5.95)		0.76 (0.96)	

Figures in the parentheses are standard errors.
Standard errors are calculated from country time averages for each region.

A.6: Population growth rate (%) by region

Region	Sub-period (interval)	Population growth rate (%)	
		Sub-period	**Entire period**
1. Africa, West	1	2.28 (0.99)	2.57 (0.53)
	2	2.83 (0.49)	
2. Africa, Central	1	2.46 (0.53)	2.63 (0.33)
	2	2.79 (0.24)	
3. Africa, East	1	2.75 (0.36)	2.59 (0.55)
	2	2.44 (0.81)	
4. Africa, South	1	2.55 (0.57)	2.56 (0.43)
	2	2.58 (0.47)	
5. North Africa and Middle East	1	3.12 (0.92)	2.88 (0.78)
	2	2.65 (0.74)	
6. America, North	1	2.84 (0.67)	2.48 (0.67)
	2	2.15 (0.74)	
7. America, South	1	2.49 (0.70)	2.15 (0.69)
	2	1.84 (0.84)	
8. Caribbean	1	1.63 (0.88)	1.47 (0.72)
	2	1.32 (0.64)	
9. Asia, Central	1	2.62 (0.00)	2.35 (0.00)
	2	2.09 (0.00)	
10. Asia, East	1	2.20 (0.64)	1.67 (0.49)
	2	1.17 (0.37)	
11. Asia, South East	1	2.58 (0.37)	2.35 (0.28)
	2	2.15 (0.44)	
12. Asia, South West	1	2.47 (0.32)	2.34 (0.33)
	2	2.21 (0.45)	
13. Europe, Eastern	1	0.97 (0.02)	0.69 (0.27)
	2	0.44 (0.50)	
14. Europe, Western	1	0.77 (0.39)	0.61 (0.27)
	2	0.47 (0.24)	
15. Oceania	1	2.21 (0.24)	1.85 (0.46)
	2	1.51 (0.73)	

Figures in the parentheses are standard errors.
Standard errors are calculated from country time averages for each region.

A.7.1: Critical value of σ < 1 (σ_L^c) by country (depreciation rate = 0.04)

Country	$\sigma_L^c < 1$	**σ**
Benin	0.0597711	0.332678
Burundi	0.1263528	
Central African Republic	**0.2208936**	**0.093348**
Congo, Dem. Rep.	0.2157893	
Ethiopia	**0.3208854**	**0.082016**
Gambia, The	0.2270898	0.278348
Haiti	0.2094736	
Madagascar	0.445044	0.564631
Mauritania	0.074316	0.098026
Mozambique	0.4619328	
Niger	0.0317994	0.171831
Nigeria	0.0310554	0.177224
Rwanda	0.4059577	
Sierra Leone	0.4221908	
Uganda	0.5528431	

(Values of σ significant at least at 10% level are reported)

A.7.2: Critical value of σ > 1 (σ_H^c) by country (depreciation rate = 0.04)

Country	σ_H^c >1	σ	Country	σ_H^c >1	σ
Algeria	19.252579		Italy	3.4668534	0.115633
Australia	4.4412428	0.230056	Jamaica	3.0283525	
Austria	3.179765	0.231658	Japan	2.8246355	0.330502
Barbados	7.5121797		Luxembourg	3.5298226	
Belgium	4.8411651	0.238228	Mexico	5.2996931	0.087222
Botswana	3.767404		Namibia	12.742866	
Brazil	4.5805242	0.126055	Netherlands	3.3793968	0.164692
Canada	6.8020772	0.236155	New Zealand	5.2035029	
Cape Verde	42.008126		Norway	1.9551068	0.76199
Chile	6.6193989		Peru	2.6632254	
Congo, Republic of	2.0989857		Portugal	7.0172398	0.488343
Cyprus	3.1212831		Puerto Rico	3.1143755	
Denmark	4.2151198	1.321511	Romania	2.1183477	
Ecuador	2.1291552	0.126152	Singapore	1.6684352	0.538795
Fiji	21.43457		South Korea	3.4997177	1.440629
Finland	3.380817	0.196602	Spain	3.2593888	0.126667
France	6.0313398	0.216632	Sweden	12.59185	1.197656
Greece	17.760609		Switzerland	5.9652882	0.154932
Guinea-Bissau	7.5445314		Tanzania	6.7689653	
Guyana	3.587578		Thailand	2.8042986	0.196835
Hong Kong	2.3784279	2.184898	Tunisia	8.4075561	
Iceland	3.7114775	0.23863	United Kingdom	251.56127	
Iran	15.465144		Uruguay	14.499303	
Israel	6.6137735	0.135631	Venezuela	5.5427171	0.261887
			Zimbabwe	2.8411369	

(Values of σ significant at least at 10% level are reported)

A.7.3: Critical value of $\sigma^c < 0$ by country (depreciation rate = 0.04)

Country	$\sigma^c < 0$	σ	Country	$\sigma^c < 0$	σ
Angola	-0.155434643		Kenya	-1.424586494	
Argentina	-22.63476089		Lesotho	-5.00837612	0.833528
Bangladesh	-0.661257795		Malawi	-1.378379192	
Bolivia	-0.797667719		Malaysia	-25.79592396	1.52205
Burkina Faso	-0.368897178		Mali	-0.162207322	
Cameroon	-0.032299815		Mauritius	-5.29630123	0.687388
Chad	-0.714371022	0.783858	Morocco	-7.738932204	
China	-102.6885012	0.548428	Nepal	-1.156496564	0.563015
Colombia	-1.311435635	0.146666	Nicaragua	-0.652384918	
Comoros	-0.080853353		Pakistan	-0.989376445	
Costa Rica	-0.954382135	0.114007	Panama	-4.771682145	0.264794
Cote d'Ivoire	-0.046955477		Papua New Guinea	-1.604652141	
Dominican Republic	-1.731345522	0.503423	Paraguay	-1.19553967	1.279504
Egypt	-0.011291037		Philippines	-5.624267457	0.07539
El Salvador	-0.040134935		Senegal	-0.042961474	
Equatorial Guinea	-1.480698214		Seychelles	-5.867717552	0.872653
Gabon	-4.212460952		South Africa	-2.545047022	
Ghana	-0.800897597		Sri Lanka	-0.426908518	0.428039
Guatemala	-0.242772095	0.088517	Syria	-1.278753401	
Guinea	-1.792041308		Taiwan	-3.890932242	1.282201
Honduras	-0.851113805	0.112279	Togo	-0.040632408	
India	-1.167373833		Trinidad &Tobago	-1.103560501	
Indonesia	-2.654998475	1.138845	Turkey	-4.422391774	0.685593
Ireland	-139.1475222	0.684165	USA	-11.58237732	0.643052
Jordan	-0.615892279	0.331228	Zambia	-6.556422334	0.133313

(Values of σ significant at least at 10% level are reported)