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GUIDE 7

A GENERAL PROGRAM FOR EVALUATING THE PROPERTIES OF SCINTILLATION AND ČERENKOV COUNTER OPTICAL SYSTEMS

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ABSTRACT

GUIDE 7 is a design tool for optical systems which are constructed predominantly from reflecting surfaces. A wide range of surface forms and qualities may be chosen and there is provision for the user to extend the range of applicability of the program. Available facilities include estimation of light collection efficiencies, simulation of the space-time intensity distributions of photons arriving at a detector, optimization of the parameters of focusing systems. The report gives a complete description of the program, including user-defined routines. A large-area time-of-flight counter and ray-tracing for a parabolic mirror are treated as examples, detailing the input data and showing the corresponding output. The derivation of the vector algorithms used in the program is also included.

CONTENTS

1. INTRODUCTION

The optical response of a particle detector depends on the mechanism by which the light is produced, on the transmission properties of the propagating medium, and on the quality and shape of the reflecting surfaces. GUIDE 7 allows these features to be defined in a flexible way so as to evaluate absolute light collection efficiencies for a wide range of geometries and counter characteristics, to obtain histograms and scattergrams of quantities which are of interest in designing a detector, and to output the coordinates of successive reflection points for ray-tracing. The contents of this program description are organized as follows:

2. *Definitions:* The meaning of a counter system within the context of this program. This section serves also to indicate its possible uses.
3. *Operation of the program:* How it is organized, its size, and some operation time estimates.
4. *Data cards*
5. *Some examples of the input data and the corresponding output*
6. *User-defined routines:* To superpose the response of the detector to many particle tracks; to map the image from a source which is extensive in three dimensions or to obtain the response from a trajectory which is non-linear; to extend the set of quantities which may be entered in histograms, etc.
7. *Glossary of symbols in COMMON:* The more the USER routines are used, the more this section will be found useful.
8. *Vector algorithms*

2. THE DEFINITION OF A COUNTER SYSTEM

A counter system consists of a light-producing medium which is excited by an incident particle, and then an optical system to transfer this light to a light-sensitive detector, the photomultiplier. The components of the system must be specified in terms of both their optical and geometrical properties, as described below.

2.1 The light-source geometry

For testing design ideas, and also in some practical cases, the light source may be considered to be a point. More often it is a straight line particle trajectory with either constant or varying light output. In the latter case the particle may stop in the counter. One is usually interested in the response of the system to a single track. Options described in Section 6 allow the user to write more complicated cases by making also the position and direction of the trajectories random variables in the Monte Carlo integration. Examples are: the mean response of the counter to a distant source of particles; the response to non-linear trajectories; neutron counter mean response and efficiency; and gas Čerenkov imaging and light collection.

2.2 The response of the medium

This refers to the properties of the light produced by a particle in the medium. If the medium is a scintillator then the light is produced isotropically, with a certain time distribution. It has been found experimentally that a scintillator has up to three different decay constants; that is, the light intensity as a function of time is given by

$$I(t) = A\ e^{-t/B} + C\ e^{-t/D} + E\ e^{-t/F}\ .$$

In a particular calculation, one may want to know the time response of the optical system alone: in this case I(t) must be a delta function.

The other important response type is that of the Čerenkov counter, where the light is emitted in a cone with half-angle θ defined by the mass and momentum of the particle and the refractive index of the medium:

$$\cos\theta = \frac{1}{\beta n}\ ,$$

where β is the velocity of the particle and n is the refractive index. The light is plane-polarized with its electric vector lying in the plane defined by the incident particle direction and the emitted photon direction, but to avoid loss in efficiency this program does not take into account the dependence of reflection properties on the polarization.

In the case of Čerenkov radiation, it is assumed that the source is a line along which the velocity remains constant. The line may cross the whole counter width, or it may be defined to be so short that it is effectively a point in order to see the response of the system to light from different parts of the incident track.

2.3 The optical guiding system

Light transmission from the source to the detector depends on the characteristics of the material through which it travels, and of the boundaries. The material is characterized by a group index n_g = (velocity of light *in vacuo*)/(group velocity of light in the medium) and by its attenuation.

Because the media are dispersive, the group velocity is in general different from the wave velocity, being characterized by a group index

$$n_g = n(\lambda) - \frac{\lambda\ dn\ (\lambda)}{d\lambda}\ ,$$

where n is the refractive index and λ is the wavelength. Typically, at a wavelength λ = 4200 Å, the group index of scintillator is $n_g = 1.73$ compared with a refractive index of $n = 1.5$.

It is common practice to quote an attenuation length for the material, and the program assumes exponential light attenuation. However, since both Čerenkov and scintillation signals are produced in a dispersive medium, each wavelength has a different attenuation length so that the 'blue' end of the spectrum is filtered out first with a shorter attenuation length, leaving the yellow light with a longer attenuation length. Light transmission as a function of distance is not exponential, and if these effects need to be included, then this can usually be effected in the USER routine.

The light must be guided by a series of geometrical boundaries, which may have different shapes and reflecting properties. Guiding surfaces included in the program are the commonly used ones: planes, cylinders, cones, bent surfaces, and conic sections. Plane surfaces are defined as rectangles, and several of them may be used together to form the closed surface of part of a counter. Cylindrical and conical guides are already closed surfaces, and the light is incident on them from the interior. A bent guide is a section of a cylinder cut parallel to its axis. The light may be incident on either face, depending on the problem. Since this is an open surface it will always be associated with at least one other open surface (plane or bent) to form a section of the guiding system. Conic sections are surfaces formed by the rotation of the ellipse, parabola, and hyperbola about their axes. Frequently the first series of surfaces forms a rectangular cylinder, and the final surface in a light-guide is a circular cylinder.

Three types of reflecting surface are found in counters: mirrors, diffusing surfaces, and dielectric boundaries:

Mirrors. The angle of reflection is equal to the angle of incidence, and the light is reflected for all angles of incidence. The mirror may not be perfect, so a reflection coefficient, assumed independent of the angle of incidence, is associated with it.

Diffusing surfaces. Here the angle of reflection is independent of the angle of incidence and the differential reflection probability per unit solid angle is

$$\frac{dP}{d\Omega} = \frac{R}{\pi} \cos \theta \ ,$$

where R is a reflection coefficient less than 1, and θ is the polar angle relative to the normal to the reflecting surface.

Dielectric boundaries. The light is assumed to propagate in a dense material immersed in a rare medium. If the angle of incidence is greater than the critical angle θ_c, ($\sin \theta_c = 1/n$), then the reflection coefficient is assumed 100%, otherwise it is assumed zero. (For angles less than the critical angle, the reflection coefficient depends on the polarization of the incident light and on the angle of incidence. This behaviour is not included in the program.)

2.4 Gates

All systems must have at least one restriction or 'gate', in that the photon must reach the light-sensitive area. Other restrictions are discontinuities in the cross-section area of the counter or discontinuities in its physical properties. Gates often separate groups of guiding surfaces, and this aspect is used as a means of improving the program efficiency by specifying the sequence of guides between successive gates.

Two types of gate are defined: rectangular and circular. The last circular gate of the system is, by convention, always the photomultiplier. The light must cross all gates.

The physical nature of each gate is described by the refractive indices of the materials before and after the gate, and by the attenuation length and group velocity of the light in the new medium. If the light is totally reflected by the gate, it is assumed lost, otherwise the transmission is assumed 100%. (Again this is an approximation -- since angle and polarization-dependent transmission should be used.)

2.5 Focusing devices

Elements such as lenses and special mirrors have not been included explicitly in the program. Often such elements are the last stage before the detector, and in such cases provision is made for the user to write his own routine to continue the calculation after the final gate.

2.6 Output signal characteristics

The spatial and temporal distributions of photons arriving at the photomultiplier depend on the position and transit time of the incident particle through the counter, the scintillation decay constant when applicable, the geometrical shape and size of the system, and the values of the group velocity of the light in the various parts of the system.

The polishing of the light-guides may not be perfect, so it may be interesting to know how many reflections occur. The uniformity of the photocathode illumination may be required, and also the correlation between position in the detector and transit time, and so on. Such studies and many others may be carried out by using the histogram facilities and the USER routines.

3. OPERATION OF THE PROGRAM

3.1 Organization

Data is input in sets consisting of one or two key words which may be followed by several cards of numerical data. These key words control the program flow and identify the data. Sufficient data must be supplied to describe fully the type and geometry of the source, the guiding system, the gates, the statistical accuracy needed, and the options for the detailed operation of the program, and to specify the quantities to be output as histograms.

When a particular case has been calculated, it is possible to erase part or the whole of this data using a series of RESET commands so that a new case, either similar or different to the previous one may be set up by reading further data cards. Data for which reset options are not provided may simply be overwritten.

The word RUN initiates the photon tracking routine which, if the user is not providing his own routines, proceeds as follows.

A random point is chosen on the incident particle track and then a photon emission is chosen with angular distribution and time delay distribution characteristics of scintillation light or Čerenkov light as the case may be.

Each photon is tracked by extrapolating its direction vector to intersect all active surfaces and gates and then choosing the intersection point which is nearest. A SEQUENCE command allows the problem to be broken up into independent guiding sections so that the number of intersection points to be calculated at each step is reduced.

At the intersection point, tests are made to see if the photon is reflected and, if so, with what characteristics of emission. In each step of the tracking, account is kept of the reflection and attenuation losses, the propagation time, and the number of reflections.

The new photon direction is extrapolated and the process is repeated until the photon hits the photomultiplier, is lost, or accumulates too great a time delay to be useful in contributing to the photomultiplier pulse.

If the photon is accepted, a vector of interesting quantities (RESVEC) is established, and a call is made to routine USER, where the user may modify and extend this results vector. Those elements of this vector which have been specified on the input data are then entered in the appropriate histograms.

The number of photons which are tracked may be limited in three ways:

i) by specifying a maximum number of photons leaving the source,
ii) by specifying a maximum number of photons arriving at the photomultiplier, and
iii) by imposing a time limit for the calculation.

The above description together with the data-card description and the examples should allow most problems to be calculated. If more sophisticated particle trajectories are required or if a more complicated imaging system has to be constructed, then the flow diagram shown in Fig. 1 will be found useful. Note that in the standard operation, NTRACK = 1 and

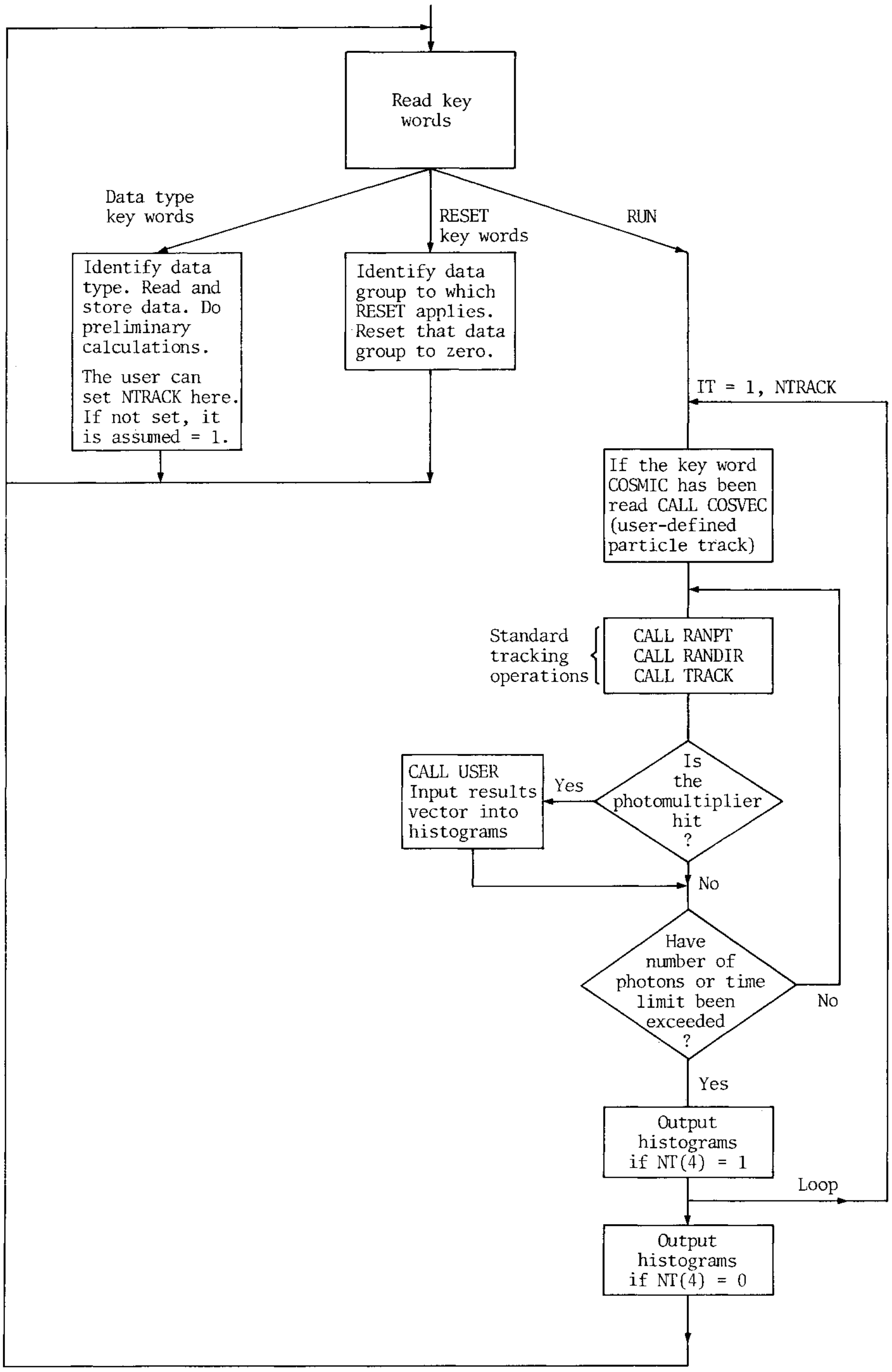


Fig. 1 Simplified flow diagram of GUIDE 7

the histogram output is called only once. When NTRACK is greater than 1, the flag NT(4) defined in the HOW section of the data cards controls the histogram output.

3.2 Storage and operation-time requirements

Results are given for the CERN CDC 7600 computer.

The program and associated private routines occupy 17535_8 words of memory. HIST adds an extra 14536_8 words, and the total storage space including all routines is 43014_8 = = 17932 words.

A typical compilation time is 3.3 seconds.

No universal formula can be given for execution times because these depend so much on the properties of the optical system being investigated and on the number of histograms being constructed. As examples may be quoted the execution times for the cases which will be described in Section 5.1.

One case is a full counter system consisting of:

1 initial rectangular cylinder
12 plane-guiding surfaces
2 rectangular gates
1 conical surface
1 circular gate
1 final circular cylinder,

that is, a total of 18 elements. One scattergram is constructed. The photon generation rate was 1000/sec giving 90 photons/sec accepted at the photomultiplier.

The second case is a simple ray-tracing involving one parabolic mirror and the final detection gate, and no scattergrams are constructed. In this case the generation and acceptance rates were both 4500/sec.

3.3 Routines, files, and common blocks used

USER ENTRY NAME	INTERNAL ENTRY NAMES	LOGICAL FILES REFERENCED	EXTERNAL ROUTINES REFERENCED	COMMON BLOCKS USED
GUIDE 7	SUB	INPUT	USER	/FAST/ 3
	DOT	OUTPUT	UZERO	/CONSEC/ 234
	SCALE	TAPE 7	UCOPY	/SEQ/ 17
	RANPT		TIMING	/SLATE/ 50
	HITA		CROSS	/ / 5231
	TRACK		VMODUL	
	RECTUB		HIST	
	CIRTUB		RNDM	
	SRAN		DATEZB	
	SEQUEN			
	CONSEC			

4. DATA CARDS

4.1 General conventions

The input data is organized into subsets, each of which has

a) a card containing BCD information in the form of one or two key words in FORMAT(2A10);
b) a series of cards containing numerical information.

There is one of these subsets for each reflecting surface and for each gate. Other subsets describe the way in which the program is to run and how the light is produced. The order of the subsets has been made as flexible as possible but is subject to the following rules:

i) The last circular gate is the photomultiplier.
ii) The last card of the data set must be the RUN card.
iii) Any series of surfaces of the same type which is implied to be in sequence by a SEQUENCE definition must be in sequence in the input data, although data subsets of other types may be inserted. (SEQUENCE is a facility to divide the optical system into independent parts to speed up execution.)

Fixed- and floating-point quantities are defined implicitly by their name as in FORTRAN.

Unless otherwise stated, formats are 2A10 or 3F10.

Distances should be in centimetres and attenuation constants in cm^{-1}. The units of mass/momentum are arbitrary provided that they are self-consistent.

Care Beware of rounding errors. For example, if a gate is placed at the exit of an initial cylinder, then rounding errors may cause the exit point to be just after the gate so that the gate is never hit and the photon is incorrectly lost. If the situation is doubtful, place the gate a few microns further along the optical system.

A restriction The maximum number of any one type of gate or guide is 20, except for conic sections where it is 10.

4.2 List of key words

SOURCE	POINT
SOURCE	LINE
SOURCE	STOP
SOURCE	COSMIC
RESPONSE	SCINTILLATOR
RESPONSE	CERENKOV
GUIDE	PLANE
GUIDE	CYLINDER
GUIDE	CONICAL
GUIDE	BENT
CONSECTION	
GATE	RECTANGULAR
GATE	CIRCULAR
FIRST	RECCYLINDER
FIRST	CIRCCYLINDER
LAST	CIRCCYLINDER
SEQUENCE	PLANE
RESET	CYLINDER
RESET	CONICAL
RESET	BENT
RESET	RECTANGULAR
RESET	CIRCULAR
RESET	FIRST
RESET	LAST
RESET	
HOW	
RECAP	
RUN	
ENDFILE	
ENDALL	

4.3 Details of the data cards

SOURCE	POINT		
A_1	A_2	A_3	Coordinates of the point.

SOURCE	LINE		(Specific ionization is constant.)
A_1	A_2	A_3	Coordinates of the start of the line.
A_4	A_5	A_6	Vector length of the line.
A_7			Velocity of the incident particle/velocity of light. This is used to obtain the time delay due to the distance which a particle travels in the detector before a particular photon is emitted.

SOURCE	STOP		(Specific ionization varies)
A_1	A_2	A_3	Coordinates of the start of the line.
A_4	A_5	A_6	Vector length of the line.
A_7	N_1		A_7 = velocity of incident particle/velocity of light. N_1 = number of values of dE/dx tabulated. Format (I5). The maximum value is $N_1 = 100$.
D(1), D(2), D(3), D(4), D(5), D(6), D(7), etc., ... , D(N_1)			The line (A_4, A_5, A_6) is assumed divided into N_1 equal segments. D(I) is the mean value of dE/dx in the I^{th} interval.

Format (5F10.0). The first segment starts at zero distance along the track. Note that in spite of the name, the track may, but does not *have to* stop in the counter. STOP really signifies that the specific ionization varies. For evaluating time delays, the velocity is assumed constant, so put a mean value for A_7 to get a better approximation.

SOURCE	COSMIC	Used for trajectories not included above.
The data cards are defined by the user.		Section 6 describes how to write your own routine.

RESPONSE SCINTILLATOR

A_1 A_2 A_3 A_4 A_5 A_6 Format (6F10.0)

The time distribution of the light from the scintillator decay is given by $I(t) = A_1 \exp(-t/A_2) + A_3 \exp(-t/A_4) + A_5 \exp(-t/A_6)$, where t is in nanoseconds. If $A_1 = 0$, a delta function $I(t) = \delta(t)$ is assumed.

A_1, A_3, and A_5 must be positive.

RESPONSE CERENKOV

A_1 A_2 A_3

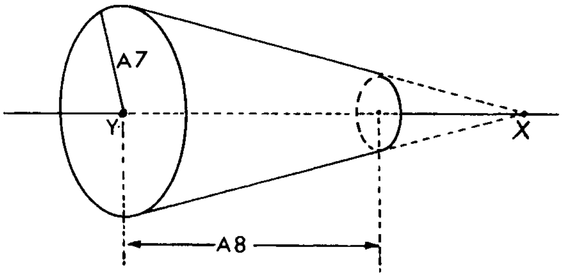
Mass and momentum of the particle, refractive index of the medium. A delta function time distribution is assumed for light emission.

GUIDE	PLANE		i.e. a plane rectangle.
A_1	A_2	A_3	Coordinates of one corner.
A_4	A_5	A_6	Vector length of one edge adjacent to the corner.
A_7	A_8	A_9	Vector length of the second edge adjacent to the corner.
A_{10}	A_{11}	A_{12}	Reflection properties: A_{10} = refractive index of the inner material; A_{11} = refractive index of the outer material; A_{12} = reflection coefficient.

If A_{12} is +ve, the reflecting surface is a mirror with reflection coefficient A_{12} (assumed independent of angle); A_{10} and A_{11} are not relevant, but set $A_{10} > A_{11} > 1.0$ to avoid trouble. If A_{12} is -ve, the reflecting surface is a diffusing surface with an isotropic re-emission of light. The integral of the total reflected energy is $|A_{12}|$ of the incident energy (set $A_{10} > A_{11} > 1.0$). If A_{12} is zero, then the reflecting surface is a boundary between two dielectrics, and the numbers A_{10} and A_{11} are relevant. If the angle of incidence is greater than the critical angle, 100% reflection is assumed, otherwise it is assumed zero.

GUIDE	CYLINDER		Light is propagating inside the cylinder.
A_1	A_2	A_3	Centre of one end.
A_4	A_5	A_6	The vector length of the axis (changed to a unit vector in the program).
A_7			The radius.
A_8	A_9	A_{10}	The reflection properties (as for PLANE).

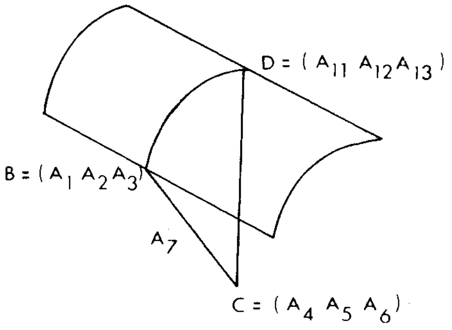
GUIDE	CONICAL		A truncated cone with the light propagating inside it.
A_1	A_2	A_3	The vertex (X below).
A_4	A_5	A_6	The centre of the large open end (Y below).
A_7	A_8		Radius of the large end and truncated length.
A_9	A_{10}	A_{11}	The reflection properties (as for PLANE).



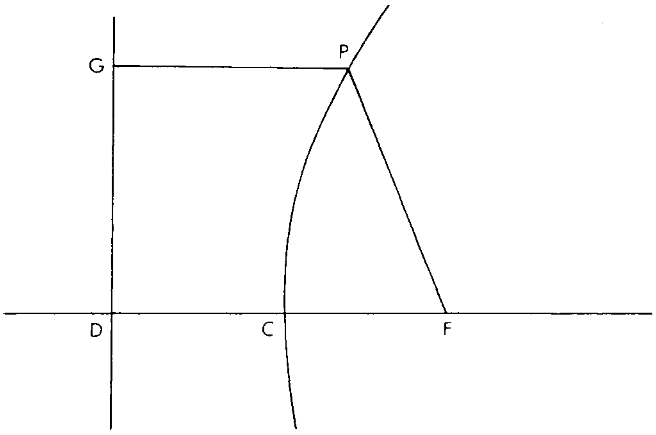
GUIDE	BENT		See the following diagram. Note that DCB is perpendicular to the axis of the cylinder.
A_1	A_2	A_3	Start of the bend = B.
A_4	A_5	A_6	Centre of curvature = C.
A_7			Radius of curvature.
A_8	A_9	A_{10}	Reflection properties (as for PLANE).
A_{11}	A_{12}	A_{13}	End of the bend = D.

N.B. The angle BCD must be less than 180°.

This is a section of a cylinder. It differs from the cylinder described previously in that the light may be incident from either the interior or the exterior of the surface, and the length of the cylinder is infinite. Extra guiding surfaces will usually be used to complete the guide.



CONSECTION			See the diagram below.
A_1	A_2		A_1 = eccentricity; A_2 = 1.0 if the light is incident on the concave surface, and A_2 = -1.0 if the light is incident on the convex surface.
A_3	A_4	A_5	Coordinates of the centre C.
A_6	A_7	A_8	Coordinates of the focus F.
A_9	A_{10}	A_{11}	Reflection properties (as for PLANE).



Recall. A_1 = eccentricity = FP/GP. $A_1 > 1$ for hyperbola, $A_1 = 1$ for a parabola, $A_1 < 1$ for an ellipse, and $A_1 = 0$ for a circle.

The reflecting surface is formed by rotation about the axis CF. In the case of a sphere, the program assumes an ellipse with very small eccentricity (10^{-6}) thus allowing the same routine to be used for all four cases.

GATE	RECTANGULAR		
A_1	A_2	A_3	One corner.
A_4	A_5	A_6	First edge.
A_7	A_8	A_9	Second edge.
A_{10}	A_{11}	A_{12}	Reflection properties.
A_{13}			Group index after the gate.

The first three sets of numbers are the same as for a PLANE guide, A_{10} and A_{11} are the refractive indices before and after the light passes the gate. A_{12} is the attenuation constant after the gate. (The attenuation constant before the first gate is defined in the HOW data set.)

GATE	CIRCULAR		
A_1			Radius.
A_4	A_5	A_6	Position of the centre.
A_7	A_8	A_9	Unit vector parallel to the normal to the gate.
A_{10}	A_{11}	A_{12}	Reflecting properties as for RECTANGULAR gate.
A_{13}			Group index after the gate.

Note. The convention is that the last gate defined must be circular, and is considered as the photomultiplier. However, two gates separated by a very small distance may be defined so as to obtain other shapes for the detector. Every photon arriving at the last circular gate is detected independent of the reflection properties stated. To simulate losses if there is the possibility of total internal reflection back from the last gate, another gate may be added a very small distance in front of it.

Often the first piece of a counter consists of a simple rectangular or circular cylinder, and there is usually a final cylinder to connect the counter to the photomultiplier. To improve program efficiency, there are more efficient routines for these special cases, but their use is more restricted. In particular, the axis of these components must always be parallel to the z axis of the coordinate system which is being used. These special cases are described below.

FIRST	RECCYLINDER		i.e. rectangular cylinder.
FAREND	(NATURE)		This describes the type of surface which closes the cylinder at the end furthest away from the phototube. The word (NATURE) may be BLACK, SILVERED, or POLISHED. Black absorbs all photons, SILVERED reflects with reflection coefficient A_9, and POLISHED reflects by total internal reflection (indices A_7 and A_8).
A_1	A_2	A_3	Coordinates of the point on the axis furthest from the phototube.
A_4	A_5	A_6	Width (in x direction), height (in y direction), length (in z direction).
A_7	A_8	A_9	Reflection properties of the walls (as for PLANE).

FIRST	CIRCCYLINDER		An initial circular cylinder.
FAREND	(NATURE)		Options as described above.
A_1	A_2	A_3	Coordinates of the point on the axis furthest from the phototube.
A_4	A_5		Radius and length.
A_6	A_7	A_8	Reflection properties (as for PLANE).

LAST	CIRCCYLINDER		
FAREND	(NATURE)		Data are as for FIRST CIRCCYLINDER.
A_1	A_2	A_3	
A_4	A_5		
A_6	A_7	A_8	

Note. 1) The circular cylinder option may only be used once in any one case. It may be either the FIRST or the LAST cylinder.

2) If it is the last cylinder, the FAREND card is not used, but it must be present in the deck.

3) The LAST CIRCCYLINDER routine is called after the photon has been accepted at the final gate. Hence the last circular gate must be positioned at the *entrance* to this cylinder.

4) The attenuation constant used is the value just before the final circular gate.

SEQUENCE See below for explanation.

N

I2 I2 I3 I4 I5 I6 I7 I8 I9 I10 I11 I12

J1 J2 J3 J4 J5 J6 J7 J8 J9 J10 J11 J12

⋮ ⋮

N1 N2 N3 N4 N5 N6 N7 N8 N9 N10 N11 N12

SEQUENCE introduces a sequencing to improve the program efficiency. In unsequenced operation, after each reflection all surfaces in the optical system are tested to see if they are candidates for the next reflection and then the nearest reflection point is selected. By dividing the system into groups of surfaces separated by gates, the sequence directive causes only those surfaces associated with a particular gate to be calculated.

N is the number of groups of surfaces. The indices I1 to N12, in Format (12I5), define the sequences of program flow. There is one card for each group of surfaces. After the photon is first produced, the gate which terminates the first part of the optical system is indicated by its gate number I1 for a rectangular gate or I2 for a circular gate. Only one of these indices I1 or I2 may be non-zero. The remaining indices on the card indicate the surfaces which are active and are used as tabulated below:

Type of surface	Surface number	
	Minimum	Maximum
Plane	I3	I4
Cylinder	I5	I6
Conical	I7	I8
Bent	I9	I10
Conic section	I11	I12

The minimum and maximum surface numbers in each class must be specified. These surface numbers are assigned by the program and are just the order in which the data is read in. Thus the fifth curved surface to be read in will have order number 5 independent of how many surfaces of other types have been read in previously. The RESET command operates by setting the appropriate sequence numbers to zero.

If there are no surfaces active in a particular class, then the values of its flow control indices must be set to zero.

Note that the FIRST and LAST cylinder options are outside the control of the SEQUENCE specifications.

In the first sequence, the photon will be tracked until it crosses the gate specified or until it is lost. If the gate is crossed, then the program will use the J values specified on the next card to define the new gate and the new surfaces to be tested.

Once SEQUENCE has been specified it cannot be cancelled. It can, however, be redefined. Further, the properties of the guides and gates may be redefined (e.g. in size or optical properties) without redefining the sequence, provided that the new system will have the same sequence topology.

```
RESET     PLANE
RESET     CYLINDER
RESET     CONICAL
RESET     BENT
RESET     RECTANGULAR
RESET     CIRCULAR
RESET     FIRST
RESET     LAST
```

These will reset the number of elements in the appropriate class to zero, allowing either continuation of the program or the reading in of new data.

Note. SOURCE, RESPONSE, HOW, SEQUENCE, FIRST, and LAST may be overwritten by reading in new sets of the appropriate card groups.

HOW This card introduces the following sets of data which describe program and output options:

Group 1. NT(1), NT(2), ..., NT(40) in format 40I2. One card.
These flags have the following effect:

NT(1) = 1 causes the random number generator to be reset after each case. (If you are using your own routine COSMIC, note that this would cause the random number generator to be reset after each step in the loop IT = 1, NTRACK.)

NT(2) = 1 means that light produced will always have a positive z component of velocity. Thus by suitable choice of geometry, the efficiency of the program can be increased by a factor of two. *Care:* Do not forget this factor of two in absolute intensity calculations.

NT(3) = 1 omit the histogram codes described below, so as to obtain cumulative histograms.

NT(4) = 1 print histograms after each value of IT.

NT(4) = 0 print histograms only after NTRACK values.
If you do not redefine NTRACK in your COSMIC routine, the program operation is the same for both values.

NT(5) = 1 causes diagnostic printing which may be used for ray-tracing. The output consists of the time and position of emission of the photon followed by one line of printing each time a guiding surface or gate is struck. The coordinates of the point, the time delay, and the product of the attenuation factors are output. Under the column PM HIT is recorded the radial position at which the photon strikes the photomultiplier.

NT(6) = 1 causes a diagnostic message NOTHING HIT when a photon escapes from the system without hitting the photomultiplier.

NT(7) = 1 causes ray-tracing data to be output on TAPE 7. Output is written using WRITE(7), IFL, X, Y, Z. X, Y, Z are the coordinates of the point and IFL is a flag with the following significance:
IFL = +1 is the first point on a new track,
IFL = 0 are successive points,
IFL = -1, (X, Y, Z) = (0, 0, 0) is added after the last point if the photon is accepted at the detector,
IFL = -2, (X, Y, Z) = (0, 0, 0) is added after the last point if the photon is not accepted at the detector.

The remaining components of NT are available for use in the user's routine.

Group 2. N1 N2 A1 N3 A2 A3 A4 Format (2I5, F5.0, I5, 3F10.0). One card.

N1 = maximum number of photons emitted for each case (i.e. for each value of IT if NTRACK has been reset in COSMIC). This is a safety device to prevent wasting computer time.

N2 = maximum number of photons to be recorded at the photomultiplier (for each value of IT). This determines the accuracy of the result.

A1 = maximum running time of the program for each case (i.e. for each value of IT), in seconds -- another safety device.

N3 = the number of histograms to be recorded. This must be an even number ≤ 10.

A2 = the maximum time-delay in nanoseconds between the particle's incidence on the counter and the photon's arrival at the detector. This is another safety device to avoid using computing time to track photons which arrive too late to be useful.

A3 = group index of the medium in which the light is produced.

A4 = 1/(attenuation length in cm) of the medium in which the light is produced.

Group 3. N1 N2 N3 ... N11 N12 ... N_{N3}

Format 10I5. Up to 10 values. This is a list of indices of the quantities to be plotted. The quantities are stored in RESVEC. If you do not redefine HIST, all the graphs are two-dimensional plots with projected histograms. N1 is plotted against N2, N3 against N4, and so on. The same quantity may enter into as many plots as desired.

The indices of locations in the vector RESVEC are:

1 = Total distance travelled by the photon.

2 = Total time-delay of the photon including the time-delay of the incident particle before the photon is emitted, the scintillation decay time if relevant, and the photon propagation time.

3 = Number of reflections.

4 = Probability that the photon arrives (i.e. product of all attenuation factors and reflection coefficients).

$\left.\begin{array}{l}5\\6\\7\end{array}\right\}$ = Coordinates of the point where the photon strikes the last gate.

8 = Radial position on the last gate (photomultiplier).

$\left.\begin{array}{l}15\\16\\17\end{array}\right\}$ = The coordinates of the source of light.

$\left.\begin{array}{l}18\\19\\20\end{array}\right\}$ = Unit vector in the direction of the photon striking the last gate or reaching the end of a final cylinder when the LAST option is used.

9 to 14 and 21 to 30 may be defined as you wish using routine USER.

Note. All scatter diagrams, except those containing RESVEC(4), are weighted with 10 times the probability that the photon arrives at the phototube, i.e. are weighted with RESVEC(4) × 10.

Group 4. These are the data cards for the histogram plotting routine, CERN library routine J504. They are:

```
 1.  bb0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ.-**

 2.  b - - - - - Y(H100)-X(H100)-PLOT(60x60),A=10-19,B=20-29,Z.GE.260 - - - - -

⎧3.  1b60b5◄──────────Heading──────────────────────────────────────────
⎪                                        41         51            61           71
⎨4.  ──────Heading────────────────────►1. ────────Lower(1) ───────Width(1) ─────2.
⎪    1                 11
⎩5.  Lower(2)──────────Width(2)

⎧6.  b60bb5◄──────────Heading──────────────────────────────────────────
⎨7.  ────Heading──────────────────────►3. ────────Lower(3) ───────Width(3) ───── 4.
⎩8.  Lower(4) ─────────Width(4)

 ▲
 │   Further sets of three cards.
 ▼

Last - 2. ⎫
Last - 1. ⎬ Three blank cards.
Last.     ⎭
```

b means blank space.

Cards 1 and 2 are fixed.

Cards then follow in sets of three, one set for each scatter diagram, and are terminated by three blank cards.

In a set of three cards, for example the first set, 1b60b5b may be left invariant (60 means a 60 × 60 bin scatter diagram).

Heading is any BCD heading which should be written above the scatter diagram.

1. and 2. in columns 41 and 71 of card 4 mean that RESVEC(N1) and RESVEC(N2) are to be plotted on this scatter diagram (N1 and N2 were defined in group 3 above).

Lower(1) is the smallest value of RESVEC(N1) to be plotted on the scatter diagram.

Width(1) is the bin width for RESVEC(N1) and similarly for lower(2) and width(2).

Note. If you have your preferred histogramming routine, the most convenient way is to write a dummy HIST in your deck to transfer the data to your own routine. The calls made to HIST are:

CALL HIST (6, 0.0, 0.0, 0.0, 0, 0) to input the scales and histogram numbers.

CALL HIST (3, 0.0, 0.0, 0.0, 0, 0) to output the histograms.

CALL HIST (2, W, X1, X2, I, J) to enter the data into a scattergram.

W = 10*attenuation factor. The factor of 10 is because fractions do not always show up clearly on a scattergram, but the program requires "attenuated" photons to be plotted.

X1 and X2 are the two quantities to be plotted, namely RESVEC(NI) and RESVEC(NJ).

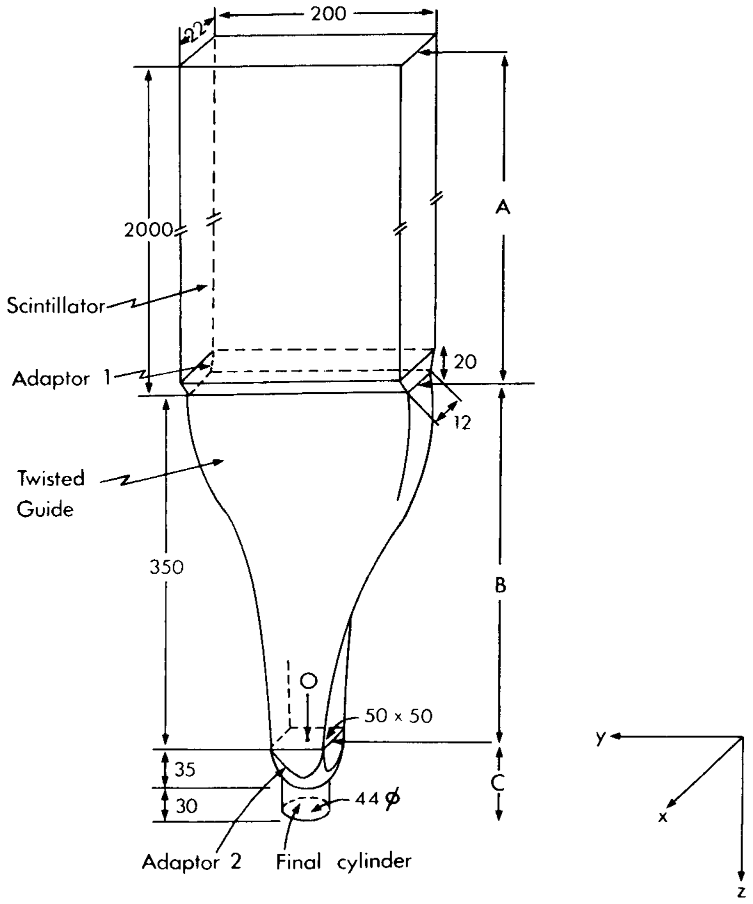
I and J are the histogram numbers associated with these two quantities.

RECAP	This requests printing of all stored data and is useful for checking that data has been modified correctly for the next case. (Note that the exterior refractive index of a surface is overwritten by the value of $\cos^2 \theta$, where θ is the critical angle.)
RUN	This card initiates the calculation. When the calculation is terminated, the program tries to read new data.
ENDFILE	causes an end of file to be written on TAPE 7. This is effective immediately after the card has been read.
ENDALL	causes the program to terminate with the FORTRAN instruction STOP.

5. EXAMPLES OF THE INPUT DATA AND THE CORRESPONDING OUTPUT

5.1 A large-area time-of-flight counter

Figure 2 shows a counter which was designed for the double-arm spectrometer, at the DORIS storage ring[1]. There is one difficulty in simulating this geometry: the twisted



(Dimensions in mm)

Fig. 2 A large-area time-of-flight counter guide B which makes the shape transformation. If the light is guided perfectly by B, then we can perhaps adapt the following strategy:

1) Obtain the intensity and time distributions of photons leaving section A (see Fig. 2). Note that the cross-section area of the scintillator is 44 cm^2. In Adaptor 1 the width of the counter is reduced by a factor of 0.546.
2) Calculate a pseudo-section A which is 9.16 × 5 cm^2, has the same reduction factor 0.546 to reach a 5 × 5 cm^2 area, and has the same angle of the sloping edges of Adaptor 1.
3) Compare the data from 1 and 2 to be sure that they are compatible.
4) Calculate the response of the whole counter.

In this example, we will show stages 2 and 4 of this procedure.

5.1.1 *Input data*

Figure 2 indicates the coordinate system; 0 is the origin.

Definition of stage 2.

```
SOURCE   POINT                  Note that only the first four letters of the words are
0,       0,       -243,32       needed.
RESP     SCIN
0,                         }
FIRS     RECC              }
FARE     BLAC              }    The scintillator.  Use "first rectangular cylinder"
0,       0,       -243,32  }    option.
9,16     5,       200,     }
1,5      1,0      0,       }
GUIDE    PLANE             }
4,58     2,5      -43,32   }
9,16                       }    One side of Adaptor 1.
0,       0,       8,32     }
1,5      1,                }
GUIDE    PLANE             }
-4,58    -2,5     -43,32   }
9,16                       }    The other side of Adaptor 1.
0,       0,       8,32     }
1,5      1,                }
GUIDE    PLANE             }
4,58     2,5      -43,32   }
0,       -5,               }    One sloping side of Adaptor 1.
-2,08    0,       8,32     }
1,5      1,                }
GUIDE    PLANE             }
-4,58    -2,5     -43,32   }
0,       5,                }    The other sloping side of Adaptor 1.
2,08     0,       8,32     }
1,5      1,                }
GATE     RECT              }
-2,5     -2,5     -35,     }
5,                         }    The exit area of Adaptor 1 (not really needed here
0,       5,                }    but it will be used for sequencing later).
1,5      1,5               }
1,7                        }
GATE     CIRC              }
10,                        }
0,       0,       -34,99   }    Define a large "photomultiplier" to accept all pho-
0,       0,       1,       }    tons reaching the end of Adaptor 1.
1,5      1,5               }
1,7                        }
HOW
 1 1
50000 200020,     230,        1,7        0,  }  The output will describe these
   1    2                                    }  options.
```

```
  0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ.-**
 -----Y(H100)-X(H100)-PLOT(60X60),A=10-19,B=20-29,Z.GE.260-----
1 60 5   DASP INITIAL STAGE APPROX BY FATTER COUNTER. 1=DISTANCE. 2=TIME.
                                               1.       200.      2.       2.
10.        0.2
```

Histogram definitions.

```
RUN
```

Start the calculation.

Now continue the calculation for the complete counter. Keep all except the photomultiplier, then add on the remaining guides and gates.

```
RESET     CIRC
```

Delete the photomultiplier.

```
GUIDE     PLANE
-2.5      -2.5      -35.
5.
0.        0.        35.
15.       1.
GUIDE     PLANE
-2.5      -2.5      -35.
0.        5.
0.        0.        35.
1.5       1.
GUIDE     PLANE
2.5       2.5       -35.
-5.
0.        0.        35.
1.5       1.
GUIDE     PLANE
2.5       2.5       -35.
0.        -5.
0.        0.        35.
1.5       1.
```

Use a 5 × 5 cm² rectangular guide instead of the twisted guide. These four planes define it.

```
GATE      RECT
-2.5      -2.5      0.
5.
0.        5.
1.5       1.5
1.7
```

This is the exit face of the "twisted guide"; it is used for sequencing.

```
SEQUENCE
  3
    1     0     1     4
    2     0     5     8
    0     1     9    12     0     0     1     1
```

Note that sequence can be defined anywhere. Here section C has not yet been defined.

```
GUIDE     PLANE
-2.5      -2.5      0.
5.        0.        0.
0.        0.        50.
1.5       1.        0.
GUIDE     PLAN
-2.5      -2.5      0.
0.        5.        0.
0.        0.        50.
1.5       1.        0.
GUIDE     PLANE
2.5       2.5       0.
-5.0      0.        0.
0.        0.        50.
1.5       1.        0.
GUIDE     PLAN
2.5       2.5       0.
0.        -5.       0.
0.        0.        50.
1.5       1.        0.
```

The plane faces of section C. Note that they are defined *<u>50 cm</u>* *long in the z direction. It does not matter because the program action is to always select the nearest surface which the photon direction intersects.*

```
GUIDE     CONI
0.        0.        9.25
0.        0.        0.
3.54      3.5
1.5       1.        0.
```

The conical shape. Note that the origin is centred on its entrance face. This choice of origin would be convenient for investigating different shapes for the section.

```
GATE       CIRC
2.2
0.         0.          3.5
0.         0.          1.
1.5        1.5         0.
1.7
```

The last circular gate -- but we are using the "last circular cylinder" option so --

```
LAST       CIRCC
FAREND     BLACK
0.         0.          3.5
2.2        3.
1.5        1.          0.
```

Define the final cylinder of C.

```
HOW
 1 1
50000 200020.      230.        1.7        0.
    1    2
```

See the output.

```
  0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZ.-**
 -----Y(H100)-X(H100)-PLOT(60X60),A=10-19,B=20-29,Z.GE.260-----
1 60 5     DASP TOTAL COUNTER. FAT APPROX. 1= DISTANCE. 2= TIME.
                                                1.        200.      2.        2.

13.        0.2



RUN
```

Routine HIST requires all these cards to be read again.

5.1.2 *Output*

```
 .396E+01
```

← *Time elapsed in seconds.*

```
SOURCE GEOMETRY
POINT SOURCE AT  0.          0.          -.243E+03
 .396E+01

RESPONSE OF DETECTOR.
SCINTILLATOR WITH ISOTROPIC LIGHT EMISSION
DECAY IS A1 EXP(-T/A2)+A3 EXP(-T/A4)+A5 EXP(-T/A6),VALUES ARE
A1=  0.          A2= -0.          A3= -0.          A4= -0.          A5= -0.          A6= -0.
 .396E+01
 THERE IS A SIMPLE INITIAL GUIDE
```

Zero values mean that a delta function is used.

```
 THERE IS AN INITIAL RECTANGULAR CYLINDER.
 CENTRE OF STARTING END   =      0.                 0.                 -.24332E+03
 WIDTH IN THE X DIRECTION =        .91600E+01
 WIDTH IN THE Y DIRECTION =        .50000E+01
 LENGTH IN THE Z DIRECTION=        .20000E+03
 INNER REFRACTIVE INDEX   =  1.5000000
 OUTER REFRACTIVE INDEX   =  1.0000000
 REFLECTION COEFFICIENT   =      0.
 CRITICAL ANGLE. . . . .  =        .41813E+02
 THE END DISTANT FROM THE PHOTOTUBE ABSORBS ALL LIGHT
  .397E+01
```

This is the scintillator.

```
NUMBER      1 PLANE REFLECTING SURFACE.
CORNER . . . . . . . . . . =    .458E+01  .250E+01 -.433E+02
SIDE 1 . . . . . . . . . . =    .916E+01-0.         -0.
SIDE 2 . . . . . . . . . . =  0.          0.         .832E+01
INTERIOR REFRACTIVE INDEX  =    .150E+01
EXTERIOR REFRACTIVE INDEX  =    .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =  0.         -.100E+01 0.
  .397E+01
```

One side of Adaptor 1.

```
NUMBER       2 PLANE REFLECTING SURFACE.
CORNER . . . . . . . . . . . =  -.458E+01 -.250E+01 -.433E+02
SIDE 1 . . . . . . . . . . . =   .916E+01-0.        -0.
SIDE 2 . . . . . . . . . . . =  0.         0.        .832E+01
INTERIOR REFRACTIVE INDEX   =   .150E+01
EXTERIOR REFRACTIVE INDEX   =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =  0.        -.100E+01 0.
   .397E+01
```

The other side of Adaptor 1.

```
NUMBER       3 PLANE REFLECTING SURFACE.
CORNER . . . . . . . . . . . =   .458E+01  .250E+01 -.433E+02
SIDE 1 . . . . . . . . . . . =  0.        -.500E+01-0.
SIDE 2 . . . . . . . . . . . =  -.208E+01 0.         .832E+01
INTERIOR REFRACTIVE INDEX   =   .150E+01
EXTERIOR REFRACTIVE INDEX   =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =  -.970E+00 0.         -.243E+00
   .397E+01
```

One sloping side of Adaptor 1.

```
NUMBER       4 PLANE REFLECTING SURFACE.
CORNER . . . . . . . . . . . =  -.458E+01 -.250E+01 -.433E+02
SIDE 1 . . . . . . . . . . . =  0.         .500E+01-0.
SIDE 2 . . . . . . . . . . . =   .208E+01 0.         .832E+01
INTERIOR REFRACTIVE INDEX   =   .150E+01
EXTERIOR REFRACTIVE INDEX   =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =   .970E+00 0.         -.243E+00
   .398E+01
```

The second sloping side of Adaptor 1.

```
NUMBER       1 RECTANGULAR GATE
CORNER . . . . . . . . . . . =  -.250E+01 -.250E+01 -.350E+02
SIDE 1 . . . . . . . . . . . =   .500E+01-0.        -0.
SIDE 2 . . . . . . . . . . . =  0.         .500E+01-0.
INTERIOR REFRACTIVE INDEX   =   .150E+01
EXTERIOR REFRACTIVE INDEX   =   .150E+01
NEW ATTENUATION CONSTANT    = -0.
NORMAL . . . . . . . . . . . =  0.         0.         .100E+01
NEW GROUP INDEX. . . . . . . =   .170E+01
   .398E+01
```

The exit face of Adaptor 1.

```
NUMBER       1 CIRCULAR GATE
RADIUS . . . . . . . . . . . =   .100E+02-0.        -0.
POSITION OF CENTRE . . . . . =  0.         0.        -.350E+02
NORMAL . . . . . . . . . . . =  0.         0.         .100E+01
INTERIOR REFRACTIVE INDEX   =   .150E+01
EXTERIOR REFRACTIVE INDEX   =   .150E+01
NEW ATTENUATION CONSTANT    = -0.
NEW GROUP INDEX. . . . . . . =   .170E+01
   .398E+01
```

The large "photomultiplier" to terminate the photon path.

```
HOW SECTION.
THE CASE WILL BE RUN FOR THE SMALLER OF   50000 PHOTONS EMITTED
                                     OR    2000 PHOTONS DETECTED
                                     OR   20.00 SECONDS
MAXIMUM TRANSIT TIME ACCEPTED =   .300E+02 NSEC
INITIAL GROUP INDEX          =   .170E+01
INITIAL ATTENUATION CONSTANT  =   0.
RANDOM NUMBER IS RESET FOR EACH CASE
ONLY PHOTONS WITH A +VE Z COMPONENT ARE GENERATED.
SCATTERGRAMS WILL BE MADE OF THE FOLLOWING RESVEC LOCATIONS.
(    1 ,      2) (
NT=  1 1-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0
   .399E+01
```

From here on, NT flags may be set in the routine USER.

```
 DASP INITIAL STAGE APPROX BY FATTER COUNTER, 1=DISTANCE, 2=TIME,                                                     01/10/76
 UF,      SCALE54,4    HI  1      I-----Y(H 10)-X(H 20)-PLOT(60X60),A=10-19,B=20-29,Z,GE,260---I  UF,      SCALE84,4    HI  2
200,                              I                                                            I 10,0
202,                              I                                                            I 10,2
204,                              I                                                            I 10,4
206,                              I                                                            I 10,6
208, 770 **************           I         Z                                                  I 10,8
210,1250 **********************   I         ZQ                                                 I 11,0
212,1090 ********************     I          Z                                                 I 11,2
214,1070 *******************      I          ZZ                                                I 11,4
216,1040 *******************      I           Z                                                I 11,6
218, 840 ***************          I           ZZ                                               I 11,81850 *********************
220, 740 *************            I            Z                                               I 12,01940 **********************
222, 870 ***************          I            UZ                                              I 12,21810 *********************
224, 750 *************            I             ZC                                             I 12,41410 ****************
226, 790 **************           I              Z                                             I 12,61380 ****************
228, 670 ************             I              ZO                                            I 12,81340 ***************
230, 680 ************             I               Z                                            I 13,01150 *************
232, 650 ***********              I               ZZ                                           I 13,21110 *************
234, 630 ***********              I                Z                                           I 13,41010 ***********
236, 600 ***********              I                OZ                                          I 13,6 890 **********
238, 560 **********               I                 Z                                          I 13,8 830 *********
240, 460 ********                 I                  Z                                         I 14,0 750 ********
242, 510 *********                I                  ZH                                        I 14,2 850 **********
244, 530 *********                I                   Z                                        I 14,4 540 ******
246, 420 *******                  I                   VT                                       I 14,6 460 *****
248, 410 *******                  I                    Z                                       I 14,8 320 ***
250, 450 ********                 I                    NZ                                      I 15,0 460 *****
252, 500 *********                I                     Z                                      I 15,2 320 ***
254, 280 *****                    I                     DX                                     I 15,4 250 **
256, 380 ******                   I                      ZH                                    I 15,6 210 **
258, 240 ****                     I                       X                                    I 15,8 150 *
260, 240 ****                     I                       NJ                                   I 16,0 100 *
262, 150 **                       I                        U                                   I 16,2 150 *
264, 270 ****                     I                        GT                                  I 16,4 110 *
266, 240 ****                     I                         X                                  I 16,6  90 *
268, 170 ***                      I                         BU                                 I 16,8  60
270, 180 ***                      I                          QA                                I 17,0  70
272, 150 **                       I                           O                                I 17,2  50
274, 110 **                       I                           IB                               I 17,4 140 *
276, 170 ***                      I                            Q                               I 17,6  40
278, 100 *                        I                            BH                              I 17,8  50
280,  70 *                        I                             G                              I 18,0  30
282,  40                          I                              D                             I 18,2  20
284,  70 *                        I                              FA                            I 18,4  10
286,  70 *                        I                               G                            I 18,6  30
288,  90 *                        I                               GB                           I 18,8  10
290,  80 *                        I                                H                           I 19,0  10
292,  50                          I                                AD                          I 19,2
294,  50                          I                                 E                          I 19,4
296,  20                          I                                  B                         I 19,6
298,  40                          I                                  D                         I 19,8
300,  50                          I                                   E                        I 20,0
302,  40                          I                                   BB                       I 20,2
304,  30                          I                                    C                       I 20,4
306,  60 *                        I                                     F                      I 20,6
308,  70 *                        I                                     G                      I 20,8
310,  10                          I                                     A                      I 21,0
312,  40                          I                                      D                     I 21,2
314,  20                          I                                       B                    I 21,4
316,  40                          I                                       CA                   I 21,6
318,  10                          I                                        A                   I 21,8
 OF,  90          SUM  20000      I11111111111111111111111111111111111111111111111111222222222I  OF,          SUM  20000
                                   000001111122222333334444455555666667777788888999990000011111

                                   ............................................................
                                   024680246802468024680246802468024680246802468024680246802468
GENERATED  ACCEPTED  WEIGHTED ACCEPTS  ACCEPT/GENERATED  WEIGHTED ACCEPTS/GENERATED
   14306     2000        .20000E+04        .13980E+00         .13980E+00
 .136E+02
```

A summary of the total acceptance is printed immediately below the histogram. Note that if further rejection is made in the routine USER, this will affect the weighted *events both in the histograms and in the "weighted accept" number but it will not influence the number of "accepted" events. "Accepted" refers to photons crossing the last circular gate (and being transmitted through the LAST circular cylinder if it has been defined). "Accepted" and "weighted accepts" are equal here because the attenuation constant is zero and there is no USER routine.*

```
NUMBEP OF  CIRC GATES RESET TO ZERO
  .136E+02
```

Cancel the photomultiplier and define the sections B and C.

```
NUMBEP      5 PLANE REFLECTING SURFACE.
CORNER . . . . . . . . . . . =  -.250E+01 -.250E+01 -.350E+02
SIDE 1 . . . . . . . . . . . =   .500E+01-0.         -0.
SIDE 2 . . . . . . . . . . . =  0.        0.         .350E+02
INTERIOR REFRACTIVE INDEX  =   .150E+02
EXTERIOR REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =  0.        -.100E+01 0.
  .136E+02

NUMBEP      6 PLANE REFLECTING SUPFACE.
CORNEP . . . . . . . . . . . =  -.250E+01 -.250E+01 -.350E+02
SIDE 1 . . . . . . . . . . . =  0.         .500E+01-0.
SIDE ? . . . . . . . . . . . =  0.        0.         .350E+02
INTERIOP REFRACTIVE INDEX  =   .150E+01
EXTERIOR REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =   .100E+01 0.        0.
  .136E+02

NUMBEP      7 PLANE REFLECTING SURFACE.
CORNEP . . . . . . . . . . . =   .250E+01  .250E+01 -.350E+02
SIDE 1 . . . . . . . . . . . =  -.500E+01-0.         -0.
SIDE ? . . . . . . . . . . . =  0.        0.         .350E+02
INTERIOR REFRACTIVE INDEX  =   .150E+01
EXTERIOR REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . =  0.         .100E+01 0.
  .136E+02

NUMBEP      8 PLANE REFLECTING SURFACE.
CORNEP . . . . . . . . . . . =   .250E+01  .250E+01 -.350E+02
SIDE 1 . . . . . . . . . . . =  0.        -.500E+01-0.
SIDE 2 . . . . . . . . . . . =  0.        0.         .350E+02
INTERIOR REFPACTIVE INDEX  =   .150E+01
EXTERIOP REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE= -0.
NORMAL TO THE PLANE. . . . = -.100E+01 0.        0.
  .136E+02
```

Section B.

```
NUMBEP      2 RECTANGULAR GATE
CORNER . . . . . . . . . . . =  -.250E+01 -.250E+01 0.
SIDE 1 . . . . . . . . . . . =   .500E+01-0.         -0.
SIDE 2 . . . . . . . . . . . =  0.         .500E+01-0.
INTERIOR REFRACTIVE INDEX  =   .150E+01
EXTERIOR REFRACTIVE INDEX  =   .150E+01
NEW ATTENUATION CONSTANT   = -0.
NORMAL . . . . . . . . . . . =  0.        0.         .100E+01
NEW GROUP INDEX. . . . . . . =   .170E+01
  .136E+02
```

Exit gate of section B.

This is how sequencing works. Sections A, B, and C of the system correspond to step numbers 1, 2, and 3.

```
THE PHOTON TRACKING HAS BEEN DIVIDED INTO  3 STEPS (EXLUDING INITIAL AND FINAL
SIMPLE GUIDES), THE FOLLOWING TABLE GIVES THE  3 SECTIONS OF THE OPTICAL SYSTEM,
AN ENTRY ZERO INDICATES THAT NO GATES OR GUIDES IN THAT CLASS ARE ACTIVE,

------------------------------------------------------------------------------
       , TERMINATES AT GATE,, ,   REFLECTING SURFACES WHICH ARE ACTIVE IN THE STEP,
       ,----------------------------------------------------------------------
STEP   , RECTANGULAR,CIRCULAR ,  PLANE   , CYLINDER, CONICAL ,BENT PLANE,CONIC SECT,
NUMBER,                       ,FROM TO  ,FROM TO  ,FROM TO  ,FROM TO   ,FROM TO
------------------------------------------------------------------------------
  1    ,      1      ,    0    ,  1   4 ,  -0   -0 ,  -0   -0 ,  -0   -0   ,  -0   -0
  2    ,      2      ,    0    ,  5   8 ,  -0   -0 ,  -0   -0 ,  -0   -0   ,  -0   -0
  3    ,      0      ,    1    ,  9  12 ,   0    0 ,   1    1 ,  -0   -0   ,  -0   -0
  ,136E+02

NUMBER     9 PLANE REFLECTING SURFACE,
CORNER . . . . . . . . . . . =   -,250E+01 -,250E+01 0,
SIDE 1 . . . . . . . . . . . =    ,500E+01 0,        0,
SIDE 2 . . . . . . . . . . . =   0,        0,         ,500E+02
INTERIOR REFRACTIVE INDEX    =    ,150E+01
EXTERIOR REFRACTIVE INDEX    =    ,100E+01
REFLECTION COEFFICIENT/TYPE=     0,
NORMAL TO THE PLANE, . . . =     0,       -,100E+01 0,
  ,136E+02

NUMBER     10 PLANE REFLECTING SURFACE,
CORNER . . . . . . . . . . . =   -,250E+01 -,250E+01 0,
SIDE 1 . . . . . . . . . . . =   0,         ,500E+01 0,
SIDE 2 . . . . . . . . . . . =   0,        0,         ,500E+02
INTERIOR REFRACTIVE INDEX    =    ,150E+01
EXTERIOR REFRACTIVE INDEX    =    ,100E+01
REFLECTION COEFFICIENT/TYPE=     0,
NORMAL TO THE PLANE, . . . =      ,100E+01 0,        0,
  ,136E+02

NUMBER     11 PLANE REFLECTING SURFACE,
CORNER . . . . . . . . . . . =    ,250E+01  ,250E+01 0,
SIDE 1 . . . . . . . . . . . =   -,500E+01 0,        0,
SIDE 2 . . . . . . . . . . . =   0,        0,         ,500E+02
INTERIOR REFRACTIVE INDEX    =    ,150E+01
EXTERIOR REFRACTIVE INDEX    =    ,100E+01
REFLECTION COEFFICIENT/TYPE=     0,
NORMAL TO THE PLANE, . . . =     0,         ,100E+01 0,
  ,136E+02

NUMBER     12 PLANE REFLECTING SURFACE,
CORNER . . . . . . . . . . . =    ,250E+01  ,250E+01 0,
SIDE 1 . . . . . . . . . . . =   0,        -,500E+01 0,
SIDE 2 . . . . . . . . . . . =   0,        0,         ,500E+02
INTERIOR REFRACTIVE INDEX    =    ,150E+01
EXTERIOR REFRACTIVE INDEX    =    ,100E+01
REFLECTION COEFFICIENT/TYPE=     0,
NORMAL TO THE PLANE, . . . =     -,100E+01 0,        0,
  ,136E+02
```

Section C plane guides.

```
NUMBER      1 CONICAL REFLECTING SURFACE
VERTEX OF THE CONE, . . . =  0,        0,         ,925E+01
CENTRE OF THE OPEN END    =  0,        0,        0,
RADIUS OF THE OPEN END    =   ,354E+01
LENGTH OF TRUNCATED PART  =   ,350E+01-0,
INTERIOR REFRACTIVE INDEX =   ,150E+01
EXTERIOR REFRACTIVE INDEX =   ,100E+01
REFLECTION COEFFICIENT/TYPE=  0,
  ,136E+02
```

Section C cone.

```
NUMBER      1 CIRCULAR GATE
RADIUS . . . . . . . . . . . =   .220E+01-0,       -0.
POSITION OF CENTRE . . . . = 0.          0.          .350E+01
NORMAL . . . . . . . . . . = 0.          0.          .100E+01
INTERIOR REFRACTIVE INDEX  =   .150E+01
EXTERIOR REFRACTIVE INDEX  =   .150E+01
NEW ATTENUATION CONSTANT   = 0.
NEW GROUP INDEX. . . . . . =   .170E+01
  .136E+02
THERE IS A SIMPLE FINAL COUPLER TO THE PHOTOMULTIPLIER.
```

Terminates the photon tracking.

```
THERE IS AN INITIAL OR FINAL CYLINDER.
CENTRE OF STARTING END     =      0.             0.
RADIUS AND LENGTH ALONG Z  =        .22000E+01      .30000E+01
INTERIOR REFRACTIVE INDEX  =   .150E+01
EXTERIOR REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE=  0.
CRITICAL ANGLE. . . . . .  =        .41813E+02
 THE END DISTANT FROM  THE PHOTOMULTIPLIER ABSORBS ALL LIGHT.
  .136E+02
```

.35000E+01

Extends the tracking through the last cylinder.

Ignore this last comment for a final cylinder.

```
HOW SECTION.
THE CASE WILL BE RUN FOR THE SMALLER OF  50000 PHOTONS EMITTED
                                     OR   2000 PHOTONS DETECTED
                                     OR  20.00 SECONDS
MAXIMUM TRANSIT TIME ACCEPTED =  .300E+02 NSEC
INITIAL GROUP INDEX          =   .170E+01
INITIAL ATTENUATION CONSTANT  =  0.
RANDOM NUMBER IS RESET FOR EACH CASE
ONLY PHOTONS WITH A +VE Z COMPONENT ARE GENERATED.
SCATTERGRAMS WILL BE MADE OF THE FOLLOWING RESVEC LOCATIONS.
(    1 ,      2)  (
NT=  1 1-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0
  .136E+02
```

```
     DASP TOTAL COUNTER, FAT APPROX, 1= DISTANCE, 2= TIME,                                                              01/10/76
  UF,     SCALE47,4   HI   1      I-----Y(H 10)-X(H 20)-PLOT(60X60),A=10-19,B=20-29,Z,GE,260---I UF,     SCALE79,6    HI  2
 200,                             I                                                            I 13,0
 202,                             I                                                            I 13,2
 204,                             I                                                            I 13,4
 206,                             I                                                            I 13,6
 208,                             I                                                            I 13,8
 210,                             I                                                            I 14,0 250 ***
 212,                             I                                                            I 14,21410 *****************
 214,                             I                                                            I 14,41830 **********************
 216,                             I                                                            I 14,61440 ******************
 218,                             I                                                            I 14,81390 *****************
 220,                             I                                                            I 15,01240 ***************
 222,                             I                                                            I 15,21080 *************
 224,                             I                                                            I 15,41060 *************
 226,                             I                                                            I 15,61190 **************
 228,                             I                                                            I 15,8 870 **********
 230,                             I                                                            I 16,0 790 *********
 232,                             I                                                            I 16,2 620 *******
 234,                             I                                                            I 16,4 700 ********
 236,                             I                                                            I 16,6 600 *******
 238,                             I                                                            I 16,8 510 ******
 240,                             I                                                            I 17,0 470 *****
 242,                             I                                                            I 17,2 370 ****
 244,                             I                                                            I 17,4 450 *****
 246,                             I                                                            I 17,6 330 ****
 248,  30                         I    C                                                       I 17,8 210 **
 250, 750 ***************         I    VZ                                                      I 18,0 150 *
 252, 760 ****************        I     Z                                                      I 18,2 240 ***
 254,1090 **********************  I     LZ                                                     I 18,4 170 **
 256,1020 *********************   I      ZP                                                    I 18,6  80 *
 258, 780 ****************        I       Z                                                    I 18,8 140 *
 260, 860 ******************      I       ZZ                                                   I 19,0 160 **
 262, 750 ***************         I        Z                                                   I 19,2  70
 264, 720 ***************         I        ZZ                                                  I 19,4  60
 266, 660 *************           I         Z                                                  I 19,6  70
 268, 730 ***************         I         MZ                                                 I 19,8  80 *
 270, 570 ************            I          ZH                                                I 20,0  90 *
 272, 560 ***********             I           Z                                                I 20,2  50
 274, 600 ************            I           ZR                                               I 20,4  20
 276, 700 **************          I            Z                                               I 20,6  40
 278, 650 *************           I            ZZ                                              I 20,8 100 *
 280, 460 *********               I             Z                                              I 21,0  10
 282, 480 **********              I             GZ                                             I 21,2  10
 284, 400 ********                I              ZB                                            I 21,4
 286, 310 ******                  I               Z                                            I 21,6
 288, 480 **********              I               ZS                                           I 21,8
 290, 410 ********                I                Z                                           I 22,0  10
 292, 270 *****                   I                JQ                                          I 22,2  10
 294, 390 ********                I                 Z                                          I 22,4  10
 296, 280 *****                   I                 DX                                         I 22,6
 298, 260 *****                   I                  Z                                         I 22,8
 300, 300 ******                  I                  AZ                                        I 23,0
 302, 250 *****                   I                   RG                                       I 23,2  10
 304, 160 ***                     I                    P                                       I 23,4
 306, 280 *****                   I                    NN                                      I 23,6
 308, 180 ***                     I                     R                                      I 23,8
 310, 250 *****                   I                     ML                                     I 24,0
 312, 210 ****                    I                      U                                     I 24,2
 314,  70 *                       I                       G                                    I 24,4
 316, 140 **                      I                       N                                    I 24,6
 318, 110 **                      I                        K                                   I 24,8
  OF,1480         SUM  18400      I11111111111111111111111111111111111222222222222222222222222222I OF,   10        SUM  18400
                                   333334444455555666667777788888999990000011111222223333344444
                                   ............................................................
                                   024680246802468024680246802468024680246802468024680246802468
GENERATED  ACCEPTED  WEIGHTED ACCEPTS  ACCEPT/GENERATED  WEIGHTED ACCEPTS/GENERATED
    21934     1840        .18400E+04       .83888E-01        .83888E-01
  .337E+02
```

5.2 Ray-tracing for a parabolic mirror

This is a simple example to illustrate the conic section facility and the diagnostic printing.

5.2.1 *Input data*

```
RESPONSE  SCINT                      An isotropic light source.  Delta function time distribution.

GATE      CIRCULAR
1000.
0.        0.         0.              Terminate the tracing with a large plane at z = 0.
0.        0.         1.
1.5       1.5        0.
1.7
CONSECTION
1.        1.
0.        0.         10.             The parabola.
0.        0.         5.
1.        1.         1.
HOW
 1 1 0 0 1 1
   10   100.1      0100.        1.7        0.00001
SOURCE    POINT
0.        0.         5.              A point at the focus.
RUN
SOURCE    LINE
0.        0.         0.
0.        0.7        1.
1.
RUN
SOURCE    LINE
0.        0.         0.
0.        0.1        1.
1.                                   Various line sources will be tried with the same mirror
RUN                                  geometry.
SOURCE    LINE
0.        0.         0.
0.        0.2        1.
1.
SOURCE    LINE
0.
0.        2.0        1.
1.
RUN
```

5.2.2 *Output*

```
                              GUIDE2, VERSION JUNE 1976
  .396E+01

RESPONSE OF DETECTOR.
SCINTILLATOR WITH ISOTROPIC LIGHT EMISSION
DECAY IS A1 EXP(-T/A2)+A3 EXP(-T/A4)+A5 EXP(-T/A6),VALUES ARE
A1= -0.       A2= -0.       A3= -0.       A4= -0.       A5= -0.       A6= -0.
  .396E+01

NUMBER      1 CIRCULAR GATE
RADIUS . . . . . . . . . . . =   .100E+04-0.       -0.
POSITION OF CENTRE . . . . . =  0.          0.      0.
NORMAL . . . . . . . . . . . =  0.          0.       .100E+01
INTERIOR REFRACTIVE INDEX    =   .150E+01
EXTERIOR REFRACTIVE INDEX    =   .150E+01
NEW ATTENUATION CONSTANT     =  0.
NEW GROUP INDEX. . . . . . . =   .170E+01
  .397E+01
```

```
THE      1 CONIC SECTION OF ROTATION IS PARABOLA   WITH ECCENTRICITY       .10000E+
RAYS ARE INCIDENT ON THE CONCAVE SIDE.
CENTRE(ON AXIS OF ROTATION)=  0.          0.           .100E+02
FOCUS . . . . . . . . . . . =  0.          0.           .500E+01
INTERIOR REFRACTIVE INDEX  =   .100E+01
EXTERIOR REFRACTIVE INDEX  =   .100E+01
REFLECTION COEFFICIENT/TYPE=   .100E+01
AXIS VECTOR(L=ECCENTRICITY)=  0.          0.          -.100E+01
  .397E+01

HOW SECTION.
THE CASE WILL BE RUN FOR THE SMALLER OF      10 PHOTONS EMITTED
                                     OR      10 PHOTONS DETECTED
                                     OR     .10 SECONDS
MAXIMUM TRANSIT TIME ACCEPTED =  .100E+03 NSEC
INITIAL GROUP INDEX          =   .170E+01
INITIAL ATTENUATION CONSTANT  =   .100E-04
RANDOM NUMBER IS RESET FOR EACH CASE
ONLY PHOTONS WITH A +VE Z COMPONENT ARE GENERATED.
NT=  1 1 0 0 1 1-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-
  .397E+01

SOURCE GEOMETRY
POINT SOURCE AT  0.          0.           .500E+01
  .397E+01
------------------------------------------------------------------------------------
IT,NTRACK=          1          1                              Output of plotting data.
 NEXT PHOTON                                                               |
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.
-.102E+01   .341E+01   .937E+01   .563E+01   .319E+00   .100E+01        -R
-.102E+01   .341E+01   .568E-13   .150E+02   .850E+00   .100E+01   .356E+01
 NEXT PHOTON
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.                                Changes only
 .118E+01   .165E+01   .979E+01   .521E+01   .295E+00   .100E+01   .356E+01  when PM hit.
 .118E+01   .165E+01   .568E-13   .150E+02   .850E+00   .100E+01  |.203E+01|<----'
 NEXT PHOTON
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.
-.687E+01  -.215E+01   .741E+01   .759E+01   .430E+00   .100E+01   .203E+01
-.687E+01  -.215E+01   .284E-13   .150E+02   .850E+00   .100E+01   .720E+01
                          ^
    Four photons omitted  |
                          v
 NEXT PHOTON
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.
 .875E+01   .794E+00   .614E+01   .886E+01   .502E+00   .100E+01   .850E+01
 .875E+01   .794E+00   .284E-13   .150E+02   .850E+00   .100E+01   .878E+01
 NEXT PHOTON
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.
 .723E+01  -.335E+01   .683E+01   .817E+01   .463E+00   .100E+01   .878E+01
 .723E+01  -.335E+01   .284E-13   .150E+02   .850E+00   .100E+01   .797E+01
 NEXT PHOTON
     X           Y           Z       DISTANCE     TIME     INTENSITY  RADIUS ON PHOTOMULT
0.          0.          .500E+01    0.0     0.
-.128E+00  -.557E+01   .845E+01   .655E+01   .371E+00   .100E+01   .797E+01
-.128E+00  -.557E+01   .568E-13   .150E+02   .850E+00   .100E+01   .557E+01
GENERATED  ACCEPTED  WEIGHTED ACCEPTS  ACCEPT/GENERATED  WEIGHTED ACCEPTS/GENERAT
     10        10          .99985E+01          .10000E+01          .99985E+00
  .399E+01
------------------------------------------------------------------------------------
SOURCE GEOMETRY
UNIFORM LINE
STARTING POINT =  0.          0.          0.
VECTOR LENGTH  =  0.          .700E+00   .100E+01
PARTICLE SPEED =   .100E+01 TIMES C
  .400E+01
```

```
IT,NTRACK=           1           1
 NEXT PHOTON
    X           Y            Z      DISTANCE      TIME     INTENSITY  RADIUS ON PHOTOMULT
 0.          .557E+00    .796E+00     0.0       .324E-01
 -.797E+01   .737E+01    .411E+01   .110E+02    .655E+00    .100E+01   .557E+01
 -.684E+01   .610E+01    .284E-13   .154E+02    .907E+00    .100E+01   .917E+01
 NEXT PHOTON
    X           Y            Z      DISTANCE      TIME     INTENSITY  RADIUS ON PHOTOMULT
 0.          .276E-01    .395E-01     0.0       .161E-02
 -.107E+02 -.331E+01     .377E+01   .118E+02    .669E+00    .100E+01   .917E+01
 -.899E+01 -.280E+01     .284E-13   .159E+02    .904E+00    .100E+01   .942E+01
 NEXT PHOTON
    X           Y            Z      DISTANCE      TIME     INTENSITY  RADIUS ON PHOTOMULT
 0.          .624E+00    .891E+00     0.0       .363E-01
 -.184E+01 -.121E+02     .245E+01   .130E+02    .773E+00    .100E+01   .942E+01
 -.170E+01 -.113E+02     .142E-13   .156E+02    .919E+00    .100E+01   .115E+02
 NEXT PHOTON
    X           Y            Z      DISTANCE      TIME     INTENSITY  RADIUS ON PHOTOMULT
 0.          .112E+00    .160E+00     0.0       .649E-02
 -.419E+01   .315E+01    .863E+01   .992E+01    .569E+00    .100E+01   .115E+02
 -.118E+01   .785E+00    .568E-13   .194E+02    .110E+01    .100E+01   .142E+01
 NEXT PHOTON
```

↓ etc.

Examples of how the output could be plotted are shown in Figs. 3 and 4. For a complicated optical system the output would be written on file TAPE 7, which could then be used as input for an automatic plotting program.

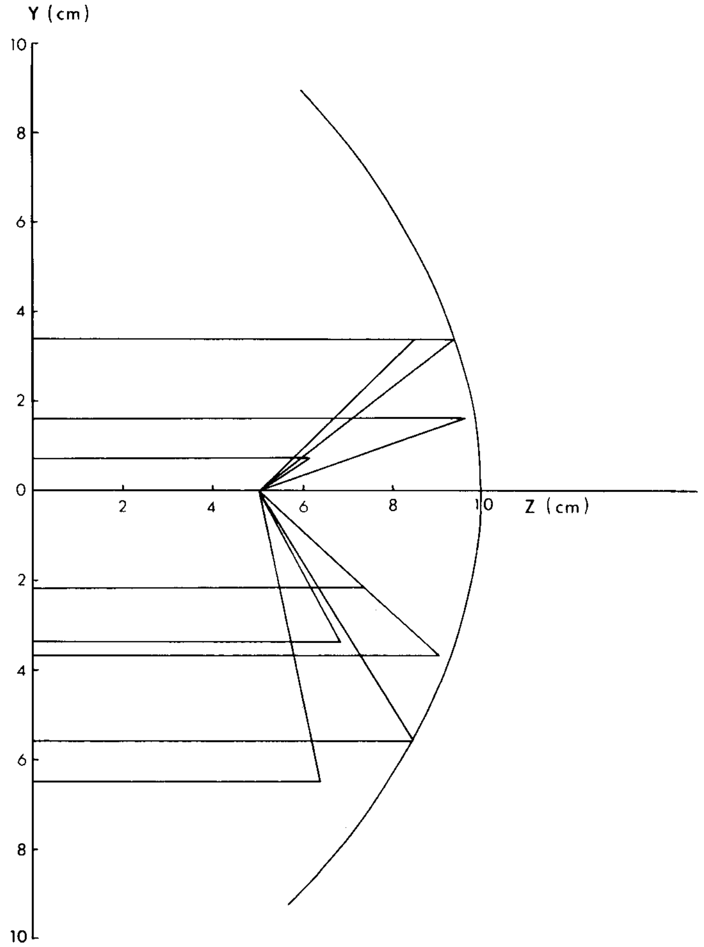


Fig. 3 Ray tracing from the focus of a parabolic mirror

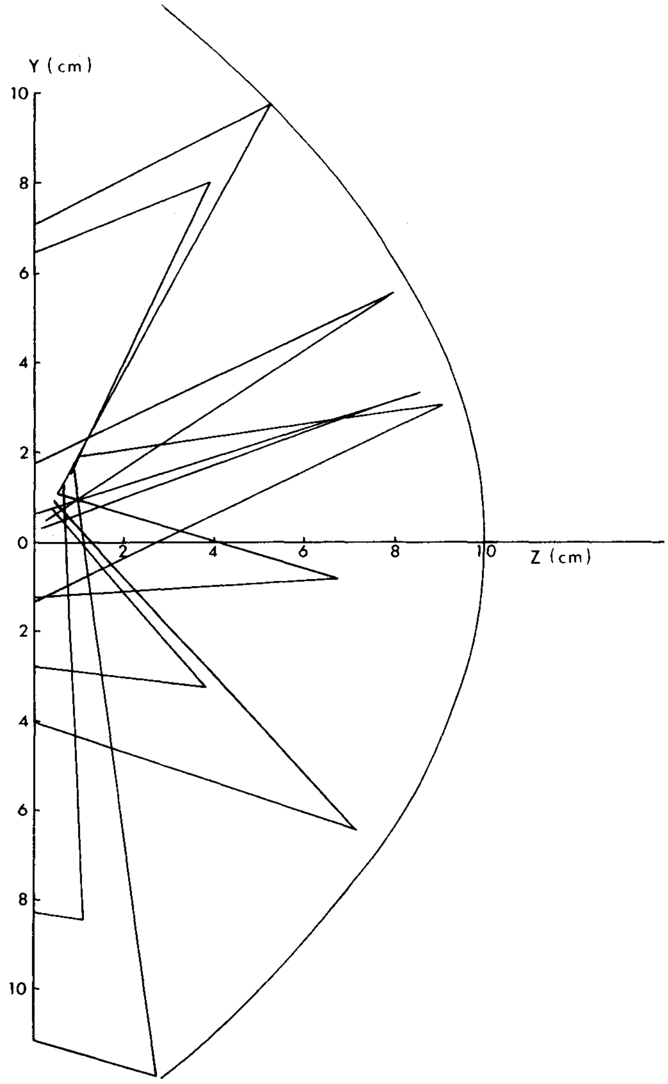


Fig. 4 Ray tracing from a line source in a parabolic mirror

6. USER-DEFINED ROUTINES

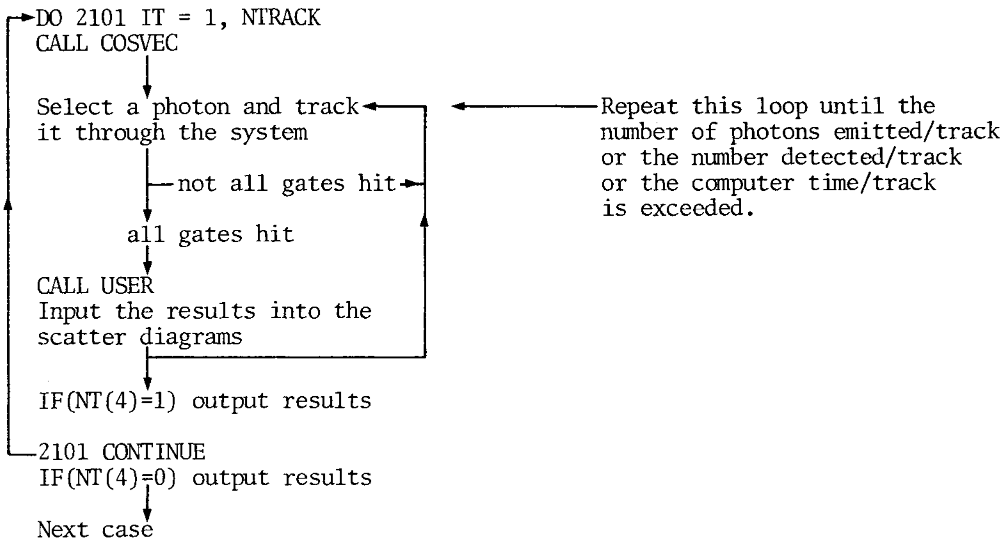
User-written routines may be included as entry points in SUBROUTINE USER. Most of the storage allocation is in blank COMMON and is defined at the end of this Section. In the standard program version, SETCOS, RECOS, and COSVEC are defined as entry points in the subroutine USER.

6.1 SETCOS The key word COSMIC in the data input means that the user is supplying his own geometry for the source of light. The code words SOURCE COSMIC on the data cards cause a call to SETCOS. At entry point SETCOS you should input all data which you require. The data should follow on cards immediately after the SOURCE COSMIC card. Set NTRACK here (see 6.3 below).

6.2 RECOS After reading the data and making any preliminary calculations, you should continue the program at entry point RECOS where a summary of the data should be output. Then RETURN to the main program.

The key word RECAP causes a summary of all stored data to be output. During this recapitulation, a call is made to RECOS to print the data from your routine.

6.3 COSVEC COSVEC redefines the geometry of the source of light. The relevant part of the program is illustrated below (see also Fig. 1).



In normal operation, the program assumes NTRACK = 1 unless it has been redefined in routine SETCOS. Note that IT and NTRACK are in COMMON/ / .

At each entry to COSVEC, you should set NCANG = 1 for a point source and NCANG = 2 for a line source. You may thus simulate your track configuration by a series of points or by a series of line elements.

For NCANG = 1, place in (AS(1), AS(2), AS(3)) the (x, y, z) coordinates of the point.

For NCANG = 2, AS are the coordinates of the start of the line, and (VS(1), VS(2), VS(3)) should be set equal to the components of the vector length of the line source.

Finally, set ALVS = the time in nanoseconds for the incident particle to travel along vector VS. If you do not want to include time spreads due to the finite track length of the incident particle, set ALVS = 0.

Note: 1) Any of these quantities which are constant may be set in SETCOS, since they are never modified by the program unless a new SOURCE key word is read in.

2) More complicated time, space, and intensity corrections should be introduced when USER is called, since this call is made only for photons which successfully reach the final gate.

6.4 USER USER is a complement to the above routines, but also has two further important uses:

1) To prepare new quantities to be included in the histograms. These may be placed in RESVEC locations 9 to 14 and 21 to 30, as indicated in Group 3 of the HOW cards described in Section 4.
2) To continue the ray-tracing along a further segment of counter which is of a type not available in the main structure of the program.

For most purposes, the values stored in RESVEC (described below the HOW data card description in Section 4) will be sufficient. If not, Section 7 gives the definitions of all the quantities available in blank COMMON.

6.5 Storage available for use in COSVEC, SETCOS, RECOS, and USER

The following stores are available in blank COMMON and are not used in the main program.
SINT(100), VC(3,3), THCOS(100), COSMIN(100), AMIN(10), DEL(10), NCH(10), N5,N6,N7, RESID, THT(20).
Stores V1, V2, ..., V9 may be used, but will be overwritten by the main program.

6.6 Random numbers. You may use any system of random numbers in setting up AS and VS. However, there is a smoothed random number routine available in the program. The routine has the property that in successive sets of 10 random numbers there will be one and only one in each of the intervals 0 to 0.1, 0.1 to 0.2, etc. This provides a form of stratified sampling. Up to 10 of these sequences may be used, and one sequence is reserved for each random variable in the calculation.

The routine is a function subroutine SRAN(L), where L is the sequence number. It uses the library function RNDM.

Normal allocation of L:

L = 1 Reserved for routine RANPT to choose a random point on vector VS. (If you have defined a point source, then you may use L = 1 in your private routine.)

L = 2 Reserved for RANDIR -- it gives the azimuthal angle of the photon.

L = 3 Used in RANDIR for the polar angle of the photon if the counter is a scintillator. (If you have specified Čerenkov response, you may use L = 3.)

$\left.\begin{array}{l}L = 4\\ L = 5\\ L = 6\end{array}\right\}$ May be used for COSVEC.

6.7 An example. Suppose that we wish to make the gas Čerenkov counter cell shown in Fig. 5 and design an optical system to collect the light most efficiently. One way would be to input a series of data cards using the SOURCE LINE option, but it would be more useful to obtain directly the mean response of the counter to a uniform illumination.

The USER routine shown below will do this. With this modification there must appear somewhere before the RUN card the two cards containing:

```
SOURCE     COSMIC
A1         A2         A3         A4     (Format 4F10.0),
```

where A_1, A_2, A_3, and A_4 are defined in Fig. 3.

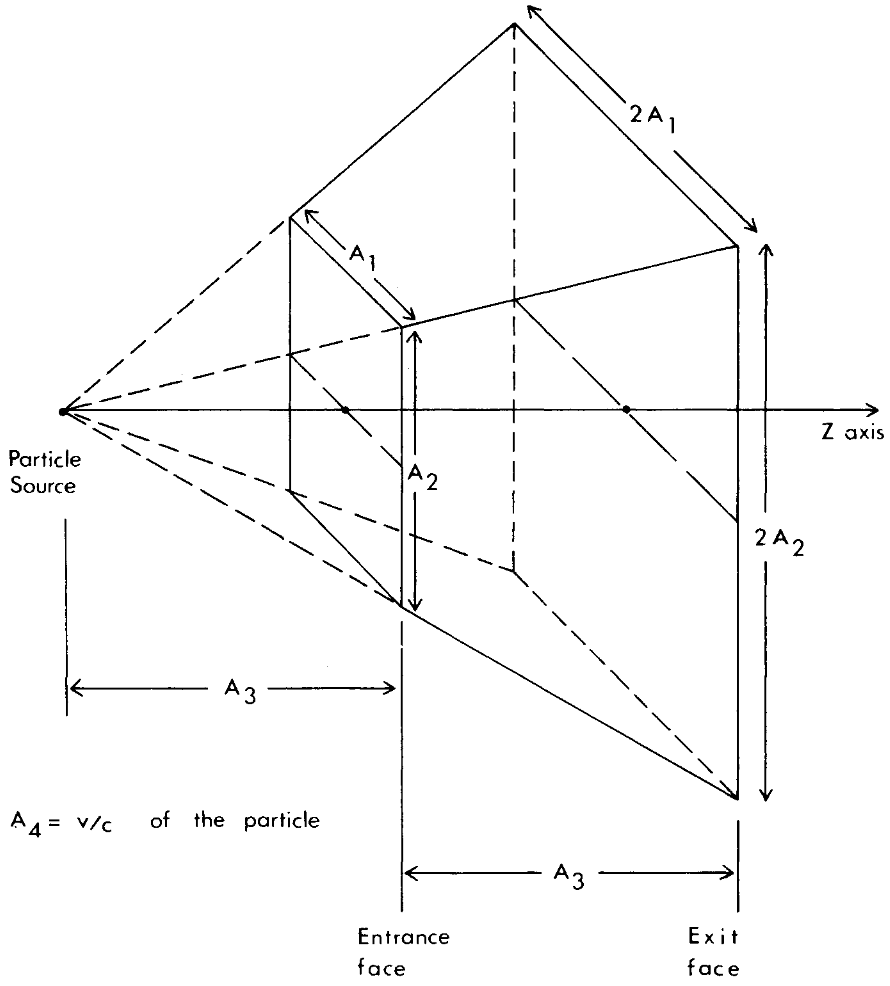


Fig. 5 The gas Čerenkov cell used for the USER routine example

Afterwards the type of gas, the geometry, and the optical system can be varied by changing data cards in the usual way.

Note that velocity needs to be defined twice when a Čerenkov counter is being used. The quantity A_4 defined above is used to give the transit time-delay of the incident particle. The values of mass and momentum which are input in the RESPONSE CERENKOV group determine the angle of the light. It is sometimes useful to be able to force these two quantities to be different.

```
 SUBROUTINE USER
 COMMON AP(20,3,6),ANG,ATYPE,AMIN(10),AS(3),AL(100),SINT(100),AMASS
1,AMOM,AST(3),ALF,ANOR(3),ALONG,ALPHA,ACON,ALVS,BVEC(3),BP(20,3,5),
2BTYPE,BETAIN,CVEC(3),COSMIN(100),CRSIN,CRCOS,CRANG,CON,DVEC(3),DTY
3PE(14),DISMAX,DEL(10),ETYPE(5),EVEC(3),FTYPE(2),FVEC(3),GTYPE(5),
3GV
4EC(3),GOOD,HTYPE(13),HITE,IT,I,J,K,L,M,MREM,NGA,NGB,NGC,NGD,NGT,NG
5U,N3,N4,N5,N6,N8,NTHROW,NHIT,NHIST,NCD(10),NCH(10),NSTEP,NCANG,NIN
6,NTRY,NREM,PI,PHI,PNEW(3),RDN,REFIN,RESID,REFR,REFIN2,RESVEC(30),S
71(3),S2(3),S3(3),S4(3),TIME,THCOS(100),THT(20),U1,U2,U3,U4,U5,U6,
7U7,RV(6),THIT(20),RADIUS,AINT,PNOW(3)
8,U8,U9,UHIT(20),VP(20,3,5),VS(3),VC(3,3),VST(3),WP(20,3,4),XP(20,3
9,4),YP(20,3,6),NT(40),EVP(20,4),EAP(20,4),Z1,Z2,Z3,Z4
9,A1,A2,A3,A4,A5,A6,A7,A8,CHUR,NTRACK,WR7
 LOGICAL WR7
```

```
C
C     HERE YOU MAY CONTINUE THE TRACKING IN A PRIVATELY DEFINED
C     EXTENSION TO THE OPTICAL SYSTEM AND/OR PLACE INFORMATION IN
C     FURTHER LOCATIONS IN RESVEC.
C
      RETURN
C
C     THE FOLLOWING ENTRY POINTS ARE TO BE USED WHEN THE PROGRAM IS
C     RUNNING UNDER THE COSMIC OPTION.
C
      ENTRY COSVEC
C
C     SELECT A PARTICULAR POINT OR LINE SEGMENT ON A PARTICULAR
C     PARTICLE TRAJECTORY.
C
      AS(1)=SINT(5)+SINT(1)*SRAN(4)
      AS(2)=SINT(6)+SINT(2)*SRAN(5)
      AS(3)=SINT(3)
      DO 4 I=1,3
    4 VS(I)=AS(I)
      AS(3)=0.
      ALVS=0.03333*VMODUL(VS)/SINT(4)
      RETURN
      ENTRY SETCOS
C
C     INPUT DATA TO DEFINE THE SET OF POSSIBLE INCIDENT PARTICLE
C     TRAJECTORIES.
C
      READ 101,(SINT(I),I=1,4)
      SINT(5)=-0.5*SINT(1)
      SINT(6)=-0.5*SINT(2)
C     SET NCANG=1 FOR A POINT SOURCE AND =2 FOR A LINE SOURCE.
      NTRACK=200
      NCANG=2
      RETURN
      ENTRY RECOS
C
C     OUTPUT A DESCRIPTION OF HOW THE SET OF POSSIBLE INCIDENT
C     PARTICLE TRAJECTORIES IS DEFINED.
C
      PRINT 102,(SINT(I),I=1,6)
  101 FORMAT(4F10.0)
  102 FORMAT(" WIDTH AND HEIGHT OF ENTRANCE,LENGTH OF COUNTER,BETA,X AND
     1Y EDGES OF ENTRANCE="/1H 6E10.3)
      RETURN
      END
```

7. GLOSSARY OF SYMBOLS IN COMMON

7.1 Blank COMMON

ACON	The attenuation constant currently in use. It is defined in the HOW definition and is redefined as the photon crosses each successive gate. It is not reset by the last circular gate nor is it changed by the LAST CIRCC final cylindrical guide. Thus the final cylindrical guide normally uses the attenuation constant of the previous section of guide, and the same constant may be used for USER extensions.
AINT	The current attenuation undergone by the "photon". It is placed in RESVEC(4) after the event is accepted by the final gate.

AL(100)	The values of the mean dE/dx in the NSTEP segments of a stopping track.
ALF	Used in HITA. It is the length of track to hit a guiding surface. It is subsequently placed in HITE if the track hits the fiducial region of the surface.
ALONG	The length of the fiducial region of the most recent cylinder or cone which has been tested.
ALPHA	The length of the last segment of track before the last circular gate. (A LAST CIRCC does not change it.)
ALVS	The transit time in nanoseconds for the incident particle along its whole track length in the counter.
AMASS	Mass of the incident particle, used for Čerenkov response.
AMIN(10)	Available for USER.
AMOM	Momentum of the incident particle, used for Čerenkov response.
ANG	= 57.293
ANOR(3)	A normal to a surface. Used in various places as scratch storage, but always in this context.
AP(20,3,6)	AP(I,J,K) is the information defining rectangular gate number I. K = 1 gives the coordinates of one corner D; K = 2 gives B, the vector length of side 1; K = 3 gives C, the vector length of side 2; K = 4 gives μ_{inner}, μ_{outer}, and the new attenuation constant; K = 5 gives the normal; K = 6 gives $(\mu_{inner}/\mu_{outer})$, cos $(\theta_{critical})$, and the new group index.
AS(3)	The position of a point source of light or the initial point of a line source.
AST(3)	The starting point or the end point of a track segment depending on context. At entry to USER AST(3) gives coordinates required for further tracking. See also VST.
ATYPE	This is the first of a pair of key words read from a data card (BTYPE is the second).
A_1 to A_6	These are input as the constants of the scintillator decay (see RESPONSE SCINTILLATOR). Before use, A_1 is reset as $A_1A_2/(A_1A_2 + A_3A_4 + A_5A_6)$, and A_2 as $(A_1A_2 + A_3A_4)/(A_1A_2 + A_3A_4 + A_5A_6)$.
A_7, A_8	Scratch variables.
BETAIN	(Velocity of the incident particle)/(velocity of light). Used for the transit time-delay.
BP(20,3,5)	BP(I,J,K) is the information defining circular gate number I. K = 1 gives the centre; K = 2 gives the unit normal; K = 3 is not defined; K = 4 gives μ_{inner}, μ_{outer}, and the new attenuation length; K = 5 gives $(\mu_{inner}/\mu_{outer})$, cos $(\theta_{critical})$, and the new group index.
BTYPE	See ATYPE.

BVEC(3)	A scratch vector.
CHOR	The square of the length of the minimum chord which defined the fiducial region for the present or most recently tested bent guide.
CON	The cosine of the half angle of the present or most recently tested cone.
COSMIN(100)	Available for USER.
CRANG	The Čerenkov angle.
CRCOS	COS(CRANG).
CRSIN	SIN(CRANG).
CVEC(3)	A scratch vector.
DEL(10)	Available for USER.
DISMAX	The maximum distance allowed for a photon track.
DTYPE(14)	Contains a list of the following key words in format A4: SOUR, RESP, GUID, GATE, NOW , RECA, RUN , RESE, FIRS, LAST, SEQU, CONS, ENDF, ENDA.
DVEC(3)	A scratch vector.
EAP(20,4)	Numbers derived from AP for a rectangular gate EAP(I,J) refers to gate I and gives the following quantities: J = 1 gives $\vec{D}.\vec{B}$; J = 2 gives $1.0/\vec{B}.\vec{B}$; J = 3 gives $\vec{D}.\vec{C}$; J = 4 gives $1.0/\vec{C}.\vec{C}$. B, C, and D are defined above under the AP description.
ETYPE(5)	Contains a list of the following key words in format A4: POIN, LINE, STOP, COSM, NEUT. NEUT is not used in this version of the program.
EVEC(3)	A scratch vector.
EVP(20,4)	Numbers derived from VP for a plane guide. EVP(I,J) for a plane guide is strictly analogous to EAP(I,J) for a rectangular gate.
FTYPE(5)	Contains a list of the following key words in format A4: SCIN, CERE, the remainder being undefined.
FVEC(3)	A scratch vector.
GOOD	A numerical flag. It remains equal to 1 unless for any reason the photon is to be rejected when control returns to the main program. In that case it is set equal to 0.
GTYPE(5)	Contains a list of the following key words in format A4: PLAN, CYLI, CONI, BENT.
GVEC(3)	A scratch vector.
HITE	See ALF.
HTYPE(13)	Contains a list of the following key words in format A4: RECT, CIRC; the remainder are undefined.
I,J,K,L,M	Indices.
IT	The current track number if several tracks are being superposed.

MREM	The index of the guide giving the lowest value of ALPHA so far in the present step of the tracking. (ALPHA is the track-segment length. The type of the guide is given by NREM.)
NCANG	A flag. Set equal to 1 for a point source. Set equal to 2 for a line source.
NCD(10)	Contains a list of the indices of the RESVEC locations which contain quantities to be entered in the scattergrams.
NCH(10)	Available for USER.
NGA	The number of plane guides.
NGB	The number of cylindrical guides.
NGC	The number of conical guides.
NGD	The number of bent guides.
NGT	The number of rectangular gates.
NGU	The number of circular gates.
NHIST	The number of histograms to be generated (two per scattergram).
NHIT	The maximum number of photons to be accepted by the final gate (plus final cylinder if it is present).
NIN	The number of photons which have so far been accepted by the final gate (and final cylinder if it is present). This number is not modified by the result of the USER routine unless it is specifically modified within USER. NIN is reset after each charged particle track. There is a cumulative number NINR, but it is not in COMMON.
NREM	Remembers the type of guide or gate giving the lowest photon track length before intersection. The index of the guide within the type set is given by MREM.
NSTEP	The number of track segment steps for a stopping track.
NT(40)	Flags controlling the program operation. Described in the HOW section.
NTHROW	The maximum number of photons which can be tracked for one charged particle track.
NTRACK	This is set equal to 1 for normal operation. It may be set differently in routine USER. NTRACK is the number of particle tracks to be generated per calculation.
NTRY	Records the number of photons which have actually been generated for the present charged particle track. It is reset at the start of each loop IT = 1, NTRACK. A cumulative number NTRYR is used in GUIDE 2 but it is not in COMMON.
N3	Source type. Takes values 1 to 5 depending on which word from ETYPE it is.
N4	Response type. 1 for scintillator, 2 for Čerenkov.
N5,N6,N7	Available for USER.

N8	Flag = 1 normally. Set = 2 during the RECAP phase to guide control through the output sections of the various data groups.
PHI	The random azimuthal angle for Čerenkov light. It is defined in the coordinate system with S2(3) as x axis, S3(3) as y axis, and S4(3) as z axis.
PI	= 180.0/RDN.
PNEW(3)	= latest intersection point of a photon with a guide or gate. When all possibilities have been tested it is transferred to AST(3).
PNOW(3)	is used as an intersection point of a photon with a guide or gate.
RADIUS	is the radius of the present or most recent cylindrical or conical guide tested.
RDN	= 1/57.293.
REFIN	= Group index of the starting medium.
REFIN2	= Group index of the present medium. Hence on entry to USER it refers to the last medium encountered before the last gate. This value is also used for the LAST circular cylinder.
REFR	Refractive index read after the CERENKOV key word.
RESID	Available for USER.
RESVEC(30)	The results vector. This is described in the HOW section of the data-card description.
RV(6)	RV(1) to RV(3) are copied from line 4 of the appropriate group of data describing a guide or gate. RV(4) to RV(6) are copied from line 6 of the group of data describing a rectangular gate or line 5 of the group describing a circular gate. Thus on entry to USER they contain the optical properties of the final gate.
SINT(100)	Available for USER. It is used in the example given in 6.7.
S1(3)	The unit vector $(1,1,1)/\sqrt{3}$.
S2(3)	The unit vector along S1 × S4.
S3(3)	The unit vector along S4 × S2.
S4(3)	The unit vector along VS. S2, S3, and S4 are used as the coordinate axes for Čerenkov light emission.
THCOS(100)	Available for USER.
THIT(20)	A vector initially filled with zeros at the start of each photon tracking. It is set equal to 1 each time the corresponding rectangular gate is hit.
THT(20)	Available for USER.
TIME	The maximum running time in seconds allowed for each track.
UHIT(20)	As THIT but referring to circular gates.
U1 to U9	Scratch stores.
VC(3,3)	Available for USER.

VP(20,3,5)	VP(I,J,K) is the information defining plane guide number I. K = 1, 2, 3 have the same significance as for a rectangular gate; K = 4 gives μ_{inner}, $\cos(\theta_{\text{critical}})$, and the reflection type or coefficient; K = 5 contains the normal to the plane.
VS(3)	The vector length of the line in a line source.
VST(3)	The present value of the photon's unit direction vector. See also AST.
WP(20,3,4)	WP(I,J,K) is the information defining data for cylindrical guide I. K = 1 refers to the initial point on its axis; K = 2 gives the vector length of the axis; K = 3 gives the $(\text{radius})^2$ and the length of the axis; K = 4 gives μ_{inner}, $\cos(\theta_{\text{critical}})$, and the reflection type/coefficient.
WR7	A logical flag. Value .TRUE. if output on TAPE 7 is required.
XP(20,3,4)	XP(I,J,K) is the information defining data for conical guide I. K = 1 gives the vertex; K = 2 gives the centre of the large open end; K = 3 gives the radius of the large open end, the length of the truncated part, and the cosine of the cone half angle; K = 4 gives μ_{inner}, $\cos(\theta_{\text{critical}})$ and the reflection type/coefficient.
YP(20,3,6)	YP(I,J,K) contains information on bent guide I. K = 1 gives a point on the first edge; K = 2 gives the centre of curvature; K = 3 gives the $(\text{radius of curvature})^2$, the $(\text{length of the minimum chord})^2$ with J = 3 undefined; K = 4 gives μ_{inner}, $\cos(\theta_{\text{critical}})$ and the reflection type/coefficient; K = 5 gives a point on the second edge; K = 6 gives the unit cylinder axis.
Z1 to Z4	Scratch stores. The also carry EVP(I,1) to EVP(I,4) from routine TRACK to routine HIT.

7.2 COMMON/FAST/

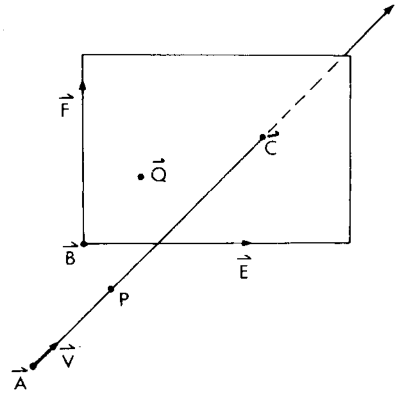
NGRC = 1 if an initial rectangular cylinder is present.
NGCC = 1 if an initial circular cylinder is present.
NGLA = 1 if a final circular cylinder is present.

7.3 COMMON/CONSEC/

NGE	The number of conic sections.
NGENOW	The index of the current or most recent conic section tested.
HITF	The length of the photon trajectory from the present or most recent step in a conic section.
EP(10,3,5)	EP(I,J,K) contains information on conic section I. K = 1 defines the type of conic section; J = 1 contains the eccentricity ε; J = 2 contains +1 if the light is incident on the concave surface and -1 if the light is incident on the convex surface. K = 2 gives the coordinates of the centre C. K = 3 gives the coordinates of the focus F. K = 4 contains the reflection properties as in YP above. K = 5 contains the utility vector $(\vec{F} - \vec{C})\varepsilon/\lvert\vec{F} - \vec{C}\rvert$.

8. VECTOR ALGORITHMS USED

8.1 Intersection of a line with a rectangle



C is the intersection point.

The vector equation of the line is

$$\vec{P} = \vec{A} + \alpha\vec{V} \ ,$$

where

$\vec{A}$ is the starting point of the line,
$\vec{V}$ is a unit vector, and
α is the length of the line.

The vector equation for a point Q in the plane is

$$(\vec{Q} - \vec{B}) \cdot \hat{n} = 0 \ ,$$

where $\hat{n} = \vec{E} \times \vec{F}/|\vec{E} \times \vec{F}|$ is the unit normal to the plane.

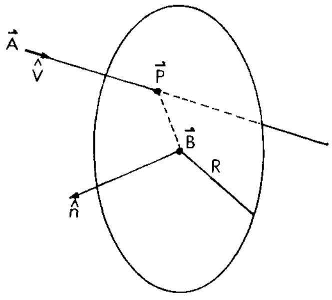
Hence $\vec{C}$ is given by:

$$\vec{C} = \vec{A} + \frac{(\vec{B} - \vec{A})\cdot\hat{n}}{\hat{V}\cdot\hat{n}} \hat{V} \ .$$

If C is inside the rectangle, then

$$0 < (\vec{C} - \vec{B})\cdot\frac{\vec{E}}{|E|^2} < 1 \quad \text{and} \quad 0 < (\vec{C} - \vec{B})\cdot\frac{\vec{F}}{|F|^2} < 1 \ .$$

8.2 Intersection of a line with a circle



The equation of the line is $\vec{P} = \vec{A} + \alpha\hat{V}$.

The equation of a point on the plane is

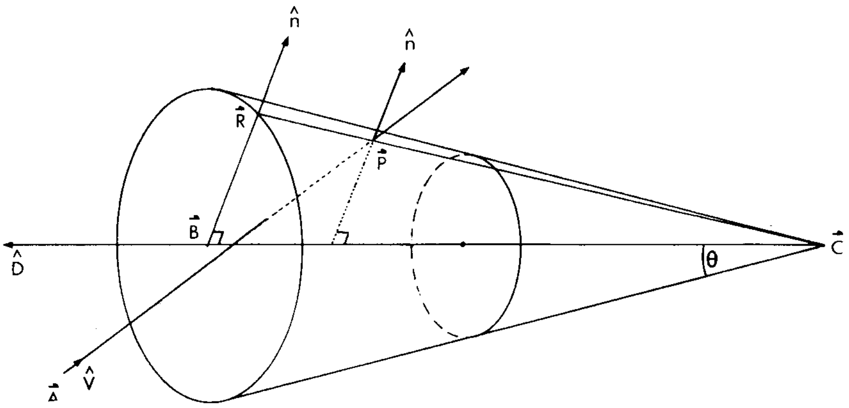
$$(\vec{P} - \vec{B}) \times \hat{n} = 0 \ .$$

The solution is

$$\vec{P} = \vec{A} + \frac{(\vec{B} - \vec{A})\cdot\hat{n}}{\hat{V}\cdot\hat{n}} \hat{V} \ .$$

If P is inside the circle, $|\vec{P} - \vec{B}| < R$.

8.3 Intersection of a line with a truncated cone



The equation of the line is $\vec{P} = \vec{A} + \alpha\hat{V}$.

The equation of the cone is

$$\frac{(\vec{B} - \vec{C})\cdot(\vec{P} - \vec{C})}{|\vec{B} - \vec{C}|\,|\vec{P} - \vec{C}|} = \cos\,\theta\ .$$

Let

$$\frac{\vec{B} - \vec{C}}{|\vec{B} - \vec{C}|} = \hat{D}$$

$$\vec{A} - \vec{C} = \vec{E}\ .$$

Then, the intersection point is given by the solution of

$$\hat{D}\cdot(\vec{E} + \alpha\hat{V}) = \cos\,\theta|\vec{E} + \alpha\hat{V}|\ .$$

Let

$$\hat{D}\cdot\vec{E} = V_1 \qquad \vec{E}\cdot\hat{V} = V_3$$

$$\hat{D}\cdot\hat{V} = V_2$$

$$\cos^2\,\theta = V_4$$

Squaring and collecting terms:

$$\alpha^2(V_2^2 - V_4) + 2\alpha(V_1V_2 - V_3V_4) + V_1^2 - V_4|\vec{E}|^2 = 0$$

$$\alpha = \frac{-(V_1V_2 - V_3V_4) \pm \sqrt{(V_1V_2 - V_3V_4)^2 - (V_2^2 - V_4)(V_1^2 - V_4|E|^2)}}{V_2^2 - V_4}\ .$$

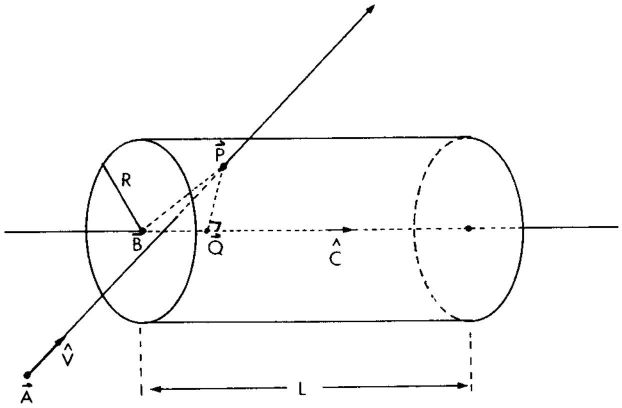
The relevant solution is the one with the smallest positive value of α, and the intersection point must be within the truncated section. To obtain the normal through $\vec{P}$, note that it is parallel to the normal $\vec{BR}$. $\vec{P} - \vec{C}/|\vec{P} - \vec{C}|$ is the unit vector along CR, so point $\vec{R}$ is given by

$$\vec{R} = \vec{C} + \left\{(\vec{B} - \vec{C})\cdot\frac{(\vec{P} - \vec{C})}{|\vec{P} - \vec{C}|}\right\}\frac{(\vec{P} - \vec{C})}{|\vec{P} - \vec{C}|}\ .$$

Then forming $\vec{R} - \vec{B}$ gives a vector parallel to the normal:

$$\hat{n} = \frac{\vec{R} - \vec{B}}{|\vec{R} - \vec{B}|} .$$

8.4 Intersection of a line with a cylinder



The equation of the line is $\vec{P} = \vec{A} + \alpha\hat{V}$.

The equation of the cylinder is $|(\vec{P} - \vec{B}) \times \vec{C}| = R$ solving for α,

$$|(\vec{A} - \vec{B}) \times \hat{C} + \alpha\hat{V} \times \hat{C}| = R .$$

Let

$$V_6 = |(\vec{A} - \vec{B}) \times \hat{C}|^2$$

$$V_5 = \{(\vec{A} - \vec{B}) \times \hat{C}\}\cdot\{\hat{V} \times \hat{C}\}$$

$$V_7 = |\hat{V} \times \hat{C}|^2 ,$$

so

$$V_7\alpha^2 + 2V_5\alpha + V_6 - R^2 = 0$$

$$\alpha = \frac{-V_5 \pm \sqrt{V_5^2 - V_7(V_6 - R^2)}}{V_7} .$$

If $\vec{P}$ is within the defined length of the cylinder, then

$$0 < (\vec{P} - \vec{B})\cdot\vec{C} < L .$$

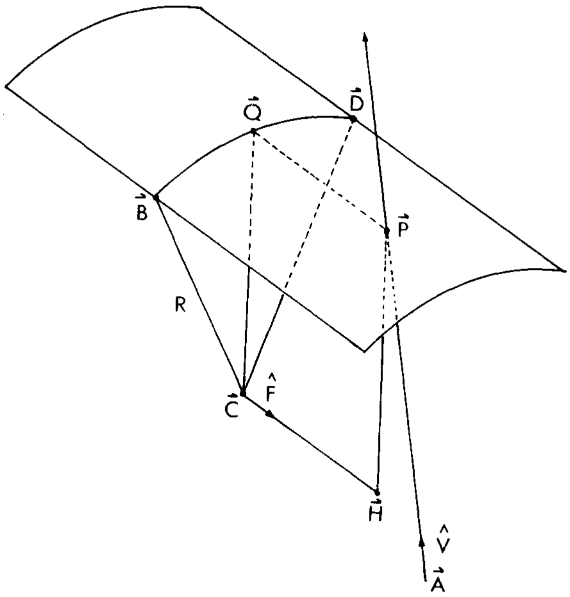
The normal is along

$$\vec{QP} = (\vec{BP} - \vec{BQ})$$

$$= (\vec{P} - \vec{B}) - \{(\vec{P} - \vec{B})\cdot\vec{C}\}\ \vec{C} ,$$

$$\hat{n} = \frac{\vec{QP}}{|\vec{QP}|} .$$

8.5 Intersection of a line with a bent guide



The equation of the line is

$$\vec{P} = \vec{A} + \alpha\hat{V} \ .$$

The equation of the cylinder is

$$|(\vec{P} - \vec{C}) \times \hat{F}| = R \ ,$$

where $\hat{F}$ is the unit vector parallel to $(\vec{D} - \vec{C}) \times (\vec{B} - \vec{C})$ and so is along the axis of the cylinder. Therefore the solution for P is

$$|(\vec{A} - \vec{C}) \times \hat{F} + \alpha\hat{V} \times \hat{F}|^2 = R^2 \ .$$

Using

$$\vec{G} = (\vec{A} - \vec{C}) \times \vec{F} \quad \text{and} \quad \vec{E} = \vec{V} \times \vec{F} \ ,$$

$$|\vec{G} + \alpha\vec{E}|^2 = R^2 \ ,$$

$$\therefore \quad \alpha^2 \, |\vec{E}|^2 + 2\alpha \, \vec{G}\cdot\vec{E} + |\vec{G}|^2 - R^2 = 0 \ .$$

Let

$$V_1 = |\vec{E}|^2 \ , \quad V_2 = \vec{G}\cdot\vec{E} \ , \quad V_3 = |\vec{G}|^2 - R^2 \ ,$$

$$\therefore \quad \alpha = \frac{-V_2 \pm \sqrt{V_2^2 - V_1 V_3}}{V_1} \ .$$

If P is in the surface defined by $\vec{B}$, $\vec{D}$, and $\vec{C}$, then $\vec{Q}$ must lie between $\vec{B}$ and $\vec{D}$ if $\vec{Q}$ is the projection of $\vec{P}$ on plane BCD:

$$\vec{Q} = \vec{P} - \{(P - C)\cdot\hat{F}\} \ \hat{F} \ .$$

The problem is now two-dimensional, and provided that the angle BCD is less than 180°, then $\vec{P}$ is in the defined area provided that

$$|\vec{Q} - \vec{B}|^2 < |\vec{B} - \vec{D}|^2$$

and

$$|\vec{Q} - \vec{D}|^2 < |\vec{B} - \vec{D}|^2 \ .$$

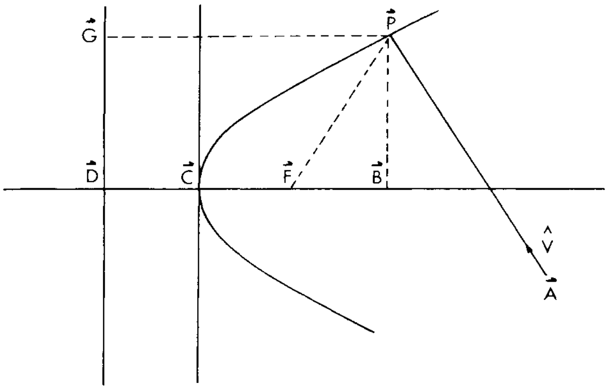
The normal at $\vec{P}$ is parallel to $\vec{P} - \vec{H}$

$$\vec{P} - \vec{H} = (\vec{P} - \vec{C}) - (\vec{H} - \vec{C})$$

$$= (\vec{P} - \vec{C}) - \{(\vec{P} - \vec{C})\cdot\hat{F}\}\ \hat{F} \ ,$$

$$\hat{n} = \frac{\vec{P} - \vec{H}}{|\vec{P} - \vec{H}|} \ .$$

8.6 Intersection of a line with a conic section of rotation



The equation of the line is $\vec{P} = \vec{A} + \alpha\hat{V}$.

The equation of the conic section is $|\vec{P} - \vec{F}| = \varepsilon|\vec{P} - \vec{G}|$. $\vec{D}$ is given by $\vec{D} = \vec{C} - (\vec{F} - \vec{C})/\varepsilon$, and α is given by the solution of

$$|\vec{A} - \vec{F} + \alpha\hat{V}|^2 = |\varepsilon(\vec{P} - \vec{D})\cdot(\vec{F} - \vec{C})/(F - C)|^2 \ .$$

Let

$$R_4 = \hat{V}\cdot(\vec{F} - \vec{C})\varepsilon/|\vec{F} - \vec{C}|$$

$$R_5 = \hat{V}\cdot(\vec{A} - \vec{F})$$

$$R_6 = \vec{A}\cdot(\vec{F} - \vec{C})\varepsilon/|F - C|$$

$$R_7 = |\vec{A} - \vec{F}|^2$$

$$R_8 = \vec{D}\cdot(\vec{F} - \vec{C})\varepsilon/|F - C|$$

$$R_1 = 1 - R_4^2$$

$$R_2 = R_4(R_6 - R_8) - R_5$$

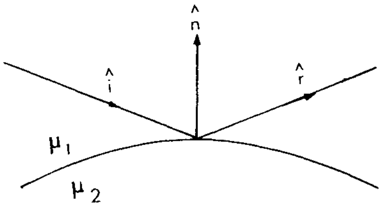
$$R_3 = R_7 - (R_6 - R_8)^2 \ .$$

Then

$$\alpha = \frac{R_2 \pm \sqrt{R_2 - R_1R_3}}{R_1} \ ,$$

where the sign which gives a positive value of α is chosen.

8.7 Direction of the reflected ray



$\hat{n}$ is the unit normal vector. $\hat{\imath}$ and $\hat{r}$ are unit vectors along the incident and reflected rays.

Law of reflection: Component parallel to $\hat{n}$ changes sign.
Component perpendicular to $\hat{n}$ is unchanged.

$$\therefore \quad \hat{r} = \hat{\imath} - 2(\hat{\imath}\cdot\hat{n})\hat{n}.$$

Note: $\hat{r}$ is independent of the sign of $\hat{n}$.

8.8 Condition for internal reflection

$$|\hat{\imath}\cdot\hat{n}| < \sqrt{1 - (\mu_2/\mu_1)^2}\ .$$

8.9 Direction of the refracted ray

$\hat{n}$ is the unit normal,
$\hat{\imath}$ and $\hat{r}$ are the unit incident and refracted vectors.
$\hat{t}$ is the unit tangent vector in the plane of incidence:

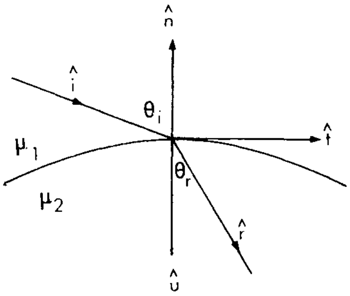
$$\hat{t} = \{\hat{\imath} - (\hat{\imath}\cdot\hat{n})\hat{n}\}/|\hat{\imath} - (\hat{\imath}\cdot\hat{n})\hat{n}|$$

$\hat{u}$ is the unit normal with which $\hat{r}$ subtends an acute angle:

$$\hat{u} = \hat{n}[\mathrm{sign}(\hat{\imath}\cdot\hat{n})]$$

$$\sin\theta_r = \mu_1/\mu_2 \sin\theta_i\ .$$

The refracted ray is $\hat{t}\sin\theta_r + \hat{u}\cos\theta_r$.



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1) B. Braunschweig, E. Königs, W. Sturn and W. Wallraff, Nuclear Instrum. Methods 134, 261 (1976).