GTES: An Optimized Game-Theoretic Demand Side Management Scheme for Smart Grid

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***Abstract*—Demand Side Management in smart grid has emerged as a hot topic for optimizing energy consumption. In conventional research works, the energy consumption is optimized from the perspective of either the users or the power company. In this paper, we investigate how energy consumption may be optimized by taking into consideration the interaction between both the parties. We propose a new energy price model as a function of total energy consumption. Also, we propose a new objective function, which optimizes the difference between the value and cost of energy. The power supplier pulls consumers in a round-robin fashion, and provides them with energy price parameter and current consumption summary vector. Each user then optimizes his own schedule and reports it to the supplier, which in turn updates its energy price parameter before pulling the next consumers. This interaction between the power company and its consumers is modeled through a two-step centralized game, objective of which is to reduce the peak-to-average power ratio by simultaneously optimizing users energy schedules and lowering the overall energy consumption in the system. The performance of the proposed game theoretic demand-side management approach is evaluated through computer-based simulations.**

Index Term - Smart grid, game theory, real-time pricing, energy optimization.

I. Introduction

Recently, smart grid has emerged as a hot research topic and attracted government, industry, and academia alike [1], [2], [3], [4]. For the successful deployment of smart grid, demand-side management or demand response [5] is crucial. Demand-side management refers to the planning and implementation of the electric utility activities, designed to influence the customers' consumption of electricity in such a fashion that produces desired changes in the shape of loads of the utility company. While demand-side management aims at producing a change in the load-shape, it needs to balance the requirement of the utility provider (i.e., the power company) and that of the customers.

Traditionally, the demand-side management technique may either shift or reduce the energy consumption. Shifting the energy consumption can effectively mitigate the aggregate energy load during the peak hours (which is the main reason for power outage and load shedding events). In this vein, the ratio of the highest peak time of energy consumption to the average consumption in a whole day, referred to as the Peak-To-Average Ratio (PAR), is used to measure the imbalance in load-shape of daily energy consumption [12]. By shifting energy consumption from the peak hours to off-peak hours, it is possible to reduce the PAR. Another demand-side management technique is to reduce the energy consumption by encouraging energy-aware consumption patterns and constructing more energy efficient housing [9]. Instead of entirely modifying the existing power infrastructure of the buildings (which may be difficult and/or time-consuming), our work considers shifting the energy use and reducing PAR for optimizing the energy consumption.

In addition, recent research studies and initiatives indicated that dynamic pricing is an efficient way to implement demand-side management in smart grid. As its name implies, in dynamic pricing, the cost of energy varies dynamically over time. The energy price may change depending on the wholesale market (to reflect the fluctuation of energy price) or the energy consumption level. By considering an adequate dynamic pricing plan as per the desired objective, smart grid customers may be provided with incentives for participating in such collaborative scheduling in return of monetary incentive. For example, they may receive a reduction of electricity bills or an amount of money for their contributed curtailment of power usage while the utility company is significantly benefited from the apparently small contribution from individual users. In the survey conducted in [6], approximately ninety percent of the customers in a smart grid initiative demonstrated money saving as the prime reason for their participation. Therefore, our work also considers monetary incentive as the principal motivation for the smart grid users.

Scheduling energy consumption for all home appliances requires considering numerous parameters and constraints. Therefore, optimizing energy consumption for several users, at the same time, is difficult in terms of computation, running time, and even convergence guarantee of the optimization algorithm. While a fully centralized optimization is not feasible for scheduling energy consumption in smart grid, an entirely distributed approach may also not be attractive due to practical issues. Because, unlike a communication network, a power grid should satisfy some specific features. Since direct connections among the users are not desired as redundant communication is unnecessary and might invite security problems, the smart grid users should only be able to communicate with the control center of the utility company. In our paper, we consider a practical smart grid architecture, whereby a two-step game [7]-based approach for a centralized optimized energy consumption schedule is proposed. The proposed game aims at reducing the system PAR by optimizing energy schedules of the users and also lowering the total energy consumption at the same time. In our proposal, we consider different objectives and conduct an extensive analysis on users' preferences about their perceived value of energy.

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In addition, the interactions between the power supplier and consumers are also considered.

The remainder of this paper is structured as follows. Section II surveys relevant research work on solving the energy optimization problem in smart grid involving real time pricing. Section III presents our considered smart grid energy consumption model. Our optimal game-theoretic centralized algorithm is proposed in Section V. The performance of the proposed algorithm is evaluated in Section VI. Finally, Section VII concludes the paper.

II. Related Work

The analysis in [6] presents case studies of dynamic pricing programs offered by electric utilities in a pilot smart grid project conducted in the United States until 2010. Smart meters were distributed to the participants of the project, and their energy consumption patterns had been studied under various dynamic pricing schemes including time of use, real-time pricing, critical peak pricing, and critical peak rebates. All smart grid customers in the project had to manually set up their schedules as per the energy price. The utility company announced new prices based on the prediction of market fluctuation and system peak load. Although the project achieved good improvement in terms of PAR reduction, it did not take into consideration how reaction from the user side could, in turn, affect (i.e., improve) these prices.

In addition, the survey and analysis in [6] also demonstrated that the biggest motivation of the participants was monetary incentive. This indicates that the attitude of the users toward the energy price is, indeed, important. Furthermore, it was suggested that the energy pricing performance could be significantly improved with the aid of automations, e.g., by having users equipped with smart thermostat to automatically reduce consumption of air conditioning and central heating during peak hours.

In the work in [8], an optimal and automatic residential energy consumption schedule framework was introduced. The framework is capable of forecasting the fluctuated energy price. The work is based on the idea that even though real-time pricing has several potential advantages, its benefits are currently limited due to lack of efficient automation systems in the building as well as users' difficulty in manually responding to time-varying prices. Thus, this work implied the potential of exploiting smart pricing in smart grid.

The work conducted in [9] introduced the use of utility function in an energy schedule optimization, where the energy generation capacity (during different hours) is limited. By applying the utility function, the power supplier is able to obtain the preference of users toward energy consumption and impose an appropriate price to limit the power usage. Also, a different energy pricing scheme for different users was introduced in [9]. However, this resulted in psychological unfairness amongst the customers. In order to overcome this issue, the work in [10] added load uncertainty to the pricing scheme.

In [11], Mohsensian-Rad *et al.* considered an energy consumption schedule game, which aims at reducing PAR by shifting energy use. Their approach comprises a totally distributed algorithm, as every user connects with one another and reviews the schedules. The game runs continuously so that if a user has a sudden change in his schedule, then the whole process recurred to find the equilibrium again. However, the connection between the users is not desired in smart grid and the schedule should be made for an interval of time ahead, e.g., for a whole day, for which a centralized control is more suitable.

In literature, centralized optimization for scheduling energy consumption exists in [12]. The scheme in [12] relies on a social welfare function in terms of the users preference toward energy consumption, which is the difference of the utility and cost of the energy. However, analysis on this approach is basic, particularly since it involves two quantities of different units (i.e., utility of energy and cost of energy). This work was not extended to analyze how to select appropriate utility functions and take into account monetary incentive. In contrast, a different approach was introduced in [13] that does not focus on the system improvement such as load balancing or energy consumption scheduling in smart grid. Instead, that work attempted at capturing the game (i.e., interactions) amongst different agents of smart grid, namely consumers, retailers, and the energy market. In the next section, we present an existing system model of smart grid for power, energy cost, and load control modeling [11].

III. Existing System Model

We employ the smart grid infrastructure as the basis for our system model as shown in Fig. 1. For further details of the smart grid infrastructure, interested readers are referred to [14], [15]. In our model, we consider a scheme with one energy supplier (i.e., the power company) and multiple consumers (i.e., users). The consumers are equipped with smart meters, each of which is assumed to have the capability of scheduling the energy consumption of the respective consumer-residence. The smart grid users are connected to the power company's control centers. The bi-directional communication between the center and its consumers is possible through the smart meters. We assume that the smart meters are able to monitor and collect all the data of electrical appliances plugged into the grid. The smart meters also have the ability to turn on/off and choose the level of energy consumption for these appliances if necessary. In addition, the smart meters are capable of informing the power company or the supplier about users' energy consumption schedules.

The existing smart grid model comprises three aspects, namely power system modeling, appropriate energy cost function modeling, and load control on consumer-end modeling. These are described below.

1) Power system modeling: The work in [11] constructs the following power system model. The model assumes a set of consumers in the considered smart grid obtaining electric power from the power company (e.g., as shown in Fig. 1) as $\mathcal{N}$. Assume that the number of consumers is $N \triangleq |\mathcal{N}|$. For every user, $n \in \mathcal{N}$, let l_n^h indicate the total load during hour $h \in \mathcal{H} \triangleq \{1, .., H\}$ where $H = 24$. Then, let the daily load

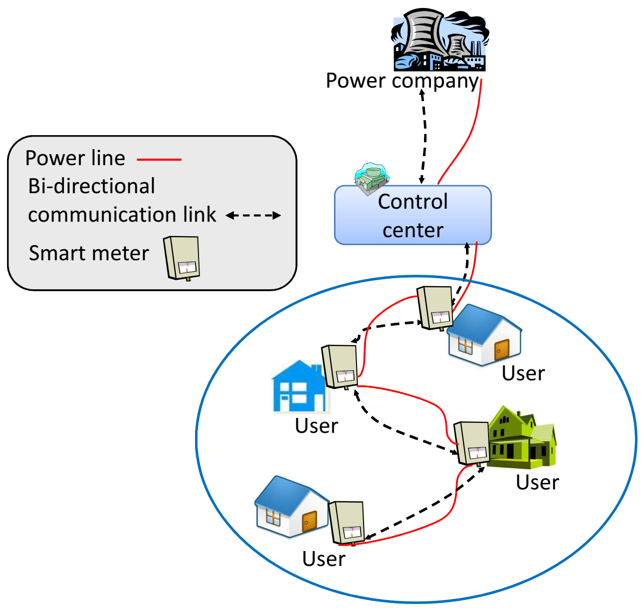

Fig. 1. Considered system architecture of the smart grid.

for the n^{th} user be denoted by the energy consumption vector, $\mathbf{l}_n \triangleq \{l_n^1, ..., l_n^H\}$. L_h, which represents the overall load (for all the users) during every hour of a day (i.e., $h \in \mathcal{H}$), may be computed as follows.

$$L_h \triangleq \sum_{n \in \mathcal{N}} l_n^h \tag{1}$$

The daily peak and average load levels are calculated as

$$L_{peak} = \max_{h \in \mathcal{H}} L_h, \tag{2}$$

and

$$L_{avg} = \frac{1}{H} \sum_{h \in \mathcal{H}} L_h, \tag{3}$$

respectively. Therefore, the Peak to Average Ratio (PAR) in the load demand is given by:

$$\mathbf{PAR} = \frac{L_{peak}}{L_{avg}} = \frac{H \max_{h \in \mathcal{H}} L_h}{\sum_{h \in \mathcal{H}} L_h}. \tag{4}$$

2) Modeling an appropriate energy cost function: The hourly energy price varies with real time, proportional to the system energy consumption during that hour. In this way, the consumers will have incentives to refrain from using electricity at peak hours, resulting in a lower PAR. The cost function is an increasing function, i.e., the energy cost increases along with the total energy consumption, L_h. For every $h \in \mathcal{H}$, we have $C_h(L_h^1) < C_h(L_h^2), \quad \forall L_h^1 < L_h^2$. This assumption is made for the higher energy consumption to have more impact on the increase on the energy price. Also, the energy cost functions are assumed to be strictly convex. In other words, for every $h \in \mathcal{H}$, $C_h(\theta L_h^1 + (1-\theta)L_h^2) < \theta C_h(L_h^1) + (1-\theta)C_h(L_h^2)$. Here, L_h^1, L_h^2, and θ are real numbers such that $L_h^1, L_h^2 \geq 0$ and $0 < \theta < 1$.

3) Modeling load control on consumer-end: Let A_n indicate the set of residential electrical equipment (e.g., air conditioner, heater, kitchen appliances, television, fridge, and so forth) for the consumer $n \in \mathcal{N}$. For every appliance for the n^{th} user, an energy consumption scheduling vector is constructed as follows.

$$x_{n,a} = [x_{n,a}^1, ..., x_{n,a}^H], \tag{5}$$

where the component $x_{n,a}^h$ denotes the corresponding one-hour energy consumption, which is scheduled for the appliance a by consumer n during hour h. Let l_n^h, L_h, and $L_{-n,h}$ denote the total energy consumed by the n^{th} consumer, all consumers, and all users except the n^{th} consumer. These can be computed as follows.

$$l_n^h = \sum_{a \in A_n} x_{n,a}^h, \quad h \in H. \tag{6}$$

$$L_h = \sum_{n \in N} l_n^h, \quad h \in H. \tag{7}$$

$$L_{-n,h} = L_h - l_n^h, \quad h \in H. \tag{8}$$

Then, at each consumer-residence, the task of the smart meter is to determine the optimal schedule of energy consumption vector, $x_{n,a}$, for all the appliances belonging to that consumer. The schedule for any appliance has several constraints, such as power consumption, minimum and maximum energy requirement to finish operation, starting time of the schedule, and stopping time of the schedule. The feasible set for energy consumption scheduling vector is defined to satisfy these conditions.

For each user $n \in N$ and each appliance $a \in A_n$, we denote the minimum daily energy consumption as $E_{n,a}^{min}$ and the maximum daily energy consumption as $E_{n,a}^{max}$. In order to shift and reduce energy consumption at the same time, there should be some bound on the energy consumption vector for all the appliances of the residence. The users also need to select the time interval, $\mathcal{H}_{n,a}$, during which the appliances can be scheduled. Let the beginning and end time instants of this scheduling interval be denoted by $\alpha_{n,a} \in \mathcal{H}$ and $\beta_{n,a} \in \mathcal{H}$, respectively (i.e., $\alpha_{n,a} < \beta_{n,a}$). The scheduling interval must be equal to or longer than the normal time required for completing the operation for each appliance. For an appliance with schedulable operation, the scheduling interval will be more than the normal requirement time. On the other hand, for an appliance with non-schedulable operation, its scheduling interval is either a whole day with constant energy consumption (e.g., refrigerator) or equal to the normal requirement time in order to avoid further change to the plan.

The power level of each appliance, $a \in A_n$, also needs to be constrained by the minimum standby power level $\gamma_{n,a}^{min}$, and the maximum power level $\gamma_{n,a}^{max}$. Then, it is clear that:

$$\gamma_{n,a}^{min} \leq x_{n,a}^h \leq \gamma_{n,a}^{max}, \quad \forall h \in \mathcal{H}_{n,a}. \tag{9}$$

$$x_{n,a}^h = 0, \quad \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}. \tag{10}$$

Finally, the feasible energy consumption scheduling set corresponding to user n is defined as follows.

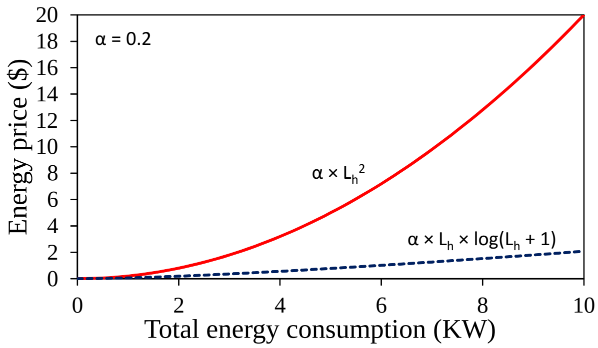

Fig. 2. The increase of energy price with the total energy consumption in case of the proposed energy price function and that in the quadratic function.

$$\begin{aligned}\chi_n = \{x_n | E_{n,a}^{min} \leq \sum_{\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h \leq E_{n,a}^{max}, \\ \gamma_{n,a}^{min} \leq x_{n,a}^h \leq \gamma_{n,a}^{max}, \quad \forall h \in \mathcal{H}_{n,a}, \\ x_{n,a}^h = 0, \quad \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}\}.\end{aligned} \tag{11}$$

In the following section, to improve the assumptions of the existing model, we propose a number of modifications to it, and also formulate an optimization problem.

IV. Considered Modifications and Optimization Problem Formulation

In contrast with piece-wise and quadratic linear functions described in [11], [12], the difference in energy price between peak hours and off-peak hours is significantly large. For instance, PAR value of "three" indicates that the load during peak-hours is three times the average load of the entire day. Then, the difference between the price of energy would be nine times. Due to tight schedules (especially during peak-hours), this large difference would pose inconvenience to the customers.

Therefore, a proportional increase in the energy cost in accordance with the total load is crucial to encourage users' participation in balancing the PAR. In [6], the energy prices were chosen carefully by taking into consideration the users' reaction, and not allowing the differences between energy prices to exceed three times.

It is worth stressing that in the first glance, we may intuitively consider that the more drastically the function changes, the better PAR reduction we might get. After all, the energy cost of the consumer is related with the imbalance of energy consumption distribution. However, the computation time increases significantly as the energy cost function varies drastically. Therefore, there is a trade-off for the selection of energy price. In our work, we propose the following energy price function.

$$C_h(L_h) = \alpha L_h \log(L_h + 1), \tag{12}$$

where α is referred to as the "price parameter". The supplier can manipulate this parameter to change the energy price of the whole day to control the energy consumption (which will be described further in Section V). The price difference between the peak and off-peak hours still remains unchanged. More details about this parameter are discussed in the end of this section. The logarithmic function in eq. (12) gives a near linear shape as demonstrated in Fig. 2. Fig. 2 also illustrates the comparison between the proposed energy price function and the conventional quadratic price function. Further comparison of these two price functions is delineated in Section V.

In eq. (11), the energy consumption schedule of all users is not optimized at once. Instead, each smart meter will optimize the schedule of its user according to that user's need. Clearly, the energy consumption schedule vector must belong to the feasible set defined in eq. (11). The objective is to optimize the users' pay-off. More precisely, it aims to maximize the users' benefit in consuming energy. The unit of objective function is in terms of monetary units (e.g., in US dollars) because we consider that the users are interested in monetary incentive or reward. The objective function is a function of energy consumption schedule vector to represent the users' payoffs by consuming that amount of electricity from the supplier. We propose adopting the following utility function.

$$\begin{aligned}W(x_1, ..., x_{24}) &= Value\ of\ Energy - Cost\ of\ Energy \\ &= V(\textstyle\sum_{h=1}^{24} x_h) - P(x_1, ..., x_{24}),\end{aligned} \tag{13}$$

where $(x_1, ..., x_{24})$ denotes the energy consumption scheduling vector of that user, and $(x_1 + ... + x_{24} = X)$ represents the total energy consumption of the user. $V(x)$ indicates the value of that amount of energy, and $P(x)$ refers to the cost of obtaining the energy from the supplier.

In the remainder of the section, we present an analysis of these two functions (i.e., $V(x)$ and $P(x)$) to explain the users' preferences and utility.

A. Value of energy perceived by users

Let $V(X)$ represent how much energy value is given toward the users. However, each user in the power system is likely to have a different energy consumption pattern. Their energy consumption schedule will change based on their own parameters. Even if some users have the same energy consumption, their attitude toward energy value might still be different due to their characteristics and habits. Therefore, it is not easy to capture the response and energy demand of different users toward the same energy price. However, we can analytically model the users' preferences toward energy consumption by adopting the concept of utility function from microeconomics [16].

Utility describes the measurement of "usefulness" that an agent obtains from the available resources. It is the way that the agent values how much he can make the best use of the resources. According to utility theory, a legitimate utility function should satisfy the three following characteristics.

1. A utility function should be upper-bounded:

$$u(x) \leq M, \qquad M > 0. \tag{14}$$

2. The utility function should be an increasing function. More money or resource means higher value of utility. This could be mathematically expressed as follows.

$$\frac{du(x)}{dx} > 0. \tag{15}$$

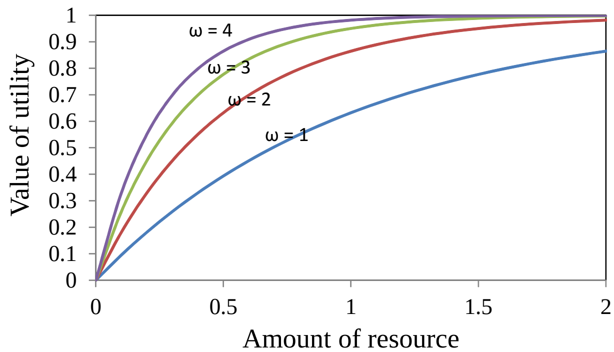

Fig. 3. Value of utility for different resources.

3. The utility function should be decreasing marginal or concave. Consider a user having one hundred dollars. The benefit of gaining one more dollar will be smaller compared to the situation when the user has nothing. This is referred to as the decreasing marginal characteristic. This, again, can be expressed as:

$$\frac{du^2(x)}{dx^2} < 0. \quad (16)$$

Functions satisfying the condition in eq. (16) belong to the **concave functions** class.

There are several utility functions satisfying these characteristics, e.g., quadratic and exponential functions. According to the work in [17], the exponential function could give better model of the user's preference. Here, we use the exponential function as its value domain has been normalized between (0,1) [16]:

$$u(X) = 1 - e^{-\omega X}. \quad (17)$$

A plot of utility functions is illustrated in Fig. 3, where ω is a parameter representing the users' tolerance toward energy consumption curtailment. As evident from the figure, a utility function with a relatively larger value of ω approaches the upper-bound in a rather slow manner. The users with large values of ω are stricter with the energy curtailment, i.e., their utility values are lower than those of the other users even with the same amount of energy consumption. Since the value of ω is a private information of the users, energy optimization algorithms should not leak out this information.

Then, we define the value function of energy as follows:

$$V(X) = pX_{max}u(X) = pX_{max}(1 - e^{-\omega X}). \quad (18)$$

Here, $X = \sum_{h=1}^{24} x_h$ is the user's overall energy consumption and X_{max} denotes the maximum amount of energy that the user can consume. p denotes the average price for a unit of energy that displays the value (in money) of acquiring that unit of energy, regardless of other elements such as time of day, level of energy consumption, the extra cost for peak hours, extra cost of delivery, and so forth.

The utility function we chose has the maximum value of one. By scaling it up with the product of average energy price and the maximum amount of energy consumption that a user can consume, the maximum value of $V(x)$ is equal to the value of the maximum energy consumption. It implies that the user will value his consumption amount not greater than the average money he needs to pay to satisfy his maximum demand in monetary units.

B. Cost of energy

The cost of energy for the user based on the energy cost function in eq. (12) can be rewritten for the n^{th} user as follows.

$$P(x_1, ..., x_{24}) = \alpha \sum_{h=1}^{24} x_h(x_h + L_{-n,h}) log(x_h + L_{-n,h} + 1). \quad (19)$$

Here, the supplier can manipulate α to change the energy price of the system to influence energy consumption of the users. More details on this behavior are provided in the end of this section.

By substituting eqs. (18) and (19), the objective function of the n^{th} user can be rewritten as:

$$W(x_1, ..., x_{24}) = pX_{max}(1 - e^{-\omega \sum_{h=1}^{24} x_h}) - \alpha \sum_{h=1}^{24} x_h(x_h + L_{-n,h}) log(x_h + L_{-n,h} + 1). \quad (20)$$

By normalizing the utility function into monetary units, the objective function also adopts monetary units. Thus, it can be exploited to represent each user's preference toward an amount of energy consumption, or more precisely his pay-off for consuming an amount of energy supplied by the power company. The user attempts at maximizing his own pay-off by solving the optimization problem expressed by eq. (20). Because this function is concave, it can be solved by using Interior-Point-Method (IPM) [18], [19].

C. Decision of energy pricing scheme from the supplier

It is important to investigate the way the supplier decides the energy pricing scheme. In the dynamic pricing, the energy price changes according to various factors. In our proposed model, the energy price changes according to the total load during different hours. For our proposed price function, the energy price is proportional to $\big(L_h \log(L_h + 1)\big)$. This property is exhibited by defining the energy price function as $\big(C_h(L_h) = \alpha L_h \log(L_h + 1)\big)$, where L_h is the total load at hour h. Then, we have an energy price vector, denoted by $\mathcal{C}(\mathcal{L})$, for the entire day as follows.

$$\mathcal{C}(\mathcal{L}) = \alpha \cdot \big(L_1 \log(L_1 + 1) + ... + L_{24} \log(L_{24} + 1)\big). \quad (21)$$

Therefore, the supplier can manipulate the parameter α to influence the whole price vector, and impose, in turn, some constraints on the users' energy consumption. The users tend to curtail their consumption if they consider a substantial increase in the energy price. The supplier needs to consider this phenomenon. As a consequence, without any constraint for choosing an appropriate value of α, it is difficult to illustrate the effect of the users' dissatisfaction. Furthermore, it is also important to consider the real value of the energy cost for deciding the dynamic energy price. The supplier needs to consider the average energy price to obtain a close equivalent of the dynamic pricing scheme. Next, we discuss the choice of the parameter α from the supplier's side.

Let us consider a fixed price scheme as the baseline. The energy price is fixed for every unit of energy regardless of the time of day or total energy consumption of the system. Indeed, this fixed price scheme is adopted by the contemporary power grids. In case of Japan, Tokyo Electric Power Company (TEPCO) [20] charges 17.87 yen for the first 120 KW (kilowatts) and 22.86 yen for the next 180 KW for the light residential user with 20 amp power line. So, we assume that the supplier has the average price for the energy that it sells to its customers. This average price was referred to as the parameter p earlier in the users' objective function in eq. (18). When the supplier uses the dynamic price scheme, we assume that the total cost they charge for the whole system is equal to the fixed price scheme with the same load. This can be represented by the expression below.

$$\begin{aligned} p\sum_{h=1}^{24} L_h &= \alpha \sum_{h=1}^{24} L_h C_h(L_h) \\ &= \alpha \sum_{h=1}^{24} L_h^2 log(L_h+1). \end{aligned} \tag{22}$$

Therefore, α can be evaluated as follows.

$$\alpha = \frac{p\sum_{h=1}^{24} L_h}{\sum_{h=1}^{24} L_h^2 log(L_h+1)}. \tag{23}$$

This is how the supplier may compute the energy price parameter α, and also the energy price for different hours in a day. We investigate a little more about this price scheme to have some insight into the difference between the fixed price and dynamic price schemes.

If we suppose that the energy consumption of the whole system is equal for all the hours (i.e., $L_1 = L_2 = ... = L_{24} = \frac{L}{24}$), then this implies that there is no imbalance in the distribution of the energy consumption in different hours and PAR is equal to one. Then, the energy price for every hour will be equal to $\left(\alpha\frac{L}{24}log(\frac{L}{24}+1)\right)$. As a result, the value of α can be computed by using eq. (24).

$$\alpha = \frac{p\sum_{h=1}^{24}\frac{L}{24}}{\sum_{h=1}^{24}(\frac{L}{24})^2 log(\frac{L}{24}+1)} = \frac{p}{\frac{L}{24}log(\frac{L}{24}+1)}. \tag{24}$$

Therefore, the energy price for every hour becomes equal to the average price (p) of the fixed price scheme. Therefore, the imbalance in the load distribution between the hours is the reason for the differences in the energy prices. As the load distribution becomes more imbalanced, the gap of the energy prices increases more. The users would have to pay additional money for consuming energy during the peak-hours. So, the best approach for the users is to curtail their energy consumption according to their objective functions which will also result in lower energy consumption for the whole system.

In the next section, we propose a game played by the consumers with the power company so that they may curtail their energy consumption in an optimized fashion.

V. Proposed Game-Theoretic Energy Schedule (GTES) Algorithm

In this section, we propose a game-theoretic approach for optimizing energy consumption. We refer to our approach as Game-Theoretic Energy Schedule (GTES) algorithm. The game is played between the power company and its consumers. The game aims at attaining two objectives at the same time, namely (i) to reduce system PAR by optimizing energy schedule, and (ii) to lower the total energy consumption. The optimization process can be modeled as a two-stage game [7] as depicted in Algorithms 1 and 2.

1) The users will try to maximize their pay-offs by optimizing functions (shown in eq. (20)) using IPM.
2) The supplier will then adjust the energy price parameter consistent with the user's energy consumption schedule according to eq. (23).

When the game reaches an equilibrium state, neither users nor supplier will change their strategies. At the same time, the system PAR and total energy consumption are reduced.

Algorithm 1 Power company's (control center's) game.

```
Begin: Gather original schedule from all users
    All users initialize their schedules from feasible sets
End
Calculate initial energy price parameter α according to eq. (23)
Repeat
    Randomly choose user n, n ∈ N
    Signal user n to run algorithm 2
    Update the new schedule vector 𝓛_n from user n
    Update energy price parameter α according to (23)
Until no user wants to change schedule
```

Algorithm 2 Users' game.

```
Begin: Receive signal from supplier
    Request α, vector 𝓛
    User n optimizes schedule by solving the problem in eq. (20) by using IPM
    If x_n changes compared to current schedule Then
        Inform the center of the new schedule vector 𝓛_n
    End If
End
```

Algorithms 1 and 2 describe the moves made by the power company (typically the control center) and the consumers, respectively. At the start of the day (e.g. at 12 am), when the control center will start the algorithm, it sends messages to all consumers to request their default energy consumption schedules. The users have to initialize energy consumption schedules, which satisfy all constraints described in Section IV. After receiving all the basic information from the users, the control center calculates the initial value for energy price parameter (α) and broadcasts it to all the users. Then, the loop in Algorithm 1 is executed until the algorithm converges. The supplier randomly selects a user to run the optimization in the loop. The selected user is not to be chosen again until all other users have executed the loop. The selected user receives a message with the total energy consumption schedule vector, $\mathcal{L}_n$, of the whole system. Based on this information, together with the energy price parameter α, the user is able to optimize his objective function to find the best schedule for himself. Each smart meter is assumed to be equipped with the program to solve the local optimization problem using IPM for convex optimization as shown in Algorithm 2. Since IPM has a high degree of accuracy, each user makes the best possible response. If the user's schedule changes, his smart meter updates the new schedule and announces new energy consumption schedule vector (i.e., the updated $\mathcal{L}_n$) to the center. The center, therefore, has to adjust the energy price parameter again based on the new system state.

The supplier center, thus, serves as a data collecting entity. It receives update schedules from the users, provides them with general information: system energy consumption schedule vector $\mathcal{L}_n$. It has to update the energy price parameter α, but the update is also simple and does not require much computation. For users, solving the local optimization problem in IPM is fast and highly efficient. Also, the users do not have to reveal all the details about their schedules. In other words, reporting only the total energy consumption schedule vector ($\mathcal{L}_n$) is sufficient for them. Normally, this type of information are already monitored by the supplier's control center. The knowledge of the parameter ω, which represents the users' tolerance toward energy consumption curtailment, is not required, and this is not revealed to others as this is to remain private (as discussed earlier in Section IV-A). In other words, in the proposed algorithm, the users do not have to worry about revealing the sensitive information to the control center. For interested readers, the proof of convergence of the proposed algorithm is presented in the remainder of the section.

First, we analyze the value range of α. We assume that the total energy consumption of the whole system is constant:

$$\sum_{h=1}^{24} L_h = \mathcal{M} \tag{25}$$

Because α is decided according to eq. (23), we need to consider the following function.

$$f(L_h) = (L_h)^2 \log(L_h + 1) \tag{26}$$

We can easily confirm that $f(L_h)$ is strictly convex as its second derivative contains only positive components for all $L_h > 0$. Therefore, $f(L_h)$ also satisfies the second assumption in Section III-2, that is for each $h \in \mathcal{H}$, any real number $L_h^1, L_h^2 \geq 0$, and any real number $0 < \theta < 1$, we have:

$$f(\theta L_h^1 + (1-\theta)L_h^2) < \theta f(L_h^1) + (1-\theta) f(L_h^2) \tag{27}$$

Therefore, we have:

$$\begin{aligned} \sum_{h=1}^{24} \frac{1}{24} \times (L_h)^2 \log(L_h+1) &\geq (\frac{1}{24}\sum_{1}^{24} L_h)^2 \log(\frac{1}{24}\sum_{1}^{24} L_h + 1) \\ \sum_{h=1}^{24} (L_h)^2 \log(L_h+1) &\geq \frac{\mathcal{M}^2}{24} \log(\frac{\mathcal{M}}{24} + 1) \end{aligned} \tag{28}$$

TABLE I
ENERGY APPLIANCES AND THEIR AVERAGE CONSUMPTION ON A DAILY BASIS.

Appliances	Average Consumption per day [KW]
Clothes Dryer	2.47
Dishwasher	0.99
Lighting	3.29
Refrigerator	5.89
Washing Machine	0.28

On the other hand, we have:

$$\begin{aligned} \sum_{h=1}^{24} (L_h)^2 \log(L_h+1) &\leq (\sum_{1}^{24} (L_h)^2) \times \max_{p \in \mathcal{H}} \log(L_p + 1) \\ &\leq (\sum_{1}^{24} L_h)^2 \times \max_{p \in \mathcal{H}} \log(L_p + 1) \\ &\leq \mathcal{M}^2 \log(\mathcal{M} + 1) \end{aligned} \tag{29}$$

Therefore, we can derive the bound for energy price parameter α from eqs. (28) and (29).

$$\frac{24p}{\mathcal{M}\log(\frac{\mathcal{M}}{24}+1)} \geq \alpha \geq \frac{p}{\mathcal{M}\log(\mathcal{M}+1)} \tag{30}$$

Here, α reaches its maximal value when $L_1 = L_2 = ... = L_{24}$ and $PAR = 1$. α reaches its minimal value in the worst case where all energy consumption concentrates in only one hour. The more imbalanced the load shape becomes, the bigger α approaches its lower bound. On the other hand, the lower the peak energy consumption or lower PAR, the nearer α is to its upper bound. In other words, α is inversely proportional with PAR.

In a general game, if all users' objective functions are strictly concave, then the equilibrium of the game exists and is unique. Since our users objective functions are strictly concave also, the equilibrium for our system exists.

Still, the proof of equilibrium existence for our case can be obtained as follows. Since α is bounded and users could not consume more than their maximum consumption levels, the value of users' objective functions are upper bounded. Moreover, users only upgrade their energy consumption schedules if they could improve their objective functions. Users' objective functions are upper bounded and increasing, therefore, they all converge to their limits. On the other hand, the energy price parameter α is upper-bounded, too, and every time some user optimizes its objective function, α is increased due to more balanced load shape. Because users plan their energy schedule by shifting consumption from peak hours, a lower PAR is achieved. So, the energy price parameter α also converges to an upper limit. Both user-side's and supplier-side's games converge to their equilibriums. Therefore, there exists an equilibrium for our proposed game. Moreover, since the limit is mathematically unique, the equilibrium of this game is also unique.

In the next section, we present the performance evaluation of our proposed GTES.

VI. Performance Evaluation

In this section, we evaluate the performance of our proposed GTES approach for reducing energy consumption by

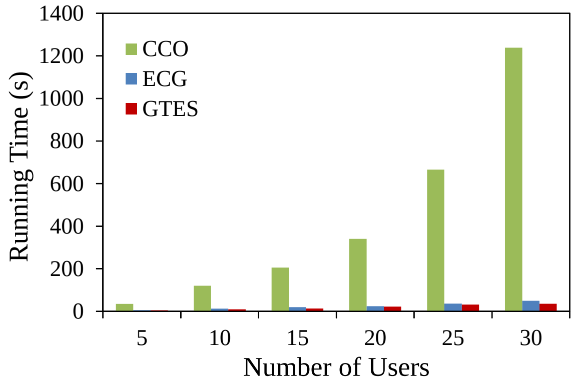

Fig. 4. Comparison of the running time between the proposed GTES, and conventional CCO and ECG algorithms for various number of users.

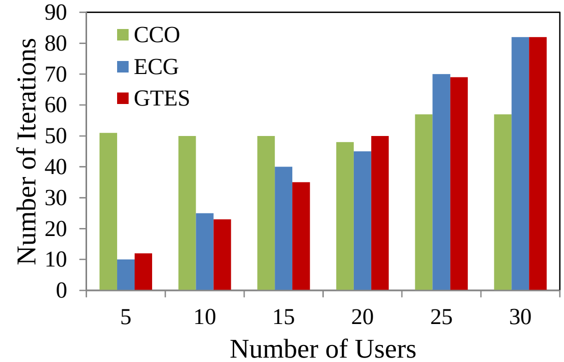

Fig. 5. Number of iterations until convergence in case of CCO, ECG, and the proposed GTES for different numbers of users.

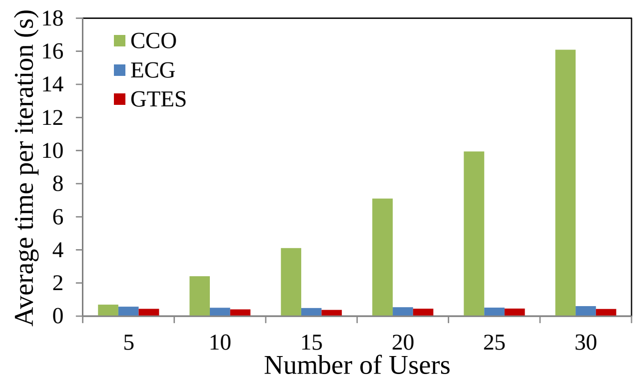

Fig. 6. Average time needed for each iteration in case of CCO, ECG, and the proposed GTES for different numbers of users.

comparing it with a number of conventional methods. The first conventional method is to gather all parameters and constraints from every user and schedule energy consumption by applying convex optimization to find schedules for all the users at once. For quick reference, let this approach be referred to as Centralized Convex Optimization (CCO). The second conventional method for comparison is the autonomous energy-theoretic optimization algorithm in [11] that is referred to as the Energy Consumption Game (ECG) for ease of notation.

The simulation environment is constructed with MATLAB [21]. We consider a smart grid infrastructure with a supplier and a population of consumers. All the consumers connected to the supplier are equipped with smart meters. The number of consumers is varied from 5 to 500. Each consumer has ten to fifteen schedulable appliances and ten to fifteen unschedulable appliances. The schedulable appliances include residential electrical appliances with a flexible schedule such as washing machines, dish washers, and plug-in hybrid electric vehicles. On the other hand, unschedulable appliances need to consume energy continuously or have a fixed schedule. Examples of unschedulable appliances include fridge, light bulbs/lamps, heaters, and so forth. A list of some typical home electrical appliances are displayed in Table I based on [22]. All these settings are randomly generated every time the simulation is conducted. Note that these settings remain the same for comparing the different methods in a specific situation. First, we conduct simulations to compare the running time of the three methods with up to 30 users. Then, we evaluate the performance of proposed method, also giving some comparison with the distributed game-theoretic ECG. Finally, we also include some comparisons between quadratic energy price function and our proposed price function.

Fig. 4 demonstrates that the running time of CCO increases quite fast, almost at an exponential rate, while the two game-theoretic methods require significantly low completion time as the number of users increases. The large number of parameters seriously affects the running time of convex optimization (i.e., CCO), even when the number of users remains rather small. Furthermore, it can affect even the convergence guarantee. In our conducted experiment, CCO often fails to converge when the number of consumers exceeds 30. Compared with this simulation, the number of appliances for each user in real life is even higher, which is the reason why a fully centralized control such as CCO may not be practical. Figs. 5 and 6 illustrate more analysis about this.

In Figs. 5 and 6, the number of iterations needed for each algorithm to converge and the average time per iteration are plotted, respectively. The running time in the previous analysis is the product of these two parameters. Fig. 5 demonstrates how the number of iterations required for CCO and both the game-theoretic algorithms (including our proposed GTES) increases gradually with the number of consumers. However, the average time needed for each iteration (as shown in Fig. 6) changes slowly for the game-theoretic algorithms while it rises drastically in case of CCO. This is expected as the game-theoretic approaches only require solving the local optimization for each consumer. These results indicate that CCO is not scalable enough for a large number of consumers in contrast with its game-theoretic counterparts, i.e., ECG and GTES. As a consequence, in the remainder of this section, the performances of ECG and the proposed GTES are taken into consideration for comparison.

Figs. 7, 8, and 9 demonstrate the load shape, convergence of system PAR, and supplier price, respectively, for the proposed GTES approach for a scenario of 50 users. Fig. 7 illustrates the system load shape for the non-scheduled scheme and that for the scheduling under the proposed GTES scheme over 24 hours. It is evident that the energy consumption had been shifting and adjusting at the same time, resulting in a more balanced load shape and a lower total load. The system PAR

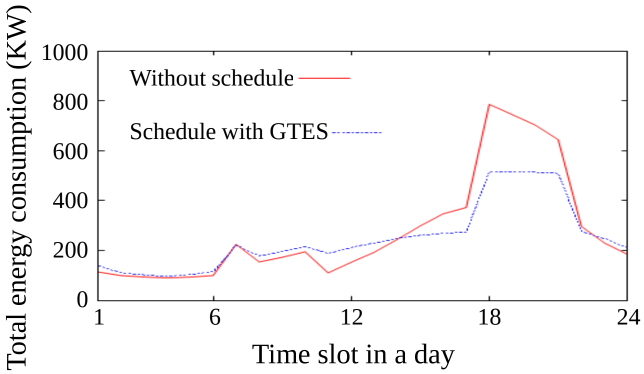

Fig. 7. Load shapes for the schemes without and with our proposed GTES scheme, respectively, illustrating how the total energy consumption becomes smooth with the proposal.

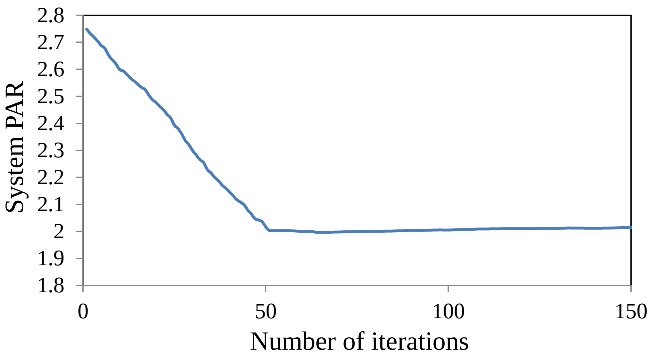

Fig. 8. Convergence of system PAR in case of the proposed GTES.

changes from 2.258 to 1.837. In addition, the total system energy consumption also reduces from 3707.8 KW to 3447.7 KW. In Fig. 8, it is remarkable that the convergence happens significantly fast, and the system PAR dropped drastically after the first round (when all the consumers had run the algorithm once). After that, the system PAR changes slowly and stabilizes after 2 to 3 rounds. This confirms that game-theoretic optimization would converge within $O(n)$ if every player (i.e., each of the consumers as well as the power company) follows the best move. Because, GTES uses IPM to solve the local problem for each user, all the users are able to find their optimal schedules, and therefore, the system approaches equilibrium state quite rapidly. Fig. 9 demonstrates the convergence of the energy price parameter α. As shown in the figure, this parameter convergence also happens substantially fast, similar to that on the user-side. In this way, both the games on the user-side and the supplier-end converge to equilibrium.

Fig. 10 illustrates the number of iterations needed for the convergences of ECG and the proposed GTES. Since both of these methods are based on game theory, they converge very fast in proportional with the number of consumers. Furthermore, as the number of consumers grows larger, the ratio between the number of iterations and number of consumers decreases a little. This can be explained by the fact that the bigger the system becomes, the less effect is inflicted by changing a single consumer's schedule. Thus, we confirm that game-theoretic optimization approaches have high convergence speeds, and they scale well with the increase in the number of consumers.

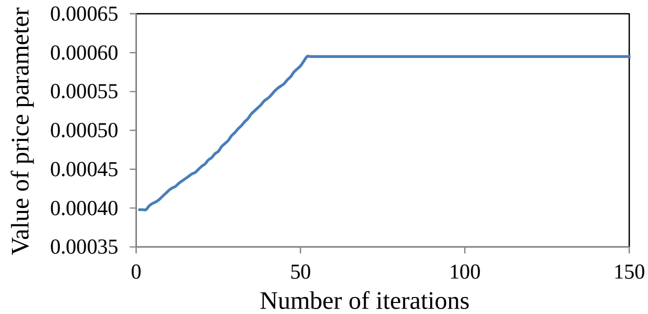

Fig. 9. Convergence of supplier price in case of the proposed GTES.

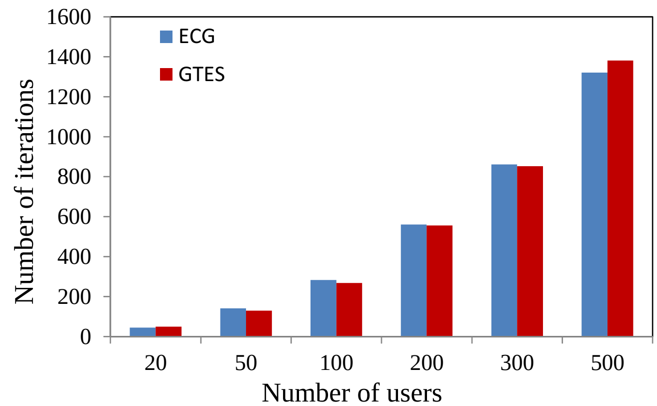

Fig. 10. Number of iterations required for ECG and the proposed GTES for a large number of users (up to 500).

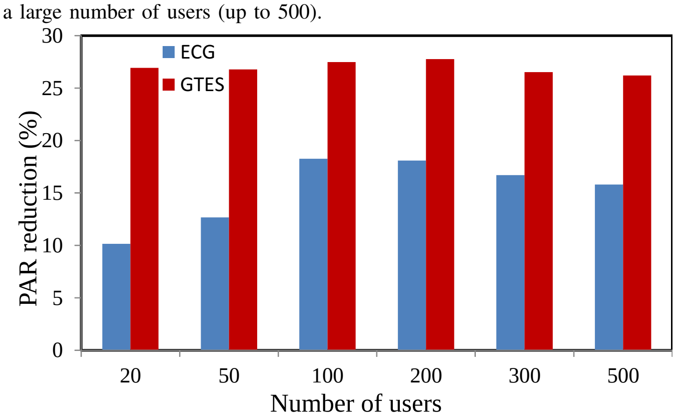

Fig. 11. PAR reduction in ECG and the proposed GTES for different numbers of users.

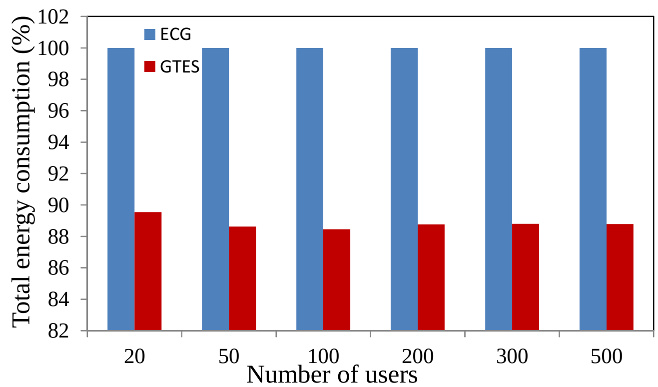

Fig. 12. Total energy consumption (in percent) in case of ECG and the proposed GTES for different numbers of users.

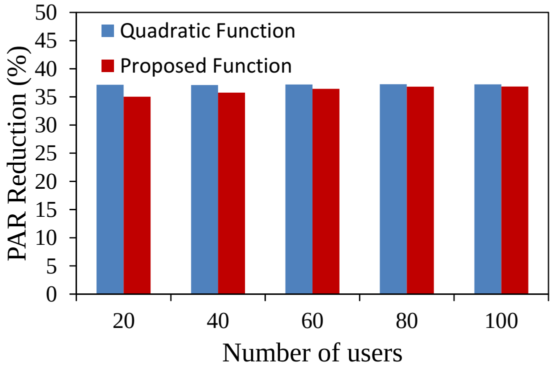

Fig. 13. Improvement of PAR due to the proposed price function in contrast with the conventional quadratic function.

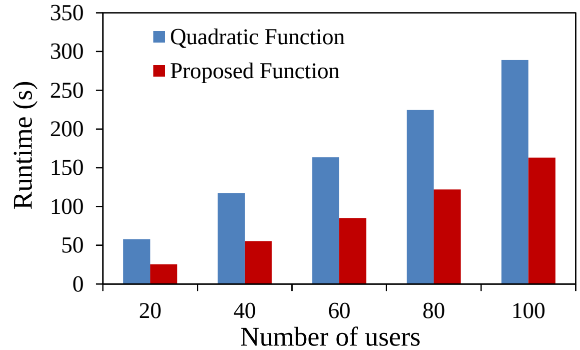

Fig. 14. Running time of the proposed price function and the conventional quadratic function.

In Fig. 11, we compare the PAR reduction between the ECG and the proposed GTES. Fig. 11 illustrates that our proposed GTES method results in higher PAR reduction in contrast with ECG. This good performance of the proposed method can be attributed to its consideration of not only shifting energy consumption while scheduling, but also its ability to adjust energy consumption levels at different hours. So, by adding more to the off-peak hours and decreasing a little at peak hours, PAR is reduced even further in GTES.

Fig. 12 demonstrates the total energy consumption after running the ECG and GTES algorithms. ECG is found to be only shifting energy consumption. As a consequence, ECG does not change the amount of energy consumption. In contrast with that, the proposed GTES method results in an average 12% reduction of the energy consumption.

Finally, we conduct a simulation to compare the differences between energy price functions. As we have mentioned in Section IV, choosing between the more drastic quadratic function and our logarithmic based function is a trade-off between the system performance and computation time. In order to compare these two functions, we use the game-theoretic algorithm with only one objective in ECG, namely shifting energy consumption to reduce PAR. The consumers try to minimize their energy cost by shifting their consumption to "cheaper" hours. Clearly, unlike our proposed GTES method, the total energy consumption will be fixed in ECG, and the same holds for the energy price parameter. The results are depicted in Figs. 13 and 14. As the results demonstrate, the quadratic function displays a small improvement in PAR reduction in contrast with that in our energy price function and the difference becomes more insignificant when the number of users increases. However, the running time of quadratic function remains larger (in fact nearly double) compared to the proposed energy price function used in GTES. This result indicates that the performance of the proposed logarithmic-based price function is better than the existing energy price function.

VII. Conclusion

In this paper, we considered a practical smart grid infrastructure, where the power company and its consumers are proposed to play their own games to optimize their energy schedules. In our proposed two-stage game in smart grid, the objective is to achieve a reduction in both the system PAR and the total energy consumption. We apply a pricing scheme, whereby the energy price changes according to the whole system energy consumption during each hour so that all the consumers may have appropriate monetary incentives to follow the system. Since the number of parameters to be optimized is rather high, a fully centralized control by using convex optimization is not found to scale well with a high population of users. To overcome this, we proposed a game-theoretic centralized optimization scheme. The simulation results demonstrated that the proposed algorithm converged in $O(n)$ iterations, and achieved good PAR reduction and energy consumption reduction. These results are desired from the power company's point of view, and providing that it has the knowledge of the consumers' energy schedules, the power company would be able to predict the total energy consumption at each hour in order to produce or estimate the necessary amount of energy (to purchase from the market). Also, every user has his electric cost reduced in an effective manner, and the values of the users' objective functions (which represent their pay-offs) are increased.

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