Pedagogical Research
2020, 5(3), em0065
e-ISSN: 2468-4929
https://www.pedagogicalresearch.com

Research Article

MODESTUM
OPEN ACCESS

Guidelines on the Use of Teaching Materials for Teaching Fractions in the Brazilian Primary Course (1930-1970)

Barbara Winiarski Diesel Novaes [1]*, Danilene Gullich Donin Berticelli [2]**, Neuza Bertoni Pinto [3,4,5]***

[1] Federal University of Technology, UTFPR, Paraná, BRAZIL
[2] Federal University of Paraná, BRAZIL
[3] Collaborating Researcher in the Science and Mathematics Education Postgraduate Program, PPGECEM, BRAZIL
[4] Collaborating Researcher in the Amazonian Network of Science Education, REAMEC, BRAZIL
[5] Researcher and Vice-Coordinator in the Associated Group for Studies and Research on History of Mathematic Education, GHEMAT, BRAZIL
***Corresponding Author:** barbaradiesel@gmail.com
****Corresponding Author:** danilene@ufpr.br
*****Corresponding Author:** neuzabertonip@gmail.com

Citation: Novaes, B. W. D., Berticelli, D. G. D., & Pinto, N. B. (2020). Guidelines on the Use of Teaching Materials for Teaching Fractions in the Brazilian Primary Course (1930-1970). *Pedagogical Research, 5*(3), em0065. https://doi.org/10.29333/pr/8221

ARTICLE INFO

Received: 1 Mar. 2020
Accepted: 19 Apr. 2020

ABSTRACT

The present study, conducted from the perspective of cultural history (Certeau, 1982; Chartier, 1990; Chervel, 1990; Julia 2001)), aims to investigate guidelines for the use of teaching materials to teach fractions in the primary school, present in pedagogical manuals of the period from 1930 to 1970. The research is based in references such as Hofstetter and Schneuwly (2017) - professional knowledge; Valente (2016, 2019) - history of mathematical education; Carrillo Gallego and Sanchéz Jiménez (2012) - teaching materials. The analysis of these references revealed that the authors of the selected manuals, Campos (1933), Albuquerque (1958), and Porto (1967, 1968), excelled in working with professional knowledge to teach fractions, when using didactic materials before reaching abstraction. When providing guidelines for the use of teaching materials for the teaching of fractions, in tune with the principles of the New Renovated School, the examined manuals make use of training and teaching knowledge linked to children interests and understanding. Thus, the research concluded that in this sense, the analyses show that the transformation of knowledge to teach fractions occurs when the use of manipulative didactic materials decreases and the use of graphs, diagrams, equivalence tables arises from Scientific Pedagogy, in the field of Psychology, in the field of Psychology Experimental, as indicated by the pedagogical manuals of Irene de Albuquerque and Rizza de Araujo Porto.

Keywords: history of mathematics education, primary school, fractions, pedagogical manuals, teaching materials

INTRODUCTION

Recent researches on degree courses in Mathematics, mainly in the subjects of mandatory curricular internship, shows that both university students and students of basic education have a very weak understanding of the concept of fractions (Powell, 2018). A first justification may be that fractional numbers are little used in the student's context, as they are usually treated by means of decimal numbers. The reason for the student's aversion to the content of fractions may stem from his impoverished understanding of the calculation of fractions, the concepts of decimal and percentage, the use of fractions in measures and the concepts of ratio and proportion, due to the limited exposure form students to content (Van der Walle, 2009).

According to Powell (2018, p. 78) "in Basic Education, one of the most important topics is the knowledge of fractional numbers" and it is of fundamental importance, as it is the basis of algebraic thinking, of Algebra itself and of later mathematical disciplines. However, for Bertoni (2004), fractions have been pointed out as one of the most problematic contents in teaching and learning, and also, in Brazilian national assessments, they have shown a low performance index. What alternatives would the teacher have to enhance the understanding of this content in Elementary School?

With care to preserve historical distance, we can say that the use of didactic materials to teach mathematics runs through the pedagogical vacancies from the intuitive, through the New School and Modern Mathematics. A current criticism of the use of materials in the teaching of fractions, teachers seem to find in this didactic resource "the magic formula - for the problems they face in the classroom's daily life" (Fiorentini & Miorim, 1990, p. 5). And they add by statin that: "[teachers] are not always clear about the fundamental reasons why materials or games are important for the teaching-learning of mathematics and, usually, do not question whether they are necessary, and when they should be used" (Fiorentini & Miorim, 1990, p. 5).

Table 1. Pedagogical manuals mobilized for the study

Title	Author (s)	Year of publication	Publisher - / Edition
Das frações dobrando e rasgando papel	José Ferraz de Campos	1933	São Paulo: Typ. Siqueira - Salles Oliveira, Rocha & Cia. 1st edition. 70 pages.
Metodologia da Matemática	Irene de Albuquerque	1958	Rio de Janeira: Editora Conquista. 3rd edition. 206 pages.
Ver, sentir, descobrir a Aritmética	Rizza Araújo Porto com colaboração de Evelyn L. Bull	1968	Rio de Janeiro: Editora Nacional de Direito. 10th edition. 166 pages.
Frações na escola elementar	Rizza Araújo Porto	1967	Minas Gerais: Editora do Professor. 4th edition. 302 pages.

These concerns about the teaching of fractions, specially the lack of understanding and clarity in the use of materials, motivated us to search in pedagogical manuals, the way of which and how they were used to teach this mathematical content in the Brazilian primary school in the period from 1930 to 1970. To this end, the authors of this article have been looking into the historical[1] constitution of knowledge involved in teacher education under new theoretical contributions, taking into account the objectified knowledge[2]. Such analyzes have been systematized by the Research Team in the History of Education Sciences (ERHISE) at the University of Geneva, Switzerland na led by Professor Rita Hofstetter[3].

Appropriating this theoretical basis, the researchers of the Research Group on the History of Mathematical Education in Brazil (Ghemat Brasil), of which the authors are part, focus their research on professional knowledge related to the teaching of fractions, in historical dimension. In understanding professional knowledge, two concepts are addressed: *mathematics to teach and mathematics for teaching*. “The first most directly linked to the disciplinary field, to mathematics; the second, linked to the teaching profession” (Bertini et al., 2017, p. 9).

As Valente (2016) explains:

> Researchers have recently turned their attention to the analysis of mathematics in primary school from a historical perspective. This theme, until recently, does not seem to make much sense. After all, “two and two are always for” and this is the firmly established representation or mathematics present in the early school years. Any other type of contentious discussion, or better said, involving mathematical knowledge in primary school, proves to be ill-posed. “Two and two are always for”. Therefore, the result is that researchers with mathematical training are not willing to analyze the mathematics of the primary course. (Valente, 2016, p.7) (our translation)

Furthermore, the historical study of issues related to mathematical education aims to draw on the past to understand both educational movements, and to build elements to think about the current ways that bypass the educational process. When observing the present, we cannot forget the past, because in it we can find answers to rethink contemporary practices, in order to evaluate them in their historical context.

The present study, conducted in the perspective of cultural history (Chartier, 1990), aims to investigate guidelines on the use of teaching materials to teach fractions in the primary course, present in pedagogical manuals, from 1930 to 1970. With priority sources, pedagogical manuals by authors who have carried out studies on the subject, material that is intended for teacher training, have made important contributions to the teaching profession, in particular, to teachers who teach or will teach mathematics in the early school years.

According to Silva (2005), pedagogical manuals are printed differently from textbooks because they are made especially for teachers and future teachers. The product of a school culture[4], the manuals provide a set of specialized knowledge, resulting form research theoretically based on knowledge recognized by the scientific community of the network of researchers in the educational field.

The manuals (**Table 1**) were selected due to the discussion presented considering the traces of recommendations issued by the authors for the use of materials in the teaching of fractions in primary school. The study focuses on the teaching vacancy at New School with two experts - Rizza de Araújo Porto and Irene de Albuquerque - whose manuals were widely circulates among primary teachers and in normal schools. To make a counterpoint, we also analysed a 1933 work, written by José Ferraz de Campos that marks the transition from the intuitive pedagogical vacant to Escola Nova. Rizza de Araújo Porto’s books also mark the transition from New School to Modern Mathematics. The research group has adopted longer periods for historical studies to understand permanences and transformations.

[1] Michel de Certeau (1982) understands the writing of history from a historiographical operation conceived as: “a social place, of ‘scientific’ practices and of writing” (p. 66). As a result of this place, “methods are established, a topography of interests is outlined, and documents and research questions, which will be proposed, are organized” (p. 67). The study we carried out, guided by the historical-cultural perspective, has as its social place the field of mathematical education and considers as scientific practices, the set of rules that allows controlling operations aimed at producing a history of school subjects (Chervel, 1990).

[2] Objectified knowledge is knowledge that is institutionalized over time, in terms of explicit knowledge, formalized, transmitted and included intentionally in teacher education (Bertini et al., 2017, p. 7).

[3] Bertini et al. (2017) appropriate the concepts of knowledge to be taught and to teach initially proposed by this research group. For more information about the research group, access: https://cms.unige.ch/fapse/SSE/erhise

[4] About school culture Julia (2001, p. 10) considers it as “a set of norms and practices that define knowledge to teach and conducts to inculcate, and a set of practices that allow the transmission of this knowledge and the incorporation of these behaviors; norms and practices coordinated with purposes that may vary according to the times (religious, socio-political or simply socialization purposes)”.

With this work we will try to answer the following guiding question: What professional knowledge is necessary to teach fractions with the use of didactic materials?

KNOWLEDGE *TO* TEACH AND KNOWLEDGE *FOR TEACHING*

Recents studies carried out by Gatti (2014), show that in the training of teachers the spaces related to the articulation between theory and practice in terms of fundamentals and content should be expanded. In this way, training courses, especially those aimed at teachers in the first school years, should be guided by consisting "precisely in what the call 'intentional learning', that is, learning that has as its first objective to transform one's own subject" (Hofstetter & Schneuwly, 2017, p. 117) and susceptible in that way of becoming knowledge.

The knowledge *to teach*, according to Hofstetter and Schneuwly (2017), refers to the knowledge, objects of their work, produced by university disciplines, in different scientific fields, considered important for teacher training. The knowledge *for teaching* has the specificity of teaching, they are linked to that knowledge proper to the exercise of the profession, they are knowledge that are tools of their work.

More specifically, the knowledge *for teaching*, in the case of this study, knowledge related to the teaching of fractions made available in pedagogical manuals aimed at training normalists, refer to methods, procedures and devices considered appropriate for the professional activity of teaching in the early years.

In this context, it is worth alerting to conceptions that consider the knowledge of experience (knowledge of action), dominant in official discourses, in which training is articulated with everyday knowledge. In this way, "the individual thus becomes responsible for his training, for better or for worse" (Hofstetter & Schneuwly, 2017, p. 137).

According to Lussi Borer (2017), in education, knowledge of general culture and professional culture is contemplated, unlike teacher training for secondary education, in which knowledge of general culture prevails. This observation implies that the knowledge for teaching fractions in the first school years, takes into account the principles of a primary school culture.

> The relationship between the different forms of knowledge present in the formation takes place differently depending on the categories of teacher-trainers: it tends to be oriented form a predominance of knowledge as teaching tools to a predominance or knowledge as teaching objects, as we progress from the lowest to the highest degrees of the system (Hofstetter & Schneuwly, 2017, p. 141) (our translation)

In this way, the researchers at Ghemat Brazil, based on the studies of the Swiss Hofstetter group, place the formalized knowledge, the objectified knowledge at the center of the current reflections on the training of teachers who teach mathematics in the early years of schooling, considering the necessary formalization of knowledge both in education training. According to the authors Hofstetter and Schneuwly (2017), both the knowledge *to teach* and the knowledge *for teaching* must be worked on in the different teacher training courses.

In this way, the analysis of didactic materials for teaching fractions present in the pedagogical manuals aimed at the intentional training of teachers who teach mathematics (professional knowledge) in primary education are in line with the theoretical-methodological framework presented and are guided by a hypothesis of background "pedagogy in the different teaching vacancies, does not constitute a lubricant for teaching an invariant knowledge: mathematics. Different pedagogies build different elementaries and, consequently, restructure school knowledge" (Mendes & Valente, 2017, p. 9).

ABOUT TEACHING FRACTIONS

Much more than just a part-whole comparison, the word fraction has several meanings interconnected by different sub-concepts: part-whole relationship, quotient, reason and operator. A whole is continuous when it is the result of a measurement, such as a pizza, whereas the whole said to be discreet is the result of a count, like a package with several products inside (Ciryno et al., 2014).

The meaning of the term quotient is the division of an integer into a number of groups. We also have that the reason subconstruct represents a comparative index between two units (a:b). As for example, João has ten bullets and Maria twenty, the ratio of the number of bullets of João to the numbers of bullets of Maria is ___, or we can still represent him by 1:2. The operator, on the other hand, brings na idea of function, as it is something that acts on a given situation and modifies it, being able to reduce or enlarge it, being present in the idea of "what number should I multiply by five to obtains two" (Brasil, 1998, p. 103).

The understanding of these sub-constructs can make the student also develop proportional reasoning, as it will raise greater meanings for the context of fractions, thus triggering a possible understanding of other areas of mathematics and other sciences (Cyrino et al., 2014).

Lopes (2008) states that the teaching of mathematics should be more focused on meaning than on representation, since meaningful mathematics is one that makes sense to the student. He also proposes that students should have contact with situations that "make problematization possible" in contexts of comparison, because fractions also represent rate of change, measure, probability, but not all of these issues have been addressed in textbooks.

According to the National Curriculum Parameters (Brasil, 1998) of elementary school, final years, we cannot treat each of these interpretations separately for fraction, because the realization of these meanings by students is the result of systematic work, fundamental for understanding, for example, decimal numbers. We also have, according to Van der Walle (2009) that:

> Mathematical ideas are "important" if they are used for the development of other ideas, if they link to each other or serve to illustrate the discipline of mathematics as a human endeavor (Van der Wale, 2009, p. 21) (our translation).

Thus, fractions should not be taken as a theme, but as an integral part in understanding other meanings (Lopes, 2008). As Van der Walle (2009, p. 362) states, it is extremely important to establish this order, to present fractions first and then the other contents, but treating them so in isolation makes it difficult to connect the concepts. In this sense, we ask ourselves: The contents and the way fractions are worked in the early years contribute to the understanding of other meanings in later studies?

Many problems presented on the theme of fractions are classified by Lopes (2014, p. 3) as a "pseudo-didactic aberration", because for him, there is a certain urgency of teachers to make students relate the theme fractions with the content of decimal numbers, and end up using non-intuitive ideas for them to reach that. An example would be contextualized activities such as "João ate 3/17 of a cake, his brother ate 5/9 of what was left… how much was left for your sister?" (Lopes, 2008, p. 4). In other words, such situations do not promote a reflective moment, they only bring complicated operations, which are difficult for the students to understand. In this case, your only strategy would be passive, that is, repeating previous exercises.

As Lopes (2008, p. 11) suggests, curriculum should provide older students with the most diversified experiences with the topic, from elementary to high school, bringing much more than problems with complicated fractions, activities exploring mainly the visualization of figures.

Lopes concludes that

> The teaching of fractions has been practiced as if our students lived at the end of the 19th century, a teaching marked by mechanism, by the exaggeration in the prescription of rules and tricks, useless applications, obsolete concepts, "wagons", calculation by calculation (Lopes, 2008, p. 20) (our translation).

According to Van der Walle (2009) there is significant evidence to suggest that the use of models[5] for teaching fractions is important and that it is sometimes useful to do the same activity with two very different models. The author distinguishes three types of models: area or region; length and sets.

The models of area or region would be the pieces of circular pie, rectangular regions, quarters in the geoplane, drawing on meshes or dotted paper, pattern blocks and folds of paper.

The length models would be the fraction strips and Cuisenaire bars. Bars were named after their inventor - Georges Cuisenaire (1891-1976), a professor in the early grades in Belgium - publishes a book on their use in 1952 called *The Numbers in Color*. The use of bars for teaching mathematics was developed and popularized by Caleb Gattegno in many countries around the world (Van der Walle, 2009; Powell, 2018; Roberts, 2012).

In the next sections, we present the didactic guidelines for teaching fractions with the use of didactic materials for the primary in educational manuals and magazines from 1930 to 1970.

USE OF MATERIALS *FOR TEACHING* FRACTIONS IN EXPERTS CONCEPTIONS

The New School movement, which took place in Brazil in the first half of the twentieth century, widely discussed by historians of Brazilian education (Monarca, 2009; Souza, 1998, 2009; Valdemarin, 2010; Vidal, 2000) proposed a school organization that world allow students to "learn by doing" through different methodologies that emphasized the child's activity and projects according to the interest of the students. To do so, it was necessary to place teaching materials within their reach that would allow this form of knowledge construction (Carrillo Gallego & Sánchez Jiménez, 2012).

In the demarcated period, normal schools, inspired by the principles of *New School*, were making changes in curriculums and programs, in particular in the methods and processes for teaching mathematics in primary school.

An example of these changes was investigated by Silva (2017), when analyzed a curricular organization of the Normal Schools of São Paulo. Addressing institutionalization process of a professional arithmetic to form the primary teacher, between the 1920s and 1960s, or to study changes implemented in the curriculum of the training of normalists, among others, an introduction, in 1950, of the discipline of Mathematics and Notions of Statistics, discipline which knowledge offered came to expand a professional culture of future teachers.

[5] The author designates what we call teaching materials as models.

The production of new knowledge in the pedagogical field of the Brazilian primary school took place, to a large extend, through education *experts*[6] and the institutionalization of professional *expertise*[7].

This institutionalization is taking shape based on the professional trajectory of each of the authors of the pedagogical manuals[8] selected for the study: José Ferraz de Campos, Irene de Albuquerque and Rizza Araújo Porto.

OF FRACTIONS FOLDING AND TEARING PAPER - JOSÉ FERRAZ DE CAMPOS, 1933

We elucidated the use of didactic materials for the teaching of fractions through the pedagogical manual written by Jose Ferraz de Campos[9], "*Das frações dobrando e rasgando papel*[10] in 1933. On the back cover, the professor-author writes that "all the rules [will be] objectively induced by the student". In the seventy pages of the book, there is guidance to work with the trips of paper to teach about the integer, measures, quantities, operations with fractions (addition, subtraction, multiplication and division), problems solving, equivalent, fractions, comparison of fractions, among others.

The author of the book proposes reflections and guidelines on the teaching of fractions in the preliminary course[11] using strips of colored paper. An accessible teaching material with a view to overcoming the decorated teaching of fractions rules.

The author demonstrates his dissatisfaction with the teaching of fractions, stating that "it was always and continues to be deplorably poorly administered in the preliminary course" (Campos, 1933, p. 5). And he continues to use the Latin expression "*Horresco re*ferens" that means "I tremble when referring" or "I am terrified of saying such a thing". The author denounces the exacerbated use of Arithmetic rules from secondary education[12], such as multiplication and division operations.

> Feeling[13] incapable of a well-done objective teaching, of being able to explain and transmit reasoned and conscious knowledge of cases of multiplication and divisions of broken to the classes of 3rd and 4th years of the preliminary course, teachers try to induce the rules, in those cases, due to the theory found in secondary school arithmetic, which is absolutely inadequate, abstract and beyond the reach of the still incipient mentality of its students. The disaster is certain, bringing, in addition it puts a balance, in addition to the useless torture to which those innocent heads are subjected, the children's annoyance by the logical and sequential reasoning; disinterest, inattention, and, worse than anything, the idea, the certainty (!) that in view of the accumulated failures, each students is forming, that "there is no way", "there is no fall", "there is no way for Mathematics" (Campos, 1933, p. 5) (our translation).

In the period in vogue, the training of primary school teachers in Brazil still retains traces of secondary culture, as occurred in Switzerland, according to Lussi Borer (2017). One hypothesis would be that teaching the operations of multiplication and division of fractions by means of two abstract rules could be impregnated with this secondary school culture, which gave the primary teacher a certain insecurity.

> There are those who, in order to simplify, to facilitate teaching, reduce the whole theory of multiplication and division of the broken to two single rules. How? Quite simply: having each non-fractional term given the denominator 1. But, why? What is the reason for this procedure? They don't explain. It's a secret. Naturally because in this way, the respective rules decorated, the multiplication and division of fractions chapter will be dead, paid off, known… And the teacher will be able, having the certainty of the duty accomplished, quietly and consciously to pass to another part of the program… (Campos, 1933, p. 5-6) (our translation).

---

[6] "They are 'experts' because they know the teaching profession perfectly and stand out in it" (Hofstetter et al., 2017, p. 67). In times of New School, countless experts stook out, recognizes educators who spread new ideas and proposing the use of new teaching materials, contributes to the modernization of the teaching of fractions, in overcoming the abstractionism and formality of traditional teaching.

[7] The concept of *expertise* is understood as an instance, in principle recognized as legitimate, attributed too one or more specialists recognized for their knowledge, attitudes and experiences in order to examine a situation, to evaluate a phenomenon, to verify facts that are mobilized education authorities in view of the need to take a decision (Hofstetter et al., 2017, p.57).

[8] The analysed manuals and the materials for teaching fractions addressed in them, here understood as cultural objects, constitute raw material in the history of mathematics education, a specific features of the history of Brazilian education that preserving school memory characterizes a cultural history of school practices, in this case the history of school mathematics.

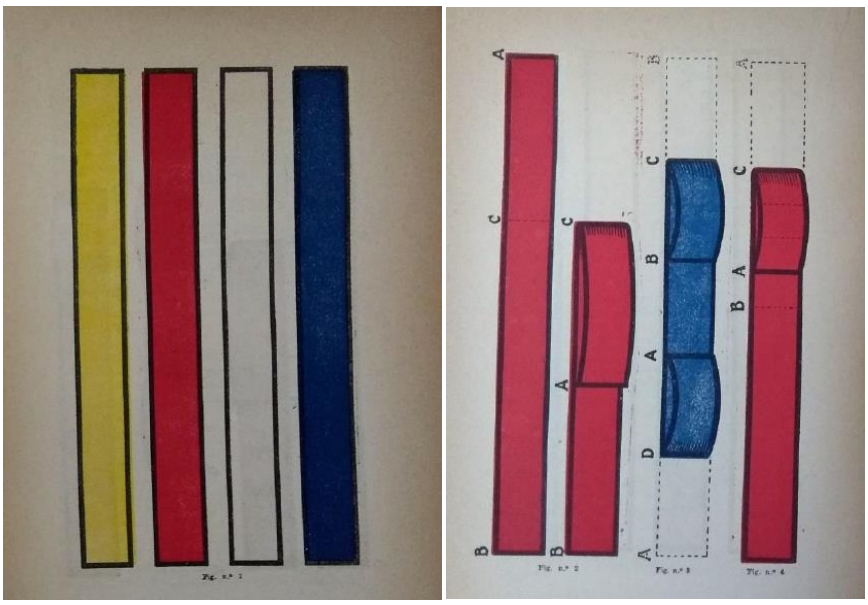
[9] Born in São Paulo on January 17, 1881. He graduated at the age of sixteen an at the age of eighteen he was already an adjunct professor at the *Luis Leite de Amparo* School Group, inside of São Paulo. Then he took over the interim direction of a school in Santos (SP) and later was appointed professor of mathematics at the Normal School of Itapetininga and then, he went on to teach physics at the *Instituto de Educação Caetano de Campos*. José Ferraz de Campos worked on publishing works on mathematics until his death on July 22, 1955. (Valente & Pinheiro, 2015).

[10] In the year 1932, an article by the author was published in the magazine Education, an organ of the General Directorate of Education of São Paulo entitled "Fractions folding and tearing paper" containing the first eight pages of the Pedagogical Manual that would be published the following year (Campos, 1932).

[11] This denomination was instituted through the first reform of public instruction carried out in the State of São Paulo, in the republished period, in 1892. The primary course comprised two courses: the preliminary and the complementary. Subsequently, with the de-characterization of complementary schools as secondary schools in primary education, this was reduced to primary education. The preliminary course was mandatory for children aged seven to twelve and comprised the first to the fourth primary grades (Souza, 1998).

[12] Secondary education in primary education was a topic addressed by Lussi Borer (2017), when she exemplified how in secondary school Switzerland (specialistas) entered the training of primary teachers (normalists).

[13] In all cases the spelling of the time has been updated to the present time.



(a) (b)

Figure 1. Strips of paper (a) and division process into equal parts (b)
Source: Campos (1932, p. 8-9)

It is noticed in the appropriation made by Campos, the presence of principles of New School, of what should happen and how it happened, in fact, in his vision. He states that there is a need for "objectification" of the work, in such a methodical and judicious way. However, most schools, according to Campos (1933), occur "the simple memorization and unconscious application of those rules, dictated without further explanation, by the teacher to the students" (Campos, 1933, p. 6). He adds that in primary education the harm is less due to the habit acquired by most teachers to start the study of fractions for a few concrete cases

> [...] by dividing the whole, a fruit or a sheet of paper, for example, in a few equal parts […] and some colleagues, very few go further, try to make the student acquire practically the sense of fractional greatness, by constant division of the unit into a given number of equal parts and subsequent collation and comparison of these parts with each other" (Campos, 1933, p. 6) (our translation).

The didactics of the subject, according to the author, can still be improved and bring advantages to the student and educator. He further states that he had been proposing a new method for a more concrete teaching of fractions for a long time.

In this way, conceiving the class as a "center of interest" to solve a problem suited to the immediate purpose, the author based "the study on the pleasure that children always feel when we provide them with an opportunity to apply their own activity and attention in a manual work of your pleasure" (Campos, 1933, p. 6-7). In this way, the activity of folding and tearing the paper, would lead in a "practical and educational" way to a

> [...] achievement of a specialized skill in these divisions, the objective realization of the fractional quantities at stake, collation and reasoned comparison of these quantities, to the concrete resolution of problems; the habit of doing so and judging, the acquisition of this rare, intimate and subconscious evaluation factually, this 'sense of greatness', the exclusive prerogative of personal experience, of a self-education that verbal teaching could never produce (Campos, 1933, p. 7) (our translation).

Result of na articulation of knowledge from the field of arithmetic (*knowledge to teach*) with the education sciences (*knowledge for teaching*), configuring an objectification, institutional formality that puts knowledge (in the case of fractions) in conditions to be teachable and legitimized as professional knowledge.

In the others words, the teaching object is presented as "thought concrete", a way of producing knowledge in the logic of the real constructed in thought (Borba & Valdemarin, 2010).

For the execution of the activity (see **Figure 1a**), it would be necessary to make available to each student three or four strips of paper of bright colors, divers, contrasting and of the same dimension (2x20 cm = 0,78x7,8 inch) that they can make themselves. The author suggests that another alternative material be used, in the absence of a better one. In addition, it would be necessary "calculation paper, pencil and gum Arabic or wax; nothing more" (Campos, 1933, p. 7).

Campos argues that generally when the teacher Works with strips of paper, the division process is done mechanically by splitting, for example, the ends of the strip when you want to divide it in half. It suggests an "education way division" in which the child, first point out a small dot in the middle of the paper and, immediately afterwards, it will check its accuracy with a strip divided in half. In this way, the child will also be able to check its accuracy in measurements, for example, the value of a decimeter. The explanation for the construction, together, of the child and the teacher follows for the division into parts such as ⅓, ¼, ⅕, 1/7, 1/9. Note that in addition to the notions of measurements, there is geometric knowledge involved in the activity, such as distance between points and line segments. For the division into five equal parts (**Figure 1b**), the orientation is to fold "the paper by the

ends, simultaneously and calculatively, so that the distance between A and B is equal to the folded proportions AD and BC" (Campos, 1933, p. 10).

We can observe that it is a simple activity and that the materials proposed for the execution of the activity are easy to acquire, even for the time. For Carillo Gallego and Sanchéz Jiménez (2012) what matters is no whether the material is little or a lot, simple or expensive, big or small, what matters is that it is adequate, alive, that is, as a result of the common work between the professor and the disciple.

Before the students operate with the paper strips, the author suggests an introduction to the fractions by means of "tasty lessons" in which "the professor will objectively divide and distribute a few goodies to the students" (Campos, 1933, p. 11). Among the "goodies" were dulce de leche, chocolate, chocolates, apples, pineapples and cookies. For example, "a dulce de leche, for two students: ½, and then each of them will share the portion received, with another classmate: ¼" (p.11). The idea is for students to make the division as evenly as possible and then measure them on a scale. Campos makes an observation that he could never "understand a preliminary class without a scale and a collection of weights" (p. 12). And then he explains: "During all this work, speaking, inquiring, analyzing and explaining, the first notions [of fractions] will be taught in a natural and concrete, suggestive and pleasant way (!)" (Campos, 1933, p. 12). The concepts worked on would be the denomination and numerical representation of the parts of the unit; the value and significance of the terms of each fraction; the necessary pieces would be the reconstitution of the whole; the appreciation of the remains in the inaccurate divisions.

In this appropriation of Campos, Ovide Decroly's concepts about globalizing education and centers of interest are found, stating that in globalized education classes, opportunities are presented so that together with numerical knowledge, "countless common notions about fruits, sugar, cocoa, cookies, money…"(Campos, 1933, p. 12). According to Valente (2019, p. 308) "the motto of the school 'for life, by life" is indicative of philosophy basis of Decroly's studies. From this perspective, the idea of the global and of the child's needs and interests is installed."

It is observed that Campos (1933) concerns were in line with the *New School* tendencies, which foresaw an education in movement and not just static, besides proposing a teaching process that started from something desired by the student for later expansion of this knowledge.

In order to do active teaching, sensorial and manual activity must be privileged and mathematics must be associated with life, in particular with other sciences and with manual work (Carrillo Gallego & Sánchez Jiménez, 2012).

In this sense, Campos (1933) mobilizes two types of knowledge: knowledge of training and knowledge of teaching, which involve knowledge *to teach* and *for teaching* (Hoftetter & Schneuwly, 2017).

In addition to the content of fractions, Campos (1933) seeks to objectify professional knowledge to teach, in the desire for education to take a new direction in particular, placing intuitive teaching in line with the principles of New School, as can be seen in "tasty lessons."

MATHEMATICS METHODOLOGY - IRENE DE ALBUQUERQUE, 1958 (3rd edition)

In the 1950s, period of consolidation of *escolanovimso*, known as the New Renovated School Movement, Irene de Albuquerque reaffirms the purpose of "giving children the pleasure of 'rediscovering' in mathematics, a right that has been denied them a the expense of successful teaching" (Albuquerque, 1958, p. 7). When the child discovers the rule and enunciates it, "this rule is known forever, and the time spent is only a few minutes" (p.7). On the contrary, if the teacher, eager to save time and effort, gives the child the ready rule to use, it becomes more difficult and uninteresting and learning will take several days. The teacher will insist again, and the child will forget because "what we never confess to ourselves is that the child forgets precisely because he never learned" (p. 8).

Irene de Albuquerque, referring to her professional experience, informs that the pages of the manual "Methodology of Mathematics" are product of individual experience, based on readings, courses and observation" (1958, p.9) and that her best teachers were the best students. According to Albuquerque (1958), his "learning was done, in part, at the expense of the sacrifice of a few dozen children who were not successful by the methods we wanted to impose on them" (Albuquerque, 1958, p. 9).

Regarding the purpose of the work, the author points out that her only merit, perhaps, was to put the mathematics teacher in line with the child's identity, his way of seeing, feeling, actin, thinking so that mathematics would become simply, easy, pleasant and alive (Albuquerque, 1958, p. 9). She makes a strong criticism about what she called "blackboard mathematics", waring that the "Mathematics that is required in primary school is alive an concrete, around us" (p. 8) and that "when that same Mathematics passes to the blackboard of a classroom, the teacher kills it, taking its life, makes it abstract" and "the child, whose reasoning cannot yet be abstract, is incapable for this 'blackboard mathematics'"(p. 8).

Results of research by American authors on the use of ordinary fractions in the United States in the 1930s showed that the fractions ""½, ⅓, ¼, ⅔ and ¾ constitute ninety percent of all fractions used in life, with the fraction ½ contributes with sixty percent" (Albuquerque, 1958, p. 147). Unlike the Americans, the author warns that in Brazil "the use of calculations with ordinary fractions is very low in the life of the Brazilian since our units of measurement obey, almost all, the decimal system" (Albuquerque, 1958, p. 147).

For the author, "all transformations of fractions would be based on the objective knowledge of equivalence" (p. 148). However, due to the entrance exams to the gym, he warns that "even we, who were called to collaborate in the programs of the Federal District, had to suggest an exhaustive study of fractions, since the entrance exams to the secondary course require them, although students have to study the same things, again, in the early years of the gym" (Albuquerque, 1958, p. 148).

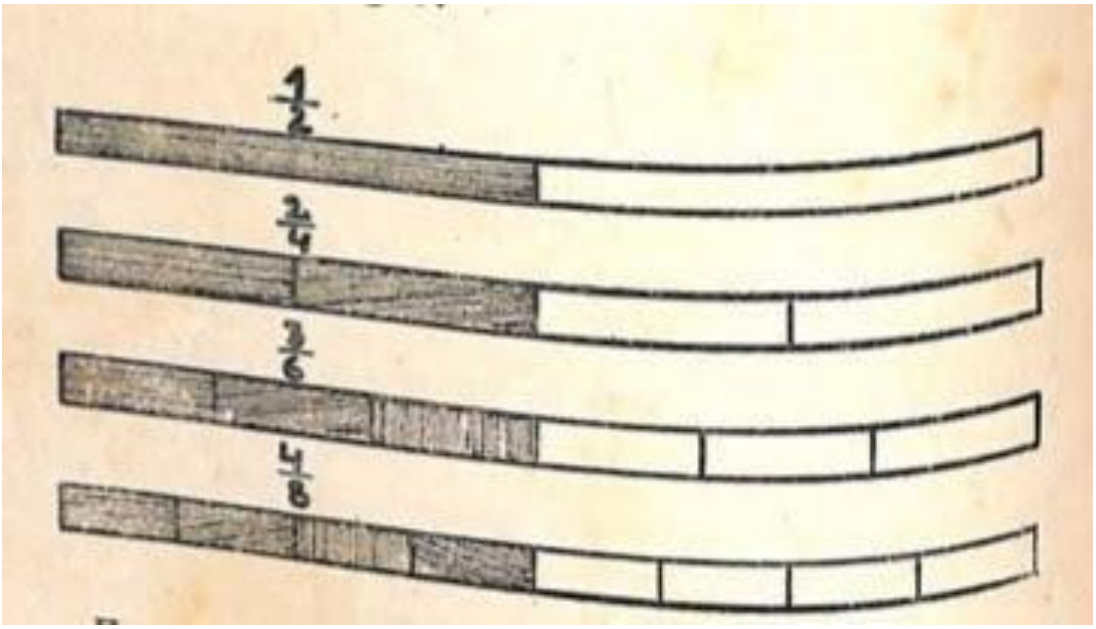


Figure 2. Example of equivalence table
Source: Albuquerque (1958, p. 150)

The learning of equivalent fractions would be essential for understanding fractions and for further studies of operations with heterogeneous fractions. The teaching of this content should take place in a practical way, with objectification and adequate exercises. For this, the child must, for example, know objectively by heart how many quarters are in a middle, how many eighth are in a quarter. As teaching materials, it suggests that they should be made and posts for consultation "tables of equivalence of the most common fractions" (Albuquerque, 1958, p. 150) according to **Figure 2**.

SEE, FEEL, DISCOVER ARITHMETICS - RIZZA DE ARAÚJO PORTO, 1968 (10th edition)

The third source analyzed was the tenth edition of the pedagogical manual "See, feel, discover Arithmetic" based on the material advised by Foster E. Grossnickle, William Metzner and Francis A. Wade, compiled by Rizza Araújo Porto with the collaboration of Evelyn L. Bull, from the Arithmetic Department of PABAEE - Brazilian-American Assistance Program of Elementary Education from Belo Horizonte, Minas Gerais.

According to Vilella et al. (2016), this book circulated from 1959 to 1968, demonstrating wide coverage in Brazil and influencing curricular proposals of the time, such as those in Rio Grande do Sul, Paraná and the Federal District, among others.

In recent studies, França and Santos (2019) show the process of production, systematization and objectification of mathematical knowledge, practiced by professor Rizza de Araújo Porto, in her professional career. Seeking to identify the constitution of their *expertise*, in teacher training courses and normal schools, its systematization and objectification, the researchers of Ghemat Brazil found, in the materials of Porto, indications of how to form an *expert* in the pedagogical field, in addition to identifying knowledge *for teaching* arithmetic in the list of knowledge considered fundamental in teacher education.

In the Preface, Porto (1968, p. 11) emphasizes that "the thought that an educational program cannot be carried out without the appropriate instructional material is now commonly accepted" and that "by discovering and verifying the rules, the child creates the habit of observation, the meticulous care of real thought" and penetrates "in the sense of what it is doing, the 'why' of what it is processing."

For the student to manipulate the concrete material, the teacher should agree with the principle "learning occurs only when the child sees, feels, manipulates, discovers, abstracts" (Porto, 1968, p. 13) and "provides the adequate means to let this quantitative thinking take place" (p. 13).

In this perspective, the classroom becomes a learning laboratory based on understanding and for this to occur "it is important that the child has a period of experimentation, of discovery" (Porto, 1968, p. 17). The exploratory activity gradually varied "makes it capable of making generalizations" (p. 17), in addition to being "stimulates by a real interest" (p. 17).

For Porto (1968), the classroom being a laboratory "does not mean that the teacher needs expensive commercial equipment, out of reach" (p. 17). The teaching material must produce "physical and mental activity that results in experimentation, if the teacher did not understand the purpose of each material to guide the child's quantitative thinking, if the teacher did no use it effectively or if this material was not handled by the children themselves" (Porto, 1968, p. 17). She points out that the materials often consist of objects that the children themselves can bring and that are related to the content.

In the manual under analysis, the author suggests and exemplifies various materials for teaching fractions such as counting discs, flannel, charts, hundred frames, fractional parts and fraction frames.

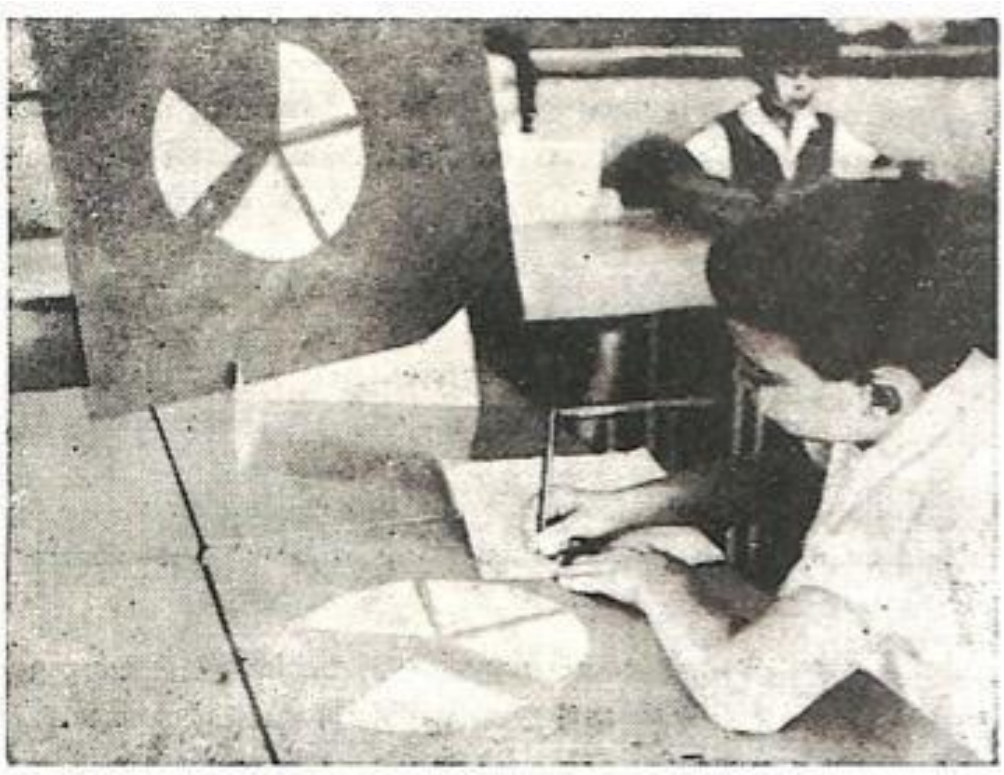


Figure 3. Example of use of teaching material – fractional parts
Source: Porto (1967, p.114)

Fractional Parts

It consists of a set made of large size flannel, with a variable number of pieces according to the need, in "that the teacher uses to demonstrate the various concepts and relationships involved in ordinary fractions, as well as all operations with fractions" (PORTO, 1968, p. 100). Each student must have an individual set of pieces to be worked on in their portfolio when looking for the solution of a problem (**Figure 3**).

According to Porto (1968, p. 100) "The use of broken discs, which represent a 'cake' or a 'cheese', is probably the best way to concretize the concept of the fractional part of the whole. The manipulation of the equal parts of a unit allows the child to discover the relationship of the part to the whole and the relationship between the parts". In this way, the child will have an easy transition from the concrete manipulation to the abstract symbols because he uses the symbols to represent the operation that he carried out concretely, "understand these symbols, see their meaning and then formulate rules that he learned, though use and understanding" (Porto, 1968, p. 102). The reduction and equivalences can be proved by the parts and in this way the student concretely penetrates these fractional relations. For Porto (1967, p. 178) "children like to compose their own rules. It is an excellent activity where they will have the opportunity to use the specific vocabulary of arithmetic."

Fraction Chart

"How many twelfths do I need to make a quarter? A quarter is equal to how many eighth?" (Porto, 1968, p. 124), questions like this were suggestions for the student to explore the fractions table (**Figure 4**) and discover the equivalent fractions.

In the referred manual, the materials introduced provide experiences for the student in order to introduce, enrich, classify and generalize abstract arithmetic concepts; develop in the child an attitude of appreciation for arithmetic; stimulate the child's interest and activity in learning; develop the habit of looking, through reasoning, for the solution to real problems, even when the formal process is no yet known, carefully considering the child's emotional well-being (Porto 1968).

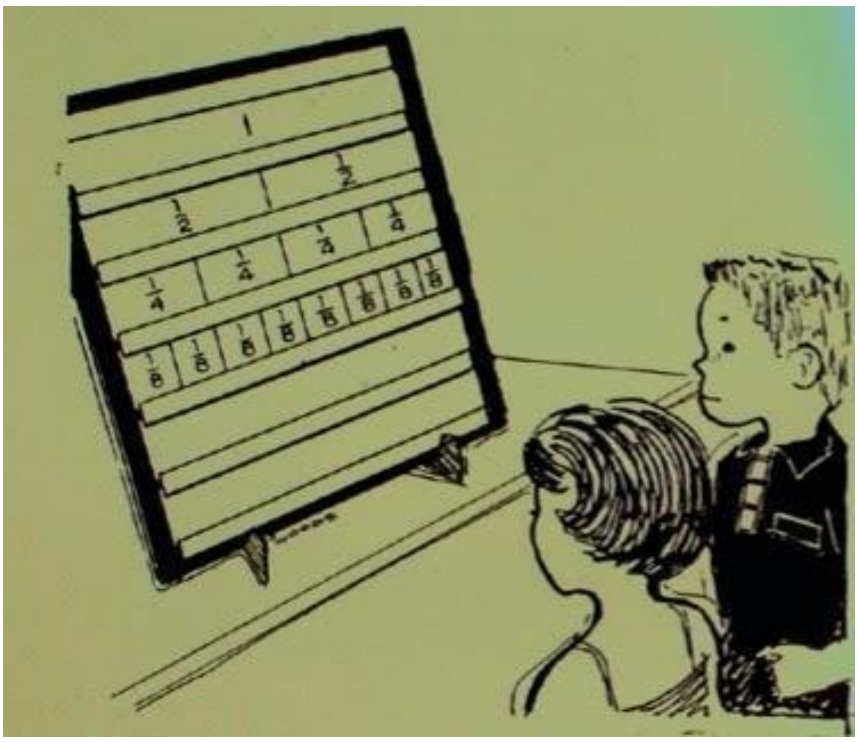


Figure 4. Fraction chart
Source: Porto (1968, p. 116)

FRACTIONS IN ELEMENTARY SCHOOL - RIZZA DE ARAÚJO PORTO, 1967 (4th edition)

In 1963, Rizza Araújo Porto wrote the book "Fractions in elementary school" in which she reported the results of "studies and observations collected in the guidance of experimental classes teachers in the Demonstration School Group of the Institute of Education" of Minas Gerais (Brazil). Supported by the basic principles already exposed above, the choice of the theme fractions is justified as "one of the areas that seems to us most affected by the traditional teaching of 'do as I'm saying'" (Porto, 1967, p. 11). Various materials are used in the book for teaching of fractions, mainly the fractional parts, flannelgraph, fractions board and posters. The work we had access dates from 1967 and is the fourth edition. According to the author:

> We try to show how we can, through questions, raise the student's thinking, how to help him by offering a wealth of experiences, which enables him to proceed from a more concrete level and fractional symbols, intelligently, to a higher standard of maturity (Porto, 1967, p. 12) (our translation).

Right in the first chapter that deals with "Fundamental aspects in the teaching of fractions", we noticed a level of refinement of the principles of the *Escolanovista* pedagogical vacancy experienced by Rizza. It argues that the teaching of the modern elementary school is perhaps the "expression of the recognition of the importance of the child as the center of the whole educational process" (Porto, 1967, P. 15). It recommends that the social objective, or social use of the experience with fractions, should be one of the criteria for this content to be part of a primary education program. Does this numerical experience integrate into the child's life? Is it necessary in your daily activities? Rizza points out that some years ago and "even today in many schools" (Porto, 1967, p. 16) children were subjected to exhaustive work with fractions with denominators greater than one hundred, with long operations that resulted in sources of errors. It raises questions: "will there be such values in the work with fractions of high denominators to justify the child's time and effort, leading, as in a large percentage of cases, to confusion, discouragement, failure and disgust for arithmetic?" (Porto, 1967, p. 17).

To justify the use of exploratory materials recalls a comment, made by Grossnickle and Brueckner, on "two reports of experimental studies showing the relative value of teaching fractions, with and without exploratory material" (Porto, 1967, p. 23). In cases where operations with fractions were done in an abstract manner, the reports showed no difference from teaching without the use of material, but studies "prove that concrete material is indispensable in teaching that takes into account learning by understanding" (Porto, 1967, p. 24).

For Porto (1967) the opportunities for the child to receive a wealth of experiences to discover the real sense of fractions occur since the first grade, with the words middle and half in three variations: half of an integer, half of a group and half of measures, interrelated and interdependent concepts that would be broken down for the purpose of planning activities, in an increasing difficulty.

Recommends that with the help of the teacher, each student can bring a circle cut out for "acquiring a new essential ideal about half: when we divide something into two parts, these parts to be called **halves** must be the **equal**" (Porto, 1967, p. 44).

Students will have similar experiences using other shapes, “either using geometric figures cut out of cardboard or using drawings” (p. 47). She considers that “the proportion that mental images are formed, the child stops looking for material when he needs to find solutions to his arithmetic problems” (p. 48). In this way, “the child sees the half in very different situations, thus avoiding linking the word only to a certain experience, or a certain material” (p. 48).

Even in the first series, the fraction of measures stars from experience that the child already had with the liter, the meter, the kilo, currency and time units. Measurement teaching is one of the most lively in the first grade because, “the child likes to measure and handle the measuring instruments, he likes to estimate” (Porto, 1967, p. 53). For the first time, the author mentions “*Cantinho da Aritmética*” (“Arithmetics Corner” - our translation) (Porto, 1967, p. 59) as a place in the classroom where children could consult materials for problem solving whenever they need it.

For the second series, the fraction worked would be a quarter of an integer, of a group and of a measurement by means of a concrete material (five measurement discs) prepared individually by the child. The greater the child’s participation in the organization of the material, the better his attitude and interest would be. The author suggests “the use of a flannelgraph with the essential fractional parts, in visible size for the whole class” (Porto, 1967, p. 65). She also recommends making posters after the ideas have been discovered by the children. With the use of fractional parts, counts in quarters, identification of fractions, comparisons between quarters, halves and integers, equivalence, fractions greater than the whole would be worked out. The concepts are presented with the use of different material such as the fraction table and the concepts are reinforced through posters with summary information.

The methodological structure for the third series remains the same, adding fractions with eighths, in addition to gradually introducing other fractions. The class is led to construct their own definitions, and “the teacher, with great caution, takes the class to choose that generalization considered more specific, more complete […] elaborated by the child and no imposed by the adult” (Porto, 1967, p. 98).

For example, a child who did not really understand the meaning of the terms of the fraction, when asked what the largest fraction is “incurs the error of showing precisely the fraction that has a larger denominator” (Porto, 1967, p. 103) and that is derived from the knowledge that they have “greater number expresses greater quantity” (p. 103). After understanding, it would be important to get children to express how they know the larger fraction and many of them “use material, so to speak, a concrete explanation” (Porto, 1967, p. 103) because they lack specific vocabulary. It is up to the teacher to encourage them, giving them necessary assistance in the composition of an explanation related to their mental maturities.

For the fourth grade, Porto (1967) clarifies that “we often use the same activities, the same materials, but with different objectives, attending different stages of the learning process” (p. 130). In this way, “the understanding stabilized in concrete terms, we must dedicate special attention to the work of elaborating concepts and generalizations, in a natural graduation of the learning process” (p. 131). But, whenever the student has “a certain doubt in explaining the arithmetic truth, he allows him to use some drawing or concrete material” (p. 132).

The last four chapters (seven to ten) are dedicated to operations with fractions. Chapter seven is dedicated to the operations of adding fractions, indicating to work with this content as from the second series. The material indicated is the fractional parts in which the child will be “adding us in concrete terms and, at the same time, preparing for abstract work” (Porto, 1967, p. 156).

Porto (1967) recommends a preparatory activity by means of individual material in which children remove fractions from an envelope representing, for example, one bedroom, two bedrooms, three bedrooms, four bedrooms, five bedrooms… to carry out a rational count that “leads the student to perceive the number of fractional parts always added by on more” (p. 157).

The graduated teaching of operations with multiplication starts in the second grade when the student tries to find a solution to questions such as: I have two half liters of milk, how many liters do I have?” or “If we put a quarter circle on the flannelgraph threes times, how much will I have in total?” (Porto, 1967, p. 224).

In view of generalization work, the author suggests that the teacher should provide the students several means to find the answer through the individual material of the fraction parts, flannelgraph, drawing, diagrams, relate to the addition and express in symbolic form. Through “all these repeated activities, it is intended to achieve the work of generalization” (Porto, 1967, p. 229) and “after the class reaches the rule, a poster is made, as the crowing of all previous work, allowing your consultation whenever necessary” (p. 230).

The author warns that “the division of fractions is considered the most difficult of operations” because it involves “very abstract concepts, its logic originates directly forma a mathematical framework of principles and interpretations that area rarely reached, with understanding by children between ten and twelve years old. On the other hand, the division of fractions does not correspond to the criterion of social use of numbers” (Porto, 1967, p. 259).

In addition, common problems involving division of fractions can be solved using their decimal equivalents. The author weaves these considerations in order to alert teachers that, recognizing the problem of the situation, they will be more cautious in planning activities. In this way, “the process, which is inherently difficult, can become worse if we do not try to rationalize it through good preparation, the use of diagrams and a considerable wealth of experiences” (Porto, 1967, p. 259-260). Except for the particularities of the division of fractions, the procedures adopted follow a didactic sequence identical to that present for the other operations.

As an appendix, the author answers a very common question among scholars of the subject, “Why do we invert the divisor and multiply?” in the fraction’s division operation. It justifies the need to answer this question because “the absence of a rational and significant answer has been very common, since all teaching has been done on mechanical bases” (Porto, 1967, p. 297).

In the middle of the twentieth century, when **New School** was already more consolidated in primary schools, several pedagogical manuals, among others, published by Afro do Amaral Fontoura[14], who, inciting criticisms of the exaggerations of some aspects of *New School* , such as the excesses of freedom and autonomy given to the child, in addition to the overvaluation of the spontaneity defended by some aspects of New School, such as the so-called *New Renovated School* .

As indicated by the guidelines present in the two books in Porto (1967 and 1968), the assumptions of this aspect seem to be permeating the author's books, with regard to alive and modern teaching, conceived as renewed, in relation to the old one.

The transfer of space to other forms of intervention, in the mid-1960s, is commented on by Souza and Garnica (2012):

> when New School although still present in educational practices, already gave way to other forms of intervention in the school routine, in the same way that, almost six decades before, the *Lessons of Things* and the new pedagogy coexisted in the same space. In the official guidelines for primary schools in this handbook widely distributed in the 1960s, the proposals of *New School* explicitly coexisted with the defense of the need for memorization and training. This reinforces our perception that no ideas will be detected, in any situation and at any time, in a pure form, apart from practices that remain in the midst of expectations of overcoming (Souza & Garnica, 2012, p. 488 - emphasis added) (our translation).

Thus, the presence of New School , is still very striking, at a time when the ideas of the modern mathematics movement (MMM) begin to penetrate school practices, notably in the process of teaching and learning mathematics, interfering in the knowledge of formation and teaching fractions.

FINAL CONSIDERATIONS

Teaching fractions in primary school in times of New School requires the teacher to master a set of knowledge *to teach* and f*or teaching,* a time when the pedagogical manuals aimed at normalists renewed the knowledge considered indispensable to teach, in view of the recognition of the student's role in the teaching and learning process. By breaking with previous practices that valued memorization, oral exposition, repetition, favrting formal education and distanced from the reality of children, the 1930s to the 1970s represented a moment of renewal of principles and innovations in pedagogical practices, making use of methods and process most appropriate to the child's interests. In this renewal, didactic materials for teaching come to occupy a prominent place in the knowledge *for teaching* fractions in primary school, objective and appropriate knowledge both for the training of teachers and for the teaching of mathematics in the first school years, therby denaturalizing the secondary culture previously incorporated in this process.

Therefore, the pedagogical manuals selected for this study, which indicate the analyzes in relation to the use of didactic materials to teach fractions, bring traces of the concern of their authors regarding the appropriate knowledge for teaching fractions in order to meet the interests and understanding of the child. In this sense, they present new possibilities for more effective teaching considering the child's active participation in the teaching and learning process.

Of the fractions folding and tearing paper, problematized by Campos (1933), the importance of the child to work with didactic materials that help to prove properties, deduce rules and understand the fractions, before reaching abstraction. The manual provides suggestions to link the reference science (mathematics) to the educational sciences, by shifting the attention from teaching to learning, with respect for the child, its interests and activities. Thus, the analyzed manual works with the idea that concrete material is indispensable for teaching fractions. Criticizes the use of materials that do not lead the child to establish relationships with his daily life and other matters of primary education.

In the Albuquerque manual (1958), recommendations for the use of materials stand out, attributing to the student the search for the rules involved in calculations with fractions, using a process of "rediscovery" in line with his way of "seeing, feeling, act and think". Therefore, the "fraction equivalence table" is recommended, considering the importance of the concept for the appropriation of knowledge about fractions.

In relation to the two manuals in Porto (1967 and 9168), there is a recurring statement about the importance of using didactic materials for the teaching of fractions, permeating the recommendations of all grades of primary school.

It is also noted that the guidelines for the use of didactic materials for the teaching of fractions present in the book "*Ver, sentir e descobrir a aritmética*" and "*Frações na escola elementar*" expressa a strong concern with the exploratory and discovery character on the part of students, striking features of New School , in its renewed form. For the author, the use of materials in learning fractions favors the overcoming of errors in situations where the child confuses properties of natural numbers with the properties of fractional numbers.

The analysis of the referred manuals made it possible to understand that the authors of the time stook out for working with didactic materials, pointing out weaknesses in their use, considering that the resources are not the solution for all problemas, their misuse can impair learning, including. In addition, they do not necessarily need to be sophisticated, on the contrary, the most important is the active participation in the use and handling by the children, in a lively way, always according to the mathematical properties of the fractions to be studied.

[14] Born in Rio de Janeiro, Afro do Amaral Fontoura (1912-1987) taught at several Normal Schools in the state. Graduated in Philosophy, he also served as a professor at several universities in Rio de janeiro. Author of numerous educational works published by Editora Aurora, where he directed the *Escola Viva* serie, from the Didactic Library of Education.

The analyzes show that knowing how to properly use materials to teach fractions is part of a set of professional knowledge, required for the teacher to teach mathematics in primary school, during the period of New School . It is noted that in the initial years of the primary course (1st and 2nd series), there is a greater concern of the authors of the manuals with the use of materials to teach fractions, by having a variety of ways of learning with the help of different teaching materials. These seem to function as mediators of the learning process with regard to the concrete/abstract relationship involved in the appropriation of mathematical knowledge. In the final grades, the use of materials is decreasing and these are replaced by diagrams, graphs, drawing, equivalence tables, in compliance with the propaedeutic purposes of a school concerned with preparing the student for the Admission Exams, in view of their admission to the high school. A dimension of professional knowledge that express the contributions strongly disseminated by Scientific Pedagogy arising from Experimental Psychology, pointing to a notorious transformation required *for teaching* fractions in the first school years.

REFERENCES

Albuquerque, I. (1958). *Metodologia da Matemática* (3th Ed.), Rio de Janeiro: Conquista.

Bertini, L. F., Morais, R. S., & Valente, W. R. (2017). *A Matemática a ensinar e a Matemática para ensinar: novos estudos sobre a formação de professores*. São Paulo: Editora Livraria da Física. Retrived on 20 December 2019 from https://repositorio.ufsc.br/xmlui/handle/123456789/160284

Bertoni, N. E. (2004). *Um novo paradigma no ensino e aprendizagem das frações*. Retrieved from http://www.sbembrasil.org.br/files/viii/pdf/15/PA01.pdf

Borba, S., & Valdemarin, V. T. (2010). A construção teórica do real: uma questão para a produção do conhecimento em educação. *Currículo sem fronteiras, 10*(2), 23-37.

BRASIL. (1998). *Parâmetros Curriculares Nacionais*. Brasília, DF: MEC/SEF.

Campos, J. F. (1932). Das frações dobrando e rasgando papel. *Revista Educação, São Paulo, 6*(1), 63-73. Retrived on 20 December 2019 from https://repositorio.ufsc.br/handle/123456789/116720

Campos, J. F. (1933). *Das frações dobrando e rasgando papel*. São Paulo: Typ Siqueira - Salles Oliveira, Rocha & Cia. Retrived on 20 December 2019 from https://repositorio.ufsc.br/xmlui/handle/123456789/183307

Carrillo Gallego, D., & Sánchez Jiménez, E. (2012). El uso de materiales en la enseñanza de la matemática escolar (1925-1936). In: *Anais… III Foro Ibérico de Museísmo Pedagógico - V Jornadas Científicas de la SEPHE*. Universidade de Murcia, ES, 181-196.

Certeau, M. (1982). A operação historiográfica. In: M. Certeau, *A Escrita da História*. Rio de Janeiro: Forense-Universitária.

Chartier, R. (1990). *A História Cultural: Entre Práticas e Representações*. Lisboa: Difel.

Chervel, A. (1990). História das disciplinas escolares: reflexões sobre um campo de pesquisa. *Teoria & Educação, Porto Alegre: Pannonica, 2*, 177-229.

Cyrino, M. C. C. T., Garcia, T. M. R., Oliveira, L. M. P., & Rocha, M. R. (2014). *Formação de professores em comunidades de prática: frações e raciocínio proporcional*. Londrina, UEL.

Fiorentini, D., & Miorim, M. A. (1990). Uma reflexão sobre o uso de materiais concretos e jogos no ensino de Matemática. *Boletim da SBEM-SP*, (7), 5-10.

França, D. M., & Santos, E. S. C. (2019) The Teaching of Arithmetic in the Teacher Training Course and the Constitution of an Expert in the Systematization of this Knowledge in Times of the New School (1950-1970). *Acta Scientiae, Canoas, 21*(Special), 27-42. https://doi.org/10.17648/acta.scientiae.v21issEid5219

Gatti, B. A. (2014). Formação inicial de professores para a educação básica: pesquisas e políticas educacionais. *Est. Aval. Educ., São Paulo, 25*(57), 24-54. https://doi.org/10.18222/eae255720142823

Hofstetter, R., & Schneuwly, B. (2017). Saberes: um tema central para as profissões do ensino e da formação. In R. Hofstetter & W. R. Valente (Eds.), *Saberes em (trans)formação: tema central da formação de professores (Coleção Contextos da Ciência* (pp. 113-172), trad. Viviane Barros Maciel e Wagner Rodrigues Valente). São Paulo: Editora Livraria da Física.

Hofstetter, R., Schneuwly, B., & Frymond, M. (2017). Penetrar na verdade da escola para ter elementos concretos de sua avaliação: a irresistível institucionalização do expert em educação (século XIX e XX). In R. Hofstetter & W. R. Valente (Eds.), *Saberes em (trans)formação: tema central da formação de professores* (Coleção Contextos da Ciência, pp. 55-112, trad. Marcos Denilson Guimarães e Wagner Rodrigues Valente). São Paulo: Editora Livraria da Física.

Julia, D. (2012). A Cultura Escolar como Objeto Histórico. *Revista Brasileira De História Da Educação, 1*(1), 9-43.

Lopes, A. J. (2008). O que nossos alunos podem estar deixando de aprender sobre frações, quando tentamos lhes ensinar frações. *Bolema, Rio Claro - SP, 21*(31), 1-22.

Lussi Borer, V. (2017). Saberes: uma questão crucial para a institucionalização da formação de professores. In R. Hofstetter & W. R. Valente (Eds.), *Saberes em (trans) formação: tema central da formação de professores* (Coleção Contextos da Ciência, pp.173-200, trad. Viviane Barros Maciel e Wagner Rodrigues Valente). São Paulo: Editora Livraria da Física.

Mendes, I. A., & Valente, W. R. (2017). *A matemática dos manuais escolares, curso primário, 1890-1970*. São Paulo: Editora Livraria da Física, 2017.

Monarcha, C. (2009). *Brasil arcaico, escola nova: ciência, técnica & utopia nos anos 1920-1930*. São Paulo: Editora Unesp.

Porto, R. A. (1967). Frações na Escola Elementar. 4. ed. Belo Horizonte: Editora do Professor Retrieved on 20 December 2019 from https://repositorio.ufsc.br/xmlui/handle/123456789/169836

Porto, R. A. (1962, November). Partes fracionárias. Revista do Ensino, Rio Grande do Sul, 89. Retrieved on 20 December 2019 from https://repositorio.ufsc.br/xmlui/handle/123456789/127657

Porto, R. A. (1968). Ver, sentir, descobrir a Aritmética. 10 ed. Rio de Janeiro: Editora Nacional de Direito. Retrieved on 20 December 2019 from https://repositorio.ufsc.br/xmlui/handle/123456789/135016

Powell, A. B. (2018). Melhorando a Epistemologia de Números Fracionários: uma Ontologia Baseada na História e Neurociência. *Revista de Matemática, Ensino e Cultura (REMATEC), 13*(29), 78-93.

Roberts, D. L. (2014). History of Tools and Technologies in Mathematics Education. In A. Karp & G. Schubring (Eds.). *Handbook on the History of Mathematics Education* (pp. 565-577). Springer, NY. https://doi.org/10.1007/978-1-4614-9155-2_28

Silva, M. R. I. S. (2017). *A Matemática para a formação do professor do curso primário: Aritmética como um saber profissional (1920 - 1960)* (Doctoral Dissertation), Universidade Federal de São Paulo, Escola de Filosofia, Letras e Ciências Humanas, Programa de PósGraduação em Educação e Saúde na Infância e na Adolescência.

Silva, V. B. (2003). Uma história das leituras para professores: análise da produção e circulação de saberes especializados nos manuais pedagógicos (1930-1971). *Revista Brasileira de História da Educação*, (6), 29-57.

Souza, L. A., & Garnica, A. V. M. (2012). Movimentos de um movimento: um estudo sobre os significados atribuídos ao New School e seus ritmos. *Educação Matemática em Pesquisa, 14*(3), 481-506.

Souza, R. F. (1998). *Templos de civilização: a implantação da escola primária graduada no Estado de São Paulo: (1890-1910)*. São Paulo: Fundação Editora da Unesp.

Souza, R. F. (2009). *Alicerces da pátria: História da escola primária no Estado de São Paulo (1890-1976)*. Campinas, SP: Mercado das Letras.

Valdemamarin, V. T. (2010). *História dos métodos e materiais de ensino: a escola nova e seus modos de uso*. SP: Cortez.

Valente, W. (2016). Introdução: da naturalização do “dois e dois são sempre quatro” para o debate teórico da matemática no curso primário. In N. B. Pinto & W. R. Valente (Eds.). *Saberes elementares matemáticos em circulação no Brasil* (v. 1, pp. 7-14). São Paulo: Editora Livraria Física.

Valente, W. R. (2019). Aritmética e interdisciplinaridade: ecos da história da educação matemática. *Educar em Revista, Curitiba, Brasil, 35*(75), 305-318. https://doi.org/10.1590/0104-4060.66837

Valente, W. R., & Pinheiro, N. V. L. (2015). Chega de decorar a tabuada! As Cartas de Parker e a Árvore de Cálculo na ruptura de uma tradição. *Educação Matemática em Revista-RS, 1*(16), 22-37.

Van de Walle, J. A. (2009). *Matemática no Ensino Fundamental: Formação de professores e aplicação em sala de aula*. Porto Alegre, RS: Artmed.

Vidal, D. (2000). Escola Nova e processo educativo. In E. M. T. Lopes, L. M. Faria Filho, & C. G. Veiga (Eds.), *500 anos de Educação no Brasil* (2 ed.). Belo Horizonte: Autêntica.

Villela, L., et al. (2016). Os experts dos primeiros anos escolares: a construção de um corpo de especialistas no ensino de Matemática. In N. B. Pinto & W. R. Valente (Ed.). *Saberes elementares matemáticos em circulação no Brasil* (v. 1, pp. 245-325). São Paulo: Editora Livraria Física.