Guaranteed Passive Balancing Transformations for Model Order Reduction

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ABSTRACT

The major concerns in state-of-the-art model reduction algorithms are: achieving accurate models of sufficiently small size, numerically stable and efficient generation of the models, and preservation of system properties such as passivity. Algorithms such as PRIMA generate guaranteed-passive models, for systems with special internal structure, using numerically stable and efficient Krylov-subspace iterations. Truncated Balanced Realization (TBR) algorithms, as used to date in the design automation community, can achieve smaller models with better error control, but do not necessarily preserve passivity. In this paper we show how to construct TBR-like methods that guarantee passive reduced models and in addition are applicable to state-space systems with arbitrary internal structure.

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1. INTRODUCTION

Model reduction has been an active research field in design automation over the past decade. In an integrated circuits context, initial interest in model reduction techniques stemmed from efforts to accelerate analysis of circuit interconnect. More recently, model reduction has come to be viewed as a method for generating compact models from all sorts of physical system modeling tools. Because of the need to obtain accurate high-order models at reasonable computational cost, the Krylov-subspace model reduction methods [1] have occupied the forefront of research over the past five years.

Recently it has become apparent that, while very suitable for analysis of large-scale systems, the models generated from the Krylov techniques such as PRIMA and PVL are not necessarily as compact (that is, small in order) as is desired [2, 3]. Therefore, another approach, already well-developed in the control literature has been receiving renewed attention in the electronic design automation community: that of Truncated Balanced Realization (TBR) [4].

Truncated balanced realization algorithms (and their close relatives that generate optimal norm approximants [5]) are of importance in their own right. For small systems, a few hundred states or so, they generate superior reduced models, as well as computable bounds on the reduction error. For large systems, direct application of the techniques used to balance and truncate the systems is computationally infeasible since the computations required have $O(n^3)$ complexity when performed directly (n is the order of the system to be reduced). Therefore the TBR methods are of more interest when combined with iterative Krylov-subspace procedures. One formulation of this method is to directly solve the large Lyapunov equations via a Krylov subspace method [6, 7]. The reduced models are obtained directly from the reduced Lyapunov equation. Another viewpoint is to obtain an initial reduced model via some initial reduction or approximation technique and then further compress it using a TBR method. This second viewpoint is somewhat more general since the initial approximation can be generated by any desired method, for example rational fitting [8] or a now standard Krylov-subspace technique [2, 9].

An issue with the TBR type methods that has not been addressed in most of the above mentioned works is that they cannot be relied on to preserve passivity. The technique in [2] uses a passivity-preserving initial reduction, but follows this reduction with a standard TBR method. There is no guarantee that the second TBR step will not destroy the passivity of the initial model. More problematic, no means is given in [2] to determine if the final model is passive – or not.

Less widely appreciated is another dilemma: Krylov methods such as PRIMA have practical issues that prevent their wide application to systems outside the class of RLC circuits. These methods rely on congruence transformations to preserve positive-realness of the matrices that are internal to the state-space representation. However, whether or not a state-space model represents a passive system is a property of the input-output transfer function, not a property of the internal representation. Many passive systems are not conveniently put into a form for which algorithms such as PRIMA are applicable: they may have asymmetric or non-positive semidefinite system and descriptor matrices. It may not be possible to perform a change of basis to a convenient form without destroying sparse structure that may be present in the system, meaning that for large-scale systems such an approach is infeasible.

Further, positive realness is not necessarily the right property to seek. If the state-space model represents scattering (S) parameters of a passive system, the system is passive if the norm of the S-parameter matrix is bounded by unity, and so even the transfer function has no relation to positive-realness. Such systems cannot

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be reduced by congruence with any passivity guarantees. To the best of our knowledge, no effective truly general-purpose passivity-preserving algorithms are now widely available.

In this paper we discuss TBR-like model reduction algorithms that can preserve system passivity, have computable error bounds, and, unlike other algorithms such as PRIMA, pose no constraints on the internal structure of the state-space model. We describe variants that preserve both positive-realness (useful for systems that represent Y or Z parameters) and bounded-realness (useful for systems that represent S parameters). These algorithms can be applied directly to a given state-space description, or can be used as the second stage of a Krylov-subspace based procedure [6, 7, 2].

2. BACKGROUND

2.1 State-Space Models

Given a state-space model in descriptor form,

$$E\frac{dx}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t) \tag{1}$$

where $E, A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times p}, C \in \mathbb{R}^{p\times n}, D \in \mathbb{R}^{p\times p}, u(t) \in \mathbb{R}^p$, model reduction algorithms seek to produce a similar system with reduced $\tilde{E}, \tilde{A} \in \mathbb{R}^{q\times q}, \tilde{B} \in \mathbb{R}^{q\times p}, \tilde{C} \in \mathbb{R}^{p\times q}$, of order q much smaller than the original order n, but for which the outputs $y(t)$ and $\tilde{y}(t)$ are approximately equal for inputs $u(t)$ of interest. Often the transfer functions $H(s) = D + C(sE - A)^{-1}B$ and its reduced counterpart $\tilde{H}(s)$ is used as a metric for approximation : if $||\tilde{H}(s) - H(s)|| < \varepsilon$ for some given allowable error ε and allowed domain of the complex frequency variable s, the reduced model is accepted as accurate.

2.2 Passivity

When modeling passive systems – those that cannot produce energy internally – non-passive reduced models may cause non-physical behavior later in circuit simulators, such as by generating energy at high frequencies that causes erratic or unstable time-domain behavior. If $H(s)$ represents the Y (admittance) or Z (impedance) parameters of a system, positive-realness of $H(s)$ implies that the underlying state-space description is a representation of a passive system [10]. The function $H(s)$ is positive-real (PR) if

$$H^*(s) = H(s^*), \tag{2}$$

$$H(s) \text{ is analytic in } \mathrm{Re}(s) > 0. \tag{3}$$

$$H(s) + H(s)^H \geq 0 \text{ in } \mathrm{Re}(s) > 0. \tag{4}$$

If $H(s)$ represents the S (scattering) parameter matrix, then to represent a passive system, it is necessary that $H(s)$ be bounded-real [10]. A function $H(s)$ is bounded-real(BR) if (2) and (3) hold and in addition

$$I - H^H(s)H(s) \geq 0 \text{ in } \mathrm{Re}(s) > 0. \tag{5}$$

2.3 Krylov Methods

Recently developed model reduction methods suitable for application to large systems are based on Krylov-subspace techniques. Mathematically, the reduced models are obtained via a projection operation

$$\tilde{E} = W^T EV \quad \tilde{A} = W^T AV \quad \tilde{B} = W^T B \quad \tilde{C} = CV. \tag{6}$$

For example, PRIMA [1] constructs $V = W$ by using the Arnoldi algorithm, thereby spanning a Krylov subspace of $A^{-1}E$. Because of the moment-matching properties of Krylov-subspaces, the reduced transfer function $\tilde{H}(s)$ will agree with the original $H(s)$ up to the first q derivatives.

The PRIMA algorithm has another interesting property. Given a starting passive model, if the original state-space model can be formulated with positive-semidefinite A and E and $B = C^T$, then the transfer function of the final reduced model will be positive-real, meaning the reduced system is also passive. This is essentially because the projection operation in (6) becomes a congruence transform for $W = V$, and since congruence transforms preserve positive-semi-definiteness, the reduced $\tilde{E}, \tilde{A}$ inherit the numerical range properties of their parents, implying that the reduced function $\tilde{H}(s)$ is positive-real. Note however that it is entirely possible to have systems with positive-real $H(s)$, and thus underlying passive models, for which the conditions necessary for using PRIMA do *not* hold. Such systems cannot be reduced in a guaranteed positive-real manner via congruence transformations. Likewise, such techniques cannot guarantee bounded-real reduced models from bounded-real starting systems.

3. TRUNCATED BALANCED REALIZATIONS

Complementary model reduction techniques are based on truncated balanced realization. We are mostly interested in applying TBR procedures as the second stage of a composite model reduction procedure [2], the first stage being reduction by a Krylov–based projection method. Note that the most of algorithms in [6, 7], are essentially equivalent to a first-stage Krylov projection followed by a second-stage TBR procedure. We first discuss the most commonly used approach before presenting passivity-preserving variants. It is common in this literature to assume $E = I$. When E is non-singular, the mapping $E \to I, A \to E^{-1}A, B \to E^{-1}B$ will put a descriptor system into this form. It is very common in electrical engineering applications to have situations where E is in fact singular and this procedure cannot be performed, but in the situations of interest to us, where an initial projection step has taken place, usually E is non-singular, and so to facilitate comparisons with the literature and somewhat simplify the computational procedure, we will assume the system can be manipulated (possibly implicitly) into a form where $E = I$. We do emphasize that it *is* possible to formulate the computational procedure to work with E directly, and this is necessary when E is singular [1] but we have chosen not to do this as a matter of convenience.

3.1 Standard Approach

The TBR procedure as first presented in [4] is centered around information obtained from the controllability Grammian W_c, which can be obtained from solving the Lyapunov equation

$$AW_c + W_c A^T = -BB^T \tag{7}$$

for W_c, and the observability Grammian W_o, which can be obtained from the dual Lyapunov equation

$$A^T W_o + W_o A = -C^T C. \tag{8}$$

Under a similarity transformation of the state-space model,

$$A \to T^{-1}AT, \quad B \to T^{-1}B, \quad C \to CT \tag{9}$$

the state-space model, and the transfer function, are invariant (only the internal variables are changed). The grammians, however, vary under the rules

$$W_c \to T^{-1}W_c T^{-T}, \quad W_o \to T^T W_o T \tag{10}$$

and so are not invariant. The TBR procedure is based on two observations about W_o and W_c. First, the eigenvalues of the product

[1] For example, Eqn. (7), becomes $AW_c E^T + EW_c A^T = -EBB^T E^T$.

Algorithm 1. **Truncated Balanced Realization (TBR)**

1. *Solve $AW_c + W_cA^T = -BB^T$ for W_c*
2. *Solve $A^TW_o + W_oA = -C^TC$ for W_o*
3. *Compute Cholesky factors $W_c = L_cL_c^T, W_o = L_oL_o^T$,*
4. *Compute SVD of Cholesky product $U\Sigma V = L_o^TL_c$ where Σ is diagonal positive and U,V have orthonormal columns*
5. *Compute the balancing transformations $T = L_cV\Sigma^{-1/2}, \quad T^{-1} = \Sigma^{-1/2}U^TL_o^T$*
6. *Form the balanced realization as $\hat{A} = T^{-1}AT, \quad \hat{B} = T^{-1}B, \quad \hat{C} = CT$*
7. *Select reduced model order and partition $\hat{A},\hat{B},\hat{C}$ conformally*
8. *Truncate $\hat{A},\hat{B},\hat{C}$ to form the reduced realization $\tilde{A},\tilde{B},\tilde{C}$*

W_cW_o are invariant. These eigenvalues, the Hankel singular values, contain useful information about the input-output behavior of the system. In particular, "small" eigenvalues of W_cW_o correspond to internal sub-systems that have a weak effect on the input-output behavior of the system and are therefore close to non-observable or non-controllable or both.

Second, since the Grammians transform under congruence, and as any two symmetric matrices can be simultaneously diagonalized by an appropriate congruence transformation, it is possible to find a similarity transformation T that leaves the state-space system dynamics unchanged, but makes W_o and W_c equal and diagonal. In these coordinates, with $W_c = W_o = \Sigma$, we may partition Σ into

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \tag{11}$$

where Σ_1 describes the "strong" sub-systems to be retained and Σ_2 the "weak" sub-systems to be deleted. Conformally partitioning the matrices as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \tag{12}$$

and truncating the model, retaining $\tilde{A} = A_{11}, \tilde{B} = B_1, \tilde{C} = C_1$ as the reduced system, therefore has the effect of deleting the "weak" internal subsystems. A complete TBR algorithm [11] is shown as Algorithm 1.

We now turn to the question of when TBR procedures produce passive reduced models.

3.2 Symmetrizable Systems

It turns out that there is a special system case, of relevance to integrated circuits applications, for which the standard TBR procedure (Algorithm 1) always produces positive-real reduced models. Suppose that the state-space model is *symmetric*, that is $A = A^T, B = C^T$, and furthermore A is negative-semidefinite. Since $\mathrm{Re}\{sI - A\} = -\frac{1}{2}(A + A^T) = -A \geq 0$, the system is positive-real. From Eqns. (7) and (8) it follows that $W_o = W_c$. From inspecting step 5 in Algorithm 1, we find that $T^{-1} = T^T$. *Thus the similarity transformation is a congruence transformation.* The reduced $\tilde{A}$ must be negative-semidefinite, and we will likewise have $\tilde{B} = \tilde{C}^T$. Therefore the reduced system is positive-real. This would seem to be a similar situation as we have in PRIMA, but it is actually far more general. The reason is that the balancing transformation is essentially unique as explained in [4], meaning that we have the following broader result:

Algorithm 2. **Positive-Real TBR (PR-TBR)**

1. *Solve Eqns. (13)-(15) for X_c and their duals for X_o.*
2. *Proceed with steps 3-8 in Algorithm 1, with X_c for W_c and X_o for W_o.*

THEOREM 1. *Suppose a state-space system is transformable under similarity to a system of the form in Eqn. (1), with $E = E^T, A = A^T, A \leq 0$. A reduced model generated via Algorithm 1 is positive-real.*

In contrast, the positive-realness preserving properties of congruence transformations depend on the coordinate system used and are *not* preserved under similarity transformations.

Some systems that fall into the symmetrizable class are RL and RC circuits in MNA form, and reductions of such forms via congruence. All systems, however, do not fall into this class, and more powerful techniques are needed to preserve passivity in TBR methods.

3.3 Positive Real Conditions

We will show in Section 5 that the TBR procedure of Algorithm 1 does not necessarily produce passive models. In making assessments about passivity, we require a tool that can assess the positive-realness of a state-space model in a global manner. One such tool is the positive-real lemma [10], which states that $H(s)$ is positive-real if and only if there exist matrices $X_c = X_c^T, J_c, K_c$ such that the Lur'e equations:

$$AX_c + X_cA^T = -K_cK_c^T \tag{13}$$

$$X_cC^T - B = -K_cJ_c^T, \tag{14}$$

$$J_cJ_c^T = D + D^T \tag{15}$$

are satisfied, and $X_c \geq 0$ (X_c is positive-semidefinite). X_c is analogous to the controllability Grammian. In fact, it is the controllability Grammian for a system with the input-to-state mapping given by the matrix K_c. It should not be surprising that there are a dual set of Lur'e equations for $X_o = X_o^T > 0, J_o, K_o$ that are obtained from Equations (13)-(15) by the substitutions $A \to A^T, B \to C^T, C^T \to B$. The dual equations have a corresponding observability quantity $X_o \geq 0$ for a positive-real $H(s)$. It is easy to verify that X_c, X_o transform under similarity transformation just as W_c, W_o (Eqn. 10), that their eigenvalues are invariant, and in fact in most respects they behave as the Grammians W_c, W_o.

3.4 Guaranteed Passive Balanced Truncations

It should be clear than the Lur'e equations can be solved for the quantities X_c, X_o which may then be used as the basis for a TBR procedure. We present this as Algorithm 2 and call it PR-TBR, as it preserves positive-realness of the transfer function.

THEOREM 2. *Algorithm 2 applied to systems with positive-real transfer functions produces reduced models with positive-real transfer functions.*

PROOF. From the form of the partitioning, (11) and (12), likewise partitioning either K_c or K_o, it is clear that the reduced system, *Algorithm* 3. **Bounded-Real TBR (BR-TBR)**

1. *Solve Eqns. (19)-(21) and their duals for X_c, X_o.*
2. *Proceed with steps 3-8 in Algorithm 1, with X_c for W_c and X_o for W_o.*

in the PR-balanced coordinates, satisfies

$$A_{11}\Sigma_1 + \Sigma_1 A_{11}^T = -K_1 K_1^T \tag{16}$$

$$\Sigma_1 C_1^T - B_1 = -K_1 J_c^T, \tag{17}$$

$$J_c J_c^T = D + D^T. \tag{18}$$

Therefore the reduced system satisfies the Lur'e equations with positive semi-definite Σ_1 ($\Sigma_1 \geq 0$ as $\Sigma \geq 0$). By the positive-real lemma, the reduced system is positive-real. $\square$

We emphasize that Theorem 2 holds regardless of the internal form of the state-space system. Again, this is not true for congruence based procedures. Finally, we note that, just as for TBR, error bounds are available for PR-TBR (see [12] where a similar technique was proposed). A discussion of those bounds is beyond the scope of this paper, but we will state that in most cases we tried, the bounds obtained are competitive with those of TBR.

3.5 Bounded-Real Conditions

To obtain equivalent TBR procedures that guarantee a final transfer function that is bounded-real, useful when working with transfer functions representing S-parameters, we need the bounded real equations

$$AY_c + Y_c A^T = -BB^T - K_o K_o^T \tag{19}$$

$$Y_c C^T + BD = -K_o^T J_o \tag{20}$$

$$J_o J_o^T = I - D^T D \tag{21}$$

and corresponding dual equations that are satisfied with $Y_c \geq 0, Y_o \geq 0$ if the system transfer function is bounded-real. Algorithm 3 performs truncated balanced realization while guaranteeing the boundedness of the reduced transfer function [2].

3.6 A hybrid approach

In many cases, while not guaranteed by construction, it is often the case that the TBR approximants produced by Algorithm 1 turn out to be positive-real. Therefore we propose Algorithm 4, which performs the TBR procedure, solves the positive-real (or bounded-real) equations on the reduced model to check its passivity, and if it turns out not to be passive, discard it and proceeds to Algorithm 2 (or Algorithm 3). There is an advantage in this procedure as often the TBR approximates are more accurate for a given order than PR-TBR. Because of the cubic scaling of cost, it is relatively cheap, compared to the cost of the TBR reduction, to check a reduced model for passivity since the reduced system is presumably of lower order. As often the TBR models are passive, the net effect of the composite algorithm is to approximately double the cost in the worst case, versus usually getting better models at smaller cost (PR-TBR "costs" more than TBR) in the more-common average case.

Algorithm 4, which appropriately combines all of the previously presented algorithms, can be used as generic flow for generating accurate guaranteed passive reduced-order models of systems with arbitrary structure.

[2]The bound does not have to be unity; it can be any positive constant.

Algorithm 4. **Hybrid TBR**

1. *Perform Algorithm 1*
2. *Using the reduced model matrices $\tilde{A}, \tilde{B}, \tilde{C}$, solve Eqns. (13)-(15) for $\tilde{X}_c$ (or Eqns. (19)-(21)).*
3. **if** *Eqns. (13)-(15) (or Eqns. (19)-(21)) are solvable and $X_c > 0$, then terminate and return $\tilde{A}, \tilde{B}, \tilde{C}$.*
   else *discard TBR-reduced model and proceed with Algorithm 2 (or 3).*

4. COMPUTATIONAL CONSIDERATIONS

Solution of the Lur'e equations and solution of algebraic Riccati equations (AREs) are closely related. An overview of basic numerically robust computational procedures is given in [13]. Extensions of the positive-real lemma are available for models in the descriptor form where E is singular such that the transfer function cannot be put into the standard form [14]. To obtain a simpler procedure, consider the matrix pencil $\lambda E - A$,

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A & 0 & B \\ 0 & -A^T & C^T \\ C & -B^T & D+D^T \end{bmatrix}. \tag{22}$$

Suppose that via some means we have computed an invariant subspace $Z \in \mathbb{R}^{(2n+p)\times n}$ that satisfies $EZ\Lambda = AZ$, $\Lambda \in \mathbb{R}^{n\times n}$, of the special form

$$Z = \begin{bmatrix} I \\ X \\ \tilde{X} \end{bmatrix} \tag{23}$$

where $I, X \in \mathbb{R}^{n\times n}, \tilde{X} \in \mathbb{R}^{p\times n}$. Then from the invariance condition $EZ\Lambda = AZ$, it can be verified that X is indeed the solution to Eqns. (13)-(15). To compute an invariant subspace of such a special form, the rank-p singularity of the pencil is first compressed [13] using a QR factorization of A to reduce the dimension of the pencil to $2n$. Then, we find an invariant subspace $Z_r \in \mathbb{R}^{2n\times n}$ (for example via the QZ method) of the form

$$Z_r = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}. \tag{24}$$

X can be computed as $X = X_2 X_1^{-1}$. For a positive-definite X, we need the subspace that corresponds to stable eigenvalues of the pencil.

Two more issues deserve our attention even though, for space limitations, we cannot discuss them in detail. The TBR technique in Algorithm 1 often leads to reduced models that present a mismatch in DC gains when compared to the original unreduced model, as the algorithm tends to perform better at high frequencies than at DC. Transformation to the *reciprocal system* [15], which maps $s = 0$ into $s = \infty$, can help produce better approximations at low frequencies, as can frequency weighting.

The second issue is related to computational cost. While the cost of all balancing procedures presented is in principle cubic (due to the need for solving Lyapunov-type equations), such cost is not overwhelming if the algorithm is being applied to a system that results from a prior reduction. Furthermore, even if that is not the case, one can still directly solve the large Lyapunov equations via a Krylov subspace method [6, 7], and obtain the reduced models directly from the reduced Lyapunov equation.

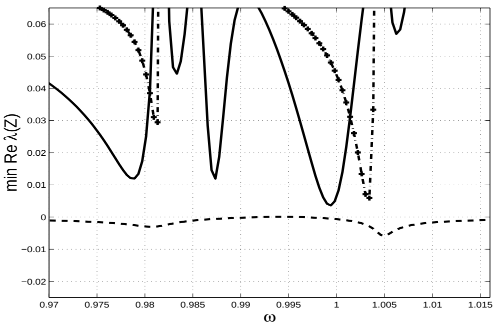


Figure 1: Minimum eigenvalue of transfer functions used to illustrate non-positive-real reduced model generated by standard TBR procedure. Solid line shows original (positive-real) order-26 transfer function, dashed line shows TBR result of order 7, dashed-dot line shows PR-TBR result of order 7. Note negative sign of TBR results indicating non-positive-realness.

5. RESULTS

In this section we show examples that illustrate the relevance of the various algorithms presented in this paper.

5.1 A non-passive ROM generated by TBR

First we demonstrate empirically that standard TBR (Algorithm 1) can generate models that are not passive by examining a 26-state lumped circuit model of a crystal filter. We generated all the possible TBR models of orders 1-26, and used the positive real lemma to inspect them for positive-realness (equivalent to passivity in this case). Several of the models were found to be non-passive (see Figure 1). We then generated all the possible PR-TBR models. All were found to be positive-real as expected.

5.2 A symmetrizable system

Our first example is a spiral inductor modeled with the magnetoquasistatic electromagnetic tool FASTHENRY. This example appeared in [2]. The initial system of around 1500 states is reduced to an initial 60-state positive-real model using PRIMA. Since this order is still considered excessive, the model is then further reduced using TBR. In [2], it was commented that the reduced models after the TBR procedure appeared to be passive, but no explanation was given. Here we have rigorously checked, using the positive-real lemma, that the models were indeed passive, and gave a proof as to why that should be the case.

5.3 A bounded-real example from rational function fitting

In the next example we consider the bounded-real variant of the TBR procedure (BR-TBR). First, a rational fitting method was used to fit a high-order model to tabulated 2-port S-parameter data originating from a full-wave EM field solver. The fitting algorithm, which has provision for automatic estimation of model order, was tuned to a conservative setting, and generated an order-42 initial model that was nearly an exact fit to the data in the given frequency range. The resulting 42-state model was much larger than desired for final simulation, so the BR-TBR procedure was used to reduce the model to six states. The results are shown in Figure 2. The reduced model had norm bounded by unity, indicating that the reduced model represented a passive element. Several models of orders six to eight were also generated by both TBR and congruence transform strategies, but all had H_∞ norms ranging from 1.05 to 1.9,

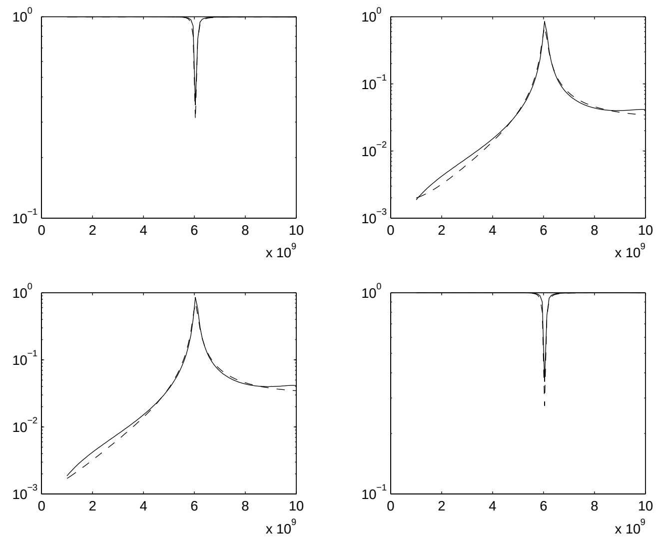
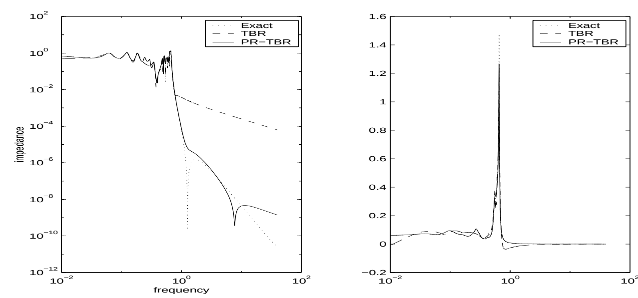


Figure 2: Magnitude of rational function fit and reduced model for *S*-parameters tabulated by full-wave field solver. Solid line shows initial data and order-42 rational fit (complete overlap). Dashed line shows order-6 reduced model obtained via BR-TBR.



a) b)

***Figure 3:** Left: Magnitude of Y_{12} for LC line. Right: Minimum eigenvalue of symmetric part of reduced model transfer function. Note that the minimum eigenvalue the TBR model drops below zero for some frequencies, indicating non-passivity.*

i.e. they were not passive.

5.4 A PEEC Connector

This example features a connector structure from Teradyne Inc. composed of 18 pins with a ground shield around and between the conductors. This structure was previously used [3] to illustrate a PEEC formulation based on PRIMA that generates passive reduced-order models. While the resulting model was indeed provably passive, disappoint reductions were reported, due mostly to the inability of the PRIMA algorithm to zero-in on the relevant modes of the system. In fact volume discretization of the interior of the conductors in order to properly model skin-effect leads to the appearance of various internal subsystems that have negligible effect in the structure impedance but which can fool the PRIMA algorithm. In [9] the same example was used to illustrate a two-step algorithm for RLC order reduction based on PRIMA followed by TBR, in an apparent attempt to solve the above problem. Significant order reductions were reported after the 2^{nd} step of reduction as TBR is able to determine that those modes are not observable nor controllable. While this clearly shows that further reduction after the PRIMA stage is possible and indeed desirable, passivity was no longer guaranteed in the final, smaller models.

Here we have used the same example and checked the passivity of reduced-order models of various orders. We believe that the modes that are being discarded by TBR are related to the internal subsystems resulting from skin-effect modeling. As such the character of the problem after the initial PRIMA reduction is predominantly RL, a type of system for which we know that TBR is passive(see Section 3.2). Once more we generated all the possible TBR models for the system obtained after the PRIMA reduction and used the positive real lemma to inspect them for positive-realness (again equivalent to passivity in this case). Due to the *almost* symmetric nature of the systems, almost all the models we obtained were found to be passive. However, models of order 19 and 29 were found to be non-passive, a problem if the model is to be used in time-domain simulations. This example shows once more that TBR can indeed lead to large reductions in model-order but can produce non-physical models. The example also presents a strong case for using the generic flow presented earlier (see Section 3.6 and Algorithm 4). Since the majority of the TBR-produced models are likely to be passive it is advantageous to obtain such a model, check it for passivity and only compute the PR-TBR algorithm if that check fails (alternatively compute another model of slightly different order and check it, which is cheap since TBR essentially produces models of all orders simultaneously).

5.5 An RLC Line

For our next example we use a 40-segment uniform RLC line that is L-dominated. The values of the line were chosen to be $R = 25$, $C = L = 0.39894$. For the purpose of comparison we computed 25^{th} order models using both TBR and PR-TBR. Figure 3-a) shows the low-frequency behavior of the exact line impedance as well as that obtained using the two models. For this particular case it turns out that PR-TBR performs much better than regular TBR in terms of the model error. More important, however is the result shown in Figure 3-b) where we plot the minimal eigenvalue of the symmetric part of the transfer function as a function of frequency. As can be seen from the plot, the minimal eigenvalue for the TBR model can go below zero at some frequencies which implies that the model is non-passive and may produce non-physical responses when used in time-domain simulations. In fact, on this example, *almost none* of the models produced by TBR were passive. Only very high order models exhibiting an almost exact match to the transfer function over the entire frequency axis were passive. In contrast, all the models produced by the PR-TBR method were found to be passive, as expected.

6. CONCLUSION

In this paper we presented a family of algorithms that can be used to compute guaranteed passive, reduced-order models of controllable accuracy for state-space systems with arbitrary internal structure.

The algorithms presented are similar to the well-known truncated balanced realization (TBR) techniques and share some of their advantages, such as computable error bounds. However, unlike standard TBR techniques, the algorithms presented have been shown to produce provably passive reduced-order models. In addition, unlike other techniques known to also produce passive reduction, the algorithms presented pose no constraints on the internal structure of the state-space. They are thus equally well applicable to systems that represent for instance Y or Z parameters as well as systems that represent S parameters. An hybrid algorithm was also presented where a TBR model is first computed, then checked for passivity and the passive-TBR algorithm is only used if that check fails.

We have experimented with our techniques in a large number of settings and have shown that they can be used as standalone procedures or as part of second step reductions for systems with a large number of unknowns, perhaps replacing the usual TBR procedure. We have thus applied our method to obtain reduced models of various structures, namely the two-port impedance of a crystal filter, a spiral inductors, a large connector and an RLC line. All models were found to be accurate and passive. All previously known techniques failed to produce acceptable models in some of the examples used.

7. REFERENCES

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