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Growth of Solid Particles in the Primordial Solar Nebula

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Some processes which are expected to lead to the growth of solid particles in the primordial solar nebula are investigated for two evolutionary phases of the nebula; one is the early phase when the nebula was contracting nearly freely, and the other is the later phase after it flattened into a gaseous disk which is rotating about the protosun.

For the free-fall phase, the collision between grains is found to be too infrequent to lead to their agglomeration for both of the two cases where the motion of grains is thermal or it shares the turbulent motion of the gas.

For the disk phase, it is found that solid particles can hardly grow to a centimeter size even in 10^9 years if only the collision is considered. However, they are found to sink towards the equatorial plane of the disk in about 10^6 years to form a high density layer where their collision is greatly accelerated. The time of this sedimentation is independent of the luminosity of the protosun.

When the density of this layer becomes greater than the Roche density, the fragmentation may occur to form the protoplanets. Finally, the effect of mass ejection from the protosun on the growth of solid particles is investigated.

§ 1. Introduction

Nowadays, it is generally believed that the sun and all the planets were formed out of the same gas cloud. Recently, a number of infrared stars have been discovered in our galaxy.[1)~4)] An attractive interpretation of them is that the central bodies are protostars which have flared up at the last stage of their dynamical contraction. For example, the spectrum of R Mon has two peaks in the visible and in the infrared regions, which have been interpreted by Low and Smith[3)] as corresponding to a central flared-up star and a surrounding dust cloud which is converting the visible radiation from the central star into the infrared radiation. Furthermore, the total mass of the solid particles in the cloud has been estimated by them to be of the order of the total mass of our planetary system. This situation suggests that R Mon is now at the early stage of forming a planetary system. It is probable that the flare-up phenomenon of FU Orion discovered by Herbig[5)] corresponds to the later stage of R Mon when the surrounding nebulosity flattened into a disk.[11)]

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Previously, many authors proposed a hypothesis that the planets were formed by the agglomeration of solid particles in the primordial solar nebula.[6),7)] This hypothesis can be summarized as follows. Small dust particles stick one another by the surface interaction and grow gradually. The solid particles which have become greater than a critical size can collect surrounding matter very efficiently by their own gravity, and finally become the planets.

Recently, the evolution of the protosun has been investigated by several authors.[8)~13)] According to Narita, Nakano and Hayashi,[11)] the luminosity of the sun was as high as $10^3 L_\odot$ when it first reached a state of gravitational equilibrium. Afterwards the luminosity decreased gradually to the present value in about 10^7 years.[9)] Before the above investigation, the luminosity of the protosun was believed to be of the order of or lower than the present value and, correspondingly, most of the theories of the origin of the solar system were based on the low luminosity of the protosun. Hence, it will be worthwhile to reconsider the problem on the basis of the new theory of the solar evolution.

In this paper, we shall investigate the agglomeration process of dust particles in the primordial solar nebula in order to find how they could grow to the size of meteorites or that of planetesimals. Here, we shall consider two evolutionary phases of the nebula; one is the early phase when the nebula was undergoing free fall until it flattened into a disk, and the other is the later phase when the nebular disk was rotating about the protosun in an equilibrium state.

In § 2, the agglomeration of grains of interstellar origin in the free-fall phase of the nebula will be studied for the two cases, where the nebula is turbulent or not turbulent. It will be found that the growth of the grains is nearly impossible for both of the cases.

In § 3, we shall investigate the equilibrium structure of the nebular disk, especially the variations of the temperature, the density and the thickness with the distance from the protosun.

In § 4, the agglomeration of grains in the disk will be studied. It will be shown that they cannot grow greater than 1 cm in radius even in 10^9 years if the effect of their sedimentation towards the equatorial plane is not taken into account. In reality, however, they sink to the equatorial plane in about 10^6 years, irrespective of the luminosity value of the protosun, and there appears a high density layer in the disk where the agglomeration is greatly accelerated. After the density of this layer becomes greater than the Roche density, the fragmentation will occur to form planetesimals, as will be outlined in § 6.

In § 5, we shall investigate the effect of mass ejection from the protosun into the nebular disk. It will be shown that the mass ejection is not necessarily important for the growth of the solid particles if we assume the rate of mass ejection as observed in T Tauri stars.

§ 2. Agglomeration of grains in the free-fall phase

It is generally believed that the solar system was formed out of the condensation of an interstellar gas cloud. According to our previous study of protostars,[8),14)] a cloud of $1M_{\odot}$ is able to contract gravitationally when its mean density is greater than about $3\times10^{-18}\,\mathrm{g/cm^3}$. During the contraction its mean temperature is kept as low as 10°K, because of efficient cooling by solid particles, as long as the cloud is transparent to radiation. Hence, even after the cloud becomes opaque, the pressure is too low to maintain gravitational equilibrium and the cloud is collapsing nearly freely.

Finally, the protosun was formed in the central region of the contracting cloud. The outer part of the cloud which has large angular momentum could not contract indefinitely because of the centrifugal force, and finally flattened into a nebular disk which was rotating about the protosun. In this section, we shall study the agglomeration process of solid particles in the gas cloud which was contracting freely.

In the interstellar clouds there are solid particles called "grain". Their physical properties have not yet been fully clarified. We assume in this paper that they are ices of H_2O, CH_4 and NH_3 containing graphite, silicates and other chemical compounds. According to Gaustad,[15)] their mean radius b_0 is about 2×10^{-5} cm and the number density is nearly given by

$$n_g=3\times10^{-13}\rho/H\,, \tag{2·1}$$

where ρ is the gas density and H is the mass of a hydrogen atom.

Now, the collision time of grains with one another is given by

$$t_c=1/n_g\sigma v\,, \tag{2·2}$$

where σ is the collision cross section,

$$\sigma=4\pi b_0^2, \tag{2·3}$$

and v is the mean relative velocity of the grains. On the other hand, the contraction of the cloud is characterized by the free-fall time,

$$t_f=\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)^{-1}=\left(\frac{1}{24\pi G\rho}\right)^{1/2}. \tag{2·4}$$

First, we consider the case where the relative velocity between the grains is given by their thermal velocity,

$$v_{th}=(9kT/4\pi b_0^3\rho_g)^{1/2}, \tag{2·5}$$

where T is the temperature of the gas and ρ_g is the density of the grains which is assumed to be $2\,\mathrm{g/cm^3}$ throughout this paper. Then, from Eqs. (2·2) to (2·5) we have

$$\frac{t_c}{t_f}=\frac{H}{3\times10^{-13}}\left(\frac{2G\rho_g}{3b_0k\rho T}\right)^{1/2}=1.0\times\left(\frac{\rho}{10^{-10}}\cdot\frac{T}{10}\right)^{-1/2}, \tag{2·6}$$

where ρ and T are in units of $\mathrm{g\,cm^{-3}}$ and °K, respectively, in the last expression. The collision occurs frequently only when $t_c < t_f$. If we take $T=10°\text{K}$, this condition is fulfilled only when ρ is greater than $10^{-10}\,\text{g/cm}^3$. However, such a high density could not be realized in the free-fall stage of the nebula, since the mean density of the nebula after it flattened into a disk is lower than $10^{-10}\,\text{g/cm}^3$ as will be shown in § 3. Hence, we can conclude that agglomeration did not occur as long as the grains collide with their thermal velocity.

Second, we consider the case where the contracting cloud was turbulent. In this case, it may be possible that the grains collide with one another with a velocity greater than the thermal velocity and thus the agglomeration may be faster.

We assume that the turbulence is in a fully developed state, because the Reynolds number is very large. The size and the turbulent velocity of the fundamental eddy are denoted by l and u, respectively. Then, the variation of turbulent velocity over the distance λ, which is much smaller than l, is given by[16]

$$v_\lambda \simeq u(\lambda/l)^{1/3}. \tag{2·7}$$

The size of the smallest eddy is given by

$$\lambda_0 \simeq l\mathcal{R}^{-3/4} \simeq l(lu/\nu)^{-3/4}, \tag{2·8}$$

where $\mathcal{R}$ is the Reynolds number and ν is the kinematic viscosity given by

$$\nu = c_m l_m/3\,. \tag{2·9}$$

Here, c_m and l_m are the mean thermal velocity and the mean free path of the molecules, respectively, which are given by

$$c_m = (8kT/\pi\mu H)^{1/2}, \tag{2·10}$$

$$l_m = \mu H/\rho\sigma_m\,, \tag{2·11}$$

where μ is the mean molecular weight of the gas and σ_m is the collision cross section of the molecules with each other.

Now, we consider the collision between two grains which are separated by a distance λ and have a relative velocity v_λ which is equal to the turbulent velocity of the gas. The grains do not necessarily collide with this initial velocity since as the grains approach their motion is slowed down by the frictional force of the gas. Thus, in order that the collision occur, the range of a grain must be greater than λ.

According to McCrea and Williams,[17] when the velocity of a spherical solid particle, v, is much smaller than the sound velocity c_m and its radius b is much smaller than the mean free path of the molecules, the frictional force exerted on the particle by the gas is given by

$$F = -(4\pi/3)\rho b^2 c_m v\,. \tag{2·12}$$

It is to be noticed that Eq. (2·12) is approximately valid even when v is nearly equal to c_m.[17] From Eq. (2·12) the range of a grain having the velocity v_λ is given by

$$l_g = \rho_g b v_\lambda / \rho c_m \,. \tag{2·13}$$

From Eqs. (2·7) to (2·13), the ratio of l_g to λ is expressed as

$$\frac{l_g}{\lambda} = A\left(\frac{u^3}{\rho l {c_m}^3}\right)^{1/2}\left(\frac{\lambda}{\lambda_0}\right)^{-2/3}, \tag{2·14}$$

$$A = \rho_g b\,(3\sigma_m/\mu H)^{1/2}. \tag{2·15}$$

The grains collide with the velocity v_λ only when l_g/λ is greater than unity, i.e.

$$A\,(u^3/\rho l {c_m}^3)^{1/2} \geq (\lambda/\lambda_0)^{2/3} \geq 1\,. \tag{2·16}$$

For this collision, the ratio of the collision time to the free-fall time is expressed as

$$\frac{t_c}{t_f} = \frac{(24\pi G\rho)^{1/2}}{n_g 4\pi b^2 v_\lambda} = B\left(\frac{l}{\rho c_m u^3}\right)^{1/4}\left(\frac{\lambda}{\lambda_0}\right)^{-1/3}, \tag{2·17}$$

$$B = (\rho/n_g 4\pi b^2)\,(24\pi G)^{1/2}\,(3\sigma_m/\mu H)^{1/4}, \tag{2·18}$$

where we have used again Eqs. (2·7) to (2·11).

From Eqs. (2·16) and (2·17) we have

$$t_c/t_f \geq B A^{-1/2}\,(c_m l/u^3)^{1/2}, \tag{2·19}$$

where the equality corresponds to the critical case, $l_g = \lambda$. If we adopt the values, $b = 2\times10^{-5}$ cm, $\rho_g = 2$ g cm^{-3}, $\mu = 2.4$, $\sigma_m = 10^{-17}$ cm^2 and $n_g = 3\times10^{-13}\rho/H$, the constants A and B defined by Eqs. (2·15) and (2·18) take the values

$$A = 0.11\,, \qquad B = 1.3\times10^{-4}\,, \tag{2·20}$$

in c.g.s. units.

All the quantities ρ, c_m, l and u appearing in Eqs. (2·16), (2·17) and (2·19) vary with time. Now, the variations of the size l and the turbulent velocity u of the greatest eddy will be estimated in the following. An eddy of size λ decays into a smaller eddy in a time of the order of λ/v_λ. For eddies whose decay time is smaller than the free-fall time, the transition to smaller eddies can be considered as steady. Then, for the greatest eddy we have

$$l/u = (24\pi G\rho)^{-1/2}. \tag{2·21}$$

Furthermore, we may put

$$u = c_m\,, \tag{2·22}$$

since the transition of the turbulent velocity to the sound velocity will be sufficiently rapid in a compressible fluid.

If we adopt Eqs. (2·21) and (2·22), we have from Eqs. (2·16) and (2·19)

$$\frac{t_c}{t_f} \geq \frac{B}{A^{1/2}}(24\pi G\rho c_m{}^2)^{-1/4} \geq \frac{B}{A^{3/2}}(24\pi G)^{-1/2} = 1.6\,, \tag{2·23}$$

where we have used the values of A and B given by Eq. (2·20). It is to be noticed that t_c/t_f takes the above minimum value, 1.6, at a stage when the three terms in Eq. (2·16) are all equal to each other, i.e. when $\rho c_m{}^2 = 24\pi GA^4$, or numerically $\rho T = 8\times10^{-18}\,\mathrm{g\,cm^{-3}\,°K}$. This value of ρT corresponds to a very early stage of the contracting gas cloud.

It might be possible that l and u take values different from Eqs. (2·21) and (2·22). For example, l and u may be given by the radius of the gas cloud and the velocity of free-fall, respectively. Thus, we may consider the two cases

$$l = (3M/4\pi\rho)^{1/3}, \qquad u = c_m \ \text{or}\ (GM/l)^{1/2}, \tag{2·24}$$

where M is the mass of the cloud which is nearly equal to one solar mass. For both of the above two cases, it has been found that the minimum value of t_c/t_f, which is calculated along the above line, is also greater than unity as long as T lies in the range between 10 and 10^3 °K.

Thus, we can conclude that the collision between grains due to turbulent motion is quite infrequent in the free-fall phase of the primordial solar nebula. Together with the result for the collision with the thermal velocity, the agglomeration of grains of interstellar origin can be neglected for the free-fall phase.

§ 3. Structure of the primordial nebular disk

According to our recent study of collapsing protostars,[11] the protosun settled to a state of gravitational equilibrium with a luminosity of about $1\times10^3 L_\odot$ and a radius of about $1\times10^2 R_\odot$. Afterwards, the protosun was contracting relatively slowly with decreasing luminosity as shown in Fig. 1. On the other hand, the remaining part of the cloud surrounding the protosun was obscuring its light for some time after the protosun had settled to equilibrium. Finally, it flattened into a nebular disk which was rotating about the protosun because of the presence of angular momentum.

Initially the gases in the disk was in turbulent motion, but the turbulence would be dissipated in a time which is several times the Kepler period of Pluto[18] because there was no energy supply to it. In this way, the nebular disk settled to an equilibrium state of circular rotation where the centrifugal force nearly counteracts the solar gravity since the pressure force is much smaller than each of these two forces.

In this section, we shall investigate the physical structure of the disk which is in thermal and gravitational equilibrium. First, we consider the balance of force in the direction z which is perpendicular to the disk. Since the density in the disk is much smaller than the Roche density as will be shown later, the gravity due to the disk itself is much smaller than the z-component of the solar gravity. Then, if we denote the half thickness of the disk by z, the balance between the gravity and the gas pressure is expressed as

$$\frac{GM_{\odot}z}{2r^3}=\frac{kT}{\mu Hz}, \qquad (3\cdot1)$$

where T is the mean temperature in the disk at the distance r from the protosun. In the following we put $\mu=2.4$*) for the mean molecular weight.

In terms of the surface density ρ_s of the disk, the mean density ρ of the gas is given by

$$\rho=\rho_s/2z\,. \qquad (3\cdot2)$$

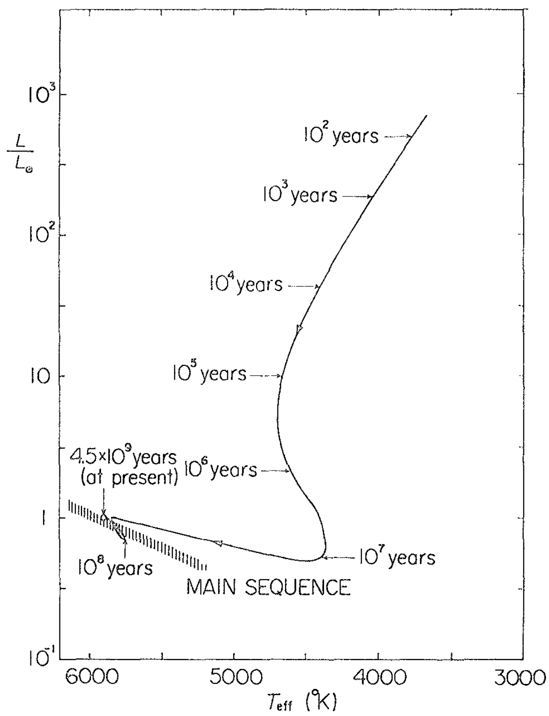

Fig. 1. The evolutionary track of the sun in the pre-main-sequence stage, according to Ezer and Cameron.[9] The time is measured from the top of the track.

A lower limit to the surface density can be estimated from the masses of the planets by comparing their chemical compositions with that of the sun, i.e. that of the primordial nebular disk.

According to Cameron,[19] the ratio of the condensable mass to the total mass was 0.003 for the inner planets, 0.1 for Jupiter and Saturn, and 0.017 for the other outer planets. A lower limit to the surface density is obtained by assuming that this total mass was distributed uniformly in a region between the circles of the radii $(r_{n-1}r_n)^{1/2}$ and $(r_nr_{n+1})^{1/2}$, where r_n is the orbital radius of the n-th planet. The values of these radii are given in Table I. The surface density must be greater to some extent, because all of the solid particles did not condense into the planets. In this paper we adopt the values of ρ_s as given in Table I, which are twice the above lower limits. Then, the total mass of the disk is $0.04M_{\odot}$.

Since the disk is very thin, T and ρ are considered as functions of r only. Besides Eqs. (3·1) and (3·2), one more relation is needed for the complete determination of the three functions, $z(r)$, $T(r)$ and $\rho(r)$. This relation is provided by the condition of steady energy transfer in the disk. Since the disk is sufficiently opaque to radiation as will be found later, the condition is given by the balance between the energy gain and loss on each surface element of the disk; the gain is due to the absorption of the solar radiation as well as the

*) In accordance with the solar chemical composition, the concentration of hydrogen by mass and that of helium are taken to be 0.70 and 0.28, respectively. For the densities and temperatures in the disk, hydrogen is all in a molecular form. Then, we have $\mu=2.4$.

Table I. Planetary data and the surface density ρ_s.

Planet	Mass[a)]	r(a.u.)	ρ_s(g/cm^2)	Radial interval (a.u.)
Mercury	0.055	0.387	1.6×10^3	0.30−0.53
Venus	0.815	0.723	1.1×10^4	0.53−0.85
Earth	1.00	1.00	7.2×10^3	0.85−1.23
Mars	0.108	1.52	2.2×10^2	1.23−2.06
Asteroids	0.0004	2.8	2.2×10^{-1}	2.06−3.82
Jupiter	318	5.20	1.5×10^3	3.82−7.04
Saturn	95.1	9.54	1.3×10^2	7.04−13.5
Uranus	14.6	19.3	3.6×10	13.5−24.0
Neptune	17.2	30.2	2.8×10	24.0−34.6
Pluto	0.9	39.8	1.0	34.6−45.7

a) relative to the Earth.

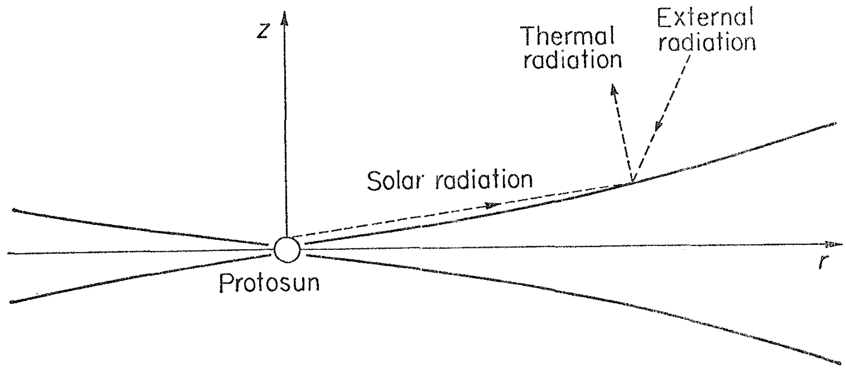

Fig. 2. The geometry of the primordial nebular disk.

external radiation such as interstellar star-light and cosmic rays, and the loss is due to the emission of the black body radiation (see Fig. 2). This problem was first studied by Safronov,[20] but our treatment is more general in that the effect of external radiation as well as the effect of radial radiation flow inside the disk are included. Nevertheless, as a final result we have obtained a very simple expression for the temperature distribution.

Since the detail of our treatment is somewhat lengthy, and, further, since the growth time of grains is almost independent of the detailed distribution of the temperature, as will be shown in § 4, the detail will be described in another paper[21] and only the final result will be given in what follows. For the protosun with the radius R and the luminosity L, the equation of energy balance is given by

$$\frac{L}{8\pi r^2}\left\{r\frac{d}{dr}\left(\frac{z}{r}\right)+\frac{4}{3\pi}\frac{R}{r}\right\}+\frac{1}{r}\frac{d}{dr}\left(\frac{acrz}{3\kappa\rho}\frac{dT^4}{dr}\right)=\frac{ac}{4}(T^4-T_{\mathrm{ext}}^4), \tag{3·3}$$

where κ is the opacity of the gas and T_{ext} is the effective temperature of the external radiation. The first and second terms on the left hand side of Eq. (3·3) correspond to the absorption of the solar radiation and the energy flow inside the disk, respectively. It has been found that the energy-flow term can be neglected for a region inside the orbit of Pluto, and that the solution of Eqs. (3·1) and (3·3) is expressed with sufficient accuracy as

$$T^4=\frac{2LR}{3\pi^2ac}r^{-3}+\left(\frac{L}{7\pi ac}\right)^{8/7}\left(\frac{2k}{\mu HGM_\odot}\right)^{4/7}r^{-12/7}+T_{\text{ext}}^4\,. \tag{3·4}$$

The grains in the disk grow considerably in a time of about 10^6 years, as will be shown in §4. Then, referring to Fig. 1, we choose an evolutionary stage of the protosun where $L=10L_\odot$ and $R=4.9R_\odot$. The values of z, T and ρ at this stage and at the orbits of the planets are shown in Table II.*) At the orbits of the inner planets the temperature is higher than 100°K so that only mineral and graphite grains exist, whereas for the position of the outer planets ice grains remain unevaporated. It is to be noticed that in the derivation of Eq. (3·1) we have neglected the gravity of the disk but this is justified because the densities shown in Table II are much smaller than the Roche densities, $3.53M_\odot/r^3$.

Table II. The structure of the disk at a stage, where $L=10\ L_\odot$ and $R=4.9\ R_\odot$.

r(a.u.)	z(a.u.)	T(°K)	ρ(g/cm³)
0.387	0.014	405	3.9×10^{-9}
0.723	0.028	274	1.3×10^{-8}
1.00	0.042	225	5.7×10^{-9}
1.52	0.070	178	1.1×10^{-10}
2.8	0.149	130	9.8×10^{-14}
5.20	0.326	97	1.5×10^{-10}
9.54	0.699	73	6.2×10^{-12}
19.3	1.75	54	6.9×10^{-13}
30.2	3.10	45	3.0×10^{-13}
39.8	4.43	40	7.5×10^{-15}

§4. Growth of grains in the disk phase

Now, we consider the agglomeration of grains in the disk which has a structure as described in the previous section. As will be shown later, the agglomeration is greatly accelerated by the sedimentation of grains towards the equatorial plane of the disk. However, to gain a clear understanding of the problem, we shall consider, first, the agglomeration process without taking into account the

*) We have put $T_{\text{ext}}=15$°K, but for this value of T_{ext} the last term in Eq. (3·4) is found to be unimportant within the Pluto's orbit. Furthermore, the first term in Eq. (3·4) is effective only within the Earth's orbit.

sedimentation, second the sedimentation process itself, and finally the combined processes.

1) *Agglomeration without sedimentation*

The mean mass and the mean radius of the grains are denoted by m and b, respectively, where

$$m=(4\pi/3)\rho_g b^3. \tag{4·1}$$

As the mean relative velocity between grains, we take their thermal velocity

$$v_{th}=(9kT/4\pi b^3\rho_g)^{1/2}. \tag{4·2}$$

Then, the growth of the grains is described by

$$\frac{1}{m}\frac{dm}{dt}=n_g\sigma v_{th}, \tag{4·3}$$

where n_g is the number density of grains and σ is the sticking cross-section, which are given by

$$n_g=3\times10^{-13}\frac{\rho}{H}\left(\frac{b}{b_0}\right)^{-3}, \tag{4·4}$$

$$\sigma=4\pi b^2 S, \tag{4·5}$$

where b_0 is the initial mean radius and S is the sticking probability.

After elimination of ρ and T with the help of Eqs. (3·1) and (3·2), Eq. (4·3) can be easily integrated and we have

$$\left(\frac{b}{b_0}\right)^{5/2}=1+\frac{5}{6}\frac{t}{t_a}, \tag{4·6}$$

where t_a is the initial growth time, i.e. the initial value of $1/n_g\sigma v_{th}$ which is given by

$$t_a=\frac{10^{13}}{9\pi}\left(\frac{\rho_g H}{2\pi b_0\mu}\right)^{1/2}\frac{t_K}{\rho_s S}, \tag{4·7}$$

where

$$t_K=2\pi(r^3/GM_\odot)^{1/2} \tag{4·8}$$

is the period of the Kepler motion. It is to be noticed that in obtaining the above solution we need not use Eq. (3·4) so that t_a is entirely independent of the luminosity and the radius of the sun. Numerically, for $\rho_g=2\ \mathrm{g\ cm^{-3}}$ and $\mu=2.4$, t_a is expressed as

$$t_a=37.3\left(\frac{b_0}{2\times10^{-5}}\right)^{-1/2}\frac{t_K}{\rho_s S}, \tag{4·9}$$

where b_0 and ρ_s are in c.g.s. units.

Now, in accordance with the temperature distribution in the disk as shown in Table II, we take $b_0=6\times10^{-6}$ cm for the region of the inner planets and $b_0=2\times10^{-5}$ cm for the outer planets.[15] The time variation of the mean radius b, which is given by Eqs. (4·6) and (4·9) where we put $S=1$ for simplicity, is shown in Table III. It is seen that the grains can hardly grow to a centimeter size even in 10^9 years if the effect of sedimentation is not taken into account.

Table III. Time variation of the mean radius (in 10^{-4} cm) of grains when sedimentation is neglected (for $S=1$).

Time (years)	0	10^3	10^6	10^9
At $r=1.0$ a.u.	0.06	5.7	91	1400
At $r=5.2$ a.u.	0.2	4.8	77	1200
At $r=19.4$ a.u.	0.2	0.52	7.9	12

2) *Sedimentation without agglomeration*

As long as the density in the disk is lower than the Roche density, the forces which act on the grains are the gravity of the protosun and the friction of the gas which is given by Eq. (2·12). The grains have the velocity of sedimentation for which the frictional force just counteracts the gravity,[17] i. e. the grains of radius b have the velocity

$$\frac{d\zeta}{dt}=-\frac{\rho_g b}{\rho c_m}\frac{GM_\odot \zeta}{r^3}, \qquad (4\cdot10)$$

at the height ζ from the equatorial plane.

Eliminating ρ and T with the help of Eqs. (3·1) and (3·2), Eq. (4·10) is expressed as

$$\frac{1}{\zeta}\frac{d\zeta}{dt}=-\frac{1}{t_s}\frac{b}{b_0}, \qquad (4\cdot11)$$

where t_s is the sedimentation time for the grain of radius b_0 which is given by

$$t_s=\frac{1}{2\pi^{3/2}}\frac{\rho_s}{\rho_g b_0}t_{\rm K}, \qquad (4\cdot12)$$

where $t_{\rm K}$ is the Kepler period given by Eq. (4·8). It is to be noticed that t_s is, again, independent of the solar luminosity and radius. Numerically, it is given by

$$t_s=2.24\times10^3\left(\frac{b_0}{2\times10^{-5}}\right)^{-1}\rho_s t_{\rm K}, \qquad (4\cdot13)$$

in c.g.s. units for $\rho_g=2$ g cm^{-3}.

3) *Agglomeration with sedimentation*

For the grains having a radius of 10^{-5} cm, t_s is much greater than t_a so that the sedimentation is not effective in the early stage. However, the grains which have grown to a certain size sink towards the equatorial plane relatively rapidly to form a region where the number density of grains is very high. In this way, the agglomeration of grains is greatly accelerated, as pointed out by McCrea and Williams[17] and by Safronov.[22] In the following the combined processes of agglomeration and sedimentation will be investigated.

The half thickness of the high density layer of grains is denoted by ζ, in contrast to that of the gaseous layer, z. The time variation of z is neglected for simplicity, since z varies at most in proportion to $L^{1/7}$ as seen from Eqs. (3·1) and (3·4). In the present case, the right-hand side of Eq. (4·4) is to be multiplied by the factor, z/ζ. Then, in terms of the new variables

$$x = b/b_0\,, \qquad y = \zeta/z\,, \tag{4·14}$$

Eqs. (4·3) and (4·11) are written as

$$\frac{dx^{5/2}}{dt} = \frac{5}{6t_a}\frac{1}{y}\,, \qquad \frac{dy}{dt} = -\frac{xy}{t_s}\,, \tag{4·15}$$

where t_a and t_s are the same as given by Eqs. (4·7) and (4·12), respectively. The initial condition is $x=y=1$ at $t=0$.

First, from Eq. (4·15) we have

$$\frac{dx^{5/2}}{dy} = -\frac{5t_s}{6t_a}\frac{1}{xy^2}\,, \tag{4·16}$$

which is integrated to give

$$x^{7/2} = 1 + \frac{7t_s}{6t_a}\left(\frac{1}{y} - 1\right). \tag{4·17}$$

Then, inserting Eq. (4·17) into the first equation in (4·15) and considering that $t_s \gg t_a$, we have

$$t = \int_1^x \frac{dx^{5/2}}{5/6t_a + (5/7t_s)\,x^{7/2}}\,. \tag{4·18}$$

The first term in the denominator of the above integrand represents the agglomeration without sedimentation, while the second term represents the effect of sedimentation. These two terms become equal to eace other at a time t_1 when y becomes 1/2 and x becomes x_1, where

$$x_1 = (7t_s/6t_a)^{2/7}. \tag{4·19}$$

Considering that $x_1 \gg 1$, we have from Eq. (4·18)

$$t_1 = 0.146\,(6t_a)^{2/7}(7t_s)^{5/7}. \tag{4·20}$$

Further, it is found that x becomes infinite at a time t_2, where

$$t_2 = 3.93 t_1\,. \tag{4·21}$$

Numerically, with the help of Eqs. (4·9) and (4·13) we have for x_1 and t_1

$$x_1 = 3.37\,(b_0/2\times10^{-5})^{-1/7}\rho_s^{4/7}S^{-2/7}, \tag{4·22}$$

$$t_1 = 682\,(b_0/2\times10^{-5})^{-6/7}\rho_s^{3/7}S^{-2/7}t_K\,. \tag{4·23}$$

The values of b_0, $b_1(=b_0x_1)$ and t_1 on the orbits of the Earth, Jupiter and Uranus are shown in Table IV, where we have used the values of ρ_s in Table I and put $S=1$ for simplicity.*) It is seen from Table IV and Eq. (4·21) that in about 10^6 years most of grains sink to the equatorial plane with a considerable growth of their size. At the same time, a high density layer of grains is formed around the equatorial plane. After the density of this layer becomes greater than the Roche density, the fragmentation of this layer into a number of planetesimals will occur, as will be described in § 6.

Table IV. Growth of grains when sedimentation is included (for $S=1$).

Distance r(a.u.)	Initially	When sedimentation becomes effective		At the Roche density		
	b_0(cm)	t_1(years)	b_1(cm)	t_R(years)	b_R(cm)	ζ_R(cm)
1.0	0.6×10^{-5}	8.6×10^4	3.8×10^{-3}	3.2×10^5	0.21	5.5×10^5
5.2	2.0×10^{-5}	1.9×10^5	4.4×10^{-3}	6.8×10^5	0.058	6.0×10^8
19.3	2.0×10^{-5}	2.7×10^5	5.2×10^{-4}	1.0×10^6	0.010	7.4×10^8

So far, we have considered only the mean size of the grains. Now, we shall briefly discuss the distribution of their size. Previously, Nakano[23] investigated the agglomeration process of gas clouds or small protostars in order to explain the mass function of stars in a star cluster. As long as the relative distribution of the size is concerned, our problem corresponds to a case which is intermediate between the cases a) and b) of his paper. From the results given in his paper it is estimated that the number of solid particles whose radius is greater than 10 times (or 100 times) the mean radius is smaller than 10^{-13} times (or 10^{-27} times) the total number of the particles. For example, if the mean radius of the solid particles is 0.1 cm near the Earth's orbit, the number of particles whose radius is greater than 10 cm is less than 10^3 in a region of radius between 0.85 and 1.23 a.u. from the sun.

§ 5. Effect of mass ejection from the protosun

It is well known that T Tauri stars are in the pre-main-sequence contraction stage. According to Kuhi,[24] they are ejecting matter to outer space with a rate of the order of $4\times10^{-8}M_\odot$ per year. In this section we investigate the effect of such mass ejection from the protosun on the growth of the grains.

The trajectory of ejected matter will be affected, to some extent, by the

*) It is to be noticed that both x_1 and t_1 are rather insensitive to the value of S.

gravity of the disk, but we assume for simplicity that it is a straight line and that the ejection is spherically symmetric around the sun. Then, from the geometry as shown in Fig. 2, the amount of mass which is captured by a part of the disk of radius between r and $r+\Delta r$ is given by

$$\Delta M_c = M_e \frac{d}{dr}\left(\frac{z}{r}\right)\Delta r\,, \tag{5·1}$$

where M_e is the mass ejected in all directions from the protosun, while the mass of this part of the disk is

$$\Delta M = 2\pi r\rho_s \Delta r\,. \tag{5·2}$$

It has been found that on the right-hand side of Eq. (3·4) the second term is predominant, i.e. T^4 is nearly proportional to $r^{-12/7}$ and, then, from Eq. (3·1) z/r is nearly proportional to $r^{2/7}$. Thus, in Eq. (5·1) we can put

$$\frac{d}{dr}\left(\frac{z}{r}\right) \simeq \frac{2}{7}\frac{z}{r^2}\,. \tag{5·3}$$

The atoms and molecules, which have been captured on the surface of the disk, diffuse into the inner layers. The time of diffusion through the depth Δz from the surface is given by

$$t_d \simeq (\Delta z)^2/l_m c_m\,, \tag{5·4}$$

where c_m and l_m are given by Eqs. (2·10) and (2·11), respectively.*) During the diffusion, the molecules of non-volatile material collide with the grains and most of them are captured in a time

$$t_c \simeq (n_g \sigma c_m)^{-1}, \tag{5·5}$$

where n_g and σ are given by Eqs. (2·1) and (2·3), respectively. Now, the diffusion length of these molecules before they are captured is given approximately by $t_d \simeq t_c$, i.e.

$$\Delta z \simeq (3\times 10^{-13}\sigma\sigma_m)^{-1/2} H/\rho\,. \tag{5·6}$$

Numerically, $\Delta z/z$ is found to be as small as $10^{-5}\rho_s^{-1}$, where ρ_s is in g/cm^2.

By capturing the molecules of non-volatile material as described in the above, the grains within the depth Δz from the surface can grow, on the average, to the radius b given by

$$\left(\frac{b}{b_0}\right)^3 \simeq \frac{\Delta M_c}{\Delta M}\frac{z}{\Delta z}\,. \tag{5·7}$$

From Eqs. (5·1), (5·2), (5·3), (5·6) and (5·7), we have finally

*) For the diffusion to the equatorial plane, $\Delta z = z$, we find from Eqs. (3·1), (3·2) and (5·3) that t_d is equal to $(1/8\sqrt{\pi})$ $(\sigma_m \rho_s/\mu H)$ t_K and this is much greater than the sedimentation time, t_s, given by Eq. (4·12).

$$\left(\frac{b}{b_0}\right)^3 \simeq (3\times10^{-13}\sigma\sigma_m)^{1/2}\frac{M_e z}{14\pi H r^3}. \tag{5·8}$$

As has been shown in § 4, the grains which are initially present in the disk sink to the equatorial plane in a time of the order of 10^5 years. Hence, as M_e in Eq. (5·8) we take the mass ejected in 10^5 years, i.e. $4\times10^{-3}M_\odot$. Then, using the values of z given in Table II, we find from Eq. (5·8) that the grains in the surface region of the disk grow to the radius, 5×10^{-4} cm, 1×10^{-3} cm and 5×10^{-4} cm on the orbits of the Earth, Jupiter and Uranus, respectively. These values of the radius are comparable to the values at 10^5 years in Table III. Then, we can conclude that, as compared with the effect of sedimentation, the mass ejection from the protosun was not important for the growth of the grains in the disk.

§ 6. Concluding remarks

In the previous sections we have examined the several processes which lead to the growth of grains in the primordial solar nebula. At present we have little knowledge on the sticking probability S of the grains, but we have found that a simple process of sticking alone does not lead to the formation of massive bodies, even if we adopt the largest possible value for S, i.e. $S=1$.

However, from the viewpoint that the planetesimals and the planets were formed out of a nebular disk which was in a state of equilibrium for a sufficiently long time, we have found that the sedimentation of the grains is very effective for the formation of massive bodies. By sedimentation a high density layer appears around the equatorial plane of the disk. When the density becomes greater than the Roche density, $\rho_R=3.53M_\odot/r^3$, the self-gravity of this layer becomes more important than the gravity of the protosun so that the fragmentation of the layer will occur, or more exactly, the sedimentation towards the center of each fragment will occur. The size of the fragments will be of the order of the thickness of the high density layer.

Using the results given in subsection 3) in § 4, we have computed the time t_R when the density of the layer becomes equal to the Roche density as well as the grain radius b_R and the half thickness of the layer ζ_R at this time. These values are shown in Table IV. It is to be noticed that t_R is very nearly equal to t_2 given by Eq. (4·21). Now, on the Earth's orbit, the mass of the fragments, $(4\pi/3)\rho_R\zeta_R^3$, is about 1.5×10^{12} g. This mass consists mostly of the grains so that, if all the grains accumulate into a solid body by the further sedimentation, the body has a radius of about 6×10^3 cm. In this way, a great number of planetesimals will be formed and the further agglomeration of them may lead to the formation of protoplanets and the planets.

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