Guard Placement For Wireless Localization

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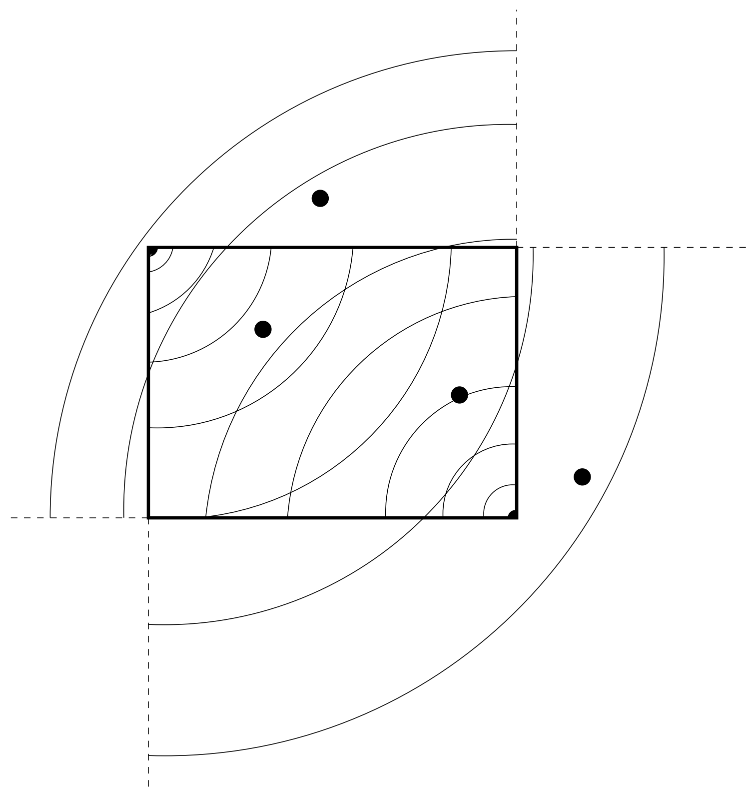
Motivating example: Cyber-Café

- Supply wireless internet to paying customers inside
- Prevent access to non-paying customers outside
- But how to tell which customers are which?

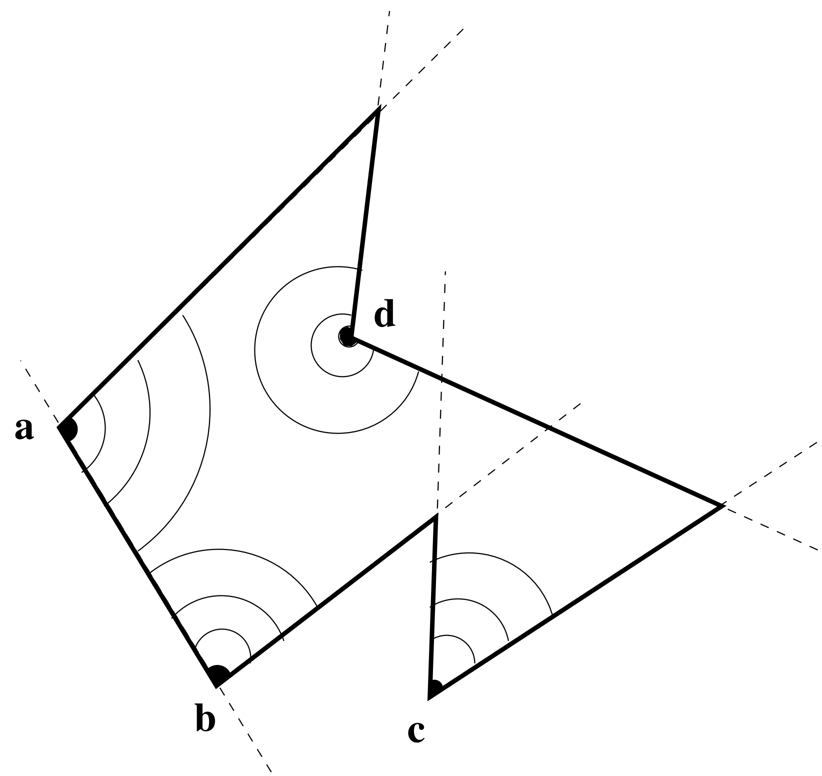
Proposed Solution



- Use multiple directional transmitters
- Customers in range of both transmitters are inside

Sculpture Garden Problem

<u>Sculpture Garden Problem</u> – the problem of placing angle guards to define a given polygon



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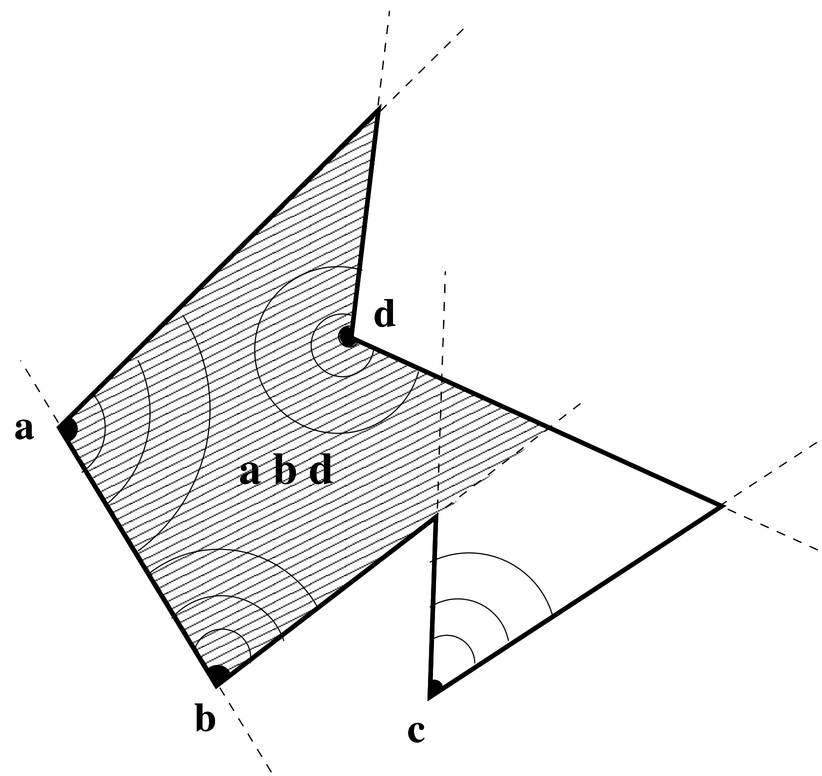

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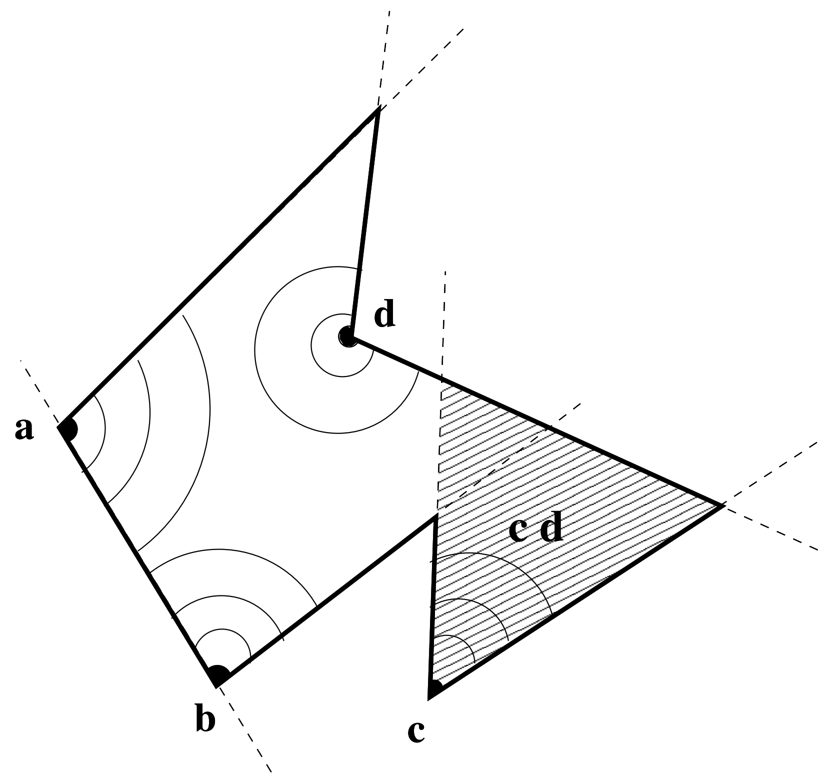

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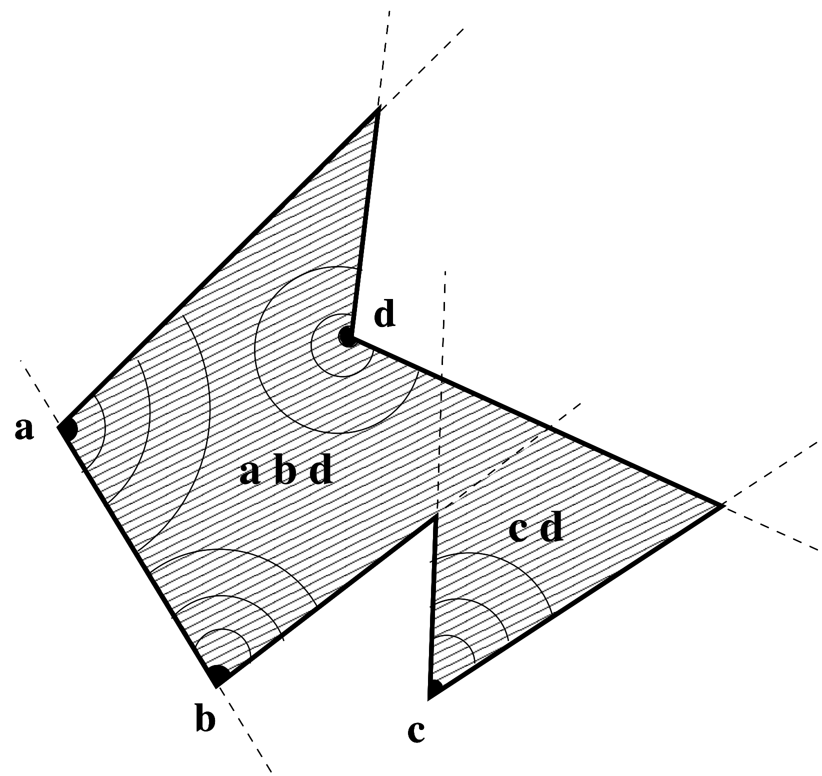
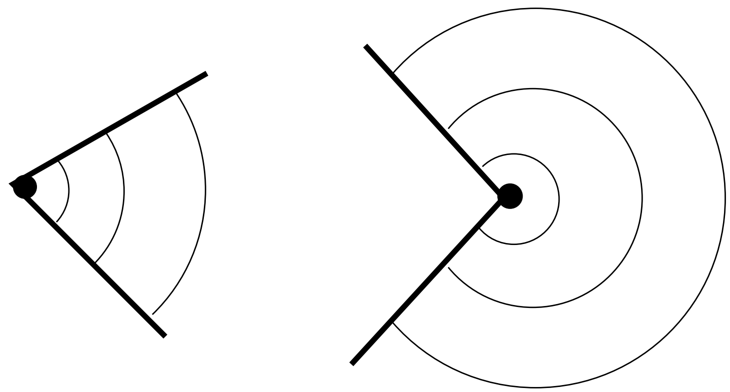


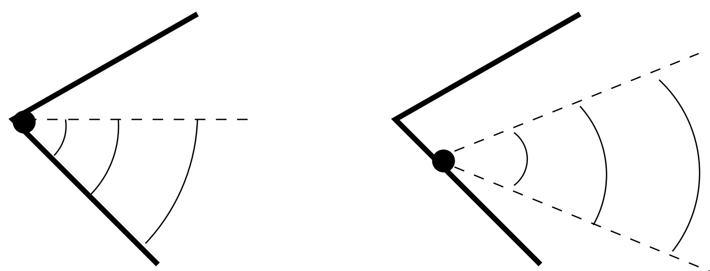
$$F = abd + cd = (ab + c)d$$

Natural Angle Guards

Natural angle guards are placed at vertices of the polygon with angle of vertex = angle of guard



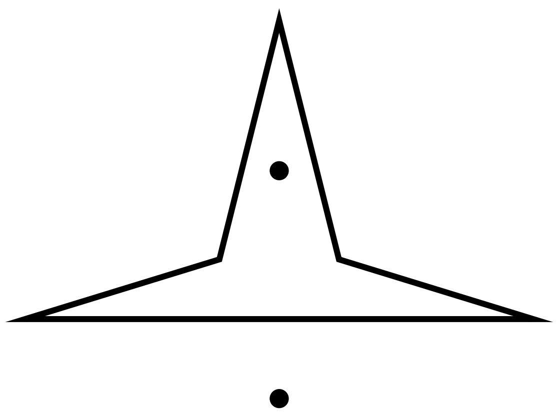
Non-natural guards:



Natural Guards Aren't Enough

Theorem. *There exists a polygon P such that it is impossible to solve SGP for P using a natural angle-guard vertex placement.*

Proof. □



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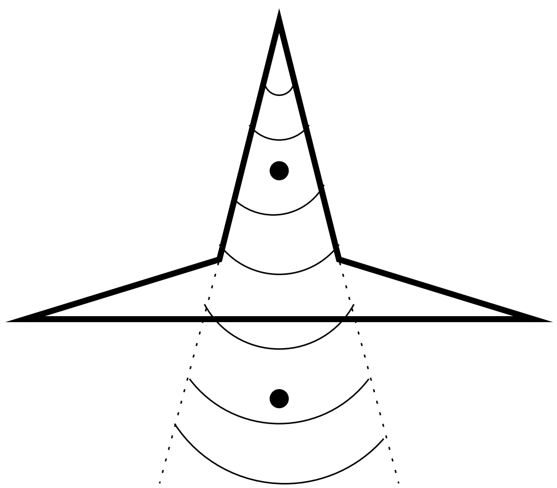

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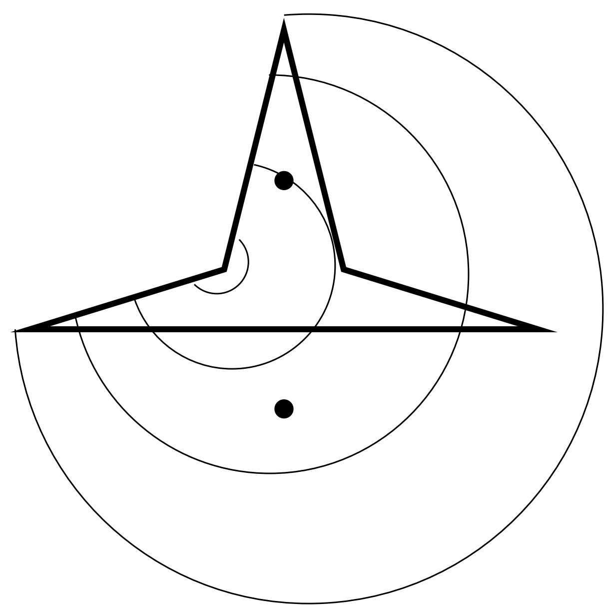

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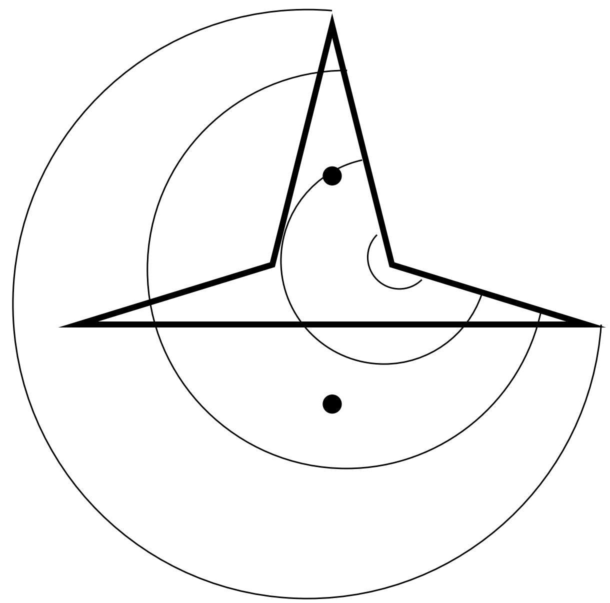

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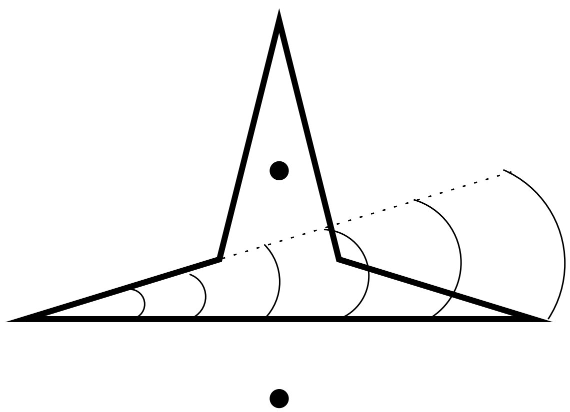

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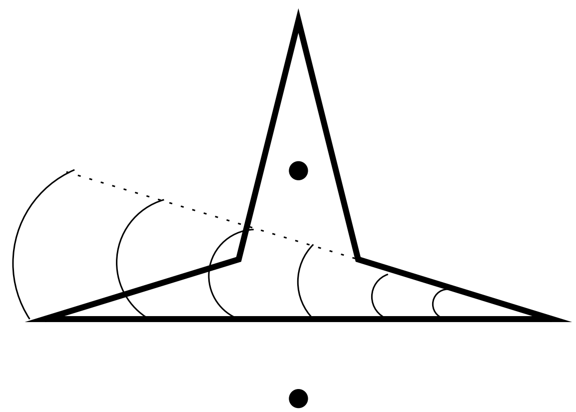
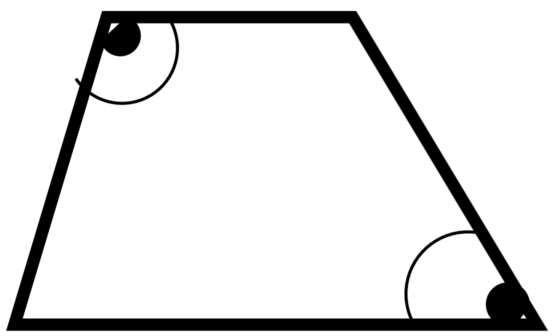
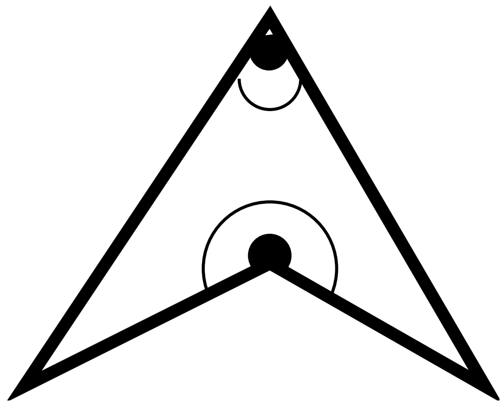


Quadrilaterals

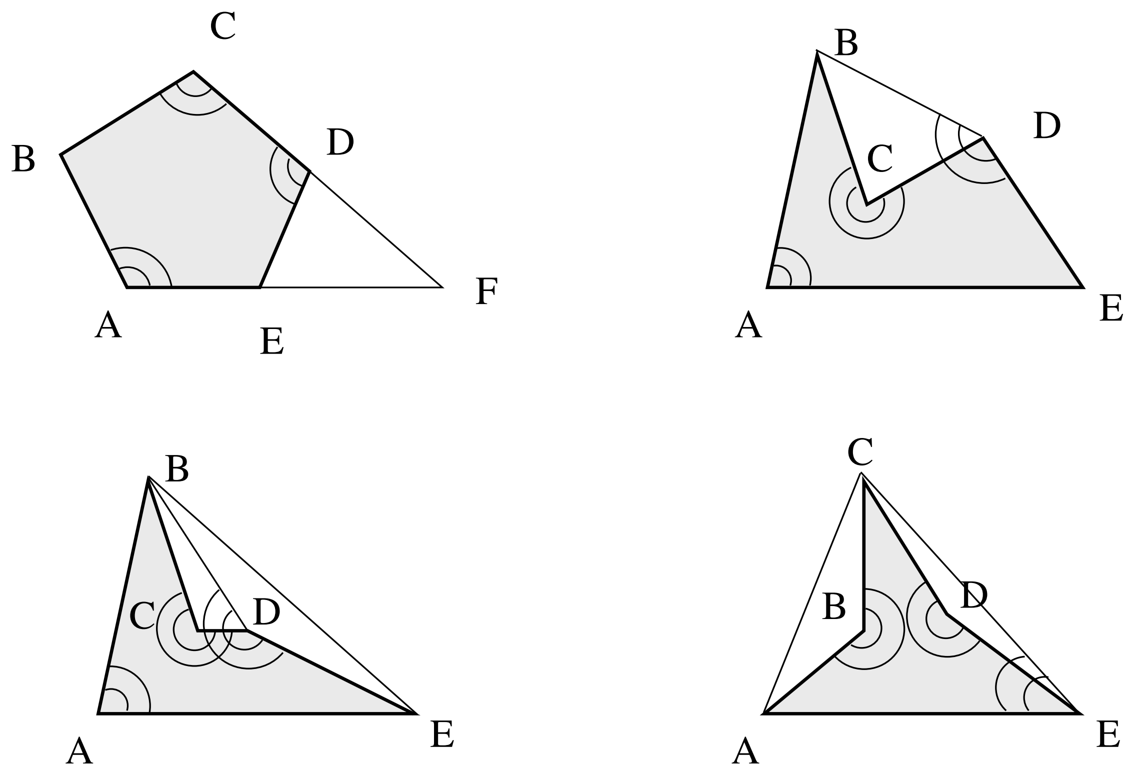
Theorem. *Any quadrilateral can be guarded with* 2 *natural angle guards.*





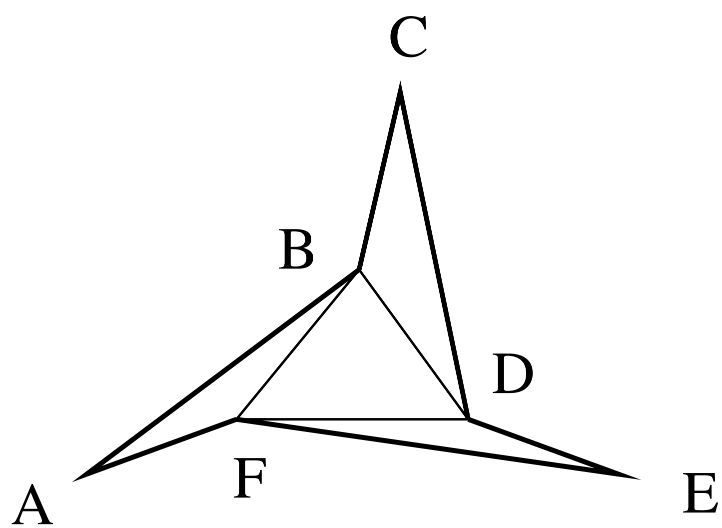
Pentagons

Theorem. *Any pentagon can be guarded with* 3 *angle guards.*



Hexagons

Theorem. *Any hexagon can be guarded with* 4 *angle guards.*



General Upper Bound

Theorem. $n + 2(h-1)$ *guards are sufficient to define any* n*-vertex polygon with* h *holes.*

Proof.

- Triangulate into $n + 2(h-1)$ triangles
- Partition triangulation into quadrilaterals, pentagons, and hexagons
- In each piece, # guards = # triangles

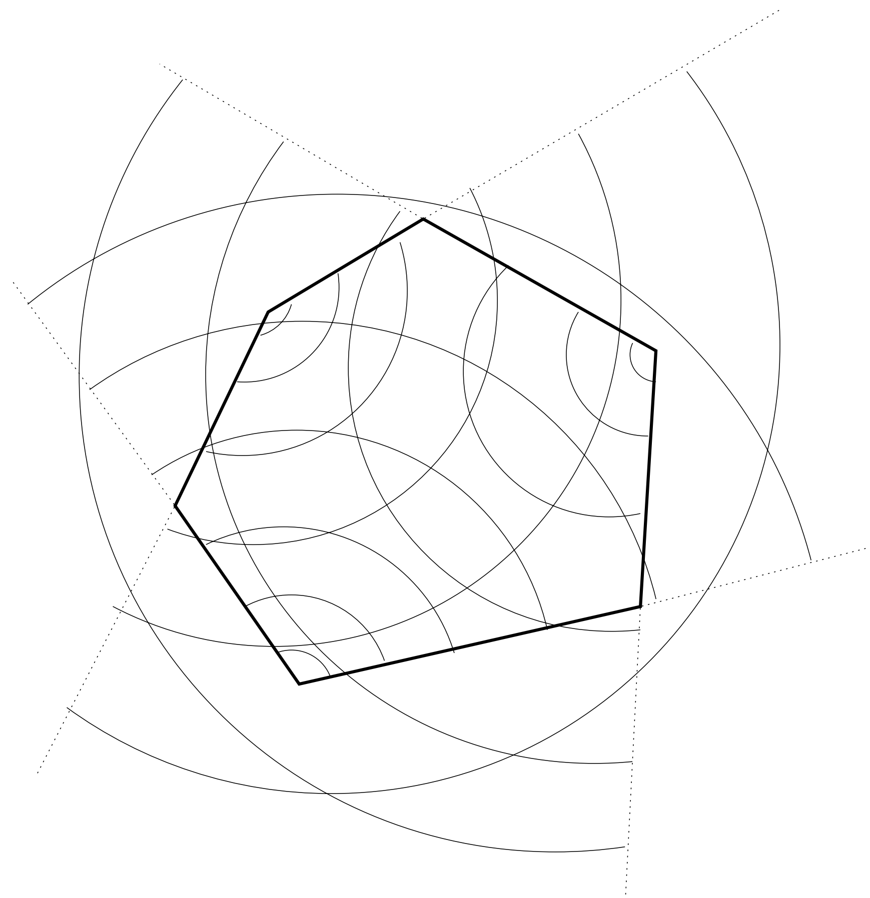
$\square$

Definition is <u>concise</u>: each region is defined by $O(1)$ guards

Lower Bounds

Theorem. *At least* $\lceil \frac{n}{2} \rceil$ *guards are required to solve the SGP for any polygon with no two edges lying on the same line.*

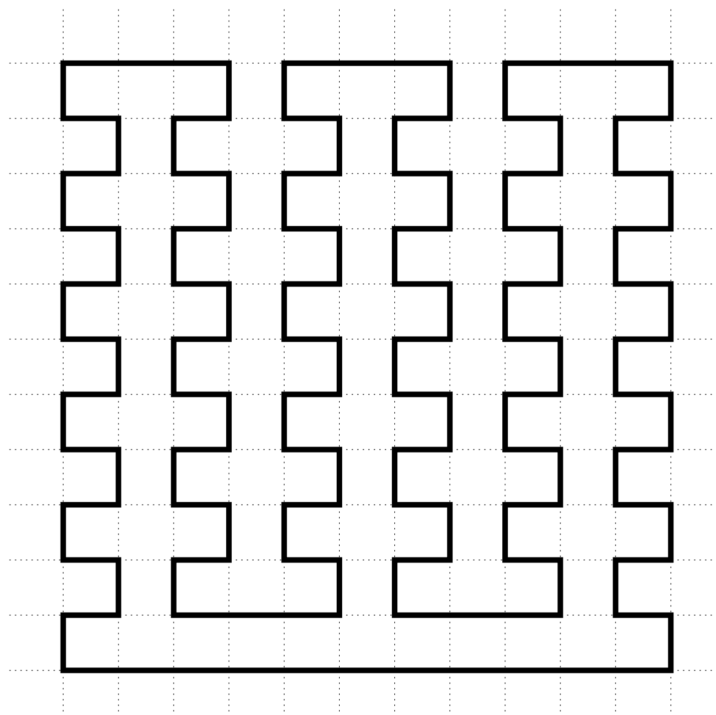
Theorem. $\lceil \frac{n}{2} \rceil$ *guards are always sufficient to solve SGP for any convex polygon.*



Lower Bounds

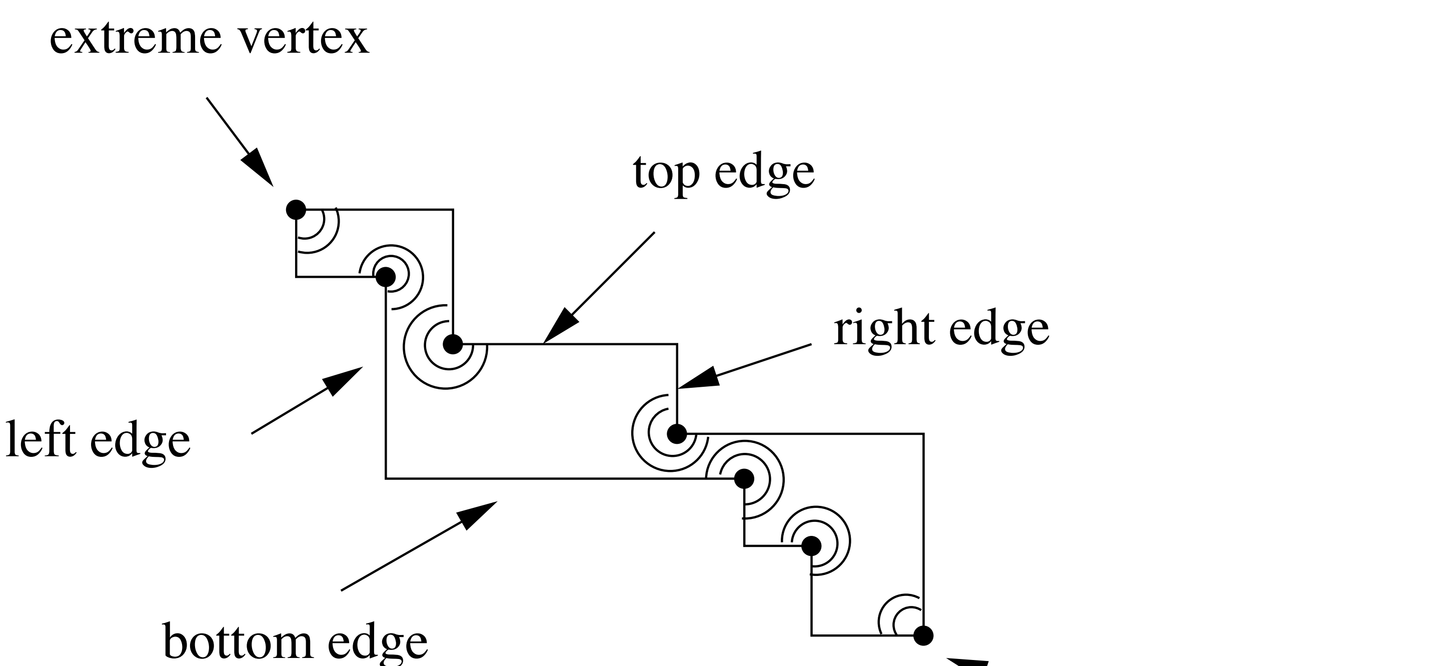
Theorem. *Any n-sided polygon requires $\Omega(\sqrt{n})$ guards.*

Theorem. *There exist n-sided simple polygons that can be guarded concisely by $O(\sqrt{n})$ guards.*



Orthogonal Polygons

Definition. *xy-monotone polygon is an orthogonal polygon which is monotone with respect to the $x = y$ line.*

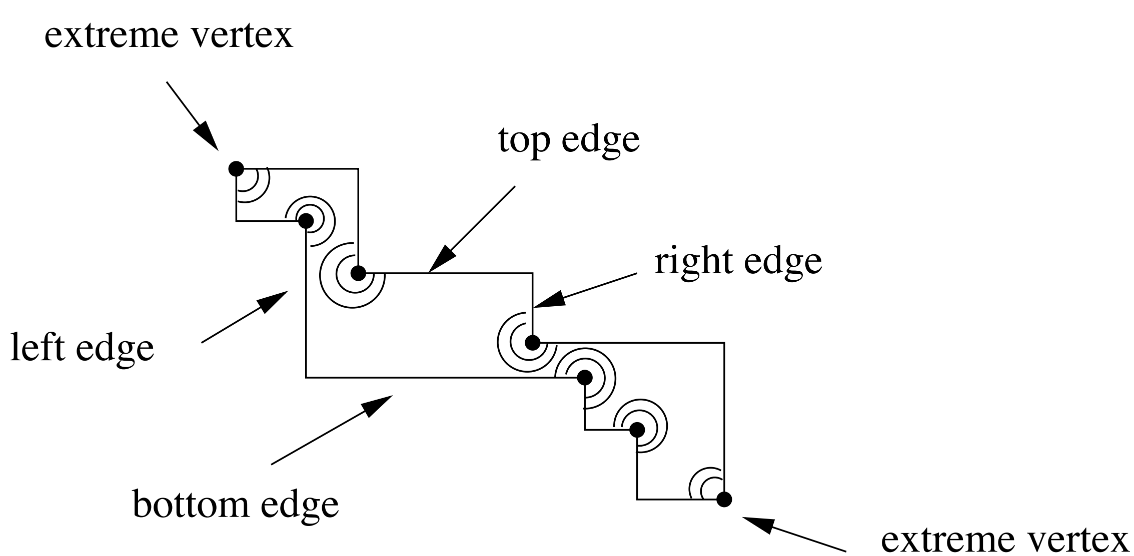


Orthogonal Polygons

Theorem. $\frac{n}{2}$ *natural guards are sufficient to solve SGP for any orthogonal polygons, by placing natural angle guards in every other vertex starting with a left vertex of some top edge.*

Proof. By induction on unguarded reflex vertices. □

Base Case



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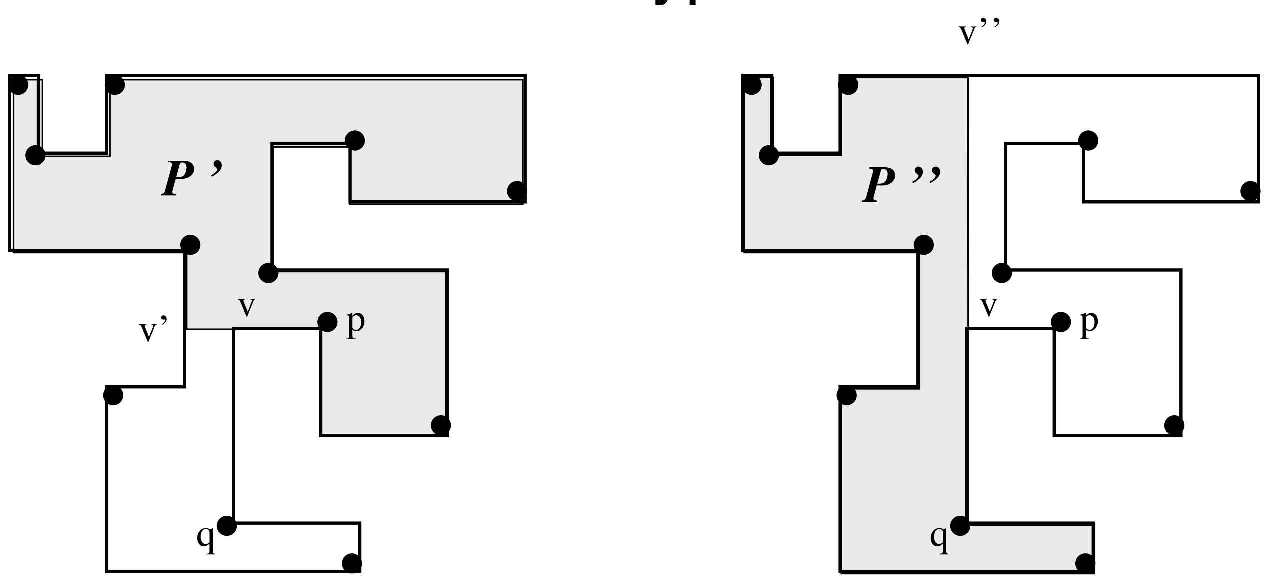
Inductive Hypothesis



Minimizing Number of Guards

Open Problem: How hard is it to find the minimum number of guards for a particular polygon?

Theorem (APPROXIMATION). *For any polygon P, we can find a collection of guards for P, using a number of guards that is within a factor of two of optimal.*

Proof.

- Place an edge guard on the line bounding each edge of P
- Use a Peterson-style constructive-solid-geometry formula
- In optimal solution, each line must be guarded
  and each guard can cover at most two lines
  so optimal # guards is at least half the guards used

□

Summary

	Sometimes required	Always Sufficient
Arbitrary	$\Omega(\sqrt{n})$	$n + 2(h-1)$
General position	$\lceil \frac{n}{2} \rceil$	$n + 2(h-1)$
Orthogonal general position	$\frac{n}{2}$	$\frac{n}{2}$
Convex	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$

Obvious open question:
close factor-of-two gap for non-orthogonal general position