Guidance and Leakage Properties of a Class of Open Dielectric Waveguides: Part I—Mathematical Formulations

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Invited Paper

Abstract—A class of open dielectric waveguides is discussed which is of direct importance to the areas of integrated optics and millimeter-wave integrated circuits. An accurate analysis of the properties of these waveguides reveals that interesting new physical phenomena, such as leakage and sharp cancellation or resonance effects, may occur under appropriate circumstances. The resulting leaky modes form a new class of such modes since the leakage, in the form of an exiting surface wave, has a polarization opposite to that which dominates in the bound portion of the leaky mode. These new effects are caused by TE–TM mode coupling, which was neglected in earlier approximate treatments. Part I presents the mathematical formulation based on a rigorous mode-matching procedure.

I. Introduction

A. General Remarks

OPEN dielectric waveguides have become increasingly important in the past few years, particularly in connection with the areas of integrated optics and millimeter-wave integrated circuits [1–3]. Optical fiber waveguides of circular cross section are, of course, central to the rapidly expanding area of fiber optics, but we shall not consider that class of structures because it has been rather exhaustively treated elsewhere. Furthermore, we restrict our concern here to those waveguides which are naturally suited for use in an integrated circuit context. One feature common to most such waveguides is the presence of a dielectric strip of rectangular cross section in conjunction with a uniform dielectric layered structure, so that the electromagnetic energy can be confined to the vicinity of the strip and be guided by it. For this class of waveguides a suitable generic name could be "dielectric strip waveguide."

The propagation characteristics of these open dielectric waveguides constitute a rich variety of phenomena, including the leakage of guided energy and leakage-related resonance effects under appropriate circumstances. The present authors were the first to predict these physical effects and to present an approximate theory describing them [4]; recent measurements [5] have confirmed their existence on a specific waveguiding structure. With respect to the leakage, it is not generally known that most modes on most of these waveguides can be *leaky*, instead of being purely bound, as is customarily assumed. On a lesser note, the hybrid guided modes on these waveguides possess six field components, and not five, as many believe. The basic reason for this incomplete understanding is that most of the published theoretical propagation characteristics have been obtained from approximate analyses [2], [6]–[11] that neglect those features which lead to the aforementioned effects. A more accurate analysis [12] is also incomplete in the sense that it furnishes the six field components but neglects any mention of the important leakage feature. It is intended in this two-part paper to offer a rigorous mathematical foundation and a clear physical picture for the explanation of these new phenomena.

This Part I of a two-part paper contains a *mathematical formulation* based on a rigorous mode-matching procedure that automatically takes into account all the features mentioned above. The key point that is neglected in the customary approximate treatments is the *coupling* produced between TE and TM waves at geometrical discontinuities. The *new physical effects* that result when the TE–TM coupling is taken correctly into account are described in Part II, together with various *numerical results* for typical waveguides which illustrate these effects quantitatively.

The new physical effects, namely, the presence of *leakage* and the appearance of sharp *resonance*, or cancellation, *effects*, are discussed in detail in Section III of Part II. The leakage, when it is present, occurs in the form of a surface wave which propagates away from the waveguide at some angle to it. The leakage effect can sometimes be used to advantage in the design of novel devices [13]. On the other hand, when this waveguide is part of an optical or millimeter-wave integrated circuit, such leakage can cause cross-talk between neighboring portions of the circuit, and deteriorate system performance. For these reasons, it is important to know in any specific case whether or not the waveguide will leak; this question is treated in detail in Section III of Part II.

The leakage to which we refer changes the guided mode from being purely bound to a *leaky mode*. There is also a point of fundamental interest here, since these leaky modes constitute *a new class* of leaky modes, in that the leakage

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portion of the leaky mode has a *polarization opposite* to that which dominates in the bound portion of the leaky mode. For example, if the electric field of the guided mode is predominantly *vertically* polarized, the leaking surface-wave portion will have its electric field *horizontally* polarized. This novel feature distinguishes these leaky modes from the usual types of leaky mode.

The modes which we discuss here are all above cutoff, in contrast to certain high-loss below-cutoff leaky-wave solutions appearing in some treatments of optical fibers (open dielectric waveguides) of circular cross section. Furthermore, we do not consider the mechanisms which give rise to certain above-cutoff "tunneling" modes which occur on such optical fibers. The class of leaky modes we treat here do not occur in open dielectric waveguides of circular cross section, and they involve a coupling mechanism which is not applicable there.

Not all dielectric strip waveguides permit leakage, and, on those which can leak, some guided modes do leak and some do not. Such questions are treated in detail in Section III of Part II, which also presents physical explanations for the leakage effect, and for the resonance, or cancellation, effect, where sharp nulls in the leakage occur for specific values of strip width. In Section IV of Part II, many numerical results are given which illustrate the new physical effects. Included are measured results which verify the theoretical predictions in a specific case. The Introduction (Section I) of Part II provides guidance for further details.

B. The Approach and the Mathematical Procedures

In Part I, the aim is to provide a *rigorous* mathematical foundation for the analysis of this class of open dielectric waveguides. Stress is placed on network representations to establish physical pictures of the wave processes and to yield insight; in addition, a systematic microwave network approach is employed. This *building-block approach* first breaks the cross-sectional geometry into constituent parts, analyzes each part rigorously, and then combines them into a transverse resonance analysis of the complete waveguide.

Some typical open dielectric waveguides are shown in Fig. 1. The waveguides in Figs. 1(a), (b), and (c) employ dielectric substrates and are intended for application to integrated optics, whereas those in Figs. 1(d), (e), and (f) are placed on metallic ground planes for use in millimeter-wave integrated circuits. It may be noted that when the dielectric constants of the strip and the film are the same, the waveguide is customarily termed a "rib waveguide" in optics (Fig. 1(b)), and a "dielectric ridge waveguide" in the millimeter-wave context (Fig. 1(e)).

It is customary in this class of waveguides to view the cross sections as consisting of a central, or *inside*, region sandwiched between two identical *outside* regions. This decomposition is to be viewed in the horizontal direction, where the inside region consists, in Fig. 1(a), for example, of the strip placed on the film on the substrate, and the outside regions have only the film on the substrate. Except for the optical slot waveguide (Fig. 1(c)), the presence of a dielectric strip in all the other structures makes the net, or effective, dielectric constant of the inside region higher

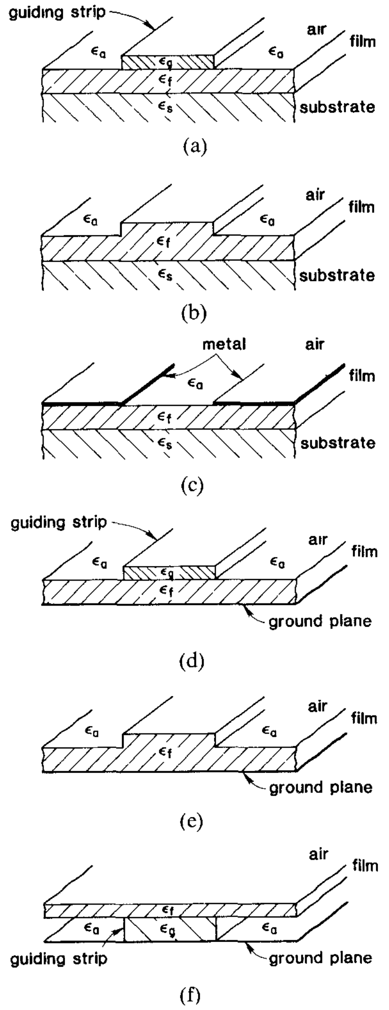

Fig. 1. Typical open dielectric waveguides for integrated optics and millimeter-wave integrated circuits $\epsilon_f, \epsilon_g \geq \epsilon_s \geq \epsilon_a$. (a) Optical dielectric strip waveguide. (b) Optical rib waveguide. (c) Optical slot waveguide. (d) Millimeter-wave dielectric strip waveguide. (e) Millimeter-wave ridge waveguide. (f) Millimeter-wave inverted strip waveguide.

than that of the outside regions. In the case of the optical slot guide, the metallic plates in the outside regions behave like an overdense plasma that possesses a predominantly negative-real dielectric constant in the optical frequency range. Thus the effective dielectric constant of the central region of the optical slot guide is higher than that of the outside regions, as in all the other structures in Fig. 1. Therefore, the electromagnetic energy is confined mostly to the inside region, with its higher effective dielectric constant. Further remarks about these waveguides are made in Section III-A of Part II.

Our approach here follows that of microwave network theory; the cross section of the waveguide is first viewed in terms of *constituent parts*, or building blocks, then each constituent is analyzed separately in its own simpler context, and finally all parts are put together to comprise the final structure of interest. In that way, the simpler parts are handled quickly, and the more difficult portions are paid the special attention required of them, and in a less cluttered context. Approximations, if they need to be made, can then be more systematically treated. When the parts are finally put together, the last step is easily handled.

The guiding of waves along the axis of these waveguides is customarily viewed in terms of surface waves which bounce back and forth inside the central region at an angle to the side walls, undergoing total reflection at each bounce; in the outside regions, the electromagnetic fields are transversely evanescent. For the structures under consideration, the waveguide side walls appear in the form of a step discontinuity, either in dielectric constant or in thickness, between two uniform dielectric-layered structures. The building blocks of the cross section are thus seen to be the inside and outside regions, which are simply portions of uniform dielectric-layered structures which support planar surface waves, and dielectric step discontinuities, or junctions, at the planes where the inside and outside regions meet.

In our building-block treatment, we first treat rigorously the separate uniform planar regions. The modes on such planar dielectric structures are well known, but we review them here both because we need them for the waveguiding problem and to explain our notation and procedure in a simpler context. The complete modal spectrum, comprising the surface waves and the non-surface waves, is discussed in Section II. In our analysis of these open structures, we follow the customary procedure of *discretizing* the continuous spectrum [12], [14], [15] by placing perfectly-conducting walls above and below (if necessary) the planar dielectric waveguide, thus replacing the open region with a partially-dielectric-filled parallel-plate waveguide, which supports, in addition to the surface waves [16], an infinite number of higher non-surface-wave modes, some propagating and the remainder nonpropagating.

The next constituent of the waveguide cross section to be analyzed is the *dielectric step junction*, or discontinuity, corresponding to the side of the dielectric strip waveguide. The rigorous mode-matching analysis is presented in Section III. Since guidance along the waveguide can be viewed in terms of surface waves bouncing back and forth at an angle to the sides of the waveguide and encountering "total reflection" at each bounce, the dielectric step junction problem involves the scattering by the step of a surface wave *obliquely* incident on it.

When the surface wave is incident normally on the step junction, the boundary-value problem is two-dimensional, and an incident TE surface wave produces reflected and transmitted waves of TE polarization only. For oblique incidence, on the other hand, the boundary-value problem becomes three-dimensional, and TE–TM coupling is produced at the step discontinuity. That means that an incident wave of a given polarization now *also* produces reflected and transmitted waves of the *opposite* polarization. This scattering problem is therefore of interest in its own right.

The rigorous analysis begins in Section III-A with a coordinate transformation which translates a TE or a TM surface wave propagating at an angle to the step discontinuity into an LSE or an LSM mode (or alternatively an H-type or an E-type mode) propagating *normally* to the step discontinuity, so that the step can be viewed as a *transverse* discontinuity. The mode-matching procedure for the boundary-value problem, which is the heart of the method, is described in Section III-B. The amplitudes of the scattered waves are determined by four infinite systems of equations, corresponding to the satisfaction of the boundary conditions. These equations, which are phrased in matrix form, must of course be truncated in practice to permit numerical results to be obtained.

The basic mode-matching procedure described in Section III-B is well known, and has been employed by others in determining the propagation characteristics of some waveguides. By utilizing certain matrix identities, however, an *alternative* matrix formulation is obtained. Both formulations yield numerically identical results when the matrices are of infinite order, but not when they are truncated. However, the alternative formulation always satisfies the conservation of power, and it is therefore useful for the development of equivalent networks for the dielectric step junction. The first formulation in truncated form does not satisfy the power conservation condition, so that its deviation from it can be used as a measure of the numerical accuracy of the scattering results.

It is then shown in Section III-C that one can readily develop an *input-admittance* formulation in terms of the matrix quantities involved in the mode-matching process. An equivalent network resulting from this formulation is employed in Section IV as a constituent of the general transverse equivalent network for the cross section of dielectric strip waveguides.

The analysis for the dielectric strip waveguides proceeds in Section IV by employing the building-block approach fundamental to microwave network theory. Since the cross section of this class of waveguides is seen to consist of two dielectric step junctions of the type just discussed connected by a length of uniform waveguide, the equivalent network for the step junction is employed in a rigorous overall transverse equivalent network from which one derives the dispersion relation for the waveguide propagation characteristics. In particular, the input admittance formulation mentioned earlier is used to obtain a rigorous *generalized transverse-resonance relation* for the determination of the waveguide properties. This transverse-resonance relation is expressed in terms of the admittance matrices looking both ways from a reference plane located at the step discontinuity. It is important to note that the procedure employed in deriving the transverse-resonance relation is general, and is independent of the detailed nature of the waveguiding structure; even the relation itself is general if one recognizes that the actual input admittance matrices will become altered when one changes from one waveguide type to another. The relation derived is in fact a generalization of the scalar transverse-resonance relation, valid when only a single mode is involved, to the matrix form for a multimode situation in which all mode-coupling effects are accounted for rigorously.

We conclude these introductory remarks by *summarizing* some of the principal features of our analyses. We employ the classic building-block approach of microwave network theory by viewing the cross sections of the dielectric strip

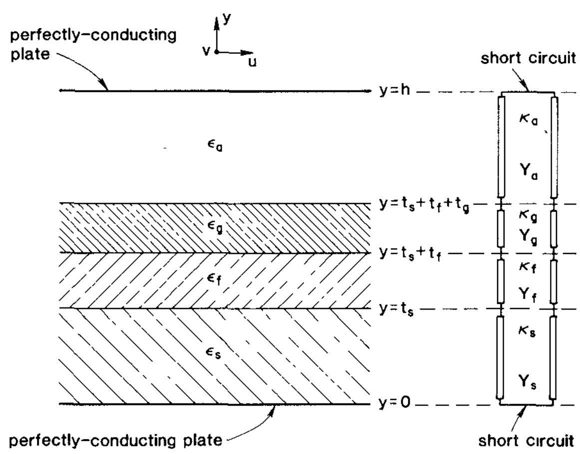

Fig. 2. Multilayer planar dielectric structure and its equivalent network representation. The physical dimensions shown are not in proportion; the perfectly-conducting plates are actually far removed from the central layers.

waveguides in terms of their constituent parts, namely, portions of planar dielectric waveguides and dielectric step junctions where they meet. The planar waveguides are well known, but the step junctions offer a major new challenge. In connection with the step junction, the oblique incidence angle transforms the scalar two-dimensional boundary-value problem that arises for normal incidence into a vector three-dimensional one. A mathematical consequence is TE–TM mode coupling at the discontinuity, and the physical consequence is that an incident wave of one polarization creates reflected and transmitted waves of the opposite polarization. The waveguide problem, which combines the constituent parts, is solved in terms of a generalized transverse-resonance relation, which represents a powerful and general approach, yielding accurate results or permitting in a systematic way various degrees of approximation. The mode-matching general procedure is phrased in admittance terms useful to those familiar with microwave networks, and equivalent networks are developed which summarize the wave processes pictorially and furnish insight into them. The TE–TM mode coupling at the step junctions corresponding to the sides of the waveguide results mathematically in complex eigenvalues under appropriate circumstances, and physically in several new effects, principally leakage and resonance, which are discussed in detail in Part II, together with many numerical results which illustrate the new physical effects quantitatively.

II. Surface Waves and Non-Surface Waves on Planar Dielectric Structures

As discussed in Section I-B, we adopt the building-block approach of microwave networks, and we recognize that the uniform regions present in the cross sections of the class of waveguides shown in Fig. 1 correspond to portions of planar dielectric layers. We therefore need to know the normalized mode functions of both the surface-wave modes and the non-surface-wave modes on these planar layered structures. These modes are well known, but they are reviewed and summarized here because they are needed later. The only new material in this section is relationship (9) involving mode functions of different types.

On these open dielectric structures, the non-surface-wave modes comprise a continuous spectrum. It is customary, however, in this class of problems [12], [14], [15] to *discretize* the continuous spectrum by placing perfectly-conducting walls above and below the planar dielectric waveguides constituting the uniform regions. The open region is thus replaced by a parallel-plate waveguide that is partially dielectric filled, which supports, in addition to a finite (small) number of surface waves, an infinite number of discrete higher modes, some of which are propagating but the remainder of which are below cutoff. These perfectly-conducting bounding walls are placed *far* above and below the guiding dielectric region so as to negligibly influence the properties of the surface waves. For the millimeter-wave structures (Figs. 1(d), (e), and (f)), which employ a ground plane, only an upper bounding plane is needed, of course.

It should also be appreciated that although the presence of these bounding planes may alter the fields far from the step discontinuity, as compared with the truly open environment, the essential physics of the scattering process is not affected. Furthermore, for the waveguide applications of concern here, the higher modes (and in fact the continuous spectrum in a truly open context) will all be below cutoff, so that all of the higher mode power excited at the step discontinuity, in the waveguide application, is completely stored and none of it is radiated.

Consistent with the method described in the preceding for discretizing the higher mode spectrum, a uniform multilayer planar dielectric structure enclosed by an oversize parallel-plate waveguide is shown in Fig. 2(a). Dimensions t_s and h are not in proportion, of course, since the perfectly-conducting plates are located far away. The guiding structure consists of four different dielectric media. For convenience, the media are designated as: air(ϵ_a), guiding strip (ϵ_g), film (ϵ_f), and substrate (ϵ_s). Such a structure is sufficiently general as a basis for the analysis of most

TABLE I
FIELD COMPONENTS OF A BASIC MODE

TE Mode	TM Mode
$E'_v = -\phi'(y)\exp(-jk'_u u)$	$H''_v = \phi''(y)\exp(-jk''_u u)$
$H'_y = \frac{k'_u}{\omega\mu_0}\phi'(y)\exp(-jk'_u u)$	$E''_y = \frac{k''_u}{\omega\epsilon_0\epsilon(y)}\phi''(y)\exp(-jk''_u u)$
$H'_u = \frac{1}{j\omega\mu_0}\frac{d}{dy}\phi'(y)\exp(-jk'_u u)$	$E''_u = \frac{1}{j\omega\epsilon_0\epsilon(y)}\frac{d}{dy}\phi''(y)\exp(-jk''_u u)$

practical dielectric-waveguide problems. For example, the outside region of a waveguide corresponds to the special case for which the thickness of the guiding strip vanishes, e.g., $t_g = 0$. For millimeter-wave applications, the lower perfectly-conducting plate may be regarded as a ground plane and, if no dielectric substrate is present, the thickness of the substrate becomes zero.

The basic modes, comprised of both surface waves and non-surface waves, of the partially-filled parallel-plate waveguide are well known in the literature [16]; some important properties of the basic modes that are relevant to the ensuing analysis are listed here.

For the rectangular coordinate system (u, y, v) indicated in Fig. 2(a), we assume that the basic modes are invariant along the v direction and propagate along the u direction. In this two-dimensional boundary-value problem, the structure supports independent TE and TM basic modes. The field components of the TE and TM modes are given in Table I. Here, we use a single prime to denote quantities for TE modes and a double prime for TM modes. It is also noted that an unprimed quantity will stand for either TE or TM modes. Such notation will be followed throughout this paper. In Table I, ϕ is the transverse mode function and k_u is the longitudinal propagation wavenumber of the basic mode. Evidently, a mode is completely determined by these two quantities.

Presented in the following are a summary of the well-known transverse-resonance technique applied to the structure of Fig. 2(a) and a brief listing of some relationships among the mode functions. Included, however, is a new result involving a mixture of TE and TM modes.

A. Transverse-Resonance Technique

Both the mode function ϕ and the propagation wavenumber k_u may be determined by the transverse-resonance technique. A transverse equivalent network for the dielectric-layer structure in Fig. 2(a) is shown in Fig. 2(b). The transmission-line parameters are known to be related to the longitudinal propagation wavenumber k_u by

$$Y_m = \begin{cases} Y'_m = \kappa'_m/\omega\mu_0, & \text{for the TE mode} \\ Y''_m = \omega\epsilon_0\epsilon_m/\kappa'', & \text{for the TM mode} \end{cases} \quad (1)$$

and

$$\kappa_m = \left(k_0^2\epsilon_m - k_u^2\right)^{1/2} \quad (2)$$

for $m = a, g, f,$ or s, designating different media, and where Y_m is the characteristic admittance and κ_m is the propagation wavenumber of the transmission line representing the mth medium. It is noted that for lossless structures κ_m can be either purely real or purely imaginary. Therefore, the sign of the square root in (2) must be properly chosen such that the radiation condition is satisfied in each medium separately. The condition for resonance of the transmission-line system can be conveniently written in the general case as

$$Y_g \frac{Y_g \tan\kappa_g t_g - Y_a \cot\kappa_a t_a}{Y_g + Y_a \cot\kappa_a t_a \tan\kappa_g t_g} + Y_f \frac{Y_f \tan\kappa_f t_f - Y_s \cot\kappa_s t_s}{Y_f + Y_s \cot\kappa_s t_s \tan\kappa_f t_f} = 0 \quad (3)$$

which determines the surface-wave (or non-surface-wave) propagation wavenumber k_u via (1) and (2). Conventionally, such a surface-wave characteristic is expressed in terms of the normalized quantity

$$n_{\text{eff}} = k_u/k_0 \quad (4a)$$

or

$$\epsilon_{\text{eff}} = n_{\text{eff}}^2 = (k_u/k_0)^2. \quad (4b)$$

Here, n_{eff} is known as the effective index of refraction and ϵ_{eff} is the effective dielectric constant. Equation (3) is commonly known as the transverse-resonance relation or the dispersion relation of the waveguide. For a given set of structure parameters and a given operating frequency, the modal propagation constant is thus determined and so are the transmission-line parameters in Fig. 2(b). For the characteristic admittance defined by (1), the mode function is then determined by the transmission-line voltage for the TE mode and the transmission-line current for the TM mode.

B. Relationships Among Transverse Mode Functions

The transverse mode functions of a partially-filled parallel-plate waveguide are governed by the Sturm-Liouville eigenvalue problem

$$\left[\frac{d}{dy}p(y)\frac{d}{dy} + q(y)\right]\phi_n(y) = \kappa_n^2 w(y)\phi_n(y) \quad (5)$$

subject to the boundary conditions

$$\phi_n(0) = \phi_n(h) = 0, \quad \text{for TE modes} \quad (6a)$$

$$\dot{\phi}_n(0) = \dot{\phi}_n(h) = 0, \quad \text{for TM modes} \quad (6b)$$

where $\dot{\phi}_n(y)$ denotes the derivative of $\phi_n(y)$ with respect to y, and p, q, and w are known functions defined by

$$p(y) = w(y) = \begin{cases} 1, & \text{for TE modes} \\ 1/\epsilon(y), & \text{for TM modes} \end{cases} \quad (7a)$$

$$q(y) = \begin{cases} k_0^2\epsilon(y), & \text{for TE modes} \\ k_0^2, & \text{for TM modes.} \end{cases} \quad (7b)$$

Such an eigenvalue problem will yield an infinite set of eigenvalues, or longitudinal propagation constants of the modes, and a corresponding set of eigenfunctions, or transverse mode functions. The explicit solutions of the mode functions can be written down by inspection from the transmission-line system in Fig. 2(b) and are therefore omitted. Instead, we summarize here the general properties of the eigenvalues and eigenfunctions.

We assume that the dielectric materials forming the waveguide in Fig. 2(a) are lossless. The Sturm–Liouville eigenvalue problem defined by (5)–(7) is Hermitian, because of the perfectly-conducting bounding plates at $y=0$ and h. Therefore, all eigenvalues κ_n^2 are real and all eigenfunctions (mode functions) can be chosen to be real. Furthermore, the mode functions of the same type (TE or TM) are mutually orthogonal. With proper normalization, they can be chosen to satisfy the orthonormality relation

$$\langle \phi_m(y)|w(y)|\phi_n(y)\rangle=\int_0^h \phi_m(y)w(y)\phi_n(y)\,dy=\delta_{ij} \quad (8)$$

for every m and n. Here, δ_{ij} stands for the Kronecker delta. On the other hand, a relationship between the mode functions of different types may be obtained, by manipulating (5)–(7) for both TE and TM modes, as

$$(k'_{um})^2\left\langle \phi'_m(y)\left|\frac{1}{\epsilon(y)}\right|\dot{\phi}''_n(y)\right\rangle + (k''_{un})^2\left\langle \phi''_n(y)\left|\frac{1}{\epsilon(y)}\right|\dot{\phi}'_m(y)\right\rangle=0. \quad (9)$$

Such a relation has not previously appeared in the literature, possibly because it has not been needed in the past. For the general case of surface-wave scattering by a step discontinuity at an oblique incidence angle, however, this particular relation will ensure that the TE and TM modes are mutually orthogonal in power, even though the mode functions of one set may not be orthogonal to those of the other set. A proof of this new relation is presented elsewhere [17].

III. Scattering of a Surface Wave Obliquely Incident on a Dielectric Step Discontinuity

It was explained earlier that a dielectric step junction, or discontinuity, corresponds to the side of a dielectric strip waveguide. Following the building-block approach outlined in Section I-B, the step discontinuity is analyzed separately as a constituent in the waveguide cross section. Furthermore, since the waveguiding process is viewed in terms of surface waves bouncing back and forth between the waveguide sides at an angle to the sides, the constituent step-discontinuity problem must require that the surface wave be incident *obliquely* on the step structure. Most treatments in the literature of the scattering of surface waves by a step discontinuity involve normally-incident waves; for this reason, the oblique-incidence case is of interest in its own right.

When a surface wave is incident normally on a dielectric step discontinuity, the boundary-value problem is two-dimensional, and all higher modes excited at the discontinuity possess the same polarization as the incident mode. When the surface wave is incident at an oblique angle, however, the resulting three-dimensional boundary-value problem requires the coupling of TE and TM modes at the discontinuity, as is shown later. A rigorous phasing of the oblique-incidence case has recently appeared in the literature [15], but we also present an input admittance formulation and an equivalent network. Both the equivalent network and the input admittance form are valuable when the step discontinuity is employed as a constituent of more complex structures. It should be added that the new physical effects that emerge when the surface wave is incident obliquely can be exhibited when *only one* surface-wave mode *of each type* is included [18]; it is necessary to include many modes only if accurate numerical results are desired.

In order to simplify the analysis, the step discontinuity is to be treated as a *transverse* discontinuity even though the surface wave is incident at an angle. The TE or TM surface wave which is obliquely incident is thus to be subject to a coordinate transformation which establishes a transmission line formulation for a mode normally incident on the step structure. The surface-wave modes are then no longer TE or TM modes, with three field components, but they become modes with five field components, which have been characterized in the literature as LSE or LSM modes, or alternatively as H-type or E-type modes.

A. LSE (or $H^{(y)}$-Type) Modes and LSM (or $E^{(y)}$-Type) Modes

A dielectric step discontinuity with a surface wave incident on it at an oblique angle is depicted in Fig. 3. The structure is characterized by the xyz coordinate system, with the discontinuity located on the $x=0$ plane. Suppose the incident surface wave is a TE mode with respect to its direction of propagation, denoted by u. The electric-field vector has only one component, in the v direction, perpendicular to u and on the xz plane. Since the transverse mode function of a surface wave is independent of its direction of propagation, the y axis remains unchanged for any angle of incidence. In the uyv coordinate system, each TE or TM mode has only three field components whose spatial variations are well known. In solving the boundary-value problem of surface-wave scattering by a step discontinuity, it is necessary to deal with the two coordinate systems: x, y, and z, to be called the structure coordinate system, and u, y, and v, to be called the eigencoordinate system; evidently, they are mutually related by a coordinate rotation about the y axis. The transformation of the electromagnetic fields of a surface wave from the eigencoordinate system to the structure coordinate system results in an increase in the number of field components from three to five.

When the two coordinate systems are rotated with respect to each other by angle θ about the y axis, they are mutually related by

$$u=x\cos\theta+z\sin\theta \quad (10)$$

$$v=-x\sin\theta+z\cos\theta \quad (11)$$

and the wavenumbers of the field components in the x, y, z

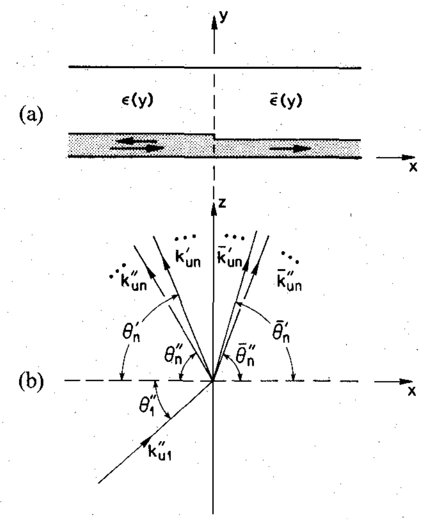

Fig. 3. Scattering of a surface wave by a dielectric step discontinuity. (a) Side view. (b) Top view.

TABLE II
FIELD COMPONENTS OF A SURFACE WAVE MODE IN THE STRUCTURE COORDINATE SYSTEM

LSE or $H^{(y)}$-Type Mode	LSM or $E^{(y)}$-Type Mode
$E'_x=\sin\theta'\exp(-jk'_x x)\phi'(y)$	$E''_x=\frac{\cos\theta''}{j\omega\epsilon_0\epsilon(y)}\exp(-jk''_x x)\frac{d}{dy}\phi''(y)$
$E'_y=0$	$E''_y=\frac{k''_u}{\omega\epsilon_0\epsilon(y)}\exp(-jk''_x x)\phi''(y)$
$E'_z=-\cos\theta'\exp(-jk'_x x)\phi'(y)$	$E''_z=\frac{\sin\theta''}{j\omega\epsilon_0\epsilon(y)}\exp(-jk''_x x)\frac{d}{dy}\phi''(y)$
$H'_x=\frac{\cos\theta'}{j\omega\mu}\exp(-jk'_x x)\frac{d}{dy}\phi'(y)$	$H''_x=-\sin\theta''\exp(-jk''_x x)\phi''(y)$
$H'_y=\frac{k'_u}{\omega\mu_0}\exp(-jk'_x x)\phi'(y)$	$H''_y=0$
$H'_z=\frac{\sin\theta'}{j\omega\mu}\exp(-jk'_x x)\frac{d}{dy}\phi'(y)$	$H''_z=\cos\theta''\exp(-jk''_x x)\phi''(y)$

system become

$$k_x=k_u\cos\theta \tag{12}$$

$$k_z=k_u\sin\theta \tag{13}$$

where k_x and k_y are the projections of the surface-wave propagation vector in the x and y directions. The field components that result from this simple coordinate transformation are listed in Table II.

The modes whose components are listed in Table II have five field components instead of the three possessed by TE and TM modes. These modes are no longer TE or TM, but they are characterizable by the absence of an electric or a magnetic field component in the y direction. Such modes are known in the literature as LSE or LSM modes [19], or as H-type or E-type modes with respect to the y direction [20], if $E_y=0$ or $H_y=0$, respectively. Thus, since the surface-wave mode in the first column of Table II possesses a y component of H but not of E, it may be designated an $H^{(y)}$-type mode or an LSE mode; similarly, the mode in the second column is an $E^{(y)}$-type mode or an LSM mode. We recall that these five-component modes enter here because we wish the transmission-line formulation to correspond to normal incidence on the step discontinuity. The physical waves are still TE or TM waves incident on the step junction at an angle.

The x variations of the field components in Table II represent the propagation of surface waves (and non-surface waves) in the forward (transmission-line) direction. In the scattering process, reflection of these modes takes place, resulting in propagation in the backward direction with the spatial variation $\exp(+jk_x x)$. Thus, the general field solution of a rotated mode must consist of both forward and backward propagating waves in the x direction. Such a general modal solution for each field component in the structure coordinate system is listed in Table III. Here, $V(x)$ and $I(x)$, with a single prime for an LSE or $H^{(y)}$-type mode and a double prime for an LSM or $E^{(y)}$-type mode, can be interpreted as the voltage and current satisfying the transmission-line equations

$$\frac{d}{dx}V(x)=-jk_x ZI(x) \tag{14a}$$

$$\frac{d}{dx}I(x)=-jk_x YV(x) \tag{14b}$$

where k_x is the propagation wavenumber and $Z(=1/Y)$ is the characteristic impedance in the x direction. Further-

TABLE III
GENERAL MODAL SOLUTIONS FOR ALL THE FIELD COMPONENTS

LSE or $H^{(y)}$-Type Mode	LSM or $E^{(y)}$-Type Mode
$E'_x = -\frac{\omega\mu_0}{k'_u}\sin\theta' I'(x)\phi'(y)$	$E''_x = \frac{1}{\omega\epsilon_0} I''(x)\frac{1}{\epsilon(y)}\frac{d}{dy}\phi''(y)$
$E'_y = 0$	$E''_y = jV''(x)\phi''(y)\frac{1}{\epsilon(y)}$
$E'_z = V'(x)\phi'(y)$	$E''_z = \frac{1}{k''_u}\sin\theta'' V''(x)\frac{1}{\epsilon(y)}\frac{d}{dy}\phi''(y)$
$H'_x = j\frac{1}{\omega\mu_0}V'(x)\frac{d}{dy}\phi'(y)$	$H''_x = -j\frac{\omega\epsilon_0}{k''_u}\sin\theta'' V''(x)\phi''(y)$
$H'_y = -I'(x)\phi'(y)$	$H''_y = 0$
$H'_z = j\frac{1}{k'_u}\sin\theta' I'(x)\frac{d}{dy}\phi'(y)$	$H''_z = jI''(x)\phi''(y)$

more, k_x, together with the propagation wavenumber along the step discontinuity in the z direction, is related to the propagation wavenumber k_u of the basic TE or TM mode by

$$k_x^2 + k_z^2 = k_u^2. \tag{15}$$

The characteristic impedance is defined by [20]

$$Z = \begin{cases} \dfrac{\omega\mu_0 k'_x}{(k'_u)^2} = \dfrac{\omega\mu_0 \cos\theta'}{k'_u}, & \text{for LSE or } H^{(y)}\text{-type modes} \\ \dfrac{(k''_u)^2}{\omega\epsilon_0 k''_x} = \dfrac{k''_u}{\omega\epsilon_0\cos\theta''}, & \text{for LSM or } E^{(y)}\text{-type modes.} \end{cases} \tag{16a}$$

Invoking (4b), we obtain the alternative form

$$Z = \begin{cases} k'_x/\omega\epsilon_0\epsilon'_{\text{eff}}, & \text{for LSE or } H^{(y)}\text{-type modes} \\ \omega\mu_0\epsilon''_{\text{eff}}/k''_x, & \text{for LSM or } E^{(y)}\text{-type modes.} \end{cases} \tag{16b}$$

It is noted that for the scattering problem under consideration here, k_z is a known constant and is related to the parameters of the incident surface wave by

$$k_z = k_u \sin\theta \tag{17}$$

where θ is the incidence angle, as depicted in Fig. 3(b). Thus, the transmission-line parameters, k_x and Z, can readily be determined from (15) and (16), and the general modal solution for each field component in Table III can readily be written down, based on the solutions of the transmission-line equations in (14). Therefore, we assume from now on that, for each mode determined in Section III-A, the general modal solution for every field component in the structure coordinate system is known, as given in Table III.

B. Boundary-Value Problem for the Step Discontinuity

The uniform multilayer structure shown in Fig. 2 can support an infinite number of modes. Let k'_{un} and k''_{un} be the propagation wavenumbers of the nth TE and TM modes, respectively. For each mode, the general solutions for all the field components in terms of the structure coordinate system are given in Table III, with k'_u and k''_u replaced by k'_{un} and k''_{un}, respectively. We formulate here the boundary-value problem of surface-wave scattering by a step discontinuity for the general case of oblique incidence.

The two uniform multilayer regions on the two sides of the discontinuity in Fig. 3(a) are characterized by the distributions of dielectric constant, $\epsilon(y)$ and $\bar{\epsilon}(y)$, respectively. As an illustration, let us consider the case for which the TM fundamental mode is incident from the left at an oblique angle θ''_1. As will be shown, all the modes in the two constituent regions will generally be excited at the step discontinuity, some propagating and some decaying away from the discontinuity. The boundary conditions at the step discontinuity require that the total tangential field components be continuous across the step discontinuity, and a necessary condition for the continuity of the tangential field components is that every mode in the two constituent regions must have the same propagation wavenumber, k_z, in the direction along the step discontinuity. From (17), we then have the Snell's law for the various modes at a step discontinuity:

$$k_z = k'_{un}\sin\theta'_n = k''_{un}\sin\theta''_n = \bar{k}'_{un}\sin\bar{\theta}'_n = \bar{k}''_{un}\sin\bar{\theta}''_n \tag{18a}$$

or

$$n''_{\text{eff}\,1}\sin\theta''_1 = n'_{\text{eff}\,n}\sin\theta'_n = n''_{\text{eff}\,n}\sin\theta''_n = \bar{n}'_{\text{eff}\,n}\sin\bar{\theta}'_n = \bar{n}''_{\text{eff}\,n}\sin\bar{\theta}''_n \tag{18b}$$

which determines the angles of reflection and transmission for every TE or TM mode, as indicated in Fig. 3(b). With the knowledge of k_z, the propagation wavenumber in the x direction can be determined from (15) by replacing k_u by k_{un} for the nth mode (either TE or TM) to yield

$$k_{xn} = \left[k_{un}^2 - k_z^2\right]^{1/2}. \tag{19}$$

For the TM surface-wave incidence shown in Fig. 3(b), the last equation can be written conveniently in terms of the effective dielectric constants as

$$k''_{xn} = k_0\left[\epsilon''_{\text{eff}\,n} - \epsilon''_{\text{eff}\,1}\sin^2\theta''_1\right]^{1/2} \tag{20}$$

where θ''_1 is the given incidence angle. After k_{xn} is determined, the characteristic impedance of the mode is then specified by (16), with k_x replaced by k_{xn} to become

$$Z_n = \frac{1}{Y_n} = \begin{cases} \dfrac{k'_{xn}}{\omega\epsilon_0\epsilon'_{\text{eff}\,n}}, & \text{for LSE or } H^{(y)}\text{-type modes} \\ \dfrac{\omega\mu_0\epsilon''_{\text{eff}\,n}}{k''_{xn}}, & \text{for LSM or } E^{(y)}\text{-type modes.} \end{cases} \tag{21}$$

Thus, with respect to the x direction, the transmission-line parameters for every mode are determined, and the general modal solutions for all the field components in the structure coordinate system are considered completely determined, as described in the preceding subsection.

Referring to Fig. 3, we observe that the tangential components of the fields at the step discontinuity consist of the y and z components, and we shall therefore consider only those components explicitly. As stated earlier, the general field solution in each constituent region may be expressed in terms of the superposition of the complete set of mode functions. For the tangential field components for the $\epsilon(y)$ region ($x<0$), we have

$$E_y(x,y)=j\sum_{n=1}^{\infty} V_n''(x)\phi_n''(y)\frac{1}{\epsilon(y)} \tag{22a}$$

$$E_z(x,y)=-\sum_{n=1}^{\infty} V_n'(x)\phi'_n(y)-\sum_{n=1}^{\infty} V_n''(x)\psi_n''(y) \tag{22b}$$

$$H_y(x,y)=-\sum_{n=1}^{\infty} I_n'(x)\phi_n'(y) \tag{22c}$$

$$H_z(x,y)=-j\sum_{n=1}^{\infty} I_n'(x)\psi_n'(y)-j\sum_{n=1}^{\infty} I_n''(x)\phi_n''(y) \tag{22d}$$

where we employ the simplifying notation

$$\psi_n'(y)=\frac{1}{k_{un}'}\sin\theta'\frac{d}{dy}\phi_n'(y) \tag{23a}$$

$$\psi_n''(y)=\frac{1}{k_{un}''}\sin\theta''\frac{1}{\epsilon(y)}\frac{d}{dy}\phi_n''(y). \tag{23b}$$

It is noted that the z dependence $\exp(-jk_z z)$ has been suppressed in (22) for clarity. A similar set with an overbar may also be written for the $\bar{\epsilon}(y)$ region ($x>0$), but is omitted here for simplicity. At the step discontinuity at $x=0$, the tangential field components must be continuous. From (22) we obtain

$$\sum_{n=1}^{\infty} V_n''(0)\phi_n''(y)\frac{1}{\epsilon(y)}=\sum_{n=1}^{\infty}\bar{V}_n(0)\bar{\phi}_n''(y)\frac{1}{\bar{\epsilon}(y)} \tag{24a}$$

$$\sum_{n=1}^{\infty} V_n'(0)\phi_n'(y)+\sum_{n=1}^{\infty} V_n''(0)\psi_n''(y)=\sum_{n=1}^{\infty}\bar{V}_n'(0)\bar{\phi}_n'(y)+\sum_{n=1}^{\infty}\bar{V}_n''(0)\bar{\phi}_n''(y) \tag{24b}$$

$$\sum_{n=1}^{\infty} I_n'(0)\phi_n'(y)=\sum_{n=1}^{\infty}\bar{I}_n'(0)\bar{\phi}_n'(y) \tag{24c}$$

$$\sum_{n=1}^{\infty} I_n'(0)\psi_n'(y)+\sum_{n=1}^{\infty} I_n''(0)\phi_n''(y)=\sum_{n=1}^{\infty}\bar{I}_n'(0)\bar{\psi}_n'(y)+\sum_{n=1}^{\infty}\bar{I}_n''(0)\bar{\phi}_n''(y). \tag{24d}$$

These four equations hold for any y at $x=0$ within the enclosure. Scalar-multiplying these equations with either ϕ_m' or ϕ_m'' and making use of the orthogonality relation (8), we then obtain

$$\boldsymbol{V}''=\boldsymbol{P}''\bar{\boldsymbol{V}}'' \tag{25a}$$

$$\boldsymbol{V}'+\boldsymbol{R}''\boldsymbol{V}''=\boldsymbol{Q}'\bar{\boldsymbol{V}}'+\boldsymbol{S}''\bar{\boldsymbol{V}}'' \tag{25b}$$

$$\boldsymbol{I}'=\boldsymbol{P}'\bar{\boldsymbol{I}}' \tag{25c}$$

$$\boldsymbol{R}'\boldsymbol{I}'+\boldsymbol{I}''=\boldsymbol{S}'\bar{\boldsymbol{I}}'+\boldsymbol{Q}''\bar{\boldsymbol{I}}'' \tag{25d}$$

where $\boldsymbol{V}'$ and $\boldsymbol{I}'$ are column vectors with the transmission-line voltage and current of the nth TE mode, $V_n'(0)$ and $I_n'(0)$, at the nth positions; similar definitions hold for $\boldsymbol{V}''$ and $\boldsymbol{I}''$ for TM modes and also for those vectors with a superbar. The P's, Q's, R's, and S's are matrices characterizing the coupling of modes at the step discontinuity; their general elements are defined by the scalar products or overlap integrals of mode functions on the two sides of the discontinuity as

$$P'_{mn}=Q'_{mn}=\langle\phi'_m|\bar{\phi}_n'\rangle \tag{26a}$$

$$P''_{mn}=\left\langle\phi_m''\middle|\frac{1}{\bar{\epsilon}(y)}\middle|\bar{\phi}_n''\right\rangle \tag{26b}$$

$$Q''_{mn}=\left\langle\phi_m''\middle|\frac{1}{\epsilon(y)}\middle|\bar{\phi}_n''\right\rangle \tag{26c}$$

$$R'_{mn}=\left\langle\phi_m''\middle|\frac{1}{\epsilon(y)}\middle|\psi_n'\right\rangle \tag{26d}$$

$$R''_{mn}=\langle\phi_m'|\psi_n''\rangle \tag{26e}$$

$$S'_{mn}=\left\langle\phi_m''\middle|\frac{1}{\epsilon(y)}\middle|\bar{\psi}_n'\right\rangle \tag{26f}$$

$$S''_{mn}=\langle\phi_m'|\bar{\psi}_n''\rangle \tag{26g}$$

for any $m,n=1,2,3,\cdots$. It is evident from either (25) or (26) that the matrices P's and Q's are responsible for the coupling among modes of the same polarization, whereas R's and S's are responsible for the cross-coupling among modes of opposite polarization.

For a given incident surface wave, the amplitudes of the scattered modes are determined by the four infinite systems of equations in (25). In practice, these infinite systems of equations must be truncated for an approximate analysis, and we shall do this in connection with the waveguide problem in Section IV.

A set of modal relations alternative to that in (25) can be derived. On use of certain matrix identities, matrices $\boldsymbol{P}$ and $\boldsymbol{S}$ are eliminated from (25), and a new set obtained which is equivalent to that in (25) if all matrices are retained to infinite order. This new set is the following:

$$(\boldsymbol{Q}'')^T\boldsymbol{V}''=\bar{\boldsymbol{V}}'' \tag{27a}$$

$$\boldsymbol{V}'+(\boldsymbol{R}')^T\boldsymbol{V}''=\boldsymbol{Q}'\left[\bar{\boldsymbol{V}}'+(\bar{\boldsymbol{R}}')^T\bar{\boldsymbol{V}}''\right] \tag{27b}$$

$$(\boldsymbol{Q}')^T\boldsymbol{I}'=\bar{\boldsymbol{I}}' \tag{27c}$$

$$\boldsymbol{R}'\boldsymbol{I}'+\boldsymbol{I}''=\boldsymbol{Q}''[\bar{\boldsymbol{R}}'\bar{\boldsymbol{I}}'+\bar{\boldsymbol{I}}'']. \tag{27d}$$

When truncations are made in order to obtain numerical values for the scattering parameters, the two different formulations in (25) and (27) no longer yield identical results, and their convergence properties are also not identical. What is more important, however, is that the alternative formulation in (27) always satisfies the condition of power conservation across the junction, regardless of the number of modes retained after a truncation. A derivation of the alternative formulation in (27) and a proof that it always satisfies the condition of power conservation are presented elsewhere [17].

The set (27) that always preserves the law of conservation of power flow forms the basis for the development of equivalent networks for the dielectric step discontinuity, whereas the other set (25) is useful for numerical analyses of the problem. In fact, because the results obtained after truncation will not satisfy the conservation of power flow, we can use the deviation from conservation as a measure of the numerical accuracy obtained. The alternative formulation in (27) is therefore used as the basis for an equivalent network representation for the dielectric step discontinuity and for an input admittance formulation, presented under Section III-C, which is then employed in the development in Section IV of a generalized transverse-resonance relation for the dielectric strip waveguides.

C. Input Admittance Matrix for the Step Discontinuity

In many practical situations, it is necessary to determine only the reflection of a surface wave by a step discontinuity. Moreover, once the reflected mode amplitudes are determined, it is straightforward to determine the transmitted mode amplitudes. For the scattering of a surface wave incident from the left in Fig. 3(a), it is sufficient to have available an input admittance characterization that takes into account the effects of the step discontinuity and the semi-infinite uniform waveguide to the right. We derive now such an input admittance matrix for the step discontinuity.

Each of the higher mode transmission lines to the right of the step discontinuity can be simply represented by its characteristic admittance, and the voltage–current relation at each terminal is then

$$\bar{I}_n' = \bar{Y}_n'\bar{V}_n' \tag{28a}$$

$$\bar{I}_n'' = \bar{Y}_n''\bar{V}_n'' \tag{28b}$$

for every mode index n. In matrix form, the last two relations may be written as

$$\bar{\boldsymbol{I}}' = \bar{Y}'\bar{\boldsymbol{V}}' \tag{28c}$$

$$\bar{\boldsymbol{I}}'' = \bar{Y}''\bar{\boldsymbol{V}}'' \tag{28d}$$

where $\bar{\boldsymbol{V}}'$ and $\bar{\boldsymbol{I}}'$ are voltage and current vectors for LSE (or $H^{(y)}$-type) modes in the outside region, with $\bar{V}_n'$ and $\bar{I}_n'$ as their nth positions, respectively, and similarly for $\bar{\boldsymbol{V}}''$ and $\bar{\boldsymbol{I}}''$ for LSM (or $E^{(y)}$-type) modes. $\bar{Y}'$ and $\bar{Y}''$ are the diagonal admittance matrices of the LSE and LSM modes, respectively, for the $\bar{\epsilon}(y)$ region. Substituting the last two equations into (27a) and (27b) and then eliminating $\bar{\boldsymbol{I}}'$ and $\bar{\boldsymbol{I}}''$ by invoking (27c) and (27d), we finally obtain

$$\boldsymbol{I}' = Y_{11}\boldsymbol{V}' + Y_{12}\boldsymbol{V}'' \tag{29a}$$

$$\boldsymbol{I}'' = Y_{21}\boldsymbol{V}' + Y_{22}\boldsymbol{V}'' \tag{29b}$$

where Y_{ij}, for $i, j = 1$ and 2, is an input admittance matrix that is related to the characteristic admittances, $\bar{Y}_n'$ and $\bar{Y}_n''$, and the mode-coupling matrices of the step discontinuity by

$$Y_{11} = Q'\bar{Y}'(Q')^T \tag{30a}$$

$$Y_{12} = Y_{11}\left[R^T - Q'\bar{R}^T(Q'')^T\right] \tag{30b}$$

$$Y_{21} = -\left[R - Q''\bar{R}(Q')^T\right]Y_{11} \tag{30c}$$

and

$$Y_{22} = Q''\bar{Y}''(Q'')^T - Y_{21}\left[R^T - Q'\bar{R}(Q'')^T\right] \tag{30d}$$

where superscript T signifies "transpose." In (29), Y_{11} and Y_{22} are responsible for the coupling of modes of the same polarization and Y_{12} and Y_{21} represent the cross-coupling between modes of opposite polarization. These input admittance matrices can be computed in the straightforward manner described in the preceding, and require only the knowledge of the modal characteristics of the two constituent uniform planar waveguides. Expressions (30) are seen to be simple in form and afford with relative ease a systematic and effective analysis of step discontinuity problems.

IV. Guidance of Waves by Dielectric Strip Waveguides

In Section III we developed a mode-matching formalism for a dielectric step discontinuity between two uniform planar dielectric waveguides. The cross section of a dielectric strip waveguide may be viewed as consisting of two or more step discontinuities connected by a length of uniform waveguide, and the equivalent network for a step discontinuity may therefore be utilized in the analysis of waveguide characteristics. In this section, we follow such a building-block approach to the dielectric strip waveguide problem and we formulate the waveguide problem rigorously in the form of a *generalized transverse-resonance relation*.

A dielectric strip waveguide is shown in Fig. 4(a). As a wave is being guided along the strip, the process may be viewed in terms of multiple reflections which take place at the two step discontinuities forming the waveguide side walls. The basic modes of each constituent region of the waveguide are presented in Section III-A. If we employ the concept of input admittance and apply the equivalent network for the step discontinuities implied by relations (29), we obtain the transverse equivalent network shown in Fig. 4(b) for the analysis of transverse resonance in the lateral direction of the waveguide. The guiding characteristics of the waveguide are completely determined by a single parameter, i.e., the longitudinal propagation wavenumber k_z. As shown in the preceding section, all the parameters of the network in Fig. 4(b) are implicit functions of k_z, and the resonance condition of the network determines the allowable values of k_z for a given waveguide structure.

In practice, most dielectric strip waveguides are symmetric in geometry, such as the one shown in Fig. 4(a). Therefore, the transverse equivalent network in Fig. 4(b) is also symmetric with respect to the center plane. Such a network may be analyzed in terms of the two simpler networks obtained from open-circuit and short-circuit bisections, as shown in Fig. 5 for symmetric and antisymmetric distributions of voltage, or electric field, in the original waveguide. For simplicity, we shall deal only with symmetric structures in this paper; the generalization for asymmetric structures is almost trivial and is omitted.

Referring to Fig. 5, the relationship between the voltages and currents at the junctions at $x=0$ may be expressed in

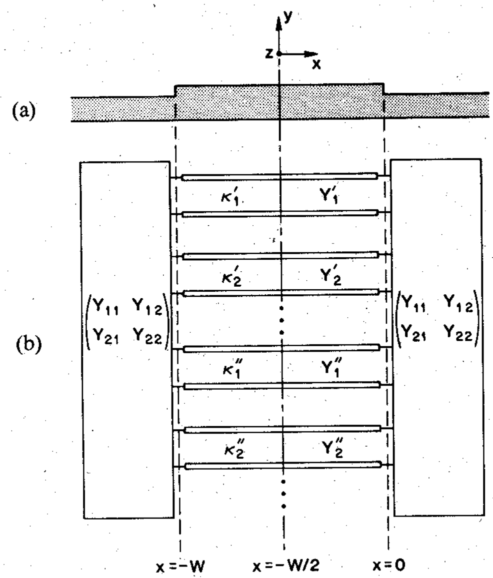

Fig. 4. Equivalent network for the transverse resonance in the lateral direction of a dielectric strip waveguide. (a) Dielectric strip waveguide. (b) Transverse equivalent network.

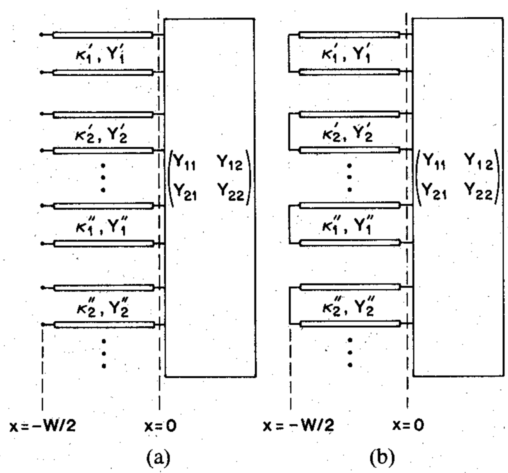

Fig. 5. Bisected transverse equivalent networks for the waveguide in Fig. 4. (a) Symmetric or open-circuit bisection. (b) Antisymmetric or short-circuit bisection.

terms of the transmission-line characteristics as

$$I' = -\overleftarrow{Y}'V' = -D(\overleftarrow{Y}_n')V' \qquad (31a)$$

$$I'' = -\overleftarrow{Y}''V'' = -D(\overleftarrow{Y}_n'')V'' \qquad (31b)$$

where $D(Y_n)$ stands for a diagonal matrix with Y_n as its element at the nth diagonal position, and therefore $\overleftarrow{Y}'$ and $\overleftarrow{Y}''$ are diagonal matrices with the diagonal elements defined by the input admittances of the transmission-line sections $\overleftarrow{Y}_n'$ and $\overleftarrow{Y}_n''$ for LSE and LSM modes, respectively. The negative signs in (31) reflect the fact that the currents in the transmission lines flow in the positive x direction, while the admittances are defined with respect to the opposite direction. Explicitly, for the two bisected networks, we have

$$\overleftarrow{Y}_n' = jY_n' \tan \kappa_n' W/2 \qquad (32a)$$

$$\overleftarrow{Y}_n'' = jY_n'' \tan \kappa_n'' W/2 \qquad (32b)$$

for the open-circuit bisection, and

$$\overleftarrow{Y}_n' = -jY_n' \cot \kappa_n' W/2 \qquad (33a)$$

$$\overleftarrow{Y}_n'' = -jY_n'' \cot \kappa_n'' W/2 \qquad (33b)$$

for the short-circuit bisection. More succinctly, the two equations in (31) may be combined to become

$$\boldsymbol{I} = -\overleftarrow{Y}\boldsymbol{V} \qquad (34)$$

where $\boldsymbol{I}$ is a super column vector with the column vectors $\boldsymbol{I}'$ and $\boldsymbol{I}''$ as its elements, and similarly for $\boldsymbol{V}$. $\overleftarrow{Y}$ is a super matrix defined by

$$\overleftarrow{Y} = \begin{pmatrix} \overleftarrow{Y}' & 0 \\ 0 & \overleftarrow{Y}'' \end{pmatrix} \qquad (35)$$

where the elements at the nth positions in the diagonal matrices $\overleftarrow{Y}'$ and $\overleftarrow{Y}''$ are given by either (32) or (33). For simplicity, $\overleftarrow{Y}$ will be called the admittance matrix looking to the left.

On the other hand, the relationship between the voltages and currents may also be expressed in terms of the input-admittance matrices for the step discontinuity as defined in (29). More succinctly, (29) may be written as

$$\boldsymbol{I} = \overrightarrow{Y}\boldsymbol{V} \qquad (36)$$

where $\overrightarrow{Y}$ will be called the admittance matrix looking to the right, and the super matrix is defined by

$$\overrightarrow{Y} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}. \qquad (37)$$

Evidently, (34) and (36) are two different equations relating the same set of voltages to the same set of currents. Equating (34) and (36), we obtain

$$(\overleftarrow{Y} + \overrightarrow{Y})\boldsymbol{V} = 0 \qquad (38)$$

which is a system of linear homogeneous equations to determine the modal voltages at $x=0$. When the solution is obtained, the modal currents $\boldsymbol{I}$ at $x=0$ can be simply determined from either (34) or (36). With the terminal voltages and currents now known, the voltages and currents everywhere in the transmission-line system can be determined by standard techniques and the electromagnetic fields everywhere within the dielectric waveguide are completely specified via Table III.

For the linear homogeneous system of equation (38), the condition for the existence of a nontrivial solution is that the determinant of the coefficient matrix vanishes, namely

$$\det(\overleftarrow{Y} + \overrightarrow{Y}) = 0. \qquad (39)$$

In other words, this is a condition under which nonzero voltages may exist in the absence of any source excitation in the network of Fig. 5(a) or (b); alternatively, the networks are in resonance. Again, the admittances in (38) are all functions of the longitudinal propagation wavenumber k_z. Therefore, (39) is the equation to determine the allowable values of k_z, and it will be simply referred to as the *generalized transverse-resonance relation* or *dispersion relation* for the dielectric waveguide.

For a single-mode case, $\overleftarrow{Y}$ and $\overrightarrow{Y}$ are scalar admittances, and (39) becomes the familiar transverse-resonance relation that states that, for a network system to be in resonance, the sum of the admittances (or impedances) looking into the two opposite directions at any point within the network system must vanish. Clearly, (39) is a generalization of the scalar transverse-resonance relation for a single-mode case to the matrix one for a multimode case, in order to account for the effect of mode coupling. Some important virtues of transverse-resonance relation (39) are 1) it is exact, 2) it is simple in form, 3) it is easily adaptable to more complex waveguide structures, and 4) it is an effective tool for a systematic numerical analysis. Furthermore, being an exact transverse-resonance relation, it can be used as a basis for developing approximation techniques that will exhibit the effect of mode coupling, and also to identify new physical phenomena that may take place in the waveguide structures.

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Guidance and Leakage Properties of a Class of Open Dielectric Waveguides: Part II—New Physical Effects

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Invited Paper

Abstract—**A class of open dielectric waveguides is discussed which is of direct importance to the areas of integrated optics and millimeter-wave integrated circuits. An accurate analysis of the properties of these waveguides reveals that interesting new physical phenomena, such as leakage and sharp cancellation or resonance effects, may occur under appropriate circumstances. The resulting leaky modes form a new class of such modes since the leakage, in the form of an exiting surface wave, has a polarization opposite to that which dominates in the bound portion of the leaky mode. These new effects are caused by TE–TM mode coupling, which was neglected in earlier approximate treatments. Part II describes the new physical effects and includes numerical results on various waveguiding structures to illustrate the new effects quantitatively.**

I. Introduction

IN THIS PART, we first describe and discuss certain *new physical effects* that follow from taking into account the *coupling between TE and TM modes* that occurs at the sides of the open dielectric waveguides. Since such coupling is ignored completely in the customary approximate treatments, those treatments miss these physical effects entirely. Later in the paper various *numerical results* are presented for typical waveguiding structures which illustrate these new physical effects quantitatively, as well as indicating for which physical properties the approximate theories are satisfactory and for which they are not.

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Part I of this paper presents in detail the general mode-matching procedure which results in an accurate analysis of the propagation behavior of these open dielectric waveguides. In that treatment, the approach and the point of view are developed in detail; we will require them in our physical discussions below, and we briefly review them in the context of our summary in Section II of the "effective dielectric constant" approximation. Following the summary of that approximate method, we show in Section II-B the physical consequences of improving that approximation by including the TE–TM mode coupling at the sides of the open dielectric waveguide. Most importantly, we thereby obtain the new physical effects to be described, but we also show how the hybrid nature of these guided modes is altered. The *mathematical* consequences of accounting for the TE–TM coupling, with the implied inclusion of higher modes, are examined in detail in Part I.

The new physical effects to which we refer are the presence of *leakage* and the appearance of sharp *resonance*, or cancellation, *effects*. As mentioned earlier, the customary approximate theories neglected the TE–TM coupling and thus never predicted these physical effects. The effects themselves are discussed in Section III. We present, in Sections III-B and -C, physical explanations for the leakage mechanism and the resonance effect. The leakage occurs in the form of a surface wave which emerges from the guiding structure at some angle to it. The exit angle of the leaking surface wave is discussed in Section III-E.

The leakage to which we refer changes the guided mode

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