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Guaranteed cost control for switched recurrent neural networks with interval time-varying delay

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Abstract

This paper studies the problem of guaranteed cost control for a class of switched recurrent neural networks with interval time-varying delay. The time delay is a continuous function belonging to a given interval, but not necessary differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals, a guaranteed cost controller is designed via memoryless state feedback control, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities and new sufficient conditions for the existence of the guaranteed cost state-feedback for the system are given in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the obtained result.

Keywords: neural networks; guaranteed cost control; switching design; stabilization; interval time-varying delays; Lyapunov function; linear matrix inequalities

1 Introduction

Stability and control of recurrent neural networks with time delay have attracted considerable attention in recent years [1–8]. In many practical systems, it is desirable to design neural networks which are not only asymptotically or exponentially stable but can also guarantee an adequate level of system performance. In the area of control, signal processing, pattern recognition and image processing, delayed neural networks have many useful applications. Some of these applications require that the equilibrium points of the designed network be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time-varying, and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. Guaranteed cost control problem [9–12] has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound. The Lyapunov-Krasovskii functional technique has been among the popular and effective tools in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such a diversity of results available, the most existing works either assumed that the time delays are constant or differentiable [13–16].

Although, in some cases, delay-dependent guaranteed cost control for systems with time-varying delays were considered in [12, 13, 15], the approach used there cannot be applied to systems with interval, non-differentiable time-varying delays. To the best of our knowledge, the guaranteed cost control and state feedback stabilization for switched recurrent neural networks with interval time-varying delay, non-differentiable time-varying delays have not been fully studied yet (see, *e.g.*, [9–12, 15–25] and the references therein). Which are important in both theories and applications. This motivates our research.

In this paper, we investigate the guaranteed cost control for switched recurrent neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. Specifically, our goal is to develop a constructive way to design a switching rule to exponentially stabilize the system. A nonlinear cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with the Newton-Leibniz formula, new delay-dependent criteria for guaranteed cost control via memoryless feedback control are established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI control toolbox.

The outline of the paper is as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main result. LMI delay-dependent criteria for guaranteed cost control and a numerical example showing the effectiveness of the result are presented in Section 3. The paper ends with conclusions and cited references.

2 Preliminaries

The following notation will be used in this paper. $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the scalar product $\langle x,y\rangle$ or x^Ty of two vectors x, y, and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$ dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A=A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A)=\max\{\operatorname{Re}\lambda;\lambda\in\lambda(A)\}$. $x_t:=\{x(t+s):s\in[-h,0]\}$, $\|x_t\|=\sup_{s\in[-h,0]}\|x(t+s)\|$; $C^1([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuously differentiable functions on $[0,t]$; $L_2([0,t],\mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$-valued square integrable functions on $[0,t]$.

Matrix A is called semi-positive definite ($A\geq 0$) if $\langle Ax,x\rangle\geq 0$ for all $x\in\mathbb{R}^n$; A is positive definite ($A>0$) if $\langle Ax,x\rangle>0$ for all $x\neq 0$; $A>B$ means $A-B>0$. The notation $\operatorname{diag}\{\cdots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following switched recurrent neural networks with interval time-varying delay:

$$\begin{aligned}\dot{x}(t)&=-A_{\gamma(x(t))}x(t)+W_{0\gamma(x(t))}f\bigl(x(t)\bigr)\\&\quad+W_{1\gamma(x(t))}g\bigl(x\bigl(t-h(t)\bigr)\bigr)+B_{\gamma(x(t))}u(t),\quad t\geq 0,\\x(t)&=\phi(t),\quad t\in[-h_1,0],\end{aligned}\tag{2.1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural, $u(\cdot) \in L_2([0,t], \mathbb{R}^m)$ is the control; n is the number of neurons, and

$$f\big(x(t)\big) = \big[f_1\big(x_1(t)\big), f_2\big(x_2(t)\big), \ldots, f_n\big(x_n(t)\big)\big]^T,$$
$$g\big(x(t)\big) = \big[g_1\big(x_1(t)\big), g_2\big(x_2(t)\big), \ldots, g_n\big(x_n(t)\big)\big]^T,$$

are the activation functions; $\gamma(\cdot) : R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = j$ implies that the system realization is chosen as the jth system, $j = 1, 2, \ldots, N$. It is seen that system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries.

$A_j = \operatorname{diag}(\overline{a}_{1j}, \overline{a}_{2j}, \ldots, \overline{a}_{nj})$, $\overline{a}_{ij} > 0$, represents the self-feedback term; $B_j \in R^{n\times m}$ are control input matrices; W_{0j}, W_{1j} denote the connection weights and the delayed connection weights, respectively. The time-varying delay function $h(t)$ satisfies the condition

$$0 \le h_0 \le h(t) \le h_1.$$

The initial functions $\phi(t) \in C^1([-h_1, 0], R^n)$, with the norm

$$\|\phi\| = \sup_{t\in[-h_1,0]} \sqrt{\big\|\phi(t)\big\|^2 + \big\|\dot{\phi}(t)\big\|^2}.$$

In this paper we consider various activation functions and assume that the activation functions $f(\cdot)$, $g(\cdot)$ are Lipschitzian with the Lipschitz constants $f_i, e_i > 0$:

$$\begin{aligned} &\big|f_i(\xi_1) - f_i(\xi_2)\big| \le f_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \\ &\big|g_i(\xi_1) - g_i(\xi_2)\big| \le e_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n, \forall \xi_1, \xi_2 \in \mathbb{R}. \end{aligned} \tag{2.2}$$

The performance index associated with system (2.1) is the following function:

$$J = \int_0^\infty f^0\big(t, x(t), x\big(t - h(t)\big), u(t)\big)\,dt, \tag{2.3}$$

where $f^0(t, x(t), x(t-h(t)), u(t)) : R^+ \times R^n \times R^n \times R^m \to R^+$, is a nonlinear cost function satisfying

$$\exists Q_1, Q_2, \quad R : f^0(t, x, y, u) \le \langle Q_1 x, x\rangle + \langle Q_2 y, y\rangle + \langle Ru, u\rangle \tag{2.4}$$

for all $(t, x, y, u) \in R^+ \times R^n \times R^n \times R^m$ and $Q_1, Q_2 \in R^{n\times n}$, $R \in R^{m\times m}$, are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (2.1) and the cost function (2.3) such that the resulting closed-loop system

$$\dot{x}(t) = -(A_j - B_j K)x(t) + W_{0j} f\big(x(t)\big) + W_{1j} g\big(x\big(t - h(t)\big)\big) \tag{2.5}$$

is exponentially stable and the closed-loop value of the cost function (2.3) is minimized.

Remark 2.1 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero; therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system.

Remark 2.2 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the delay function $h(t)$ is non-differentiable; therefore, the stability criteria proposed in [5, 6, 8, 10–12, 14–19, 22–25] are not applicable to this system.

Definition 2.1 Given $\alpha > 0$. The zero solution of closed-loop system (2.5) is α-exponentially stabilizable if there exists a positive number $N > 0$ such that every solution $x(t,\phi)$ satisfies the following condition:

$$\left\|x(t,\phi)\right\| \leq Ne^{-\alpha t}\|\phi\|, \quad \forall t \geq 0.$$

Definition 2.2 Consider control system (2.1). If there exist a memoryless state feedback control law $u(t) = Kx(t)$ and a positive number J^* such that the zero solution of closed-loop system (2.5) is exponentially stable and the cost function (2.3) satisfies $J \leq J^*$, then the value J^* is a guaranteed constant and $u(t)$ is a guaranteed cost control law of the system and its corresponding cost function.

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 (Schur complement lemma [26]) *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$, $Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

Proposition 2.2 (Integral matrix inequality [27]) *For any symmetric positive definite matrix $M > 0$, scalar $\sigma > 0$ and vector function $\omega : [0,\sigma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds*:

$$\left(\int_0^\sigma \omega(s)\,ds\right)^T M\left(\int_0^\sigma \omega(s)\,ds\right) \leq \sigma\left(\int_0^\sigma \omega^T(s)M\omega(s)\,ds\right).$$

3 Design of guaranteed cost controller

In this section, we give a design of memoryless guaranteed feedback cost control for neural networks (2.1). Let us set

$$w_{11} = -[P + \alpha I]A_j - A_j^T[P + \alpha I] - 2B_jB_j^T + 0.25B_jRB_j^T + \sum_{i=0}^{1} G_i,$$

$$w_{12} = P + A_jP + 0.5B_jB_j^T,$$

$$w_{13} = e^{-2\alpha h_0}H_0 + 0.5B_jB_j^T + A_jP,$$

$$w_{14} = 2e^{-2\alpha h_1}H_1 + 0.5B_jB_j^T + A_jP,$$

$$w_{15} = P0.5B_jB_j^T + A_jP,$$

$$w_{22} = \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T + \sum_{i=0}^{1} h_i^2H_i + (h_1 - h_0)U - 2P - B_jB_j^T,$$

$$w_{23} = P, \qquad w_{24} = P, \qquad w_{25} = P,$$

$$w_{33} = -e^{-2\alpha h_0}G_0 - e^{-2\alpha h_0}H_0 - e^{-2\alpha h_1}U + \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T,$$

$$w_{34} = 0, \qquad w_{35} = -2\alpha h_1 U,$$

$$w_{44} = \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T - e^{-2\alpha h_1}U - e^{-2\alpha h_1}G_1 - e^{-2\alpha h_1}H_1, \qquad w_{45} = e^{-2\alpha h_1}U,$$

$$w_{55} = -e^{-2\alpha h_1}U + W_{0j}D_0W_{0j}^T,$$

$$E = \operatorname{diag}\{e_i, i = 1,\dots,n\}, \qquad F = \operatorname{diag}\{f_i, i = 1,\dots,n\},$$

$$\lambda_1 = \lambda_{\min}(P^{-1}),$$

$$\lambda_2 = \lambda_{\max}(P^{-1}) + h_0\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} G_i\right)P^{-1}\right] + h_1^2\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} H_i\right)P^{-1}\right] + (h_1 - h_0)\lambda_{\max}(P^{-1}UP^{-1}).$$

Theorem 3.1 *Consider control system* (2.1) *and the cost function* (2.3). *If there exist symmetric positive definite matrices P, U, G_0, G_1, H_0, H_1 and diagonal positive definite matrices D_i, $i = 0, 1$, satisfying the following LMIs*:

$$\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.1}$$

$$\mathcal{S}_{1j} = \begin{bmatrix} -PA_j - A_j^TP - \sum_{i=0}^{1} e^{-2\alpha h_i}H_i & 2PF & PQ_1 \\ * & -D_0 & 0 \\ * & * & -Q_1^{-1} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.2}$$

$$\mathcal{S}_{2j} = \begin{bmatrix} W_{1j}D_1W_{1j}^T - e^{-2\alpha h_1}U & 2PE & PQ_2 \\ * & -D_1 & 0 \\ * & * & -Q_2^{-1} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.3}$$

then

$$u_j(t) = -\frac{1}{2}B_j^TP^{-1}x(t), \quad t \geq 0, j = 1,2,\dots,N, \tag{3.4}$$

is a guaranteed cost control and the guaranteed cost value is given by

$$J^* = \lambda_2\|\phi\|^2.$$

The switching rule is chosen as $\gamma(x(t)) = j$. Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t}\|\phi\|, \quad \forall t \geq 0.$$

Proof Let $Y = P^{-1}$, $y(t) = Yx(t)$. Using the feedback control (2.5), we consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned}
V(t,x_t) &= \sum_{i=1}^{6} V_i(t,x_t),\\
V_1 &= x^T(t)Yx(t),\\
V_2 &= \int_{t-h_0}^{t} e^{2\alpha(s-t)} x^T(s)YG_0Yx(s)\,ds,\\
V_3 &= \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s)YG_1Yx(s)\,ds,\\
V_4 &= h_0\int_{-h_0}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_0Y\dot{x}(\tau)\,d\tau\,ds,\\
V_5 &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_1Y\dot{x}(\tau)\,d\tau\,ds,\\
V_6 &= (h_1-h_0)\int_{t-h_1}^{t-h_0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YUY\dot{x}(\tau)\,d\tau\,ds.
\end{aligned}$$

It is easy to check that

$$\lambda_1\|x(t)\|^2 \leq V(t,x_t) \leq \lambda_2\|x_t\|^2, \quad \forall t \geq 0. \tag{3.5}$$

Taking the derivative of V_1, we have

$$\begin{aligned}
\dot{V}_1 &= 2x^T(t)Y\dot{x}(t)\\
&= y^T(t)\big[-PA_j^T - A_jP\big]y(t) - y^T(t)B_jB_j^Ty(t)\\
&\quad + 2y^T(t)W_{0j}f(\cdot)y(t) + 2y^T(t)W_{1j}g(\cdot)y(t)\\
\dot{V}_2 &= y^T(t)G_0y(t) - e^{-2\alpha h_0}y^T(t-h_0)G_0y(t-h_0) - 2\alpha V_2;\\
\dot{V}_3 &= y^T(t)G_1y(t) - e^{-2\alpha h_1}y^T(t-h_1)G_1y(t-h_1) - 2\alpha V_3;\\
\dot{V}_4 &= h_0^2\dot{y}^T(t)H_0\dot{y}(t) - h_1e^{-2\alpha h_0}\int_{t-h_0}^{t}\dot{x}^T(s)H_0\dot{x}(s)\,ds - 2\alpha V_4;\\
\dot{V}_5 &= h_1^2\dot{y}^T(t)H_1\dot{y}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{y}^T(s)H_1\dot{y}(s)\,ds - 2\alpha V_4;\\
\dot{V}_6 &= (h_1-h_0)^2\dot{y}^T(t)U\dot{y}(t) - (h_1-h_0)e^{-2\alpha h_1}\int_{t-h_1}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds - 2\alpha V_6.
\end{aligned}$$

Applying Proposition 2.2 and the Leibniz-Newton formula

$$\int_s^t \dot{y}(\tau)\,d\tau = y(t) - y(s),$$

we have, for $j = 1, 2$, $i = 0, 1$,

$$\begin{aligned} -h_i \int_{t-h_i}^t \dot{y}^T(s) H_j \dot{y}(s)\,ds &\le -\left[\int_{t-h_i}^t \dot{y}(s)\,ds\right]^T H_j \left[\int_{t-h_i}^t \dot{y}(s)\,ds\right] \\ &\le -\bigl[y(t) - y\bigl(t-h(t)\bigr)\bigr]^T H_j \bigl[y(t) - y\bigl(t-h(t)\bigr)\bigr] \\ &= -y^T(t) H_i y(t) + 2x^T(t) H_j y\bigl(t-h(t)\bigr) \\ &\quad - y^T(t-h_i) H_j y(t-h_i). \end{aligned} \tag{3.6}$$

Note that

$$\int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds = \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds + \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds.$$

Applying Proposition 2.2 gives

$$\begin{aligned} \bigl[h_1 - h(t)\bigr] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds &\ge \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s)\,ds\right]^T U \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s)\,ds\right] \\ &\ge \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]. \end{aligned}$$

Since $h_1 - h(t) \le h_1 - h_0$, we have

$$[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds \ge \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr],$$

then

$$-[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds \le -\bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr]^T U \bigl[y\bigl(t-h(t)\bigr) - y(t-h_1)\bigr].$$

Similarly, we have

$$-(h_1 - h_0) \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds \le -\bigl[y(t-h_0) - y\bigl(t-h(t)\bigr)\bigr]^T U \bigl[y(t-h_0) - y\bigl(t-h(t)\bigr)\bigr].$$

Then we have

$$\begin{aligned} \dot{V}(\cdot) + 2\alpha V(\cdot) \le{}& y^T(t)\bigl[-PA_j^T - A_j P\bigr] y(t) - y^T(t) B_j B_j^T y(t) + 2y^T(t) W_{0j} f(\cdot) \\ &+ 2y^T(t) W_{1j} g(\cdot) + y^T(t)\left(\sum_{i=0}^1 G_i\right) y(t) + 2\alpha\bigl\langle Py(t), y(t)\bigr\rangle \\ &+ \dot{y}^T(t)\left(\sum_{i=0}^1 h_i^2 H_i\right)\dot{y}(t) + (h_1 - h_0)\dot{y}^T(t) U \dot{y}(t) \end{aligned}$$

$$\begin{aligned}
&- \sum_{i=0}^{1} e^{-2\alpha h_i} y^T(t-h_i) G_i y(t-h_i) \\
&- e^{-2\alpha h_0} \big[y(t) - y(t-h_0)\big]^T H_0 \big[y(t) - y(t-h_0)\big] \\
&- e^{-2\alpha h_1} \big[y(t) - y(t-h_1)\big]^T H_1 \big[y(t) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1} \big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U \big[y\big(t-h(t)\big) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1} \big[y(t-h_0) - y\big(t-h(t)\big)\big]^T U \big[y(t-h_0) - y\big(t-h(t)\big)\big].
\end{aligned} \tag{3.7}$$

Using equation (2.5)

$$P\dot{y}(t) + A_j P y(t) - W_{0j} f(\cdot) - W_{1j} g(\cdot) + 0.5 B_j B_j^T y(t) = 0,$$

and multiplying both sides by $[2y(t), -2\dot{y}(t), 2y(t-h_0), 2y(t-h_1), 2y(t-h(t))]^T$, we have

$$\begin{aligned}
&2y^T(t)P\dot{y}(t) + 2y^T(t)AjPy(t) - 2y^T(t)W_{0j}f(\cdot) - 2y^T(t)W_{1j}g(\cdot) \\
&\quad + y^T(t)B_jB_j^T y(t) = 0, \\
&-2\dot{y}^T(t)P\dot{y}(t) - 2\dot{y}^T(t)AjPy(t) + 2\dot{y}^T(t)W_{0j}f(\cdot) \\
&\quad + 2\dot{y}^T(t)W_{1j}g(\cdot) - \dot{y}^T(t)B_jB_j^T y(t) = 0, \\
&2y^T(t-h_0)P\dot{y}(t) + 2y^T(t-h_0)AjPy(t) - 2y^T(t-h_0)W_{0j}f(\cdot) \\
&\quad - 2y^T(t-h_0)W_{1j}g(\cdot) + y^T(t-h_0)B_jB_j^T y(t) = 0, \\
&2y^T(t-h_1)P\dot{y}(t) + 2y^T(t-h_1)AjPy(t) - 2y^T(t-h_1)W_{0j}f(\cdot) \\
&\quad - 2y^T(t-h_1)W_{1j}g(\cdot) + y^T(t-h_1)B_jB_j^T y(t) = 0, \\
&2y^T\big(t-h(t)\big)P\dot{y}(t) + 2y^T\big(t-h(t)\big)AjPy(t) - 2y^T\big(t-h(t)\big)W_{0j}f(\cdot) \\
&\quad - 2y^T\big(t-h(t)\big)W_{1j}g(\cdot) + 2y^T\big(t-h(t)\big)B_jB_j^T y(t) = 0.
\end{aligned} \tag{3.8}$$

Adding all the zero items of (3.8) and $f^0(t, x(t), x(t-h(t)), u(t)) - f^0(t, x(t), x(t-h(t)), u(t)) = 0$, respectively, into (3.7) and using the condition (2.4) for the following estimations:

$$\begin{aligned}
f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big) &\le \big\langle Q_1 x(t), x(t)\big\rangle + \big\langle Q_2 x\big(t-h(t)\big), x\big(t-h(t)\big)\big\rangle \\
&\quad + \big\langle Ru(t), u(t)\big\rangle \\
&= \big\langle PQ_1Py(t), y(t)\big\rangle + \big\langle PQ_2Py\big(t-h(t)\big), y\big(t-h(t)\big)\big\rangle \\
&\quad + 0.25\big\langle B_jRB_j^T y(t), y(t)\big\rangle, \\
2\big\langle W_{0j}f(x), y\big\rangle &\le \big\langle W_{0j}D_0W_{0j}^T y, y\big\rangle + \big\langle D_0^{-1}f(x), f(x)\big\rangle, \\
2\big\langle W_{1j}g(z), y\big\rangle &\le \big\langle W_{1j}D_1W_{1j}^T y, y\big\rangle + \big\langle D_1^{-1}g(z), g(z)\big\rangle, \\
2\big\langle D_0^{-1}f(x), f(x)\big\rangle &\le \big\langle FD_0^{-1}Fx, x\big\rangle, \\
2\big\langle D_1^{-1}g(z), g(z)\big\rangle &\le \big\langle ED_1^{-1}Ez, z\big\rangle,
\end{aligned}$$

we obtain

$$\begin{aligned}\dot{V}(\cdot) + 2\alpha V(\cdot) &\leq \zeta^T(t)\mathcal{E}_j\zeta(t) + y^T(t)S_{1j}y(t) + y^T\big(t-h(t)\big)S_{2j}y\big(t-h(t)\big) \\ &\quad - f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big),\end{aligned} \tag{3.9}$$

where $\zeta(t) = [y(t), \dot{y}(t), y(t-h_0), y(t-h_1), y(t-h(t))]$, and

$$\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix},$$

$$S_{1j} = -PA_j - A_j^T P - \sum_{i=0}^{1} e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP + PQ_1P,$$

$$S_{2j} = W_{1j}D_1W_{1j}^T - e^{-2\alpha h_2}U + 4PED_1^{-1}EP + PQ_2P.$$

Note that by the Schur complement lemma, Proposition 2.1, the conditions $S_{1j} < 0$ and $S_{2j} < 0$ are equivalent to the conditions (3.2) and (3.3), respectively. Therefore, by conditions (3.1), (3.2), (3.3), we obtain from (3.9) that

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0. \tag{3.10}$$

Integrating both sides of (3.10) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \geq 0.$$

Furthermore, taking condition (3.5) into account, we have

$$\lambda_1\big\|x(t,\phi)\big\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t}\|\phi\|^2,$$

then

$$\big\|x(t,\phi)\big\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|, \quad t \geq 0,$$

which concludes the exponential stability of closed-loop system (2.5). To prove the optimal level of the cost function (2.3), we derive from (3.9) and (3.1)-(3.3) that

$$\dot{V}(t, z_t) \leq -f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big), \quad t \geq 0. \tag{3.11}$$

Integrating both sides of (3.11) from 0 to t leads to

$$\int_0^t f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big)\,dt \leq V(0, z_0) - V(t, z_t) \leq V(0, z_0),$$

due to $V(t,z_t) \geq 0$. Hence, letting $t \to +\infty$, we have

$$J = \int_0^\infty f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big)\,dt \leq V(0,z_0) \leq \lambda_2\|\phi\|^2 = J^*.$$

This completes the proof of the theorem. □

Example 3.1 Consider the switched recurrent neural networks with interval time-varying delays (2.1), where

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad W_{01} = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.8 \end{bmatrix},$$

$$W_{02} = \begin{bmatrix} -0.7 & 0.3 \\ 0.4 & -0.9 \end{bmatrix}, \qquad W_{11} = \begin{bmatrix} -0.4 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, \qquad W_{12} = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.7 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}, \qquad R = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.9 \end{bmatrix},$$

$$\begin{cases} h(t) = 0.1 + 1.2652\sin^2 t & \text{if } t \in \mathcal{I} = \bigcup_{k\geq 0}[2k\pi, (2k+1)\pi], \\ h(t) = 0 & \text{if } t \in R^+ \setminus \mathcal{I}. \end{cases}$$

Note that $h(t)$ is non-differentiable, therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system. Given $\alpha = 0.3$, $h_0 = 0.1$, $h_1 = 1.3652$, by using the Matlab LMI toolbox, we can solve for P, U, G_0, G_1, H_0, H_1, D_0, and D_1 which satisfy the conditions (3.1)-(3.3) in Theorem 3.1.

A set of solutions is as follows:

$$P = \begin{bmatrix} 1.5219 & -0.3659 \\ -0.3659 & 2.2398 \end{bmatrix}, \qquad U = \begin{bmatrix} 3.1239 & -0.2365 \\ -0.2365 & 3.0123 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} 1.3225 & 0.0258 \\ 0.0258 & 1.2698 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 2.2368 & 0.0148 \\ 0.0148 & 3.1121 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} 2.2189 & 0.1238 \\ 0.1238 & 1.2368 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 2.3225 & 0.0369 \\ 0.0369 & 2.1897 \end{bmatrix},$$

$$D_0 = \begin{bmatrix} 2.9870 & 0 \\ 0 & 3.2589 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 3.2698 & 0 \\ 0 & 4.3258 \end{bmatrix}.$$

Then

$$u_1(t) = 0.2579x_1(t) + 0.2589x_2(t), \quad t \geq 0,$$

$$u_2(t) = 0.1397x_1(t) + 0.2176x_2(t), \quad t \geq 0,$$

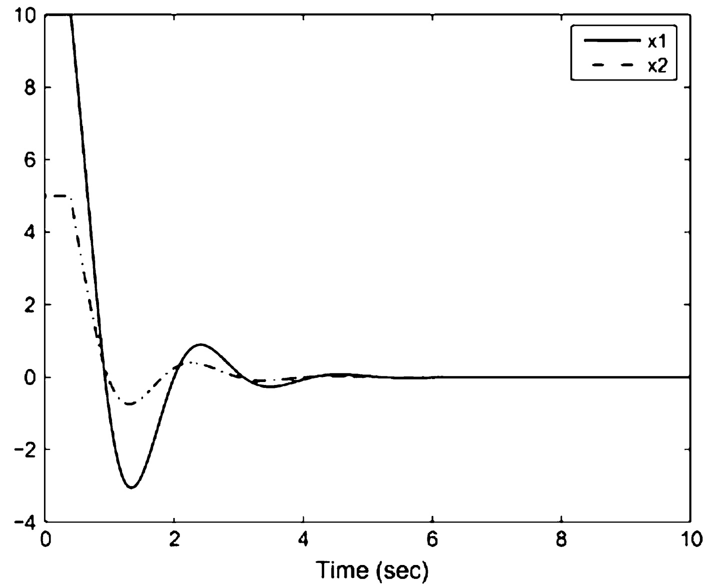

Figure 1 The simulation of the solutions $x_1(t)$ and $x_2(t)$ with the initial condition $\phi(t) = [10\ 5]^T$, $t \in [0, 10]$.

are a guaranteed cost control law and the cost given by

$$J^* = 1.1268\|\phi\|^2.$$

Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq 2.3257e^{-0.3t}\|\phi\|, \quad \forall t \geq 0.$$

The trajectories of solution of switched recurrent neural networks is shown in Figure 1, respectively.

4 Conclusions

In this paper, the problem of guaranteed cost control for Hopfield neural networks with interval non-differentiable time-varying delay has been studied. A nonlinear quadratic cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of time-varying Lyapunov-Krasovskii functionals, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities. A memoryless state feedback guaranteed cost controller design has been presented and sufficient conditions for the existence of the guaranteed cost state-feedback for the system have been derived in terms of LMIs.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

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