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STUDIES

GROWTH FUNCTIONS, SOCIAL DIFFUSION, AND SOCIAL CHANGE*

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Abstract: Owing to the spectacular currency of information and communication technologies, the diffusion of innovations has become one of the most exciting research topics in the social sciences in the past decade. This study gives an account of the most basic types of growth functions, and then inspects the broad applications of this diffusion of technological innovations. The second half of the study surveys the endeavors which seek to apply the use of growth functions to the broadest possible areas of social change via the long waves of economic development and logistic substitution processes.

Keywords: logistic function, growth functions, diffusion of innovations, bi-logistic growth, logistic substitution

INTRODUCTION

Relevance in social theory and practical use – this duality elevated studying the diffusion of innovations to the rank of one of the most curious subjects of research in the social sciences many decades ago. Certainly, the current spectacular diffusion of information and communication technologies (ICTs) also gave impetus to research interests in this field.

This study can be considered a kind of attempt to sum up related literature. In the first part I will give some introductory or elementary examples which will in turn help describe the basic types of growth functions. In the second part I will show how such means can be applied in relation to the diffusion of innovations. The third part sets the foundations for proceeding to the final part of this study, where I will gradually leave the areas of both technological and social diffusion and describe experiments which extend the application of growth functions over various fields of analyzing social change.

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GROWTH FUNCTIONS

Early Beginnings: Logistic Mapping

In nature as well as in society the processes of self-reproduction or following patterns where the actual increment of a given population somehow depends on the actual size of that population are extremely important. This idea appeared in social sciences more than two hundred years ago. In 1798 Malthus suggested that the change which occurs in the size of human population is directly proportional to the actual size of the population, which results in the following relation:

$$\dot{N} = rN. \tag{1}$$

If r is constant, then the given human population will grow according to the exponential function shaped as

$$N(t) = N_0 e^{rt}. \tag{2}$$

In fact, all processes of self-reproduction seem to follow this rule.

The history of science gives several examples of researchers who insisted fanatically on revealing cases of exponential growth in various areas. One of their most famous representatives is physicist-turned-sociologist-of-science Derek de Solla Price, who argued that all scientometric indicators seem to support the assumption that by reasonably measuring the normal rate of growth in any sufficiently large segment of science, we would get exponential growth (Price 1963).

Obviously, a population can follow this growth law which conforms to its own inherent properties only as long as stronger external factors do not interfere. Therefore the growth model described under rule (1) can have an effect only for populations which propagate in sufficiently big “domains”. Today new, i.e., still “empty” spaces usually come to be as a result of evolving ICTs; for instance, exponential growth can be observed in the development of hard disk capacity and CPU performance (Coffman and Odlyzko 1998) or Internet penetration in several countries.

Nowadays, much discussion is going on about the possible effect of the explosive diffusion of communication technologies mentioned above that it can replace or substitute people’s physical movement in many respects. Whether it will occur remains an open question. Nevertheless, the chance of such development is greatly reduced by the remarkable fact that the performance indicators of transport and communications have been following very similar paths of exponential growth concurrently, complementing rather than replacing each other, for the last 150 years. We can assume that Figure 1 refers to social processes which show great inertia and thus they are hard to change.

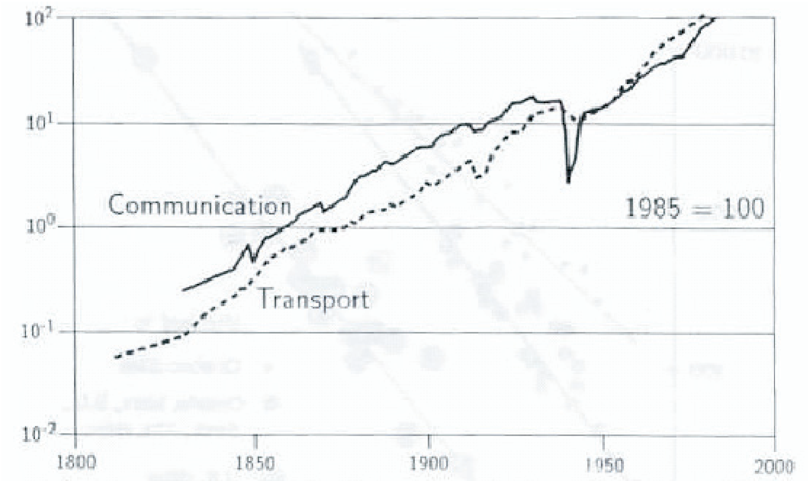


Figure 1. Transport Performance in Passenger Kilometers
and Communications Performance as No. of Messages Exchanged in France
Source: Ausubel et al. 1998

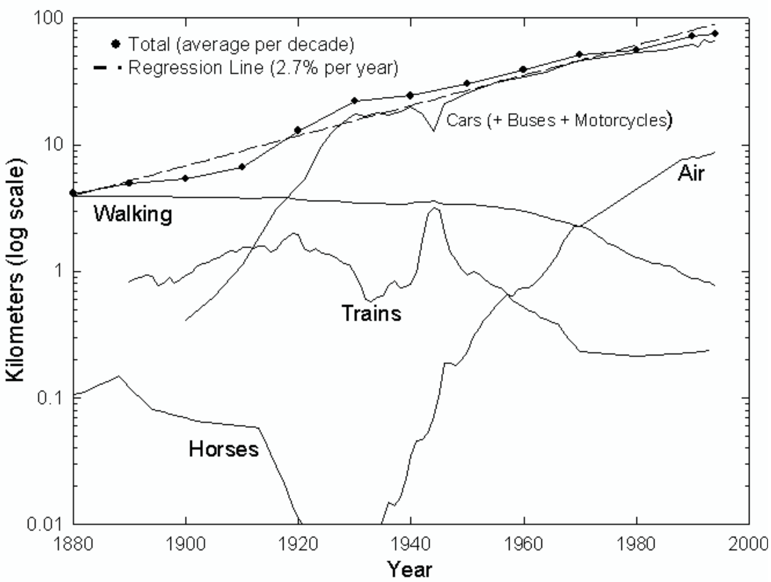


Figure 2. Daily Passenger Kilometers per Capita Taken for Various Modes of Transport
in the United States. *Source:* Ausubel et al. 1998

Obviously, for exponential growth to exist for such a long time, available space – or, more accurately, the amount of resources available – should be sufficiently large or at least it should expand over time. The latter is illustrated by *Figure 2,* which clearly shows how the successive appearance of new transport vehicles extended the "space" for transport. New vehicles allow us to set out on journeys which we could not undertake earlier, consequently, the average total daily distance taken by an individual via all possible means of transport has grown *exponentially* for the last hundred years. Research findings also reveal the existence of a similar "space-expanding" mechanism in the use of energy sources (Grübler et al 1999).

In either case, space can be initially large and it can also expand, but it will sooner or later inevitably be consumed by ceaseless exponential growth. Dennis Meadows and his associates essentially relied on this script in their 1972 report for the Club of Rome when they added a new aspect to the Malthusian proposition, saying that industry also grows exponentially due to its own inherent properties. Thirty years later they published an updated and expanded version of their book, where they claim that Earth as a space for human activities became too little because the process "shot up" beyond the necessarily existing limits to growth and thus it cannot go on (Meadows et al. 2002).

It is by no means a new idea that exponential growth cannot go on in the longer term. There were people who recognized this as early as in the nineteenth century. However, the fundamental question was how the model of exponential growth can be altered in order to embed the limits to growth. In three articles published between 1838 and 1847, Belgian mathematician François Verhulst proposed a solution which became highly appreciated later (Cramer 2004).

Assuming that every stable population has a distinct saturation level *K,* Verhulst added correction item $\left(1-\frac{N}{K}\right)$ to the model of exponential growth defined under (1):

$$\dot{N} = rN\left(1-\frac{N}{K}\right) \tag{3}$$

Verhulst tested this model already in his first article through specific applications and forecasts on the population growth of France, Belgium and Russia, while he also introduced the term "*logistic* growth", common today, in his second article, published in 1845. However, social scientists soon forgot about this model and its inventor, too. Late nineteenth-century chemists helped the above function survive as the so-called auto-catalytic function. As such, it was discovered again for the purposes of demography in 1920 by American scientists R. Pearl and L.J. Reed, although they mention Verhulst and his pioneering role only in an article published later, in 1922, and only in a single footnote. It was G.U. Yule who revived the term "logistic growth" in 1925, and finally the model set off for glory in the social sciences.

The Mathematics of Logistic Mapping

Since this success is by no means independent of the mathematics of the model and the resulting function, we will have a closer look at this mathematics below. The solution of model (3) is the three-parameter function

$$N(t)=\frac{K}{1+e^{-n-b}} \tag{4}$$

whose graph forms the well-known S-curve. However, the parameters in formula (4) can be easily substituted with other parameters. If Δt is the time needed for the process to get from saturation percentage 10 to 90 and t_m is the point in time when the inflection point occurs, then formula (4) takes following shape:

$$N(t)=\frac{K}{1+e^{-\frac{\ln 81}{\Delta t}(t-t_m)}}. \tag{5}$$

This new formula has the advantage that the three parameters included can be easily interpreted, and their value can be predicted well before the completion of the whole process on the basis of available data. Empirical applications can be further simplified through the procedure known as the Fisher–Pry transformation, which means that if $F=\frac{N}{K}$, then the following correspondence exists:

$$\ln\frac{F}{1-F}=rt+b \tag{6}$$

i.e., if N is a logistic function, then the expression $\frac{F}{1-F}$, represented on a logarithmic scale, should result in a straight line (Fisher and Pry 1971).

The discussion above shows that the logistic function gives a simple and elegant solution to the problem of describing processes which near some level of saturation. However, the same qualities may also be disadvantageous. In the eyes of scientists, the world often proved to be fundamentally simple, or sometimes perhaps elegant too, but why should it be always so? Therefore it is time to seek experiments which have a different approach to the issue of describing growth processes which near the saturation level.

Other Early Experiments

Chronologically, the first instance of such an approach can be definitely associated with self-taught British mathematician Benjamin Gompertz. His starting point is a relation which is very similar to the Malthusian formula (1):

$$\dot{N}=-rN. \tag{7}$$

However, it is different in that here parameter r can be interpreted as some hazard rate which influences the survival of individual entities or “the power of mortality”. In his study written in 1825, Gompertz starts form the premise that r increases exponentially with age, and eventually sets forth the function

$$N(t) = Ke^{-be^{-at}} \tag{8}$$

which became known later as the Gompertz survival law (Formoso 2005). Apparently, formula (8) gives a function which represents an S-shaped curve (see *Figure 3*). Unlike the logistic function, however, this curve is not symmetrical. The inflection point is reached at about one third of saturation.

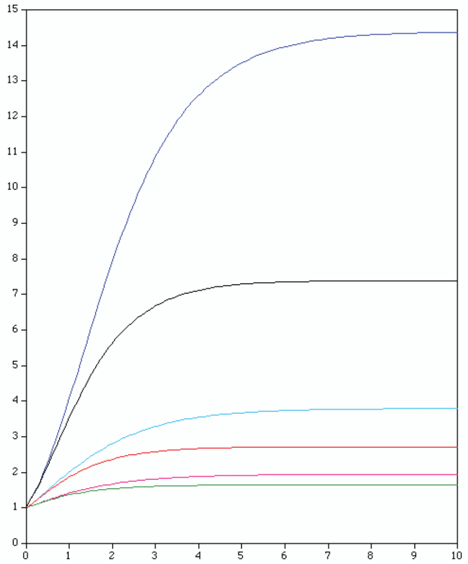


Figure 3. The Shape of the Gompertz Function for Different Parameter Values

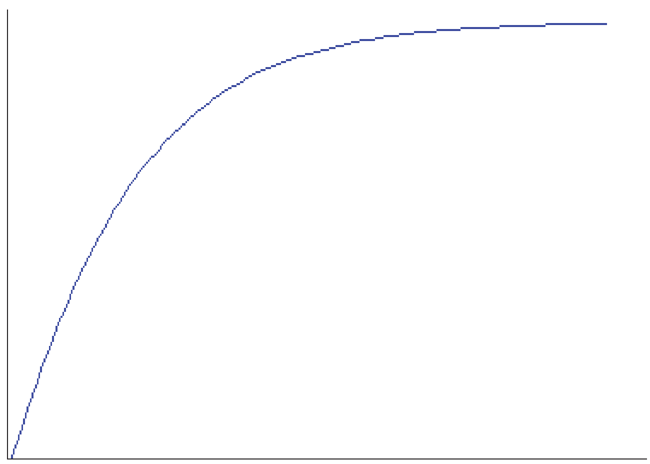


Figure 4. The Mitscherlich Function

Despite the early origin of the Gompertz function, it was more rarely applied in social sciences than the logistic function. Nevertheless, it was commonly used for modeling demographic processes, particularly the rates of fertility or age-dependent internal migration (Valkovics 2000).

In contrast with the functions described above, the birth of the following classic S-curve, the so-called von Bertalanffy function has no direct relation to the social sciences. It was developed in order to predict how the length of a shark changes as a function of its age (von Bertalanffy 1938). If we assume that the difference between length at birth and maximum length with rate constant K "decays" exponentially, then the von Bertalanffy function results the following formula:

$$N(t) = K(1 - be^{-rt}). \tag{9}$$

Now this function is applied in a wide range of areas, although it still reflects the conditions of its birth in that it is particularly preferred in population biology (Grandcourt et al. 2005).

We can get an even more special growth function through the von Bertalanffy function if we assume that the process starts from zero, which results in the following formula:

$$N(t) = K(1 - e^{-rt}). \tag{10}$$

Again, it is a growth process nearing the saturation level at a decreasing rate, but it does not represent an S-curve, since this so-called Mitscherlich function has no inflection point (see *Figure 4*). It is easy to understand that the von Bertalanffy and Mitscherlich functions derive from the differential equation

$$\dot{N} = r(K - N). \tag{11}$$

Models of Growth

Since we passed the fourth growth function above, it is worth sorting these functions out somehow. The differential equation for the logistic model, shaped as

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$

can be also generalized in the following formula:

$$\dot{N} = rN^{\alpha}\left(1 - \left(\frac{N}{K}\right)^{\beta}\right)^{\gamma} \tag{12}$$

Clearly, it is an entirely formal procedure, which is hard to justify especially because the equation above has no solution for arbitrary positive values of α, β, and γ (Tsoularis and Wallas 2002). However, it is also evident that we received a more general model this time because we can easily reach the formula for exponential growth and the equation of logistic growth from (12). Moreover, the differential equation (12) allows us to produce additional new models. Particularly, the following special case of (12), where α=γ=1, is worth noting:

$$\dot{N} = rN\left(1 - \left(\frac{N}{K}\right)^{\beta}\right). \tag{13}$$

The solution of this model, analyzed by Richards in 1959, is the function

$$N(t) = H + \frac{A - H}{\left[1 + Te^{-r(t-t_m)}\right]^{\frac{1}{T}}} \tag{14}$$

which defines an S-curve running from the bottom limit H to the top saturation level A. This so-called Richards function is also known as the general logistic function (Lei and Zhang 2004).

Several other models could be produced with the procedure described above (Tsoularis and Wallas 2002). However, we have already "manufactured" the most fundamental ones, so we do not proceed further along this line.

DIFFUSION OF TECHNOLOGICAL INNOVATIONS

The areas of application for the almost half a dozen growth functions described above range from population biology through medicine and demography to sociology. Although the logistic function has lost its monopoly in social science applications, it retained a hegemonic position, which may be due to its origins: early researchers who studied the diffusion of technological innovations gave special attention to the logistic function during their initial steps already.

Early Preliminaries

The first really empirical research projects analyzed the diffusion of agricultural innovations. The study carried out by Ryan and Gross in 1943 on the diffusion of hybrid seed corn in the state of Iowa can be considered a pioneering work (Ryan and Gross, 1943). However, a paper on a similar subject, written by Zvi Griliches in 1957, has proved to be really influential (Griliches 1957). The symmetrical S-curves presented in his study (see *Figure 5* below) have been symbolic icons of applying logistic growth in the social sciences, and the study itself has been one of the most cited works ever since its publication (David 2003).

Most early research studies on the diffusion of innovations (Mansfield 1961) revealed some sort of an S-curve and almost automatically identified this curve with the logistic function.

However, the presence of such a curve illustrates the relative slowness of initial diffusion. Recognizing this problem, Rogers (1995) assumed that people's "willingness to adopt an innovation" follows normal distribution within the human population. He also argued that a new innovation would spread along an S-curve because different categories of the population adopt it at different times. Yet it is not evident that some sociologically relevant characteristic really follows normal distribution within the human population. Suffice it to say that this is the case in so-called scale-free networks and the power law distribution which characterizes them. Nevertheless, we still have to find the answer to the question "Why would it be an S-curve?" And, if it is an S-curve, *what is it like?*

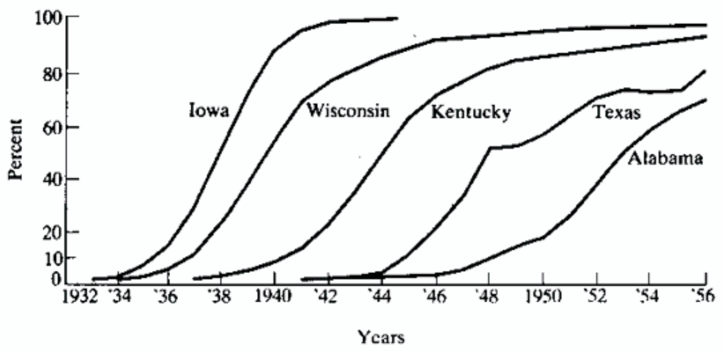


Figure 5. Percentage of Total Acreage Planted with Hybrid Corn in Different U.S. States.
Source: Griliches 1957.

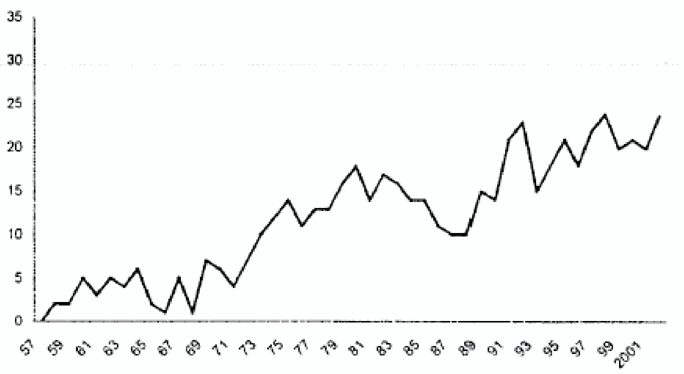


Figure 6. Citations to Griliches' Study
Source: David 2003.

Diffusion as a Process of Disseminating Information

When seeking the answers to the questions above, it is often assumed that potential users adopt the new technology as soon as they learn about it. At this point the issue of technological diffusion turns into a problem of disseminating information, so we have to examine the types of such dissemination (Geroski 2000).

The Dissemination of Information in Mass Communication

Let us first assume that information is transmitted from some central source and reaches percent of the population of size K within a unit of time. Clearly, if $\alpha=1$, then the flow of information immediately reaches all members of population and diffusion is instantaneous. If $\alpha<1$, then the spread of information can be described by the differential equation

$$\dot{N} = \alpha(K - N) \qquad (15)$$

which is, in terms of its form, identical with the Mitscherlich model introduced in (10).

The Information Process of Interpersonal Communication

It is useful to draw a distinction between the *"hardware"* and the *"software"* aspects of new technology (Rogers 1994, 2002). Much of the information on the *existence* of hardware probably spreads as it is described under formula (15). However, information which is required for the *application* of technology is a somewhat different case. Much of this information is knowledge which can be acquired in use only, and it is rarely disseminated through technical documentation or user guides. James Coleman and his associates (Coleman et al. 1966) who also found an S-curve when they studied the diffusion processes of antibiotics, concluded that the influence of pharmacies and advertisements is rather limited when compared to physicians' experience gained through interpersonal communication channels. Research studies analyzing the relationship between the worker and technology also highlighted the significance of spreading information through such channels. An operator who has been working on a machine for a longer time knows an array of unwritten tricks and techniques which are essential for running the machine safely and efficiently. Workers exchange such tricks and techniques orally or obtain them by stealth from one another (Kemény 1990).

Initially, an innovation often contradicts some standard practice, convention or social norms. Experience shows, however, that interpersonal communication is much more effective in altering existing attitudes or developing new ones than the flow of information originating in a single centre.

Obviously, this person-to-person or word-of-mouth information diffusion in which the main source of information is previous users will have a different dynamics than its counterpart which spreads from a central source. Now let us suppose that each existing user independently contacts a potential user with probability β. It is easy to see that the change in the number of actual users over time follows the rule

$$\dot{N} = \beta N(K - N)$$

when K is extracted, it results

$$\dot{N} = \beta KN(1 - \frac{N}{K})$$

then, via defining , we reach the following logistic model:

$$\dot{N} = rN(1 - \frac{N}{K}). \qquad (16)$$

Consequently, interpersonal communication allows us to discover a diffusion mechanism which produces the logistic function itself rather than a simple S-shaped curve. Of course, it cannot be the last word in this respect due to the simplifying assumptions applied, yet we have an opportunity to have a closer look at some specific diffusion studies at this point.

Recent Applications

Reasonably, here we can direct our attention to one of the information technologies which transform our work and everyday life as well as our private and public spheres: the diffusion of mobile telecommunications services.

The Diffusion of Mobile Telecommunications Services

It is well known that most OECD countries show an S-curve for mobile diffusion which, after a slow and relatively long initial stage, entered the stage of rapid growth in 1997, and reached saturation already in 2002. However, there is a considerable difference between advanced and backward countries. In 2001, mobile penetration in Luxembourg and Taiwan was 96 percent, while it was only 44 and 32 percent for the United States and Canada, respectively.

Gruber and Verboven (2001a, 2001b) used a model, accounting for these differences, to analyze the diffusion of mobile telecommunications services in the European Union. It is also well known that the diffusion of mobile services was influenced by both further innovations following the introduction of this technology and regulatory decisions made by the governments. Using the logistic model, Gruber and Verboven successfully tested the effects of *technological change* or transition from analogue to digital, the timing of first licenses granted and in turn the introduction of *market competition* on the speed of diffusion. They found that, for the then 15 member states of the EU, diffusion would become stable at 60 percent of the population.

The speed of "autonomous" diffusion implied a growth rate of about 15 percent at the inflection point. The role of switch to digital technology in accelerating diffusion can be also considered significant; however, the effect of introducing a second competitor is much weaker than those of the factors introduced earlier. This latter result seems to parallel research findings published by Parker and Röller (1997), who found that the introduction of duopoly in the U.S. had a relatively small effect on diffusion speed. It is also worth noting that the later a country granted first license, the larger diffusion speed it could reach, which implies a kind of coming-up or catching-up tendency.

The flexibility of the logistic model was shown when it proved to be applicable to the diffusion of the mobile telephone occurring under the very peculiar circumstances of Central and Eastern European countries which joined the EU in 2000 (Gruber 2001). For these countries, only 17 percent was found at the saturation level. It is a remarkable regional feature that transition to digital technology had no significant effect in these countries. However, the increase in the number of competitors exerted a significant positive effect. It is also remarkable that the effect of telephone mainlines per capita was also positive, which implies that, unlike in the 15 original member countries of the EU, mobile telephony initially had a complementing rather than substituting function in these countries. They also show the tendency for latecomers to catch up. A fifteen-year catching-up period was predicted. Since here the diffusion process began in 1990, the predicted convergence year 2005 is very close to 2006, the year of international convergence for the 15 original members. Therefore diffusion processes in the two groups of countries will eventually merge.

A Mixed Model

There is a fundamental weakness in the above application of logistic mapping. Logistic growth as a process of imitation can start only when people already have others to learn from. But who would be the first ones? They could be the innovators described by Rogers. Therefore a more accurate model should distinguish at least the innovators and the imitators. For instance, we can assume that early adopters are more sensitive to information coming from a central source than people who adopt an innovation later. In an early 1969 "social contagion" model for the diffusion of consumer durables Frank M. Bass (1969) suggested that Rogers' idea should be modified in this way.

Jang and his associates (Jang et al. 2005) attempted to test such a mixed model for the diffusion of mobile telecommunications in the 29 OECD member countries and Taiwan between 1980 and 2001. They found that the effect of the central source was not significant in the selected 30 countries, which implied that diffusion occurred mostly through networks of interpersonal communication. The S-curve for the average of all countries and the total period of twenty years showed that the first five years can be interpreted as the initial phase with slow growth rate, followed by take-off, then the process started to near the saturation level around the 15th year. When the authors examined change in diffusion for the selected countries separately, they could distinguish three basic types (see *Figure 7-8*).

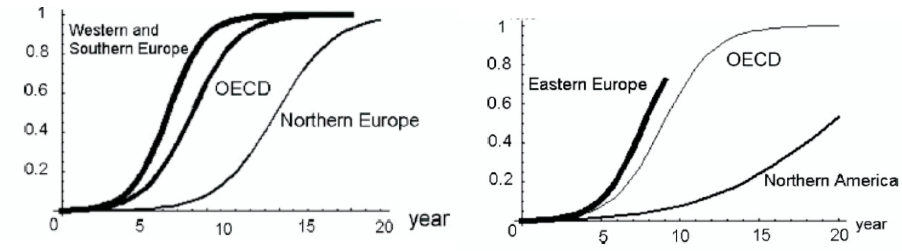


Figure 7-8. Mobile Diffusion in Various Groups of Countries
Source: Jang et al. 2005.

Northern, Western and Southern European countries show a complete S-curve, although a much slower rate of diffusion was found for the group of Northern European countries which had a pioneering role in setting off diffusion. Presently, Eastern European countries are in the phase of rapid growth, while the group of North American countries shows definitely low penetration.

SOCIAL DIFFUSION

Below we will attempt to proceed from analyzing the diffusion of technological innovations towards studying social diffusion. In preparation, we clarify some tacit assumptions and also introduce two new theoretical approaches which somewhat differ from the ones described above.

Population Characteristics

Previously, we assumed a population which is homogeneous in terms of diffusion: any of its individual members has the same chance to meet other individuals. It is by no means the case in reality. Obviously, diffusion is strongly influenced by network structure, but it would exceed the framework of this study to explore that structure. However, we can still attempt to grasp the population effect in a much simpler way.

Let us assume that we have two subpopulations of sizes K_1 and K_2 within the total population. If there is no interaction between these subpopulations at all, then the total number of actual users of an innovation will be the sum of two processes of information diffusion through interpersonal networks. Consequently, change in the number of actual users per unit time can be formulated in the following equation:

$$\dot{N} = \left[\beta_1 N_1 \left(K_1 - N_1\right) + \beta_2 N_2 \left(K_2 - N_2\right)\right]. \quad (17)$$

This formula is also relatively easy to extend over the case when there is interaction between the two subpopulations. Let us assume probability η_{12} that actual users in the first subpopulation meet potential users in the second subpopulation and probability η_{21} for the reverse case. The resulting model is as follows:

$$\dot{N} = \left[\beta_1 N_1 + \eta_{12} N_2 \left(K_1 - N_1\right) + \beta_2 N_2 + \eta_{21} N_1 \left(K_2 - N_2\right)\right]. \quad (18)$$

This model poses a really interesting case if we assume that one subpopulation adopts earlier and more rapidly than the other, because this subpopulation can serve as a source of information for the second population starting the word-of-mouth communication. Here diffusion will be the sum of two logistic growth pulses which started at the same time but one of them has a shorter diffusion time than the other and thus it reaches the saturation level more rapidly (see *Figure 9*). The aggregate diffusion curve will be an *asymmetrical* S-curve with a more or less rapid take-off and a long stage of saturation (see *Figure 10*).

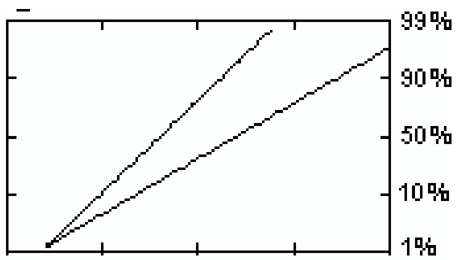


Figure 9. Two Logistic Growth Pulses Started Simultaneously (Fisher–Pry transformation)

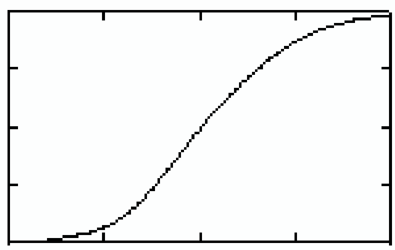


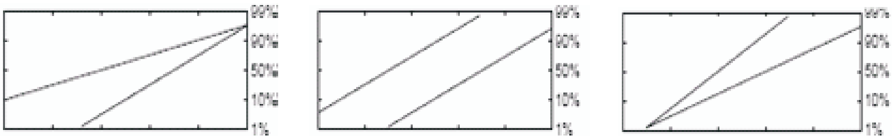
Figure 10. The Sum of the Two Logistic Growth Pulses in *Figure 8*

Limits to Growth

Figures 9 and *10* reveal a pulsing process where a new growth pulse is superposed on an earlier one. This so-called bi-logistic growth considerably broadens the scope of applying the logistic function (Meyer 1994).

Bi-logistic Growth

Bi-logistic growth can result in growth processes with a great variety of forms depending on the degree to which growth durations overlap and the nature of the relationship between diffusion speeds. In addition to the growth types depicted in *Figures 9* and *10,* three other basic types are presented in *Figure 11.*



a. b. c.

Figure 11. Variations for Bi-Logistic Growth

Examples of such processes discussed in related literature can be drawn from a great variety of areas. Japan's population change, for instance, has been shaped in the past millennium by a logistic growth process of about 800 years which can be associated with the era of the Tokugawa Shogunate and another growth process which began with the Meiji revolution in 1868, referring this case to the category shown in *Figure 11a* (Marchetti et al. 1996).

An interesting but somewhat different bi-logistic growth is exhibited in the growth process for nuclear weapons tests carried out in the United States. Here the novelty is that the two processes, like the processes in *Figure 11b,* significantly overlap and proceed, despite some delay, at basically identical speed (Meyer 1994). *Figure 12* shows an even more unusual S-curve, which apparently has two take-off phases and thus it provides a strange "meandering" shape, depicting the change in the population of England between 1541 and 1975.

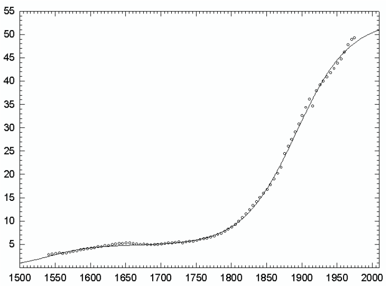


Figure 12. The Population of England, 1541–1975
Source: Marchetti et al. 1996.

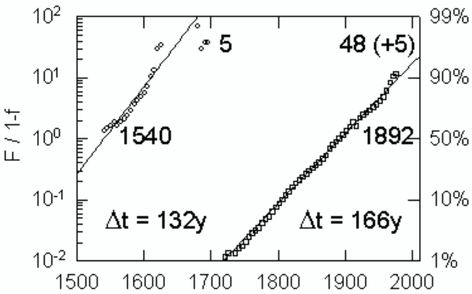


Figure 13. The Population of England, 1541–1975
(Fisher–Pry transformation)

This phenomenon is explained in *Figure 13,* which clearly shows that here we have two sequential rather than overlapping processes, one superposed on the other, with identical speeds.

Changing Limits

Having introduced the concept of bi-logistic growth, we obviously succeeded in overcoming the barrier posed by the symmetrical nature of the graph in applying the logistic function. However, we also surpassed, unintentionally, the barrier represented by saturation level K. Of course, a given logistic growth process still retains its fixed upper limit, but changing circumstances allow new, different logistic processes to emerge and in turn create new boundaries beyond the old ones.

The problem of fixed limits, however, cannot be overcome through setting new boundaries only. The saturation level itself may also change according to some defined rule. In this case, K in the formula of logistic mapping will be a status variable rather than a fixed parameter; hence we need a status equation. Two of several possible solutions can be mentioned here. According to Perrin Meyer and Jesse Ausubel (1999) saturation can be formulated as a logistic growth

$$\dot{K} = \alpha K\left(1 - \frac{K}{K_K}\right) \tag{19}$$

while John Thornley and James France (2005) suggested exponentially decaying growth shaped as

$$\dot{K} = -D(K - N). \tag{20}$$

Recent Theoretical Approaches

For the theoretical considerations above, it was assumed that diffusion is controlled by some process of disseminating information. However, many other models, which are not necessarily associated with the flow of information, can result an S-shaped curve. Below we will describe two additional versions.

Individual Decisions

Models of "social contagion" ignore the level of the individual in order to handle diffusion as a process of disseminating information. In contrast, the application of an innovation is always the result of adoption decisions made at the level of individuals, the individual members of the population.

If we try to prove that the diffusion process is based on individual decisions, then it is the simplest way to assume that individuals differ from each other in some characteristic which influences their decision to adopt an innovation. Considering an idea proposed by Granovetter (1978) we can assume that decisions made by individuals depend on whether this characteristic exceeds a certain threshold value. Clearly, the dependence of any macro-dynamics on individual decisions will be determined by the distribution of this characteristic within the population. If it follows normal distribution, and the threshold value decreases at a constant rate over time, then the change in the number of adopters over time will provide an S-curve.

This threshold value, in other words, the existence of a *critical mass* can influence the diffusion of any innovation. However, its effect can be particularly strong in the case of so-called *interactive* innovations (Mahler and Rogers 1999). We speak about *network effects* when some goods or services become more valuable for their users merely because more people use them. For such innovations, it may be important how many people adopted them already. Therefore this network effect strongly influences the eventual shape of the S-curve.

Population Density-dependence Models

At the beginning of this study, the logistic function is tacitly defined on the basis of the assumption that space available for a "propagation process" is limited and thus the logistic model is implicitly introduced as one of the *density-dependence* growth models, which are commonly used in population ecology (de Vladar 2005). As to its mathematical form, the model is entirely identical with the model of disseminating information. However, it is an important difference between them that here the rate of growth is regulated by population density, as defined by birth and death rates, rather than a process of information diffusion.

New ideas in organizational ecology represent a remarkable extension of density-dependence growth models (Hannan and Caroll 1992). This theory attempts to grasp the propagation mechanisms of organizations through two concepts, competition and legitimatization. Legitimatization continuously dissolves the barriers before new organizations, increases their "birth rates", and improves their chances of survival. At the same time, however, the increasing number of organizations strengthens the competition for resources, which can result in decreasing birth rates and increasing mortality rates. Heuristically, we can conclude that such density-dependence models also lead to some kind of S-curve.

SOCIAL DIFFUSION AND SOCIAL CHANGE

Obviously, an innovation is not only technology materialized in an artifact but it can also be an idea or action. Consequently, growth functions can provide an efficient means of understanding *social change* (Grübler and Nakicenovic 1996). To support this case, I wish to present a comprehensive yet, as to specific examples, *illustrative* tableau in the final part of this study.

Logistic Substitution Processes

The diagram on the left side of *Figure 14* was already shown in *Figure 5.* There it aimed to demonstrate how the effect of successive innovation waves can gradually expand space so as to make room for exponential growth with a constant rate. Here novelty can be seen on the right side of the figure, where we plotted the same processes as on the left side (as well as in *Figures 15* and *16*), except for absolute numbers. In *Figures 14b* to *16b* the relative *share* of various transport vehicles and energy sources within the total supply of transport and energy was plotted using the Fisher–Pry transform.

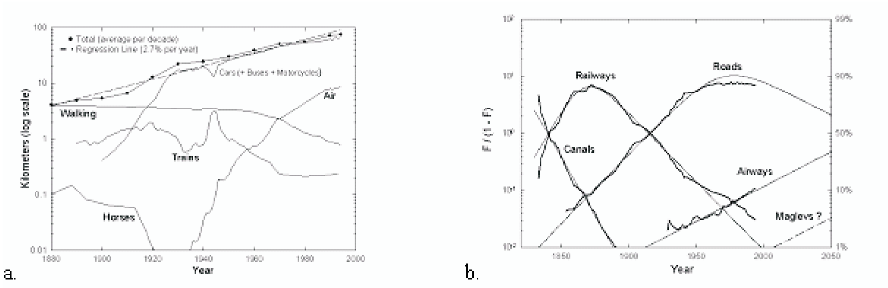


Figure14. Transport: Exponential Envelope and Logistic Substitution
Source: Ausubel et al. 1998; Marchetti 1994.

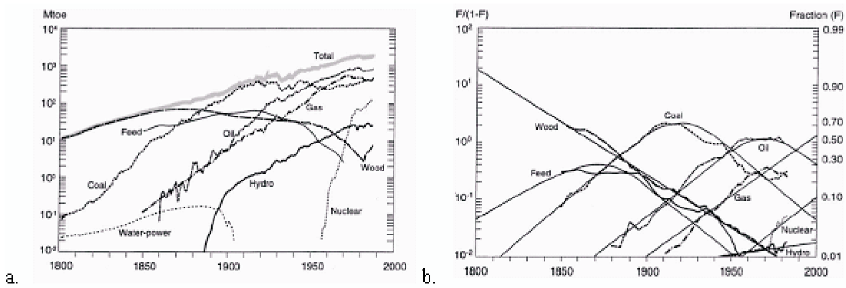


Figure 15. Primary Energy Sources: Exponential Envelope and Logistic Substitution
Source: Grübler et al. 1999.

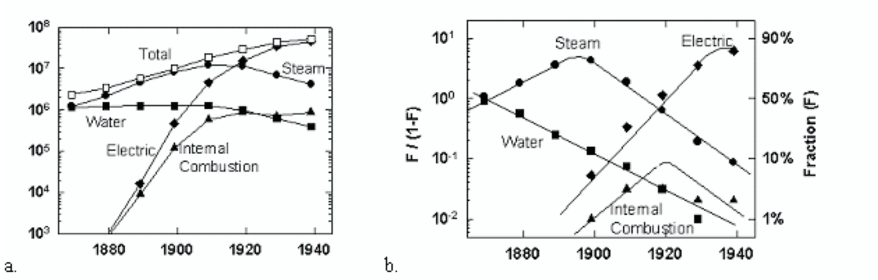


Figure 16. Electric Energy Sources: Exponential Envelope and Logistic Substitution
Source: Ausubel and Marchetti 1996.

The varied “relief” in *Figures 14b* to *16b* shows a series of overlapping or sequential *logistic substitution processes* (Marchetti and Nakicenovic 1979; Meyer et al. 1999). For example, it is highly visible in Figure 14b that around the 1920s coal, and then in the 1970s crude oil and natural gas passed their maximum shares, respectively. The surprising overlap between these turning points and the two most significant economic crises of the past century as well as the turning points of the Kondratiev cycles implies that logistic substitution processes may contribute original considerations to understanding the “long waves” of economic development.

Figure 17 shows the innovation waves for the most important transport networks over time in the United States. It is worth noting that these innovation waves followed each other in about 50-year intervals, which correspond to the Kondratiev cycles.

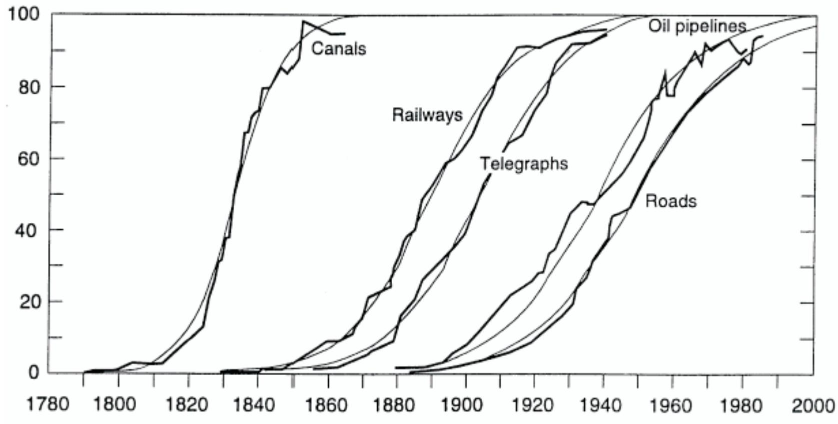


Figure 17. The Diffusion of Transport Infrastructures in the U.S.
Source: Grübler et al. 1999.

Research on long waves, which gained momentum especially during the oil crises of the 1970s, has remained intense (Marchetti 1986). Let us highlight a model developed by Gerald Silverberg and Doris Lehnert (1996) which successfully simulated the multiple processes of logistic substitution described above. In this model they enhanced Goodwin's model of growth cycles (Goodwin 1967). They assume that, on the one hand, economy consists of a large number of technologies which can be described by linear equations of production with constant coefficients; on the other hand, a continuous flow of innovations, materialized in fixed assets, enters the economy over time. The economy so constructed – and acting as a kind of filter – transforms this continuous process of innovation into a pattern of logistic substitution. Essentially, the authors set up a model which is equivalent to a large-dimensional Lotka–Volterra system with stochastically perturbed coefficients. Therefore it is particularly remarkable that, despite its multivariate nature, the model generates artificial time series (for unemployment, profitability and productivity), which carry the characteristics of low-dimensional chaos.

Omnipresent Productivity

We have already mentioned that Verhulst tested his model in his earliest papers through forecasts on population dynamics. The analysis of demographic processes has had an important role in applying logistic growth in the social sciences ever since then (Marchetti 1997a). Let us cite one interesting example only: in Norway change in life expectancy at birth followed a 113-year logistic growth which passed its midpoint in 1930 and is presently nearing saturation at 84 years of age (Marchetti et al 1996).

Marchetti and his associates received very interesting results when they examined change in the number of children in an unusual way, by the age of the father rather than the mother. They discovered very similar bi-logistic growth phenomena for Egypt and Canada (*Figure 18*). It is assumed that Moslem men in Egypt can afford to have a second wife in their household after a certain age only, and the second wave of procreation generates children who are borne by this new wife. It is really curious that an entirely different legal institution, free divorce and remarriage, led to a very similar demographic result in Canada.

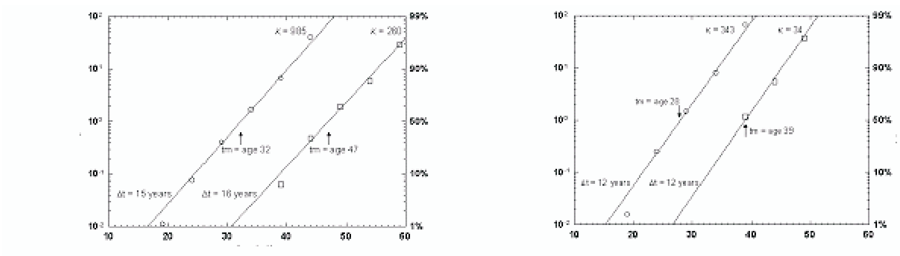


Figure 18. Number of Children by Age of Father in Egypt and Canada
Source: Marchetti et al 1996.

However, we can use "productivity" in a broader sense for instance in the case of a learning process where children acquire basic vocabulary in the first six years of their lives (see *Figure 19*).

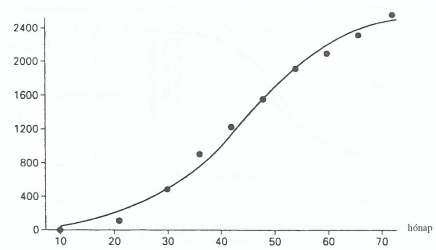


Figure 19. Number of Words Children Learn by Age
Source: Modis 1992.

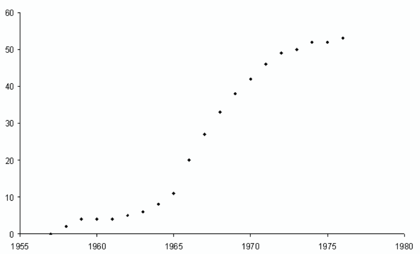


Figure 20. Number of U.S. and Soviet Satellites Sent to the Moon
Source: Fokas 1999.

Efforts similar to those of children can be also observed in the case of smaller or bigger human communities, business corporations, nations and, in fact, the populations of entire continents. An illustrative example could be the widely publicized event in the 1960s and 1970s when satellites were sent to the Moon during the contest for space between the United States and the Soviet Union (see *Figure 20*).

Another contest, the competition to conquer the Earth at the dawn of the modern age made a much more significant contribution to social history. "The culture of medieval Europe was introvertive …conspicuously, it was this continent, divided into small socio-political units, which could discover all the seas and lands of the globe and weave them into a single economic and political network within a few generations at the beginning of the modern age. The Age of Discovery demonstrates the strength of social development in medieval Europe, an organization which was deeper and more powerful than those of any other societies, past or present." (Hajnal 1988).

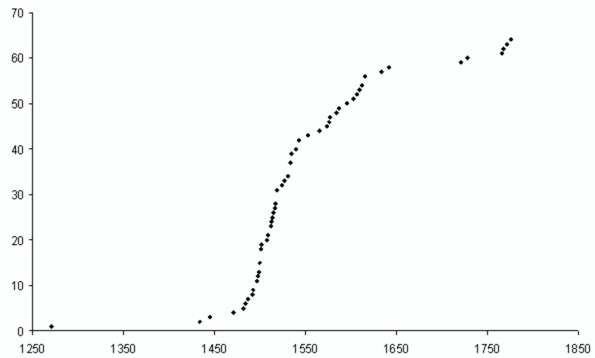


Figure 21. Cumulative Number of Geographical Discoveries Originating in Europe
Source: Fokas 1999.

Figure 21 shows that expeditions, which were launched first by Portuguese Prince Henry the Navigator in the mid-fifteenth century to explore the western coasts of Africa and, almost two centuries later, ended in the gradual exploration of Central and South America in the second half of the seventeenth century, produce an almost complete logistic curve. However, it is also true that the expansive power of European societies manifested much earlier, "mainly in the conquests in power and commerce by the Italian cities", and the expansion of the Portuguese and Spanish states was later replaced by "Dutch and English expansion, based on more sophisticated work organizations" (Hajnal 1988). Considering all these efforts, we can have a notable logistic curve which links the expansive ambitions of several countries and encompasses about 500 years!

Learning, exploration, research or the endeavors of *homo creativus* are all processes of similar nature. In a report published in 2002 under the title *Productivity versus Age,* Marchetti gives an almost hundred-page account of lifetime achievements by a host of Nobel Prize winners in three disciplines (physics, chemistry and medicine), various filmmakers and producers, famous criminals and criminal organizations, athletes (runners and javelin throwers), musicians, painters and writers, arguing that their productivity can be described by a logistic equation of growth (Marchetti 2002).

Konstandopoulos and Modis (2003) followed a similar path when they analyzed the "productivity" of the 17N (November 17) urban guerilla group in Greece. Figure 22, plotting attacks or "hits" by 17N, clearly shows that the activities of this group have come to an end.

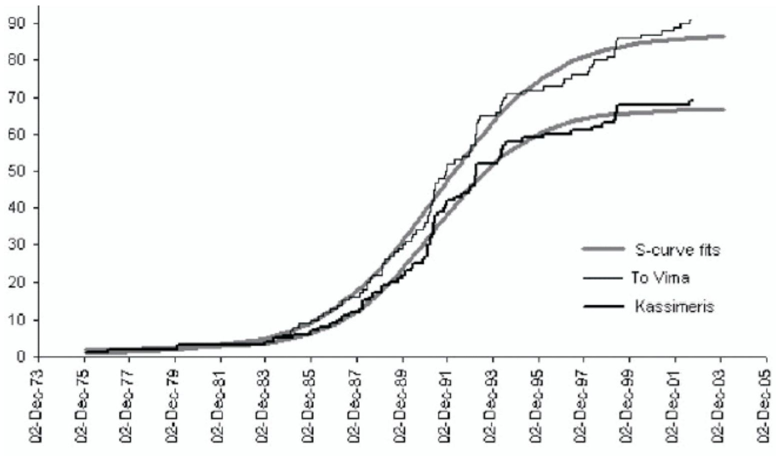


Figure 22. Number of Hits by the Guerilla Group 17N as Reported by Various Greek Newspapers
Source: Konstandopoulos and Modis 2003.

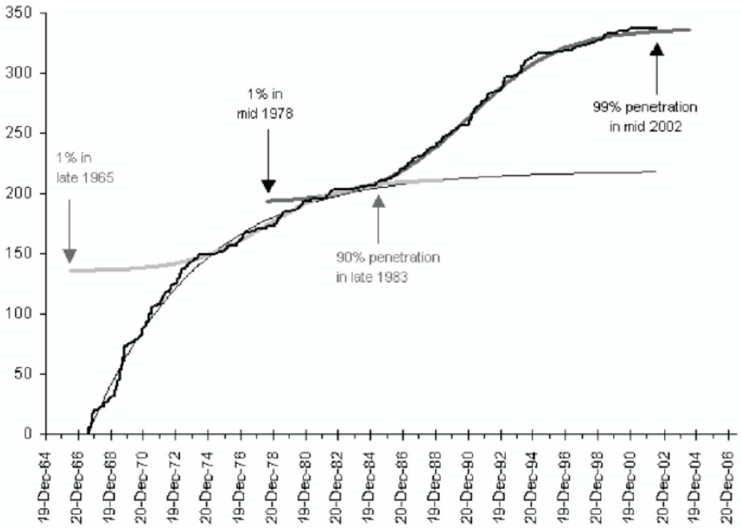


Figure 23. Urban Guerilla Activities in Greece
Source: Source: Konstandopoulos and Modis 2003.

Konstandopoulos and Modis (2003) conclude that this leftist urban guerrilla group, founded in 1974, may have been originated in resistance against the rightist military junta which reigned between 1967 and 1974. When they linked these earlier guerilla attacks to activities by the 17N, they could fit a tri-logistic function to the time series so extended (see *Figure 23*). It is really remarkable that the authors "pushed back" the start of the logistic function which represented the first wave of attacks fifty years into the past. Doing so, they used the logistic growth model to reconstruct processes in the past rather than simply describe a process or give a prognosis. Fokas and his associate made a similar attempt when they analyzed the dynamics of newspaper articles related to a scandalous statement on the radio (Fokas and Fokas 2006). The cumulative number of articles on this topic is enclosed by the square highlighted in *Figure 24.* Apparently, if we try to fit a logistic function to these data, then we have to push our function about a hundred days back into the past.

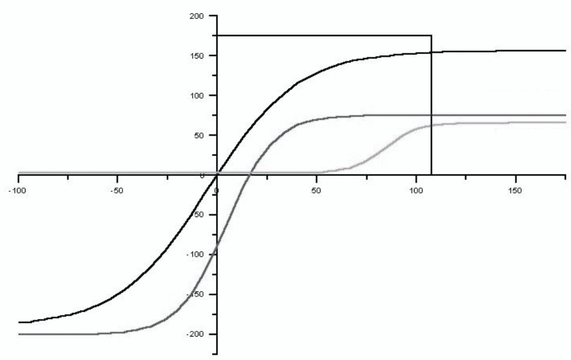


Figure 24. Dynamics for the Themes “Tilos Affair”,
”Flag-Burning” and “Writers’ Association” in the Magyar Nemzet, Logistic Fit.
Source: Fokas and Fokas 2006.

Obviously, this “withdrawal” into the past is applicable only under strict conditions. On the one hand, we have to be sure that a logistic function has an exclusive validity for this issue; on the other hand, we have to find convincing evidence that the process examined is rooted in the past. As long as these criteria have not been met, all that we proposed above can be considered a research hypothesis only. The same applies to Marchetti’s procedure, who attempted to grasp a process of two thousand years when he gave an account of how the number of saints canonized by the Catholic Church, listed according to their birthdates, changed over time (Marchetti 1997b). When analyzing the time series compiled from cumulative data, he found bi-logistic growth which included two logistic functions, spanning growth periods of 800 and 640 years, respectively; however, in order to fit the first wave, he also had to project the logistic function almost 500 years (!) backward into the past.

I am not sure if we can follow Marchetti on this path. It is not self-evident that it is possible or reasonable to compress any process of two thousand years into a few simple functions. The example above probably took us to the limits to applying growth functions. By all means, this overview clearly shows that, despite their simplicity, such functions can serve as efficient means of analyzing and understanding social change.

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